NOAA Technical Memorandum NOS CGS - 4

GENERAL INTEGRATED ANALYTICAL TRIANGULATION PROGRAM (GIANT), VERSION 3.0, USER'S GUIDE

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Rockville, MD
August 1991

U.S. DEPARTMENT OF COMMERCE / National Oceanic and Atmospheric Administration / National Ocean Service / Coast and Geodetic Survey

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UNITED STATES DEPARTMENT OF COMMERCE
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NATIONAL OCEAN SERVICE
Coast and Geodetic Survey
Rockville, Maryland 20852
May 5, 1992

CHANGE ANNOUNCEMENT

NOAA Technical Memorandum NOS CGS 4
General Integrated Analytical Triangulation Program (GIANT)
Version 3.0, User's Guide, August 1991

Change Number	:	One
Change Date	:	May 5, 1992

General Information

1. This change affects only the information and instructions contained in the user documentation dated August 1991.

2. Keep this change announcement as a permanent record.

ERRATA

PAGE 11 - Record No. 1 through No. N, GROUPS file (continued):

Change lines 14, 15, and 16 to read as follows:

78	1=solve, 0=enforce parameter 1	I1
79	1=solve, 0=enforce parameter 2	I1
80	1=solve, 0=enforce parameter 3	I1

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Coast and Geodetic Survey / R. Adm. J. Austin Yeager, Director

ENVIRONMENT

GIANT is a modular style program written in FORTRAN 77. The source code for the software is on a nine-track tape at 1600 bpi density in ASCII format.

Although originally written for the IBM 360/370 computers, the program has no machine dependent limitations when run on a virtual memory computer. The maximum size of a project that the program accommodates depends on the values of certain parameters. These are defined during the installation of the program.

AVAILABILITY

This documentation of the GIANT program accompanies the software sold by the National Geodetic Information Center (N/CG174), Coast and Geodetic Survey, National Ocean Service, National Oceanic and Atmospheric Administration (NOAA), Rockville, MD. See inside cover page for further information.

A list of the organizations and individuals who acquire the package will be maintained by NOAA. Future enhancements, corrections, or updates to GIANT will be announced and made available to those on the list.

Specific questions regarding GIANT should be addessed to:

Photogrammetric Technology Programs (N/CG213)
Nautical Charting Research and Development Laboratory
National Ocean Service, NOAA
Rockville, MD 20852

Telephone number (301-443-8985)

PREFACE

This user's guide addresses the needs of a photogrammetrist who may want to perform analytical aerotriangulation using the new version 3.0 of GIANT (General Integrated ANalytical Triangulation) program. This GIANT V3.0 is an extension of the original GIANT program (Elassal 1976) explained in the 1987 GIANT Users's Guide (Elassal 1987). However, since there are several significant changes and enhancements to the 1987 version, it was deemed necessary to rewrite the guide.

The main text of this GIANT V3.0 User's Guide is kept simple by avoiding such details as project planning, preprocessing of measured data, and related considerations. The objective is to give to a photogrammetrist sufficient information to enable him to build the data files, execute the program, and interpret the results. No unusual demand is required of the user, although interpretation of results will become more meaningful with experience and knowledge. Appendices have been added for some of the important background information relating to the GIANT program. For more information on aerotriangulation in general, the reader may wish to consult the Analytical Mapping System User's Guide (Engineering Management Series 1981). Another good reference is the Manual of Photogrammetry (American Society of Photogrammetry 1980).

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I. INTRODUCTION

A sufficiently dense control net is required to adequately control instrument settings for exploitation of photographs to generate base maps by stereocompilation, orthophoto mosaic, or other methods. Control can be established by one of the three well known photogrammetric methods: analog, semianalytical, or fully analytical. The fully analytical approach is usually used. This method has developed since the 1960's when digital computers made the associated computations both possible and economical. Its primary advantage is the flexibility to accept and enforce various photographic formats, camera focal lengths, ground control, camera station state vector, and other parameters of the data acquisition system.

The three data reduction phases of an analytical system:

- o Preprocessing or data refinement. Measured image coordinates of all the points are reduced to the plate coordinate system, centered at the principal point. Effects of all known systematic errors, such as lens distortion, are removed.
- o Triangulation. Programs such as GIANT (General Integrated ANalytical Triangulation) accept pre-processed plate coordinates, focal length, ground control, initial estimates of camera station position and orientation, etcetra, for an iterative least squares solution to solve for camera station position and orientation, and ground coordinates of all points.
- o Postprocessing. Camera station position and orientation are subsequently transformed into instrument settings, which are then used for stereomodel setup, to generate base maps and other cartographic products.

A. Functions of the GIANT program

GIANT is a computer program designed to perform analytical triangulation to solve for the ground coordinates of image points measured on two or more photographs. The basic parameters solved for by this program are the ground coordinates of each of the measured image points, and the state vector (position and attitude) parameters of the camera at each of the exposure stations in a block of photographs. GIANT V3.0 is an extension of the previous GIANT (Elassal 1987) version. It incorporates several computational refinements and a new capability to exploit kinematic (airborne) Global Positioning System (GPS) in triangulation.

In GIANT V3.0 the concept of a photo group has been introduced. A block of photographs in an aerotriangulation project is broken up into sub-blocks, henceforth called photo groups, each having a unique set of data reduction parameters (sec.II.B.2) associated with it, depending on the characteristics of the data acquisition system. Each photograph in a block belongs to only one photo group.

In GIANT V3.0 three additional parameter sets have been introduced in the mathematical model of the generalized photogrammetric least squares solution. Depending on the application and the characteristics of the photo group data acquisition system, the user can select any parameter set for each photo group. These additional parameters are introduced as observations, and are treated as either known or unknown entities in the photogrammetric adjustment. These parameters are solved simultaneously for all the photo groups in the photogrammetric adjustment.

Explanation of the three sets of additional parameters is given below. Corresponding to the three sets of additional parameters, GIANT V3.0 uses three distinct mathematical models in the generalized photogrammetric solution.

Model 1 (appendix I): The three additional parameters in this model are the principal distance, and the x and y position coordinates of the principal point (in the image coordinate system). This model is used when calibration of the camera system is desired.

Model 2 (appendix J): The three additional parameters in this model are the coefficients of a polynomial, which represents unmodelled systematic errors caused by various unaccounted for factors such as the Earth's curvature, optical abberations, and various in-flight environmental conditions. This model is used when calibration of the camera system is desired.

Model 3 (appendix K): This model solves for three additional parameters, which are the three offsets between the GPS receiver antenna and the camera node, in the image coordinate system (fig 1.1). This model is applicable to the group of photographs taken with an active GPS receiver on board an aircraft.

Null Model: None of the above models are used. No additional parameters are solved for. The solution is identical to the one provided by the previous GIANT version (Elassal 1987).

A note of caution: Most of the aerotriangulation projects will use the null model. However, if other models are used, extra care and knowledge of the parameters, their solution and significance will be necessary. Appendices I, J, and K have been introduced for explanation. The GIANT V3.0 documentation is to be used only as a guide. The decision as to what model to use is left to the user.

A switch is provided for selecting *any one or none* (null option) of the three models for each of the photo groups in a block of photographs to be triangulated. (See sec. II.B.2, record no. 1, GROUPS file.)

The program uses an iterative least squares technique. All parameters are treated as weighted observations, ranging from known to unknown (except for the three additional parameters which are either known or unknown). Observation equations are set up as functions of the parameters. The solution assumes only uncorrelated observations. All parameters and observations may be weighted to reflect *a priori* knowledge of their precision. This is particularly useful in differentially weighting control (ground or GPS determined), compensating for different sources of control and varying precison, as well as being able to utilize control with unknown components. By allowing the use of partial control points, any horizontal and vertical component, known with varying accuracies, can be used. The user may enforce known camera station positions and orientation, if they are determined by external sources, such as GPS or any geopositioning device on the aircraft. When these parameters can be determined with sufficient accuracy and enforced as observed quantities, the need for ground control is significantly lessened for comparable accuracy. In this regard, mathematical model 3 facilitates the use of kinematic (airborne) GPS-controlled photography.

The program also propagates error estimates through the solution, computes the *a posteriori* estimate of variance of unit weight and, on option, the variance-covariance matrix and standard deviation of each parameter of camera station position and orientation, and of ground coordinates. When used with a fictitious data generator, a user may predict results of various project configurations, using a set of photographs, a given control pattern, or other variables. Accuracy could be predicted, and additional or different configurations of control could be planned.

The iterative least squares approach requires an initial approximation for each unknown parameter. The user furnishes initial approximations for camera position and orientation paramters whereas, the program generates the initial estimates of the pass point coordinates and of the missing components of the ground control points. The program accepts reasonably gross approximations for these parameters.

The program expects object space coordiantes to be in a space rectangular or in a spherical/geographic coordinate system. The rectangular coordinate system is generally required for close-range photogrammetry and the spherical/geographic system for conventional mapping projects. The camera attitudes are parameterized in terms of roll, pitch, and yaw (ω, ϕ, κ) referenced to the local vertical, and express the relation between image and object coordinates.

B. Program Capabilities and Restrictions

The GIANT V3.0 employs a highly efficient algorithm for the formation, solution, and inversion of large linear systems of equations. During installation of the program, the agency using the program must determine the maximum size it will ever handle for the following parameters:

- o Maximum number of photographs or exposures N1
- o Maximum number of ground points, including all points of which ground coordinates are computed N2
- o Maximum number of ground control points N3
- o Maximum number of frames on which a unique point appears N4
- o Maximum number of photo groups N5
- o Normal equation bandwidth (appendix A) N6

Due to the virtual memory available in computers, the size of the project that can be handled is almost unlimited.

Other capabilities include the following:

- o Object space can be expressed in a space rectangular or in a spherical/geographic coordinate system. The rectangular coordinate system is generally required for close-range photogrammetry and the spherical geographic system for mapping projects.

- Camera attitudes are parameterized in terms of roll, pitch, and yaw (ω, ϕ, κ) and express the relation between image and object coordinate spaces or vice versa.

- Camera station position and attitude parameters can be constrained individually, using proper weights.

- Vertical and/or horizontal components of ground control can be utilized as full or partial ground control points.

- Photographs from any number of photo groups (not to exceed N5 as defined above) may be triangulated simultaneously.

- Data entries are grouped by photographs with the program performing all necessary cross-referencing and pass point ground coordinate estimations.

- An error propagation facility exists for detailed statistical assessment of the triangulation results.

- A facility exists for sorting the triangulation results

- Corrections applied to ground control point coordinates as a result of the triangulation are listed for reference.

- The internal defaults for estimated standard deviations of object space coordinates of control points can be declared on an additonal record (sec. II.B.1). Provision still exists for declaring individual data items (sec. II.B.5).

- The unit variance of the triangulation residuals is listed.

- Camera station position and attitude corrections for each iteration are given.

- Control points can be designated as *unheld* and used as test points. The residuals are listed separately and separate root-mean-square errors computed.

- Run time errors detected during the input phase, due to illegal format or data types, are printed showing the record number and contents of the offending record.

Expanded capabilities (sec. I.A):

GIANT V3.0 introduces new parameters for each photo group into the photogrammetric solution. The new parameters are introduced as observations and are weighted as either known or completely unknown in the least squares solution. There are three types of parameter sets to choose from, depending on the data acquisition characteristics of the photo group and the user needs. A switch is provided in the photo group file (sec. II.B.2) for selecting one or none of the three sets. Corresponding to each of the parameter sets is a mathematical model, giving additional capabilities to the GIANT V3.0 program.

II. DATA FILES USED

This section describes the input and output data files.

A. An Overview

The five input data files: COMMON, GROUPS, IMAGES, FRAMES, and GROUND, are input into GIANT and are processed to give output data files PRINT and DATA. (See figure II.1.)

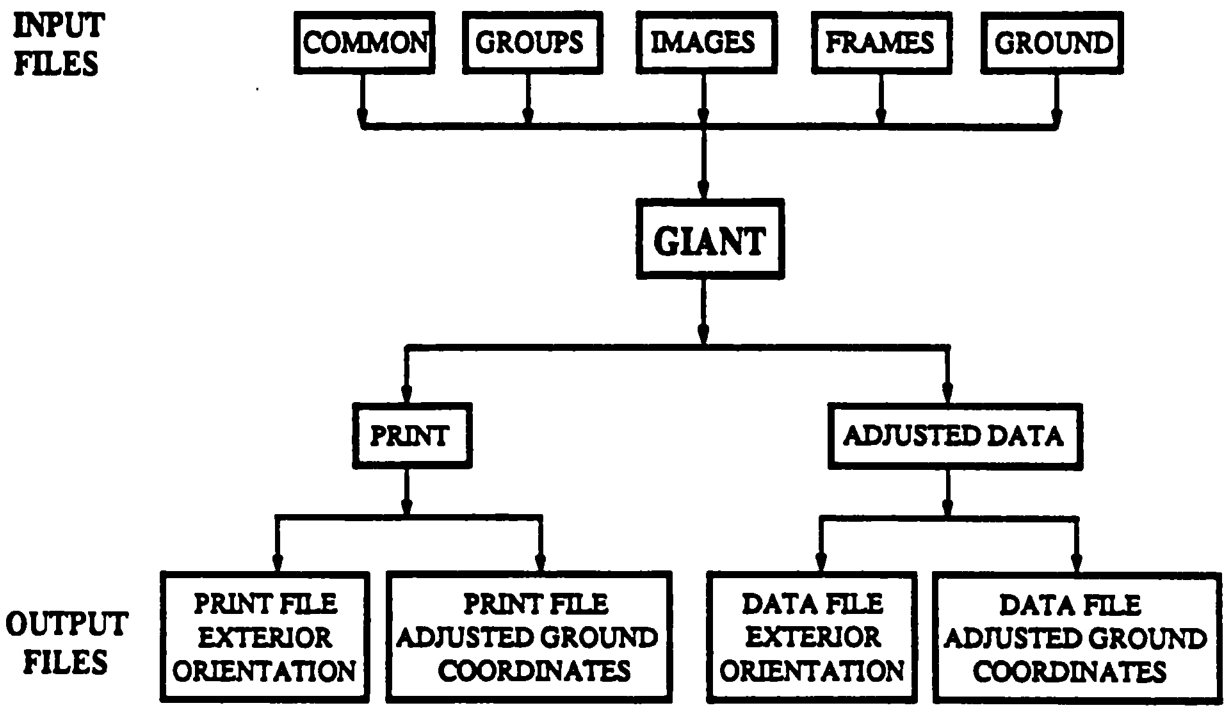

Figure II.1.--Schematic of input and output files

There are three stages of data reduction as explained in sec.I. In the preprocessing stage, the measured image coordinates are reduced from machine coordinate system to the image coordinate system, and are refined by eliminating certain systematic errors before entering the GIANT V3.0 program. The corrections made are for film deformation, lens distortion, and atmospheric refraction. For a detailed description of a typical preprocessor, refer to the Engineering Management Series (1981).

The switch for applying atmospheric refraction correction (sec. II.B.1, record no. 2) within GIANT V3.0 is turned off if the correction has already been made in the preprocessor.

The GIANT V3.0 program has an atmospheric refraction correction model applicable up to an altitude of 9,000 m (appendix F). The dynamic nature of this model makes it possible to carry out a more accurate correction for the refraction effect. This correction is based on the *altitude* and *attitude* of the camera. In the program's iterative adjustment process, the atmospheric refraction correction is carried out according to the updated state vector of the camera. The application of this model slows down the convergence of the solution with only a slight improvement in the results. This may discourage its use by production units. In the future, the model will be replaced by another more universally applicable correction model due to the 9,000 m altitude limitation.

B. Input Files and Their Organization

A description of the five input data files, their contents, and organization are given in this section.

II.B.1. COMMON File

The ouput listing will depend on the options used in the input COMMON file. Also, the various steps in the computations will be dictated by the options chosen in the job definition data record in the COMMON file.

Record No. 1: Job Title - Alphanumeric characters 1-80

These 80 characters will be printed at the top of each page of the program.

Record No. 2: Job Definition

This record contains option flags and parameters for the triangulation run.

Column No.	Content	Field/Remarks
1	Definition of object space[1] =0, Rectangular coordinates =1, Geographic coordinates	I1

[1]In all mapping applications use geographic coordinates; in close-range applications use rectangular coordinates.

All angles in Degrees, Minutes and Seconds (DMS) are in the form:

DMS field: ±DDDMMSS.SS...SS

Where DDD are degrees; MM are minutes; and, SS.SS...SS are seconds. The DMS field is interpreted by the program left to right and leading zeros may be dropped. For example, an angle with zero degrees can be expressed as: MMSS.SS...SS, but leading zeros must be included in the minutes and seconds portion for obvious reasons.

Record No.2, COMMON file (continued):

Column No.	Content	Field/Remarks
2	Type of camera station rotations switch (affecting both input and output) =0, Photo-to-ground =1, Ground-to-photo	I1
3	List input camera station parameters switch =0, list =1, do not list	I1
4	List input plate coordinates switch =0, list =1, do not list	I1
5	List input ground control switch =0, list =1, do not list	I1
6	List output triangulated ground point coordinates switch =0, list =1, do not list	I1
7	Save (as data file) output triangulated ground coordinates switch =0, save =1, do not save	I1
8	List output adjusted camera station parameters switch =0, list =1, do not list	I1
9	Save (as data file) adjusted camera station parameters switch =0, save =1, perform intersection only, holding camera position and attitude fixed	I1
10	Selected process switch =0, perform complete triangulation =1, perform intersection only, holding camera position and attitude fixed	I1

Record No. 2, COMMON file (continued):

Column No.	Content	Field/Remarks
11	Error propagation switch =0, do not perform error propagation =1, perform error propagation	I1
12	*A posteriori* unit variance adjustment flag =0, unit variance is based on completely free camera parameters =1, unit variance is based on constrained camera parameters =2, Set unit variance equals unity	I1
13	Sort triangulated ground points switch =0, perform ascending sort of ground points =1, do not perform sort	I1
14	Maximum allowable number of iterations in the least squares adjustment. If this field is left blank, the program will assign a maximum of four iterations	I1
15	Any valid alphanumeric character. Leading character(s) matching this character will be removed from name fields of camera systems, camera stations, and ground points	A1
16	Air refraction model switch =0, apply =1, do not apply	I1
17	Water refraction model switch =0, apply =1, do not apply	I1
18-19	Criterion E for convergence of least squares adjustment. Least squares solution will be considered complete if the absolute change in the weighted sum of squares for two consecutive iterations is less than E percent. If this field is left blank, the program will assume E=5	I2

Record No. 2, COMMON file (continued):

Column No.	Content	Field/Remarks
31-40	Water level (linear units) with respect to the reference ellipsoid, at the time of exposure. This value applies to the whole block for bathymetric mapping application	F10.3
41-50	Plate residual listing criteria (F, in micrometers) equal to 0: All images residuals listed; greater than 0: Only those residuals whose absolute value > F will be listed; less than 0: No residuals will be listed	I10
51-60	Semimajor axis of the Earth's spheroid in linear units. If not specified, program will assume the value of Clarke's 1866 spheroid. (=6,378,206.4 m)	F10.2
61-70	Semiminor axis of the Earth's spheroid in linear units. If not specified, program will assume the value of Clarke's 1866 spheroid. (=6,378,206.4 m)	F10.2

Record No 3: COMMON file: Default standard deviations[1] (optional) for object space coordinates of control points.

Column No.	Content	Field/Remarks
	Standard deviation (planimetry)	
1-10	X (meters), defaults to 1.0 units or λ (DMS), defaults to 0.01 DMS	F10.3

[1]Standard deviations of object space coordinates of control points can be defined in the GROUND file (sec. II.B.5). If not specified, the program will adopt default values. This record is used to modify the program's built-in default values in the GROUND file.

Record no. 3, COMMON file (continued):

Column No.	Content	Field/Remarks
11-20	Y (linear units), defaults to 1.0 units or ϕ (DMS), defaults to 0.01 DMS	F10.3
	Standard deviation (elevation)	
21-30	Z (linear units), defaults to 1.0 units or H (linear units), defaults to 1.0 units	F10.3

II.B.2. GROUPS File

This file contains information of imaging system(s) for photo groups in a triangulation block. What distinguishes a photo group from another is that each photo group has a unique set of data reduction parameters. The program allows for inclusion of several photo groups in a single triangulation run. The number of photo groups may not exceed the maximum number as defined by the parameter N5. (See sec. I.B.)

Record No. 1 through No. N: (N = Number of photo groups)

One record per photo group is required as follows:

Column No.	Content	Field/Remarks
1-8	Photo group identification: A blank identification is legal and may be used in case there is only one photo group	2A4
11-20	Photo group camera principal distance[1] (micrometers) in a right-handed coordinate system	I10
21-30	Standard deviation for image x-coordinate for the photo group. Default is 10 microns	I10

[1]Principal distance is negative if working in positive plane, and positive if working in negative plane.

Record No. 1 through No. N, GROUPS file (continued):

Column No.	Content	Field/Remarks
31-40	Standard deviation for image y-coordinate for the photo group. Default is 10 microns	I10
41-50	First parameter of model opted in column 76. If no model is opted, then leave blank	F10
51-60	Second parameter of model opted in column 76. If no model is opted, then leave blank	F10
61-70	Third parameter of model opted in column 76. If no model is opted, then leave blank	F10
76	Model type:(0=null model),(1=x_o, y_o, f model) (2=K_1, K_2, K_3 model), (3=ΔX_o, ΔY_o, ΔZ_o, GPS) See appendices I, J, and K	I1
78	1= solve, 0= enforce parameters of model type 1	I1
79	1= solve, 0= enforce parameters of model type 2	I1
80	1= solve, 0= enforce parameters of model type 3	I1

II.B.3. IMAGES File

This file contains preprocessed image measurements for all the image frames (photographs) in a block. Frames in a block may be included in this file in any order desired. Each frame (photo) has the following records.

Record No. 1: Frame header record for each frame (photo)

Column No.	Content	Format/Remarks
1-8	Frame (photo) identification	2A4

Record No. 1, IMAGES file (continued):

Column No.	Content	Format/Remarks
21-30	Assigned standard deviation of image x-coordinate (micrometers). Default option for this field is 10 micrometers	I10
31-40	Assigned standard deviation of image y-coordinate (micrometers). Default option for this field is 10 micrometers	I10
41-48	Photo group identification: (same as in GROUPS file)	2A4

In triangulation tasks, which involve one photo group, the photo group identification may be left blank. This alleviates the need to enter characters in columns 41-48 of the current record. Furthermore, if the default standard deviations for image coordinates are exercised, then columns 21-40 are left blank.

Record No.2 through No.(N+1): (N=Number of image points per frame)

One record for each image point is required as follows:

Columns	Content	Field/Remarks
1-8	Image point identification	2A4
11-20	Image x-coordinate	I10
21-30	Image y-coordinate	I10

Any number of image coordinate records can be included for each frame.

Record No. (N+2): Frame termination record

Column No.	Content	Field/Remarks
1-8	****	2A4

I.B.4. FRAMES File

This file supplies GIANT V3.0 with estimates of positions and attitudes for each frame (photo). The frames included in FRAMES file *must* also be included in the IMAGES file. However, frames in the IMAGES file may or may not be included in the FRAMES file. *Only those frames (photographs) appearing in the FRAMES file will be considered in the triangulation process.*

The order in which frames are included in this file influences the performance and efficiency of the triangulation process. This subject is discussed in appendix A.

For each frame (photo) there are two records: one for *position* and one for *attitude*.

Record No. 1: Frame (camera) position

Column No.	Content	Field/Remarks
1-8	Frame identification	2A4
9-20	Primary component of frame position: *Coordinate (X)* in space rectangular coordinate system (linear units); *Longitude (λ)* in geographic coordinate system (DMS)	F12.3
21-32	Secondary component of frame position: *Coordinate (Y)* in space rectangular coordinate system (linear units); *Latitude (ϕ)* in geographic coordinate system (DMS)	F12.3

Record No. 1, FRAMES file (continued):

Column No.	Content	Field/Remarks
33-44	Tertiary component of frame position: *Coordinate (Z)* in space rectangular coordinate system (linear units); *Elevation (h)* in geographic coordinate system (linear units)	F12.3
45-54	Standard deviation of primary coord. of frame position: *Sigma-X* in rectangular coordinate system (default option = 60,000 units); *Sigma-Longitude* in geog.coord.system (default option = 10 minutes) (DMS)	F10.3
55-64	Standard deviation of secondary coord. of frame position: *Sigma-Y* in rectangular coordinate system (default option = 60,000 units); *Sigma-Latitude* in geog.coord.system (default option = 10 minutes) (DMS)	F10.3
65-74	Standard deviation of tertiary coord. of frame position: *Sigma-Z* in rectangular coordinate system (default option = 60,000 units); *Sigma-h* in geog.coord.system; (default option = 60000 units)	F10.3

Note: DMS field is read as real field and then interpreted as degrees, minutes, and seconds.

Record No. 2: Frame (photo) attitude

Frame attitude record must be prepared in the following format.

Column No.	Content	Field/Remarks
1-8	Frame identification	2A4

Record No. 2, FRAMES file (continued):

Column No.	Content	Field/Remarks
9-20	Primary rotation angle (ω) frame attitude (DMS)	F12.3
21-32	Secondary rotation angle (ϕ) frame attitude (DMS)	F12.3
33-44	Tertiary rotation angle (κ)[1] frame attitude (DMS)	F12.3
45-54	Standard deviation of primary rotation angle (default = 90 degrees DMS)	F10.3
55-64	Standard deviation of secondary rotation angle (default = 90 degrees DMS)	F10.3
65-74	Standard deviation of tertiary rotation angle (default = 90 degrees DMS)	F10.3

[1](κ) is approximated by a clockwise angle (photo-to-ground) and counter-clockwise (ground-to-photo) measured from east to the photo (x) in the plane of the vertical photograph.

The maximum number of imaging sensor stations depends on the value of the parameter (Nl) which is defined during the installation of the GIANT program. (See sec. I.B.)

II.B.5 GROUND File

This file contains the ground coordinates of ground control points up to the maximum number specified by parameter N3, as explained in sec. I.B. The value of N3 is assigned during the installation of GIANT V3.0.

Record No. 1 through No. N: (N = number of ground control points)

Format for each ground control is as follows.

Column No.	Content	Field/Remarks
1-8	Identification of ground point	2A4

Record No.1, GROUND file (continued):

Column No.	Content	Field/Remarks
9-20	Primary component of ground control *Coordinate-X* in rectangular coord. system (linear units); *Longitude* (λ) in geog.coord.system(DMS)	F12.3
21-32	Secondary component of ground control *Coordinate-Y* in rectangular coord. system (linear units); *latitude* (ϕ) in geog.coord.system(DMS)	F12.3
33-44	Tertiary component of ground control *Coordinate-Z* in rectangular coord. system (linear units); *Elevation (h)* in geog.coord.system(DMS)	F12.3
45-54	Standard deviation of primary component of ground control coordinates: *Sigma-X* in rectangular coord. system (default to values in COMMON file) *Sigma-Long*. in geographic coord. system (default to values in COMMON file)	F10.3
55-64	Standard deviation of secondary component of ground control coordinates: *Sigma-Y* in rectangular coord. system (default to values in COMMON file); *Sigma-Lat*. in geographic coord.system (default to value in COMMON file)	F10.3
65-74	Standard deviation of tertiary component of ground control coordinates: *Sigma-Z* in rectangular coord.system (default to values in COMMON file); *Sigma-h* in geographic coord. system (default to values in COMMON file)	F10.3

Record No.1, GROUND file (continued):

Column No.	Content	Field/Remarks
80	Missing component indicator: = 1, means primary component is to be ignored. = 2, means secondary component is to be ignored. = 4, means tertiary component is to be ignored. = X, where X is the sum of any two of the above mentioned codes. The corresponding two components are to be ignored. Example: 0 = complete control X = 1+2 = 3, elevation point 4 = planimetric point	I1

II.B. 6. Sample Input Data File

The computer printout that follows shows the input data corresponding to the aerotriangulation project depicted in fig. II.2. The input data are in five distinct data files: COMMON, GROUPS, FRAMES, IMAGES, and GROUND. (See sec. II.B.1 through II.B.5.)

The sample project consists of an area covered by 18 photographs in three flight lines, five ground control points, and nine additional vertical control points. Figure II.2 shows the layout of the project.

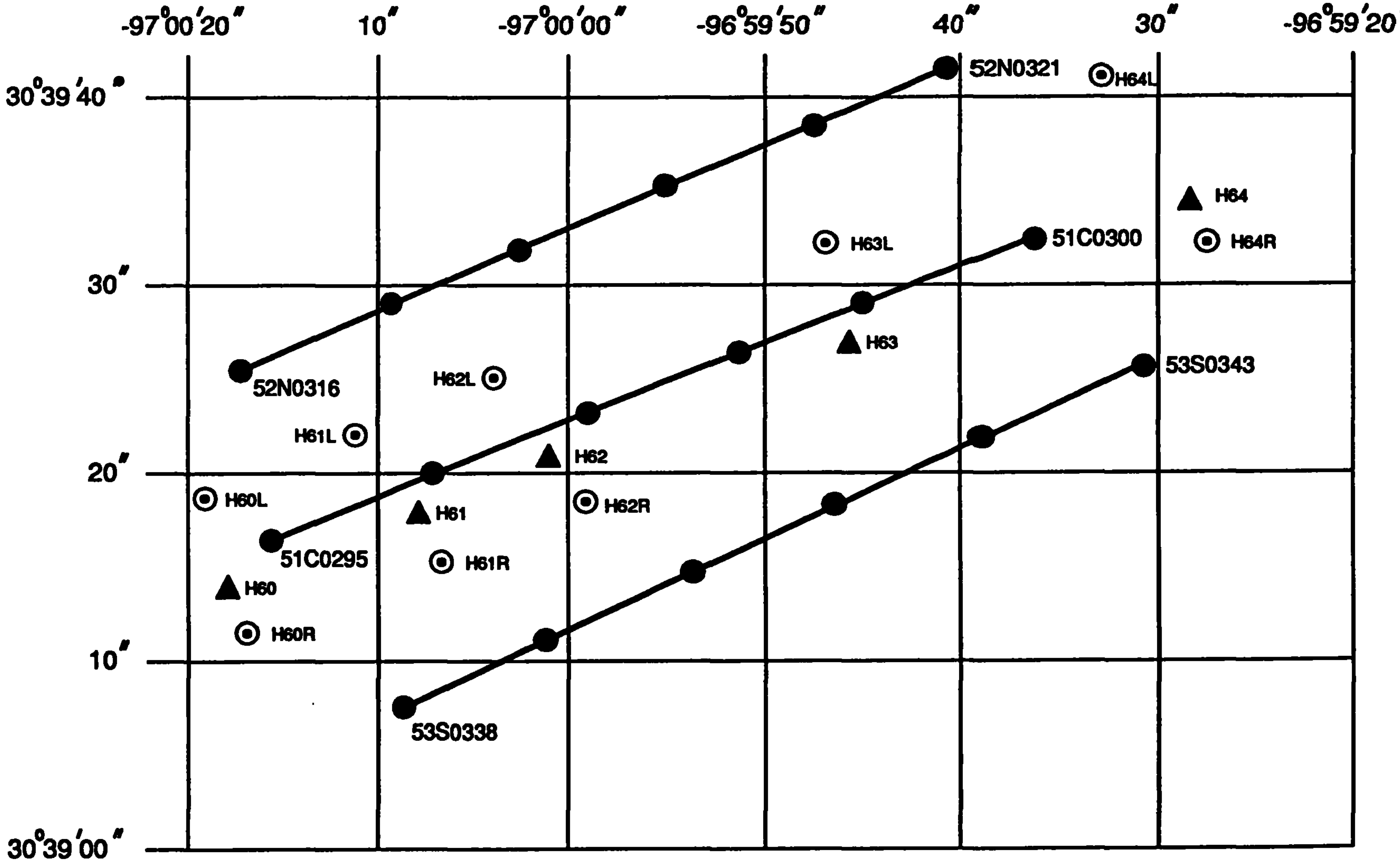

Figure II.2.--Aerotriangulation project (layout).

```
---------------------SAMPLE INPUT DATA FOR 'COMMON' FILE--------------------------

         1         2         3         4         5         6         7         8
12345678901234567890123456789012345678901234567890123456789012345678901234567890        Record Description
----------------------------------------------------------------------------------

SAMPLE RUN FOR NOAA/NOS GIANT PROGRAM                                                   #1-   Job Title
1000000000005001                                 10                                     #2-   Job Definition
                                                                                        #3-   Standard Deviations
```

```
---------------------SAMPLE INPUT DATA FOR 'GROUPS' FILE--------------------------

         1         2         3         4         5         6         7         8
12345678901234567890123456789012345678901234567890123456789012345678901234567890        Record Description
----------------------------------------------------------------------------------

CAMERA       -153280         8         8                                   0            #1-   Imaging Group System Data
```

```
---------------------SAMPLE:  INPUT DATA FOR 'IMAGES' FILE-----------------------

         1         2         3         4         5         6         7         8
12345678901234567890123456789012345678901234567890123456789012345678901234567890        Record Description
----------------------------------------------------------------------------------

 51C0295                                      CAMERA                                     #1-    Frames Header Record
     31!      -1726    -86562                                                            #2-    Image Point ID, Coordinates
    H60R     -30321    -33482                                                            .
    H60      -20116     -4515                                                            .
     33!      -8950     -1283                                                            .
    H60L     -21301     32640                                                            .
      .         .         .                                                              .
      .         .         .                                                              .
      .         .         .                                                              .
    H61       59665      1928                                                            .
    H61R      58234    -43983                                                            .
     68!      81607    -85579                                                            #(N+1)      "       "       "
********                                                                                 #(N+2)  Frame Termination
 51C0296                                      CAMERA                                     #1-    Frame Header Record
     31!     -65416    -89346                                                            #2-    Image Point ID, Coordinates
    H60R     -95757    -37092                                                            .
    H60      -86685     -7628                                                            .
     33!     -75595     -4004                                                            .
      .         .         .                                                              .
      .         .         .                                                              .
      .         .         .                                                              .
```

```
---------------------SAMPLE: INPUT DATA FOR 'FRAMES' FILE-------------------------

         1         2         3         4         5         6         7         8
234567890123456789012345678901234567890123456789012345678901234567890123456789O        Record Description
--------------------------------------------------------------------------------

51C0295 -970014.879  303916.037     618.229                                            #1-    Frame (Camera) ID, Position
51C0295                                                                                #2-    Frame (Camera) ID, Attitude
51C0296 -970007.604  303919.315     613.141                                             .
51C0296                                                                                 .
51C0297 -970000.269  303922.591     613.383                                             .
51C0297                                                                                 .
    .                                                                                   .
    .                                                                                   .
    .                                                                                   .
53S0343 -965931.803  303925.466     608.529                                             .
53S0343                                                                                 .


---------------------SAMPLE:  INPUT DATA FOR 'GROUND' FILE------------------------

         1         2         3         4         5         6         7         8
234567890123456789012345678901234567890123456789012345678901234567890123456789O        Record Description
--------------------------------------------------------------------------------

   H60  -970016.895  303914.587     109.783       0.0       0.0       0.0      0       #1-    Ground Control, Std.Deviations
   H60L       0.000       0.000     110.109       0.0       0.0       0.0      3       #2-    Vertical   "      "      "
   H60R       0.000       0.000     109.067       0.0       0.0       0.0      3        .

   H61  -970008.467  303918.974     116.955       0.0       0.0       0.0      0        .
   H61L       0.000       0.000     114.425       0.0       0.0       0.0      3        .
   H61R       0.000       0.000     116.086       0.0       0.0       0.0      3        .
    .
    .                                                                                   .
    .                                                                                   .
   H64R       0.000       0.000     124.157       0.0       0.0       0.0      3        .
```

C. Output Files

1. Printout File

Output data file, logical identification no. 6, table II.1, sec. II.D, is a formatted printout data file, the contents of which depend on the print options selected in the job definition record of the COMMON input file. The printout gives the triangulated ground coordinates of points, the images of which are measured on two or more photographs, and the camera parameters, which are the position and attitude of camera at each of the exposure stations.

A typical sample of the printout file follows. Explanations of the items appearing on the computer printout are also given.

2. Updated Frames Data File

Output data file, logical identification no. 8, table II.1, sec. II.D, contains adjusted values of parameters of each camera position and attitude. This file is created during each iteration of a successful convergent solution of GIANT V3.0. This file is used in applications such as stereocompilation and orthophoto mosaic.

Output data file, logical identification no. 20, table II.1, sec. II.D, contains updated adjusted values of parameters of each camera position and attitude, irrespective of solution convergence. This file can always be used at the user's option to restart the iterative solution from where it left off.

3. Adjusted Ground Data File

Output data file, logical identification no. 9, table II.1, sec. II.D, contains adjusted ground coordinates of all points measured on two or more photographs. This file is created during each iteration of a successful convergent solution of GIANT V3.0. This file is used in applications, such as stereocompilation and orthophoto mosaics.

4. Sample printout of a typical aerotriangulation project:

This section contains an explanation of a sample printout for a typical aerotriangulation project. Figure II.2 shows the layout of the project. Typical pages of the printout are selectively reproduced for this report. A page-by-page description of the printout follows. Three computer runs were made for the aerotriangulation:

Case No. 1:	Aerotriangulation without error propagation
Case No. 2:	Aerotriangulation with error propagation
Case No. 3:	Aerotriangulation without error propagation but with additional parameters, one set for each photo group. (See I.A.)

Explanation of Sample Printout

Case No.1: Aerotriangulation without error propagation

Page Nos. Printout/User's Guide	Case No. 1 Contents/Explanation
0 / 27	A description of the options selected in the *job definition* record of the COMMON file of the input data is printed on this page. Example: Error propagation *was not requested*, as indicated on line four. This corresponds to option = 0 in column 11 of record no.2 (job definition) of the COMMON input file (sec. II.B.1).
1 / 28	The following information appears: Photo group (group specific) parameters: Group name Principal distance (microns) Std.dev.(microns) of image x and y coordinates Auxiliary parameters (do not appear on the printout because no auxiliary model opted in GROUPS file).
2 / 29	The following information appears: Frame number: Group specific parameters: Group name Principal distance (microns) Std.dev.(microns) of image x and y coordinates Camera station position: Longitude, and std.dev.(DDMMSS.SSSS) Latitude , and std.dev.(DDMMSS.SSSS) Elevation, and std. dev. (meters) Camera attitude: (photo to ground) Omega(roll), and std.dev.(DDMMSS.SSSS) Phi (pitch) and std.dev.(DDMMSS.SSSS) Kappa (yaw) and std.dev.(DDMMSS.SSSS) Plate coordinates: ID image point x refined plate coord.(microns) y refined plate coord.(microns)

Case No.1 (continued):

Page Nos. Printout/User's Guide	Case No.1 Contents/Explanation
20 / 30	Ground control data: ID ground control point Longitude, and std.dev.(DDMMSS.SSSS) Latitude, and std.dev.(DDMMSS.SSSS) Elevation, and std.dev.(meters) Type of control--planimetry, vertical or both. (Details are given in the description for character 80, sec. II.B.5.)
? / ?	Error warnings, if there are any. Note: Section III contains a list of error warnings and their explanation.
22 / 31	Camera station corrections: This page shows the corrections to position and attitude parameters of a camera after performing an iteration in the solution. The provisional weighted sum of square also appears at the end of the page. This total is used for computing *a posteriori* estimates for the unit weight variance. The difference in its current value and its value in the previous iteration forms a criterion for convergence of the solution (character 18-19, sec. II.B.l). The printout contains the following information: Iteration number: Photo or camera station no.(51C0295) Positional corrections (*meters*) to X, Y, and Z coordinates of each camera station Attitude corrections (*radians*) to roll, pitch, and yaw, for each camera station Provisional weighted sum of squares
23 / 32	Plate coordinates residuals at the end of a least squares adjustment of the aerotriangulation run, may be printed in a manner selected by option in columns 41-50 of record no.2 of COMMON file (sec.II.B.1) The printout shows the following: Triangulated ground points residuals ID of the image measured ID of frames in which the image of the point appear and image coordinates measured Residuals (microns) in the x and y coords. of the image point

Case No.1 (continued):

Page Nos. Printout/User's Guide	Case No.1 Contents/Explanation
24 / 33	Aerotriangulation statistics: Weighted sum of squares (camera) Weighted sum of squares (ground) Weighted sum of squares (plates) Weighted sum of square (total) Degrees of freedom *A posteriori* estimates for unit weight: Variance Standard deviation The weighted sum of squares value shows the contributions of various factors to the total residuals in the solution. The *a posteriori* estimates for unit weight variance will compute to nearly one if the relative weights of the parameters entering the solution were realistic. This value is an indication of the soundness of the weight assignments.
25 / 34	This page shows the state vectors of the triangulated camera stations: ID of camera stations Position vector: Adjusted longitude (DDMMSS.SSSS) Adjusted latitude (DDMMSS.SSSS) Adjusted elevation (meters, above datum) Attitude (photo to ground): Adjusted omega (roll) (DDMMSS.SSSS) Adjusted phi (pitch) (DDMMSS.SSSS) Adjusted kappa (yaw) (DDMMSS.SSSS)
27 / 35	This page contains the position vectors of triangulated ground points: ID of object points (including ground control points indicated by *type* in front of ID) Type (vertical, horizontal, or both vertical and horizontal ground control, sec. II.B.5) Adjusted longitude (DDMMSS.SSSS) Adjusted latitude (DDMMSS.SSSS) Adjusted elevation (meters, above datum)

Case No.1 (continued):

Page Nos. Printout/User's Guide	Case No.1 Contents/Explanation
28 / 36	This page gives the corrections applied to the ground control. The *type* of ground control is considered in the least squares solution. Accordingly, weights are assigned to constrain one, two, or all three components, X, Y, and Z, or λ, ϕ, and h, of a control point.

In the printout, the following appears:

ID ground control point
Corrections to longitude and latitude (DDMMSS.SSSS) and elevation (meters) of a control point.

Note: In case it is a vertical control point the latitude and longitude corrections appear within parenthesis; hereas, in the case of a horizontal ground control, the elevation correction appears within parenthesis.

Longitude - number of components, indicates number of ground control whose longitude values are known and constrained accordingly in the least squares adjustment.

Latitude - number of components, indicates number of ground control whose latitude values are known and constrained accordingly in the least squares adjustment.

Elevation - number of components, indicates number of ground control whose elevation values are known and constrained accordingly in the least squares adjustment.

RMS - root mean square errors for longitude and latitude (DDMMSS.SSSS).

RMS - root mean square error in elevation (meters).

OBJECT SPACE REFERENCE SYSTEM IS GEOGRAPHIC

ROTATION ANGLES ARE PHOTO-TO-GROUND

COMPLETE TRIANGULATION PROCESS IS REQUESTED

ERROR PROPAGATION IS NOT REQUESTED

ATMOSPHERIC REFRACTION WILL BE INCLUDED IN THE ADJUSTMENT

WATER REFRACTION WILL NOT BE INCLUDED IN THE ADJUSTMENT

IMAGE RESIDUALS GREATER THAN 10 (MICRONS) WILL BE LISTED

LEADING '0' WILL BE ELIMINATED FROM ALL IDENTIFICATIONS

SEMI-MAJOR AXIS OF SPHEROID = 6378206.40

SEMI-MINOR AXIS OF SPHEROID = 6356583.80

TRIANGULATED GROUND COORDINATES WILL BE SAVED

ADJUSTED STATION PARAMETERS WILL BE SAVED

P H O T O   G R O U P   P A R A M E T E R E S

GROUP: CAMERA

PRINCIPAL DISTANCE = -153280   ST. D. OF X = 8   ST. D. OF Y = 8

NOAA/NOS GIANT SYSTEM ... (10/90) : SAMPLE RUN FOR NOAA/NOS GIANT PROGRAM PAGE 2

FRAME: 51C0295 GROUP: CAMERA

PRINCIPAL DISTANCE = -153280 ST. D. OF X = 8 ST. D. OF Y = 8

STATION PARAMETERS

P O S I T I O N (CAMERA)				A T T I T U D E (PHOTO TO GROUND)			
LNG =	- 97 0 14.8790	ST. D. =	0 10 0.0000	OMEGA =	0 0 0.0000	ST. D. =	90 0 0.0000
LAT =	30 39 16.0370	ST. D. =	0 10 0.0000	PHI =	0 0 0.0000	ST. D. =	90 0 0.0000
ELV =	618.2290	ST. D. =	60000.0000	KAPPA =	0 0 0.0000	ST. D. =	90 0 0.0000

PLATE COORDINATES

ID	X	Y	ID	X	Y	ID	X	Y	ID	X	Y
31!	-1726	-86562	H60R	-30321	-33482	H60	-20116	-4515	33!	-8950	-1283
H60L	-21301	32640	35!	3788	96775	36!	71389	91706	H61L	56621	42864
H61	59665	1928	H61R	58234	-43983	68!	81607	-85579			

G R O U N D   C O N T R O L   D A T A

```
               LNG = - 97  0 16.8950     ST. D. =    0  0  0.0010
    H60        LAT =   30 39 14.5870     ST. D. =    0  0  0.0010    TYPE = 0
               ELV =        109.7830     ST. D. =          0.0100

               LNG =    0  0  0.0000     ST. D. =    0  0  0.0010
    H60L       LAT =    0  0  0.0000     ST. D. =    0  0  0.0010    TYPE = 3
               ELV =        110.1090     ST. D. =          0.0100

               LNG =    0  0  0.0000     ST. D. =    0  0  0.0010
    H60R       LAT =    0  0  0.0000     ST. D. =    0  0  0.0010    TYPE = 3
               ELV =        109.0670     ST. D. =          0.0100

               LNG = - 97  0  8.4670     ST. D. =    0  0  0.0010
    H61        LAT =   30 39 18.9740     ST. D. =    0  0  0.0010    TYPE = 0
               ELV =        116.9550     ST. D. =          0.0100

               LNG =    0  0  0.0000     ST. D. =    0  0  0.0010
    H61L       LAT =    0  0  0.0000     ST. D. =    0  0  0.0010    TYPE = 3
               ELV =        114.4250     ST. D. =          0.0100

               LNG =    0  0  0.0000     ST. D. =    0  0  0.0010
    H61R       LAT =    0  0  0.0000     ST. D. =    0  0  0.0010    TYPE = 3
               ELV =        116.0860     ST. D. =          0.0100

               LNG = - 97  0  1.0780     ST. D. =    0  0  0.0010
    H62        LAT =   30 39 22.2830     ST. D. =    0  0  0.0010    TYPE = 0
```

S T A T I O N S   C O R R E C T I O N S

ITERATION 1

51C0295	POSITION	-0.000000753	0.000004388	58.6	ATTITUDE	0.055298190	0.001744533	-0.383985314
51C0296	POSITION	-0.000000431	0.000002037	49.1	ATTITUDE	0.041834562	0.006216514	-0.369735781
51C0297	POSITION	0.000000167	-0.000000919	51.9	ATTITUDE	0.013032464	0.007080181	-0.413999101
51C0298	POSITION	-0.000000290	-0.000000715	50.5	ATTITUDE	0.014613826	0.014048541	-0.413147361
51C0299	POSITION	0.000000746	0.000000344	48.3	ATTITUDE	0.012786607	-0.002606765	-0.390412525
51C0300	POSITION	0.000002198	0.000000196	46.1	ATTITUDE	-0.016243288	-0.026731762	-0.429808533
52N0316	POSITION	-0.000001066	0.000001735	28.0	ATTITUDE	0.040019893	0.046472320	-0.405401762
52N0317	POSITION	-0.000000573	-0.000001531	29.7	ATTITUDE	0.007431540	0.048198779	-0.427163027
52N0318	POSITION	-0.000001592	-0.000002272	38.9	ATTITUDE	-0.007247630	0.048288748	-0.408581207
52N0319	POSITION	-0.000000608	-0.000000798	43.3	ATTITUDE	-0.009735586	0.047310862	-0.410449640
52N0320	POSITION	-0.000001344	-0.000000915	45.5	ATTITUDE	0.000917621	0.041581086	-0.427103429
52N0321	POSITION	-0.000005214	-0.000000249	46.6	ATTITUDE	-0.000317372	0.052784328	-0.437181840
53S0338	POSITION	0.000001275	0.000001052	70.4	ATTITUDE	0.021266293	-0.016176390	-0.387188876
53S0339	POSITION	-0.000000095	0.000000567	58.7	ATTITUDE	0.060614354	0.001892756	-0.376525232
53S0340	POSITION	0.000001041	-0.000001604	56.5	ATTITUDE	0.011601995	0.000229489	-0.423520182
53S0341	POSITION	0.000001382	0.000000104	53.9	ATTITUDE	0.013552853	0.008615623	-0.425447555
53S0342	POSITION	0.000001944	0.000000533	49.6	ATTITUDE	0.027316638	-0.005538455	-0.429461629
53S0343	POSITION	0.000002984	0.000002952	41.6	ATTITUDE	0.027006126	-0.012974019	-0.436122793

PROVISIONAL WEIGHTED SUM OF SQUARES = 0.404607E+10

ITERATION 5

51C0295	POSITION	0.000000000	0.000000000	0.0	ATTITUDE	0.000000081	0.000000021	0.000000008
51C0296	POSITION	0.000000000	0.000000000	0.0	ATTITUDE	0.000000019	0.000000001	0.000000009
51C0297	POSITION	0.000000000	0.000000000	0.0	ATTITUDE	0.000000020	-0.000000010	0.000000007
51C0298	POSITION	0.000000000	0.000000000	0.0	ATTITUDE	0.000000031	0.000000009	0.000000005
.								
.								
.								
53S0340	POSITION	0.000000000	0.000000000	0.0	ATTITUDE	0.000000014	-0.000000021	0.000000007
53S0341	POSITION	0.000000000	0.000000000	0.0	ATTITUDE	-0.000000012	0.000000004	0.000000004
53S0342	POSITION	0.000000000	0.000000000	0.0	ATTITUDE	-0.000000009	0.000000017	0.000000006
53S0343	POSITION	0.000000000	0.000000000	0.0	ATTITUDE	-0.000000016	0.000000023	0.000000012

PROVISIONAL WEIGHTED SUM OF SQUARES = 126.140

T R I A N G U L A T E D   G R O U N D   P O I N T S   R E S I D U A L S

```
15!       51C0297  51C0296  51C0298  52N0318  52N0319  53S0339  53S0340  53S0341
              -10       14
                1        6

51!       53S0343  51C0299  51C0300  53S0341  53S0342  51C0298
               -4
              -17

H63 *0*   53S0343  52N0321  51C0298  52N0319  51C0299  51C0300  53S0341  53S0342  52N0320
               11
                5

23!       51C0299  52N0320  51C0298  51C0300  52N0319  52N0321
               10
                0

26!       52N0321  51C0299  51C0300  52N0320  53S0342  53S0343
               10      -10
               -8        5
```

A E R O T R I A N G U L A T I O N   S T A T I S T I C S

```
WEIGHTED SUM OF SQUARES (PHOTOS)  =          1.5
WEIGHTED SUM OF SQUARES (GROUND)  =          2.8
WEIGHTED SUM OF SQUARES (IMAGES)  =         98.2

WEIGHTED SUM OF SQUARES (TOTAL)   =        102.5
DEGREES OF FREEDOM..............  =          197


A POSTERIORI ESTIMATES FOR UNIT WEIGHT
VARIANCE =          0.520
ST. DEV. =          0.721
```

T R I A N G U L A T E D S T A T I O N S

IDENT	POSITION	ATT(PHOTO TO GROUND)
	LNG =- 97 0 14.8229	OMEGA = 0 7 11.5707
51C0295	LAT = 30 39 16.0567	PHI =- 0 19 35.8173
	ELV = 617.4481	KAPPA =- 26 23 17.8929
	LNG =- 97 0 7.5400	OMEGA = 0 31 27.5002
51C0296	LAT = 30 39 19.3364	PHI =- 0 15 10.2065
	ELV = 612.3894	KAPPA =- 24 31 16.9658
	LNG =- 97 0 0.2082	OMEGA = 0 35 11.4277
51C0297	LAT = 30 39 22.6156	PHI =- 0 8 31.2704
	ELV = 612.8024	KAPPA =- 26 32 51.5016
	LNG =- 96 59 52.8538	OMEGA = 0 56 23.2658
51C0298	LAT = 30 39 25.8800	PHI = 0 10 20.6344
	ELV = 616.9580	KAPPA =- 25 28 51.2535
	LNG =- 96 59 45.1652	OMEGA = 0 34 25.1912
51C0299	LAT = 30 39 29.1904	PHI = 0 5 6.4240
	ELV = 621.2436	KAPPA =- 24 19 29.2861
	LNG =- 96 59 37.7191	OMEGA =- 0 40 35.8729
51C0300	LAT = 30 39 32.3483	PHI =- 0 2 49.5255
	ELV = 623.6036	KAPPA =- 26 6 3.8739
	LNG =- 97 0 16.5298	OMEGA = 0 39 21.3184
52N0316	LAT = 30 39 25.4072	PHI = 0 52 11.5870
	ELV = 593.1177	KAPPA =- 25 50 57.6134
	LNG =- 97 0 9.2222	OMEGA = 0 30 49.0261
52N0317	LAT = 30 39 28.6971	PHI = 1 26 12.8647
	ELV = 588.8534	KAPPA =- 26 3 6.1477

T R I A N G U L A T E D G R O U N D P O I N T S

IDENT		POSITION		
10!		LNG =- 97 0 12.6046	LAT = 30 39 37.9637	ELV = 113.590
11!		LNG =- 97 0 7.4237	LAT = 30 39 39.3516	ELV = 115.104
13!		LNG =- 97 0 3.0676	LAT = 30 39 32.0685	ELV = 119.715
15!		LNG =- 96 59 57.3986	LAT = 30 39 23.3612	ELV = 118.378
16!		LNG =- 96 59 50.1427	LAT = 30 39 26.9175	ELV = 118.041
18!		LNG =- 96 59 56.4011	LAT = 30 39 35.2651	ELV = 125.640
20!		LNG =- 96 59 59.9227	LAT = 30 39 43.2337	ELV = 119.313
21!		LNG =- 96 59 51.6841	LAT = 30 39 45.0408	ELV = 121.642
23!		LNG =- 96 59 48.2622	LAT = 30 39 38.4656	ELV = 120.067
25!		LNG =- 96 59 43.3992	LAT = 30 39 29.7767	ELV = 115.380
26!		LNG =- 96 59 35.5795	LAT = 30 39 33.1336	ELV = 117.503
28!		LNG =- 96 59 39.7357	LAT = 30 39 41.0205	ELV = 116.537
31!		LNG =- 97 0 10.3177	LAT = 30 39 7.5779	ELV = 111.418
33!		LNG =- 97 0 15.8295	LAT = 30 39 15.4326	ELV = 109.818
35!		LNG =- 97 0 19.8105	LAT = 30 39 25.4501	ELV = 111.126
36!		LNG =- 97 0 12.0462	LAT = 30 39 28.1088	ELV = 112.975
51!		LNG =- 96 59 39.2345	LAT = 30 39 20.3639	ELV = 114.233
60!		LNG =- 96 59 32.9038	LAT = 30 39 23.2638	ELV = 118.411
65!		LNG =- 97 0 2.7306	LAT = 30 39 1.7307	ELV = 112.545
66!		LNG =- 96 59 56.1514	LAT = 30 39 3.7289	ELV = 110.238
68!		LNG =- 97 0 1.2779	LAT = 30 39 11.7038	ELV = 116.251
73!		LNG =- 96 59 53.2342	LAT = 30 39 14.7480	ELV = 112.436
75!		LNG =- 96 59 49.2063	LAT = 30 39 7.1343	ELV = 109.127
78!		LNG =- 96 59 46.6321	LAT = 30 39 19.5208	ELV = 114.258
80!		LNG =- 96 59 42.9456	LAT = 30 39 11.7442	ELV = 110.179
81!		LNG =- 96 59 33.5543	LAT = 30 39 13.7951	ELV = 114.617
90!		LNG =- 96 59 26.1837	LAT = 30 39 16.7385	ELV = 117.997
H60	*0*	LNG =- 97 0 16.8951	LAT = 30 39 14.5868	ELV = 109.782
H61	*0*	LNG =- 97 0 8.4665	LAT = 30 39 18.9740	ELV = 116.951
H62	*0*	LNG =- 97 0 1.0779	LAT = 30 39 22.2835	ELV = 118.769
H63	*0*	LNG =- 96 59 45.8981	LAT = 30 39 28.3246	ELV = 117.358
H64	*0*	LNG =- 96 59 29.3024	LAT = 30 39 35.7671	ELV = 123.912
133!		LNG =- 96 59 29.1755	LAT = 30 39 35.1733	ELV = 123.855

C O R R E C T I O N S   A P P L I E D   T O   G R O U N D   C O N T R O L

```
        LNG= -  0  0  0.0001          LNG=    0  0  0.0005          LNG=    0  0  0.0001          LNG= -  0  0  0.0011
  H60   LAT= -  0  0  0.0002    H61   LAT= -  0  0  0.0000    H62   LAT=    0  0  0.0005    H63   LAT= -  0  0  0.0004
        ELV=         -0.001           ELV=         -0.004           ELV=         -0.002           ELV=          0.001

        LNG=    0  0  0.0006          LNG=(   0  0  0.0000)         LNG=(   0  0  0.0000)         LNG=(   0  0  0.0000)
  H64   LAT=    0  0  0.0001   H60L   LAT=(   0  0  0.0000)  H61L   LAT=(   0  0  0.0000)  H62L   LAT=(   0  0  0.0000)
        ELV=         -0.001           ELV=         -0.001           ELV=          0.001           ELV=          0.004

        LNG=(-  0  0  0.0000)         LNG=(-  0  0  0.0000)         LNG=(-  0  0  0.0000)         LNG=(-  0  0  0.0000)
 H63L   LAT=(-  0  0  0.0000)  H64L   LAT=(   0  0  0.0000)  H60R   LAT=(   0  0  0.0000)  H61R   LAT=(   0  0  0.0000)
        ELV=         -0.002           ELV=          0.001           ELV=          0.000           ELV=          0.003

        LNG=(-  0  0  0.0000)         LNG=(   0  0  0.0000)
 H62R   LAT=(   0  0  0.0000)  H64R   LAT=(-  0  0  0.0000)
        ELV=          0.001           ELV=         -0.001


              LNG .... NUMBER OF COMPONENTS =    5    RMS =    0  0  0.0006
              LAT .... NUMBER OF COMPONENTS =    5    RMS =    0  0  0.0003
              ELV .... NUMBER OF COMPONENTS =   14    RMS =          0.0020
```

Explanation of Sample Printout

Case No. 2: Aerotriangulation with Error Propagation

NOTE: Printout page number 0 and three new pages (41, 53, and 54) are given here to show the enhancements to Case No. 1.

Page Numbers Printout/Text	Case No. 2 Contents/Explanation
0 / 38	A description of the options selected in the job definition record of the COMMON file of the input data is printed on this page. Example: Line four describes the option which was selected in character 11 of record no. 2 (job definition) of the COMMON input data file. Error propagation was selected with this option.
25 / 39	This page corresponds to printout page 25 (case no. 1), which gives adjusted longitude, latitude, and elevation of the triangulated camera stations. In addition to the position parameters, corresponding 3 by 3 covariance matrices are printed. Another 3 by 3 covariance matrix for the three rotational angles (ω, ϕ, κ) is also printed. The square root of the diagonal terms give the standard deviation with which the corresponding parameter has been determined. *During the computations*, all angular parameters are expressed in radians, such that the square root of a diagonal term of an angular parameter gives the standard deviation in radians.
27 / 40	This page gives the position vector of the triangulated ground points as shown on printout page 27 (case no. 1), and, in addition, shows a 3 by 3 covariance matrix and standard deviations for the three components: longitude, latitude, and elevation. The square root of each of the diagonal terms of the covariance matrix gives their standard deviation.
30 / 41	This is the last page of case no. 2 and shows the triangulated ground points. The rms errors in longitude, latitude, and elevation are displayed. These rms values are averaged over all the triangulated ground points.

OBJECT SPACE REFERENCE SYSTEM IS GEOGRAPHIC

ROTATION ANGLES ARE PHOTO-TO-GROUND

COMPLETE TRIANGULATION PROCESS IS REQUESTED

ERROR PROPAGATION IS REQUESTED

UNIT VARIANCE WILL BE BASED ON COMPLETELY FREE STATION PARAMETERS

ATMOSPHERIC REFRACTION WILL BE INCLUDED IN THE ADJUSTMENT

WATER REFRACTION WILL NOT BE INCLUDED IN THE ADJUSTMENT

IMAGE RESIDUALS GREATER THAN 10 (MICRONS) WILL BE LISTED

LEADING '0' WILL BE ELIMINATED FROM ALL IDENTIFICATIONS

SEMI-MAJOR AXIS OF SPHEROID = 6378206.40

SEMI-MINOR AXIS OF SPHEROID = 6356583.80

TRIANGULATED GROUND COORDINATES WILL BE SAVED

ADJUSTED STATION PARAMETERS WILL BE SAVED

T R I A N G U L A T E D   S T A T I O N S

IDENT	POSITION	COVARIANCE MATRIX			ATT(PHOTO TO GROUND)	COVARIANCE MATRIX		
	LNG =- 97 0 14.8229	0.219E-15	0.508E-16	0.336E-09	OMEGA = 0 7 11.5707	0.109E-07	-0.691E-09	0.113E-08
51C0295	LAT = 30 39 16.0567	0.508E-16	0.113E-15	0.150E-09	PHI =- 0 19 35.8173	-0.691E-09	0.263E-07	-0.145E-09
	ELV = 617.4481	0.336E-09	0.150E-09	0.136E-02	KAPPA =- 26 23 17.8929	0.113E-08	-0.145E-09	0.273E-08
	LNG =- 97 0 7.5400	0.720E-16	-0.685E-17	0.293E-10	OMEGA = 0 31 27.5002	0.823E-08	0.606E-09	0.579E-09
51C0296	LAT = 30 39 19.3364	-0.685E-17	0.554E-16	0.392E-11	PHI =- 0 15 10.2065	0.606E-09	0.521E-08	0.816E-10
	ELV = 612.3894	0.293E-10	0.392E-11	0.386E-03	KAPPA =- 24 31 16.9658	0.579E-09	0.816E-10	0.153E-08
	LNG =- 97 0 0.2082	0.690E-16	-0.701E-17	0.594E-11	OMEGA = 0 35 11.4277	0.818E-08	0.490E-09	0.110E-09
51C0297	LAT = 30 39 22.6156	-0.701E-17	0.535E-16	0.787E-11	PHI =- 0 8 31.2704	0.490E-09	0.498E-08	0.406E-10
	ELV = 612.8024	0.594E-11	0.787E-11	0.342E-03	KAPPA =- 26 32 51.5016	0.110E-09	0.406E-10	0.119E-08
	LNG =- 96 59 52.8538	0.808E-16	-0.307E-17	0.668E-11	OMEGA = 0 56 23.2658	0.856E-08	0.499E-09	-0.717E-10
51C0298	LAT = 30 39 25.8800	-0.307E-17	0.581E-16	0.163E-10	PHI = 0 10 20.6344	0.499E-09	0.575E-08	0.276E-10
	ELV = 616.9580	0.668E-11	0.163E-10	0.314E-03	KAPPA =- 25 28 51.2535	-0.717E-10	0.276E-10	0.106E-08
	LNG =- 96 59 45.1652	0.115E-15	0.299E-17	0.276E-10	OMEGA = 0 34 25.1912	0.113E-07	-0.319E-09	0.137E-09
51C0299	LAT = 30 39 29.1904	0.299E-17	0.879E-16	0.357E-10	PHI = 0 5 6.4240	-0.319E-09	0.101E-07	0.505E-10
	ELV = 621.2436	0.276E-10	0.357E-10	0.481E-03	KAPPA =- 24 19 29.2861	0.137E-09	0.505E-10	0.159E-08
	LNG =- 96 59 37.7191	0.189E-15	0.131E-16	-0.910E-10	OMEGA =- 0 40 35.8729	0.136E-07	0.105E-08	-0.229E-09
51C0300	LAT = 30 39 32.3483	0.131E-16	0.103E-15	-0.170E-10	PHI =- 0 2 49.5255	0.105E-08	0.147E-07	0.944E-11
	ELV = 623.6036	-0.910E-10	-0.170E-10	0.549E-03	KAPPA =- 26 6 3.8739	-0.229E-09	0.944E-11	0.196E-08
	LNG =- 97 0 16.5298	0.108E-15	-0.392E-17	0.197E-09	OMEGA = 0 39 21.3184	0.133E-07	0.122E-08	0.706E-09
52N0316	LAT = 30 39 25.4072	-0.392E-17	0.980E-16	-0.934E-10	PHI = 0 52 11.5870	0.122E-08	0.814E-08	-0.233E-08
	ELV = 593.1177	0.197E-09	-0.934E-10	0.128E-02	KAPPA =- 25 50 57.6134	0.706E-09	-0.233E-08	0.321E-08
	LNG =- 97 0 9.2222	0.291E-15	0.261E-16	0.234E-09	OMEGA = 0 30 49.0261	0.165E-07	0.628E-08	-0.280E-09
52N0317	LAT = 30 39 28.6971	0.261E-16	0.112E-15	-0.555E-10	PHI = 1 26 12.8647	0.628E-08	0.314E-07	-0.268E-08
	ELV = 588.8534	0.234E-09	-0.555E-10	0.109E-02	KAPPA =- 26 3 6.1477	-0.280E-09	-0.268E-08	0.293E-08

T R I A N G U L A T E D   G R O U N D   P O I N T S

IDENT	POSITION	COVARIANCE MATRIX			STANDARD DEV		
	LNG =- 97 0 12.6046	0.358E-16	-0.110E-16	0.103E-09	0	0	0.0012
10!	LAT = 30 39 37.9637	-0.110E-16	0.500E-16	-0.223E-09	0	0	0.0015
	ELV = 113.5897	0.103E-09	-0.223E-09	0.405E-02			0.0636
	LNG =- 97 0 7.4237	0.352E-16	-0.101E-16	0.116E-09	0	0	0.0012
11!	LAT = 30 39 39.3516	-0.101E-16	0.366E-16	-0.174E-09	0	0	0.0012
	ELV = 115.1043	0.116E-09	-0.174E-09	0.384E-02			0.0620
	LNG =- 97 0 3.0676	0.113E-16	-0.223E-18	0.105E-10	0	0	0.0007
13!	LAT = 30 39 32.0685	-0.223E-18	0.108E-16	-0.221E-10	0	0	0.0007
	ELV = 119.7151	0.105E-10	-0.221E-10	0.832E-03			0.0288
	LNG =- 96 59 57.3986	0.654E-17	-0.121E-19	0.132E-11	0	0	0.0005
15!	LAT = 30 39 23.3612	-0.121E-19	0.562E-17	-0.312E-11	0	0	0.0005
	ELV = 118.3775	0.132E-11	-0.312E-11	0.258E-03			0.0161
	LNG =- 96 59 50.1427	0.760E-17	-0.120E-18	-0.692E-12	0	0	0.0006
16!	LAT = 30 39 26.9175	-0.120E-18	0.638E-17	0.109E-12	0	0	0.0005
	ELV = 118.0407	-0.692E-12	0.109E-12	0.291E-03			0.0171
	LNG =- 96 59 56.4011	0.127E-16	-0.798E-19	0.108E-10	0	0	0.0007
18!	LAT = 30 39 35.2651	-0.798E-19	0.116E-16	-0.282E-10	0	0	0.0007
	ELV = 125.6405	0.108E-10	-0.282E-10	0.996E-03			0.0316
	LNG =- 96 59 59.9227	0.346E-16	-0.979E-17	0.967E-10	0	0	0.0012
20!	LAT = 30 39 43.2337	-0.979E-17	0.446E-16	-0.218E-09	0	0	0.0014
	ELV = 119.3131	0.967E-10	-0.218E-09	0.438E-02			0.0662
	LNG =- 96 59 51.6841	0.324E-16	-0.303E-17	0.414E-10	0	0	0.0012
21!	LAT = 30 39 45.0408	-0.303E-17	0.387E-16	-0.180E-09	0	0	0.0013
	ELV = 121.6423	0.414E-10	-0.180E-09	0.378E-02			0.0615
	LNG =- 96 59 48.2622	0.148E-16	0.510E-18	-0.555E-12	0	0	0.0008

T R I A N G U L A T E D G R O U N D P O I N T S

IDENT		POSITION		COVARIANCE MATRIX			STANDARD DEV		
		LNG =-	96 59 59.2306	0.744E-17	0.212E-18	0.621E-12	0	0	0.0006
H62R	*3*	LAT =	30 39 18.8973	0.212E-18	0.601E-17	0.578E-13	0	0	0.0005
		ELV =	116.6909	0.621E-12	0.578E-13	0.474E-04			0.0069
		LNG =-	96 59 47.4732	0.983E-17	0.351E-18	0.257E-12	0	0	0.0006
H63L	*3*	LAT =	30 39 32.1409	0.351E-18	0.773E-17	0.221E-12	0	0	0.0006
		ELV =	118.3281	0.257E-12	0.221E-12	0.478E-04			0.0069
		LNG =-	96 59 32.8211	0.236E-16	-0.152E-17	-0.306E-11	0	0	0.0010
H64L	*3*	LAT =	30 39 40.5129	-0.152E-17	0.189E-16	-0.152E-11	0	0	0.0009
		ELV =	120.9880	-0.306E-11	-0.152E-11	0.513E-04			0.0072
		LNG =-	96 59 27.3394	0.220E-16	0.162E-18	-0.342E-11	0	0	0.0010
H64R	*3*	LAT =	30 39 32.7565	0.162E-18	0.190E-16	-0.187E-11	0	0	0.0009
		ELV =	124.1565	-0.342E-11	-0.187E-11	0.514E-04			0.0072

S U M M A R Y S T A T I S T I C S F O R G R O U N D P O I N T S

RMS FOR STANDARD DEVIATIONS

COUNT =	38	LNG =	0	0	0.0010
COUNT =	38	LAT =	0	0	0.0010
COUNT =	29	ELV =			0.0523

Explanation of Sample Printout

Case No. 3: Aerotriangulation without Error Propagation and additional parameters (radial distortion, model no. 2, sec. I.A.)

NOTE: Printout page nos. 1 and 25 are unique to this run. Only these pages are explained herein. The rest of the output is similar to the one for Case No. 1.

Page Numbers Printout/Text	Case No. 3 Contents/Explanation
1 / 43	The following appears on this printout page: Title: Photo group Parameters Group name Principal distance (microns) Standard deviation of measurements of x and y image coordinates Auxiliary parameters (according to the model used): Model No. 1: Principal distance and principal point location Model No. 2: Coefficients of a polynomial to account for unmodelled radial distortions Model No. 3: Offsets between GPS receiver antenna and the outer node of the aerial mapping camera Note: In the example, three strips STRIP1, STRIP2, and STRIP3, each have a set of radial distortion (model no. 2) parameters switched on or off at the user discretion.
25 / 44	The following appears on this printout page: Title: Unmodelled error parameters (type, e.g., radial distortion) Photo group: Name: Values of parameters as obtained after the aerotriangulation adjustment. Note: Each photo group has the adjusted parameter values.

P H O T O   G R O U P   P A R A M E T E R E S

GROUP: CAMERA

PRINCIPAL DISTANCE = -153280   ST. D. OF X = 8   ST. D. OF Y = 8

AUXILIARY PARAMETERS (GPS ANTENNA OFFSET)

P1 = 0.00000D+00   UPDATING SWITCH : ON
P2 = 0.00000D+00   UPDATING SWITCH : ON
P3 = 0.00000D+00   UPDATING SWITCH : ON

UNMODELLED ERROR PARAMETERES

(GPS ANTENNA OFFSET)

PHOTO GROUP: : CAMERA

```
P1 =  0.20202D+01
P2 = -0.15407D+01
P3 =  0.62674D+00
```

D. Logical Identification Number Assignments

All of the files used by GIANT have been assigned the following logical identification numbers.

Table II.1.--Files Used By GIANT

Logical ID Number	Type	Format (F) Formatted (UF) Unformatted	Utilization	Text Page Number
1	Input	(F) sec. II.B.2	Groups data set	10
2	Input	(F) sec. II.B.3	Image coordinate data set	11
3	Input	(F) sec. II.B.4	Frame data set	13
4	Input	(F) sec. II.B.5	Ground control data set	15
6	Output	(F) sec. II.C.1	Printout	22
7	Input	(F) sec. II.B.1	Common data set	6
8	Output	(F) sec. II.C.2	Updated frames data set	22
9	Output	(F) sec. II.C.3	Adjusted ground data set	22
11	Scratch	(UF)	Temporary storage	
.	.	. .	.	
.	.	. .	.	
.	.	. .	.	
.	.	. .	.	
.	.	. .	.	
19	.	. .	.	
20	Output	(F) sec. II.C.2	Updated Frames data set	22

Output files logical ID Nos. 8 and 9 will not be generated by GIANT in case the solution fails to converge.

Output file logical ID No. 20 will always be created whether the solution converges or not. This file can always be used at the user's option to restart the iterative solution from where it left off.

III. A LIST OF DIAGNOSTIC MESSAGES PRODUCED BY GIANT

The occurrence of errors during the execution of GIANT results in the generation of appropriate diagnostic messages. Two types of diagnostics are reported, warning messages and fatal messages. If the detected error reflects the occurrence of a fatal error, execution of the job is terminated. A warning error message does not result in the abandonment of a job. The user must, however, carefully interpret the effect of all warning messages produced by the program, as they might point to problems of which the user is not aware.

The remainder of this section contains a list of the various diagnostic messages that may be produced by GIANT. The list gives the text of each message, definition parameters, resulting action taken by GIANT, and a brief explanation of the meaning. In the list, fatal error messages are identified by the prefix"**".

**ILLEGAL DMS FIELD DETECTED IN INPUT STREAM

This message will result from an attempt to read an angular field with the degrees part >360, the minutes part >60, or the seconds part >60.

**ERROR IN SUBROUTINE MODID

ADDING VARIABLE XXXX

```
VARIABLES  YYYY   YYYY   YYYY   YYYY   YYYY
             .      .      .      .      .
             .      .      .      .      .
VARIABLES  YYYY   YYYY   YYYY   YYYY   YYYY
```

This error indicates the present input data arrangement has resulted in normal equations with a bandwidth that exceeds GIANT's capacity. Usually, a rearrangement of input data will cure this problem. The present version of GIANT has a bandwidth of N6. (See sec. I.B.) This message can only occur when GIANT is running in the triangulation mode.

In the diagnostic message, the integer number (XXXX) is the input stream sequence number of the camera station that caused overflow of the normal equations storage. The integer numbers designated by (YYYY) are the sequence numbers of camera stations occupying the normal equations storage when overflow took place.

PASS POINTS APPEARING ON ONE PHOTOGRAPH

```
XXXXXXXX XXXXXXXX XXXXXXXX XXXXXXXX
   .        .        .        .
   .        .        .        .
XXXXXXXX XXXXXXXX XXXXXXXX XXXXXXXX
```

The message provides a list of alphanumeric identifications (XXXXXXXX) of ground points which appear on one photograph. All of the indicated image points will be dropped from the adjustment.

PASS POINTS APPEARING ON MORE THAN N4 PHOTOGRAPHS (See sec. I.B.)

```
XXXXXXXX XXXXXXXX XXXXXXXX XXXXXXXX
   .        .        .        .
XXXXXXXX XXXXXXXX XXXXXXXX XXXXXXXX
```

The listed alphanumeric identifications (XXXXXXXXX) are for imaged ground points appearing on more than N4 photographs. GIANT will retain only the first N4 image coordinates and drop the rest. (See sec. I.B.)

CAMERA STATIONS EXCEEDED XXXX

An attempt to adjust more than (XXXX) camera stations will result in GIANT issuing this diagnostic message. In the present version of GIANT, XXXX is Nl. (See sec. I.B.)

IMAGE POINTS EXCEEDED XXXX

The total number of image points in the triangulation exceeded (XXXX), which is limited by N2. (See sec. I.B.)

GROUND CONTROL EXCEEDED XXXX

This message will result from a ground control file which contains more than (XXXX) points. XXXX is N3 in the present version of GIANT. (See sec. I.B.)

ERROR WARNINGS
POINTS NOT PHOTOGRAPHED

```
XXXXXXXX XXXXXXXX XXXXXXXX XXXXXXXX
   .        .        .        .
   .        .        .        .
XXXXXXXX XXXXXXXX XXXXXXXX XXXXXXXX
```

This diagnostic message provides alphanumeric identifications of control points for which no image data exist. GIANT will ignore ground control coordinates of listed points.

ERROR WARNINGS
DUPLICATE CONTROL POINTS

```
XXXXXXXX XXXXXXXX XXXXXXXX XXXXXXXX
   .        .        .        .
   .        .        .        .
XXXXXXXX XXXXXXXX XXXXXXXX XXXXXXXX
```

The listed alphanumeric identifications are for control points which are repeated in the control file. GIANT will retain only first reference to the listed points and ignore all subsequent duplicates.

REFERENCES

American Society of Photogrammetry, 1980: Manual of Photogrammetry, Slama, Chester C., editor, 4, Falls Church, VA, 1056 pp.

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Elassal, Atef A., and Malhotra, Roop C., 1987: General integrated analytical triangulation (GIANT) user's guide. NOAA Technical Report NOS 126 CGS 11, 66 pp. National Oceanic and Atmospheric Administration, N/CG174, Rockville, MD 20852.

Engineering Management Series, 1981: Analytical mapping system (AMS) user's guide, II, analytical triangulation (draft), EM-7140-11, Forest Service, U.S. Department of Agriculture, Washington, D.C., 215 pp.

Lucas, J. R., 1987: Aerotriangulation without ground control, Photogrammetric Engineering and Remote Sensing, 53, pp. 311-314.

Lucas, J. R. and Mader, G. L., 1989: Verification of airborne positioning using global positioning system carrier phase measurements, Journal of Geophysical Research, 94, no. B8, pp. 171-181.

APPENDIX A.--DATA STRUCTURING BY GIANT

Input data are reordered by GIANT to facilitate the use of an efficient algorithm for the formation, solution, and inversion of the normal equations. The purpose of the data structuring process is to produce an efficient, diagonally banded matrix of normal equations.

The overall efficiency of the adjustment process depends on the arrangement of submatrices in the normal set which produces the narrowest possible bandwidth of nonzero elements. Bandwidth for the normal system structured by GIANT is a function of input data ordering.

It is true that a particular arrangement of data for the given job will imply a certain bandwidth for the normal equation matrix. A different arrangement of the data results in a different bandwidth. Since the maximum allowable bandwidth is a function of computer available storage, there may be cases where data arrangement becomes a determining factor in whether a job can be executed. By the proper arrangement of input data, the problem of exceeding the allowable bandwidth can often be prevented, or by rearrangement of the data, the problem can be rectified. The normal matrix is formed by GIANT under the following rules:

1. Modifiable parameters are divided into sets, each set being composed of three parameters. Therefore, a single camera station's parameters constitute two parameter sets, one for position and one for attitude.

2. The first reference to a ground point through one of its images results in the inclusion, in the normal matrix, of all parameter sets associated with the point. Therefore, parameter sets for camera stations observing the point will be added to the normal system if they have not already been placed in the matrix by reference from a previous point.

A simple example will best demonstrate the data structuring process used to build the normal equation matrix. Consider a strip of six photographs, each containing three image points distributed as shown in table 1.

Table A.1.--Examples of six photographs

Photograph	Image Points
1	1 2 3
2	2 3 4
3	3 4 5
4	4 5 6
5	5 6 7
6	6 7 8

Assume that input data are arranged as shown in table A.1 from photographs 1 through 6. Following the rules outlined above, the normal matrix shown in figure A.1 would be constructed as follows:

Image 1 of photo 1 is the first point encountered, resulting in the entry of the two camera parameters sets (C_1,C_1) and the ground point parameter set (P_1) into the normal matrix. Nonzero blocks occur whenever there is correlation; shown by shading the figure. The next image point encountered is point 2 which occurs on photographs 1 and 2. The camera station parameter sets for photograph 1 have already been included in the normal system, so only $C_2,C_2)$ and (P_2) must be added to the system. In the same fashion, image point 3 adds (C_3,C_3) and (P_3), etc., through photograph 6. Images 7 and 8 add no more camera parameter sets but they do add ground point parameter sets for points 7 and 8 to complete the normal system. The diagonal matrix produced is symmetric about the dashed diagonal line. Without further exploitation of the structural peculiarities for the normal matrix, the GIANT algorithm would require an amount of internal computer memory equivalent to the cross-hatched area in the figure.

GIANT logic, however, allows computer memory to be shared by a special group of unknown parameter sets. Any parameter set which is uncorrelated with the parameter sets that succeed it, is qualified as a member of this special group. Therefore, parameter sets P_1, P_2, and P_3 will share a common storage area within the cross-hatched area in the figure. This arrangement results in a reduction of the bandwidth from the original nine parameter sets to an effective bandwidth of seven parameter sets. The effective bandwidth notion applies to all possible positions of the cross hatched area along the matrix diagonal.

The reader would now be able to visualize what would happen to the normal matrix if the order of data input was changed. If photo 1 and photo 6 were exchanged in order in the previous example, the resulting effective bandwidth would be approximately double the original one.

As previously stated, the maximum allowable effective bandwidth is a function of computer system configuration. The limitation is based on the number of 3 by 3 matrix parameter sets which will fit a work area of computer memory, and it can be expressed in terms of the allowable number of photographs, either preceding or succeeding a given photograph, which may have points in common with the given photograph. If K is the number of parameter sets that will fit in the computer memory, then (K-1) camera parameter sets will be allowable in the normal matrix, using one set for the common ground point. Since two parameter sets are required for each photograph, a total of (K-1)/2 photographs may be involved. This amounts to allowing a photograph in a given input arrangement to have conjugate points with a maximum of (K-1)/2-1 photographs before or after it.

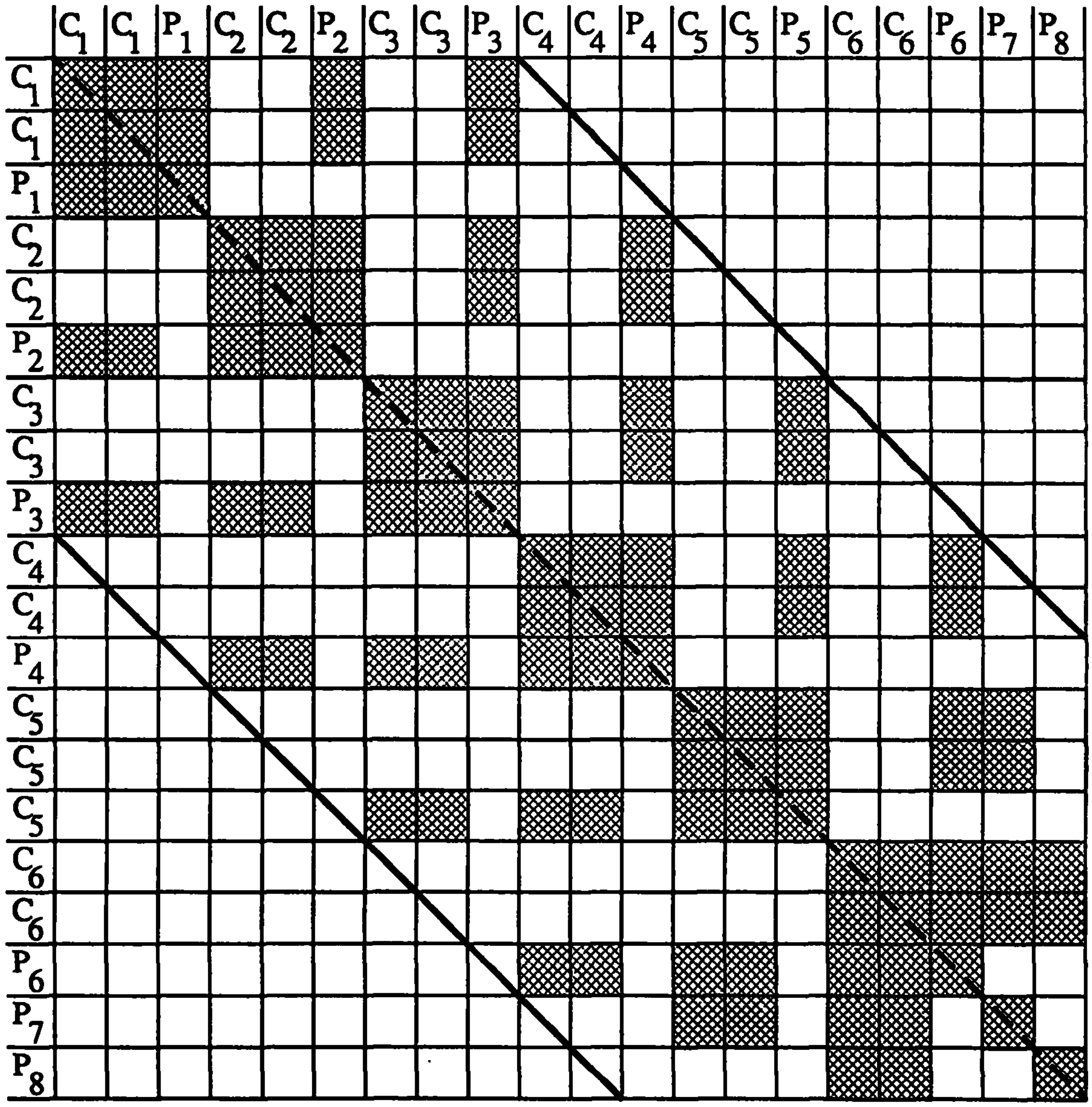


Figure A.1.--Normal matrix for the six photos defined in table A.1.

APPENDIX B.--COORDINATE SYSTEMS IN GIANT

Two object space coordinate systems are available on option (character no. 1) in to the GIANT program: 1) geographic (geodetic) and (2) rectangular (sec. II.B.1, Job Definition Data Record). The preferred option for almost all block adjustments is the geographic coordinate system. Using this system, the Earth's curvature is incorporated in the mathematical model and the interpretation of input and output is easier and more meaningful.

The Geodetic Coordinate System

This coordinate system is the conventional ellipsoidal coordinate system. Rigorously defined, geographic refers to a spheroidal coordinate system, and geodetic refers to a similar system based on an ellipsoid of revolution (fig. B.1). In ordinary usage, however, the two terms are used interchangeably. If the default values are accepted, the coordinate system chosen will be the ellipsoid of revolution (fig. B.2), using the semimajor and semiminor axes defined for the Clarke 1866 spheroid. However, it is possible to change ellipsoids by entering the values of semimajor and semiminor axes in character spaces 51-60 and 61-70, respectively. (See II.B.1, Job Definition Data Record.) This is usually used when working in other parts of the world where the basic control net is on a different spheroid.

The use of feet or meters for object space coordinates is also determined by the ellipsoid constants. If the default or another spheroid in feet is chosen, all linear components (elevations) must be in feet. If spheroid constants in meters are chosen, all input using linear measure must be in meters. Output units will be determined by the choice made for input. The values required for the Clarke Spheroid of 1866 in meters are:

Semimajor axis: $a = 6{,}378{,}206.4$ m
Semiminor axis: $b = 6{,}356{,}583.8$ m

The program does not use the geographic system directly in its adjustment but converts to geocentric coordinate system X, Y, Z (fig. B.2), using the widely accepted conversion formula. Additionally, for interpretation, the camera attitude angles are referenced to a local vertical system at the position of the camera, with Y pointing north and Z pointing up. A more complete description on orientation angles is given in appendix C.

The geocentric coordinate system may be described as a right-handed, rectangular, orthogonal coordinate system. It has three axes, usually designated X, Y, and Z. They are linear measurements, and all three must be at right angles to each other. Any point may be uniquely described by giving its X, Y, and Z coordinates. The most common system of this type is the geocentric coordinate system where X and Y are in the plane of the equator, Z is then through the North Pole, and X is through the zero longitude (fig. B.2). In this system, points in the eastern United States would be +X, -Y, and +Z, while figure B-2 points in the western United States (west of 90°W longitude or approximately New Orleans) would be -X, -Y, and +Z.

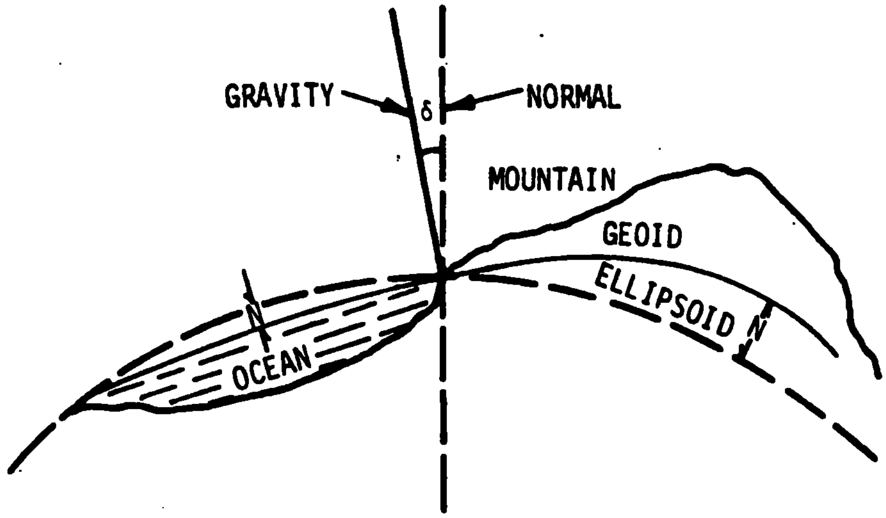


Figure B.1.--Relationship among the physical surface, the geoid, and the ellipsoid

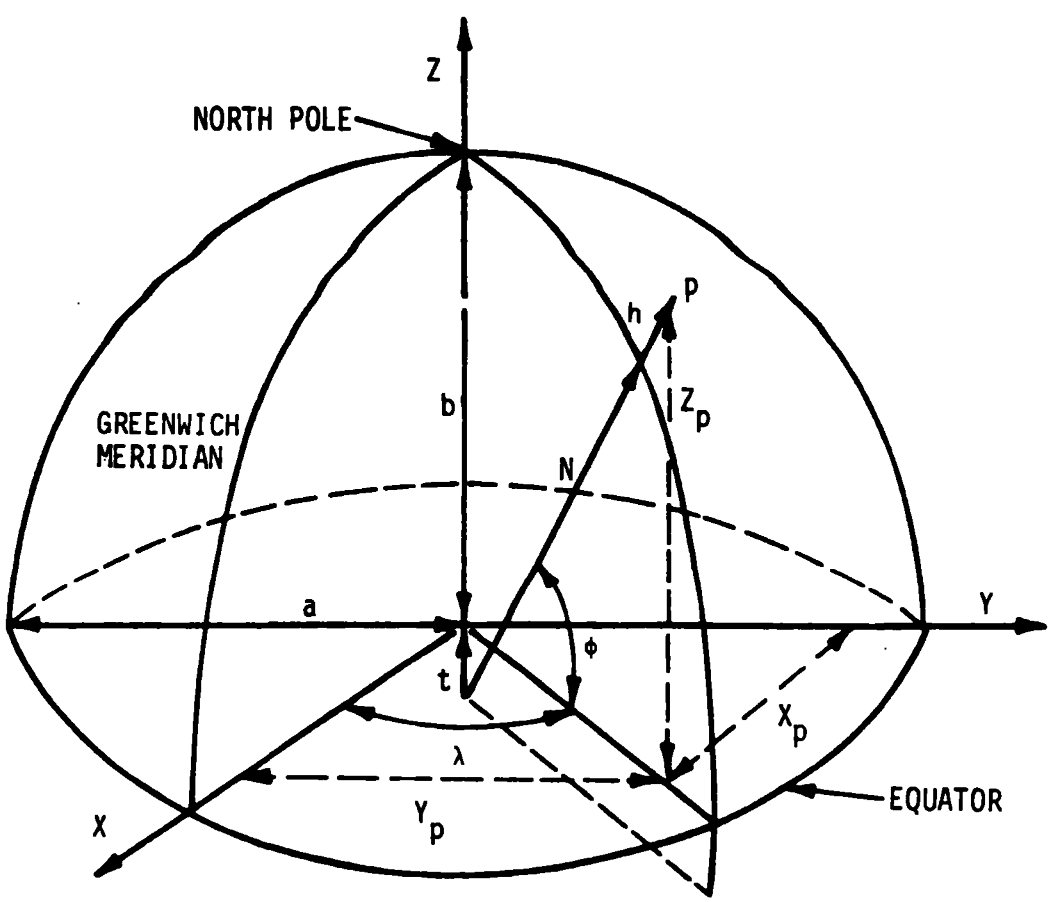


Figure B.2.--Geographic and geocenteric coordinate systems

The Rectangular Coordinate System

The other possible input coordinate system is rectangular. The primary use of this system is for "close-range" photogrammetry, although it may also be used rigorously with geocentric coordinates and with some local coordinates. The danger in this system is that the curvature of the Earth is not modeled for State plane and Universal Transverse Mercator (UTM) coordinates.

It is possible to use this option with State plane coordinates and UTM coordinates, but it is not a rigorous or correct usage. For small areas, and when the Earth's curvature is negligible, it can be used, but again, always with caution and results should be checked carefully. Interaction of the parameters can often adjust most of the errors and provide satisfactory results for small areas. Systematic error is left which cannot be correctly resolved by the least squares adjustment technique because any errors will be treated as random errors. Although the residuals themselves may appear to be satisfactory, systematic errors may remain in the solution. State plane and UTM systems, as with all map projections, are not geometrically similar to the Earth's surface and consequently they introduce distortions.

APPENDIX C.--EXTERIOR ORIENTATION

Definitions

A discussion of the exterior orientation of a photograph (fig. C.1) follows.

1. The position of the camera exposure station C in the object space coordinate system. This can be represented by the three space coordinates (X_C, Y_C, Z_C), or equivalently, by the camera station vector

$$\vec{C} = \begin{bmatrix} X_C \\ Y_C \\ Z_C \end{bmatrix}$$

2. The orientation matrix

$$M = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix}$$

which gives the angular orientation of the image coordinate system (x, y, z) with respect to the object space coordinate system (X, Y, Z).

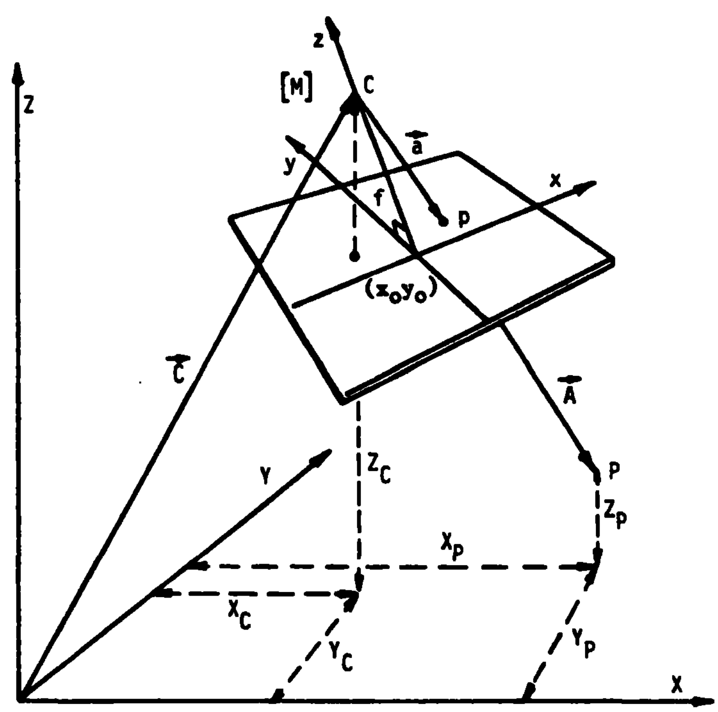


Figure C.1--Exterior orientation

The elements of M are the cosines of the nine space angles between the three axes of the image coordinate system and the three axes of the object space coordinate system; that is

$$M = \begin{bmatrix} \cos Xx & \cos Yx & \cos Zx \\ \cos Xy & \cos Yy & \cos Zy \\ \cos Xz & \cos Yz & \cos Zz \end{bmatrix}$$

where cos Xx, cos Yx, and cos Zx are the cosines of the space angles between the x-axis and the X-, Y-, and Z-axes respectively, etc. Obviously, these elements are direction cosines.

This matrix is arranged to transform object space to image space, or as it is often called, ground to photo:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \cos Xx & \cos Yx & \cos Zx \\ \cos Xy & \cos Yy & \cos Zy \\ \cos Xz & \cos Yz & \cos Zz \end{bmatrix} \cdot \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

or $x = MX$.

Sequential Rotation

The orientation matrix can be factored into three orthogonal matrices, each representing a simple rotation of the image coordinate system about a particular image coordinate axis. The sequence of the three rotations must be specified because different angles of rotation result from different sequences.

In all cases, the orientation is considered in the following way. The image coordinate system is initially coincident with the object space coordinate system. The three rotations are applied to the image coordinate system in the appropriate sequence to place the system into its final position (fig. C.2.).

The orientation matrix M is an orthogonal matrix. There is one, and only one, orientation matrix, M, for a given orientation situation.

Sequence		Axis of rotation
Primary - Roll (ω)	....	x
Secondary - Pitch (ϕ)	....	y
Tertiary - Yaw (κ)	....	z

Roll (ω) is an angle of rotation of about the x-axis. Positive roll rotates the positive y-axis toward the positive z-axis. $-180° < \omega < +180°$. (left wing up)

Assume +x is direction of flight; +y is at a right angle to +x, and the positive direction is out the left wing; +z is at a right angle to the x y plane, and the positive direction is up.

Pitch (ϕ) is an angle of rotation about the y-axis. Positive pitch rotates the positive z-axis toward the positive x-axis. $-180° < \phi < +180°$. (nose down)

Yaw (κ) is an angle of rotation about the z-axis. Positive yaw rotates the positive x-axis toward the positive y-axis. $-180° < \kappa < +180°$. (counterclockwise is positive)

(κ) is approximated by a clockwise angle (photo to ground) and counterclockwise (ground to photo) measured from east to the photo (x) in the plane of the vertical photograph.

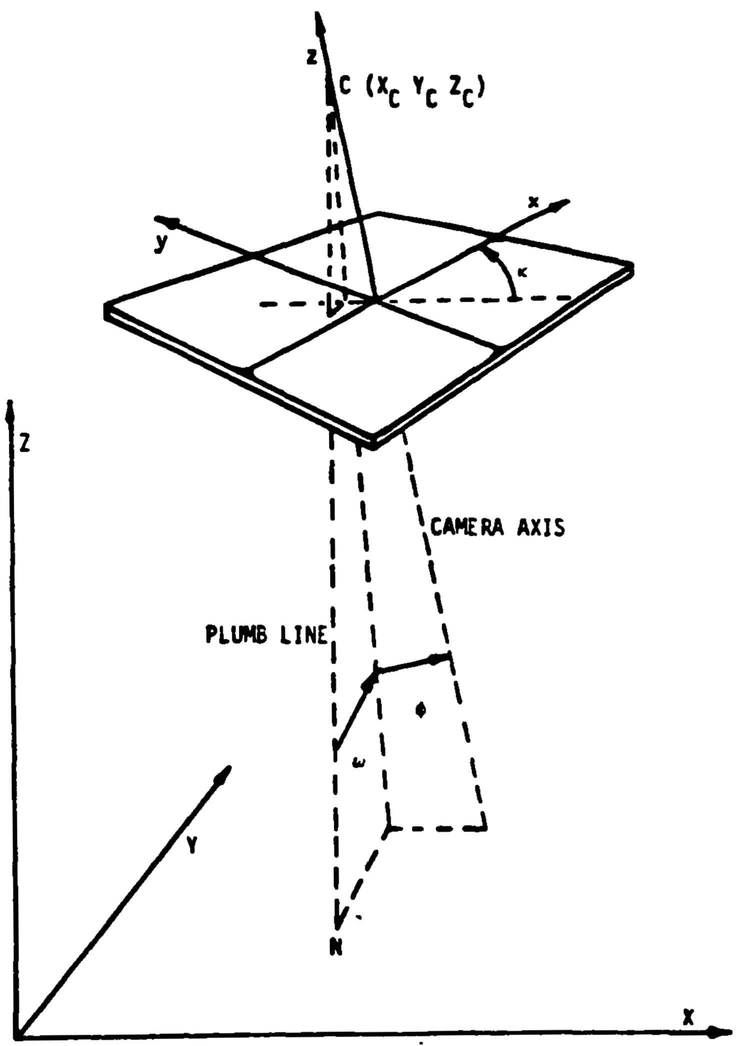


Figure C.2--Rotations

APPENDIX D.--MATHEMATICAL MODEL

Collinearity Equations

The only mathematical model in the GIANT program is collinearity. This expresses the condition that in undistorted space, a vector from the camera station to an image point is collinear with a vector from the camera station to the corresponding point in object space. Since the "real world" is not distortion-free, preprocessing has hopefully compensated for the distortions.

The orientation matrix M transforms vectors in the object space coordinate system into corresponding vectors in the image coordinate system; that is,

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = M \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

Because M is an orthogonal matrix, the length of a vector remains unchanged after the transformation. Indeed, the geometrical representation of the vector is not changed at all in length and direction by the transformation; the components of the vector are simply expressed in the image coordinate system instead of in the object space coordinate system. In fig. C.1, $\vec{A}$ = CP is such a vector. Its components in the object space coordinate system are

$$\vec{A} = \begin{bmatrix} X_p - X_c \\ Y_p - Y_c \\ Z_p - Z_c \end{bmatrix}$$

and its components in the image coordinate system are

$$\vec{a} = \begin{bmatrix} x_p - x_c \\ y_p - y_c \\ z_p - z_c \end{bmatrix}$$

Now, the vector from the camera station C to the image point p is given in the image coordinate system by:

$$\vec{a} = \begin{bmatrix} x_p - x_o \\ y_p - y_o \\ -f \end{bmatrix}$$

where x_o, y_o are the plate coordinates of the principal point (fig. C.1).

When the exterior orientation of a photograph has been correctly established, it is clear from fig. C.1 that the vector a is collinear with the vector A. The two vectors differ only in length. Thus, if they are both expressed in the same coordinate system, one is simply a scalar multiple of the other; that is

$$\vec{a} = k\vec{A}'.$$

The scalar k is called scale factor. This leads to the projective equations

$$\vec{a} = k\ M\vec{A},$$

or

$$\begin{bmatrix} x_p - x_o \\ y_p - y_o \\ -f \end{bmatrix} = k \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} \begin{bmatrix} X_p - X_c \\ Y_p - Y_c \\ Z_p - Z_c \end{bmatrix}$$

Expressed individually, these projective equations are:

$$(x_p - x_o) = k\ [m_{11}(X_P - X_C) + m_{12}(Y_P - Y_C) + m_{13}(Z_P - Z_C)]$$

$$(y_p - y_o) = k\ [m_{21}(X_P - X_C) + m_{22}(Y_P - Y_C) + m_{23}(Z_P - Z_C)]$$

$$(\ -f\ ) = k\ [m_{31}(X_P - X_C) + m_{32}(Y_P - Y_C) + m_{33}(Z_P - Z_C)]$$

Dividing the first equation by the third equation and the second equation by the third equation, and multiplying throughout by -f, we obtain:

$$(x_p - x_o) = -f\ \frac{m_{11}(X_P - X_C) + m_{12}(Y_P - Y_C) + m_{12}(Z_P - Z_C)}{m_{31}(X_P - X_C) + m_{32}(Y_P - Y_C) + m_{33}(Z_P - Z_C)}$$

$$(yp - y_o) = -f\ \frac{m_{21}(X_P - X_C) + m_{22}(Y_P - Y_C) + m_{23}(Z_P - Z_C)}{m_{31}(X_P - X_C) + m_{32}(Y_P - Y_C) + m_{33}(Z_P - Z_C)}$$

These equations express the fact that the object point P, the image point p, and the exposure station C all lie on the same straight line. They are, therefore, referred to as "collinearity equations."

Angles

If M is given, ω, ϕ, and κ can be found using the following relations:

$$\tan \omega = \frac{-m_{32}}{m_{33}}$$

$$\sin \phi = m_{31}$$

$$\tan \kappa = \frac{-m_{21}}{m_{11}}$$

Local Vertical Coordinate System

The origin of the local space coordinate system is taken at the coordinates of each camera station. The geocentric coordinates of the origin 0 are X'_o, Y'_o, and Z'_o (fig. D.1), and the geodetic coordinates of the origin are ϕ_o, λ_o, and h_o. The z axis of the local system is taken along the normal through the origin, with positive z directed away from the center of the Earth. The y axis is in the meridian through the origin, with positive y directed toward the north pole. The x, y, z axis form a right-handed coordinate system. Since only angular rotations are considered here, the value of h is of no importance. The local coordinates are x, y, and z. They are obtained from the geocentric coordinates using the following transformation:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sin\phi_0 & \cos\phi_0 \\ 0 & -\cos\phi_0 & \sin\phi_0 \end{bmatrix} \begin{bmatrix} -\sin\lambda_0 & \cos\lambda_0 & 0 \\ -\cos\lambda_0 & -\sin\lambda_0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X' - X'_0 \\ Y' - Y'_0 \\ Z' - Z'_0 \end{bmatrix}$$

or $\quad x = M_{\phi_o} \; M_{\lambda_o} \; X$

Combining the two orthogonal matrices, we get

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -\sin\lambda_0 & \cos\lambda_0 & 0 \\ -\sin\phi_0 \cos\lambda_0 & -\sin\phi_0 \sin\lambda_0 & \cos\phi_0 \\ \cos\phi_0 \cos\lambda_0 & \cos\phi_0 \sin\lambda_0 & \sin\phi_0 \end{bmatrix} \begin{bmatrix} X' - X'_0 \\ Y' - Y'_0 \\ Z' - Z'_0 \end{bmatrix}$$

or $\quad x = M_{\phi_o \lambda_o} \; X$

Orientation Matrix M

The orientation matrix $M = M_\omega \ M_\phi \ M_\kappa$

Where ω is roll, ϕ pitch, κ yaw

$$\text{and } M_\omega = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\omega & \sin\omega \\ 0 & -\sin\omega & \cos\omega \end{bmatrix}$$

$$M_\phi = \begin{bmatrix} \cos\phi & 0 & -\sin\phi \\ 0 & 1 & 0 \\ \sin\phi & 0 & \cos\phi \end{bmatrix}$$

$$\text{and } M_\kappa = \begin{bmatrix} \cos\kappa & \sin\kappa & 0 \\ -\sin\kappa & \cos\kappa & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

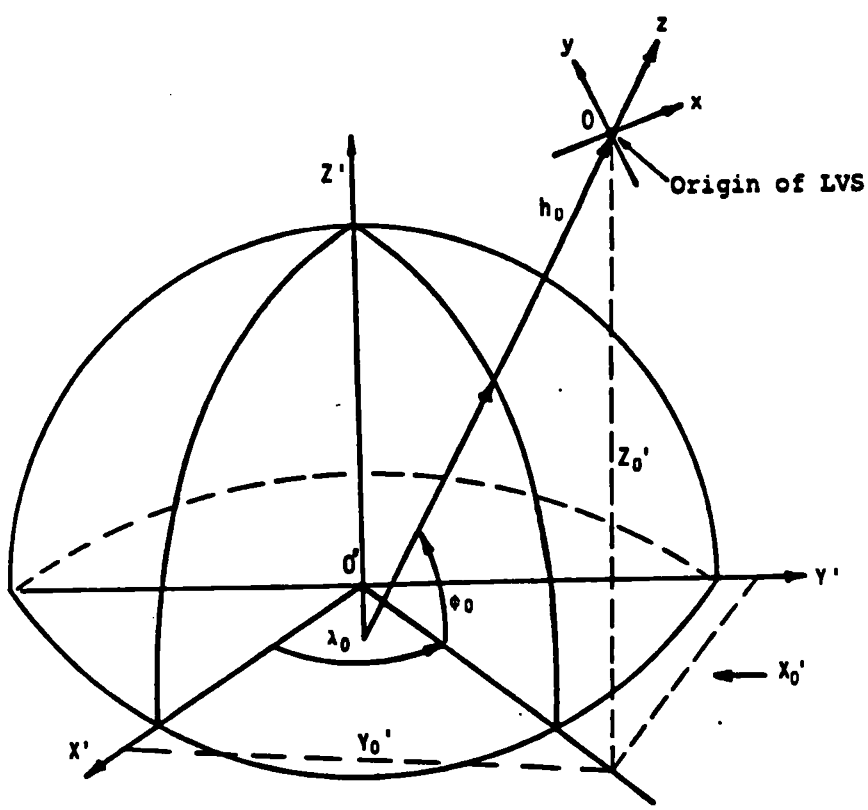


Figure D.1--Local vertical coordinate system

Composite Matrix ($M_{\kappa\phi\omega}$)

Hence

$$
= \begin{bmatrix} (\cos\kappa \cos\phi) & (\sin\omega \sin\phi \cos\kappa + \cos\omega \sin\kappa) & (-\cos\omega \sin\phi \cos\kappa + \sin\omega \sin\kappa) \\ (-\sin\kappa \cos\phi) & (-\sin\omega \sin\phi \sin\kappa + \cos\omega \cos\kappa) & (\cos\omega \sin\phi \sin\kappa + \sin\omega \cos\kappa) \\ \sin\phi & -\sin\omega \cos\phi & \cos\omega \cos\phi \end{bmatrix}
$$

or $x = M_{\kappa\phi\omega}X$ -- ground-to-photo

Computational Sequence

GIANT uses the following computational sequence:

Evaluate (compute) as derived	$M_{\kappa\phi\omega}$	Local-to-camera
Transpose forming	$M^T_{\omega\phi\kappa}$	Camera-to-local
Form	$M^T_{\phi_0\lambda_0}$	Local-to-geocentric

(Transpose of derived matrix)

Premultiply $\quad M^T_{\phi_0\lambda_0} \quad M^T_{\omega\phi\kappa}$

Note this is equivalent to:

$$M^T_{\phi_0\lambda_0} \quad M^T_{\omega\phi\kappa} \quad = \quad (M_{\kappa\phi\omega} \quad M_{\lambda_0\phi_0})^T$$

(Camera-to-local and Local-to-geocentric)^T

Yielding $\quad (M_{\phi_0\lambda_0\omega\phi\kappa})^T$

which will transform geocentric-to-camera

Either photo-to-ground or ground-to-photo angles may be used. The final results will be numerically the same for ground point positions, although differing values for ω, ϕ, κ will be obtained.

APPENDIX E.--ERROR PROPAGATION

The error propagation options (characters 11 and 12 of record 2 of COMMON file, sec. II.B.1) are expensive to exercise, requiring additional computations and printing. Based on the input and the solution, the results are estimates of how well the camera station and the ground points are determined. These options are exercised only on the last two or three runs, because the solution does not provide much information while the data are being "cleaned up" and edited.

A variance covariance matrix for each set of parameters is determined from the inverse of the normals. This is then multiplied by the estimate of variance of unit weight. The standard deviation for each element is the square root of the diagonal terms of that matrix.

Variance of unit weight (σ_o^2) may be estimated by the equation:

$$\sigma_o^2 = \frac{\Sigma(v_i w_i v_i)}{(n-u)}$$

where

v_i is the residual of the $i^{\underline{th}}$ observation

w_i is the weight

n is the number of observations

u is the number of "unknowns" or "solvable parameters"

(n-u) is the degree of freedom.

In the photogrammetric problem the number (n) of observations is equal to the numbers of plate coordinates, one for x, and one for y, or two times the numbers of image points measured. Add to this the number of measurements for ground control coordinates, one for each of the known coordinates (latitude, longitude and elevations). Depending on the external source of information, camera station position (X_c, Y_c, Z_c) and orientation elements (ω, ϕ, κ), as well, can be added to the number of observations as six times the numbers of camera stations. Although these are considered as solvable parameters, they can also be treated as weighted observations if sufficient information is available.

The unknowns or solvable parameters (u) are the ground control positions. For each unique point in the adjustment, three unknowns are counted. Camera station position (X, Y, Z) and orientation (ω, ϕ, κ) are usually considered "unknowns," giving rise to additional numbers of unknowns equal to six times the number of camera stations.

To summarize, let:

v = the output residual for each observation.

w = input weight which may be thought of as $1/\sigma^2$ for each observation. (Note it is sigma squared).

n = total number of observations.

m = 2 * number of plate measurements.

c = 1 for each ground control component.

s = 6 * number of camera stations. (Factor 6 represents the camera parameters: the position coordinates X, Y, Z and the orientation elements ω, ϕ, κ. These parameters are always treated as unknowns; however, depending on the external source of information, these may also be treated as weighted observations contributing to the number of direct weighted observation equations. When the weights of the direct observations are small, the camera parameters may be treated as completely free and no contribution is then made to the direct weighted observations).

p = 3 * number of points (X, Y, Z). (Note: one, two, or three of these components may have also been counted as observations under "c."

Again simplistically, the estimate of variance of unit weight is defined as the summation of the input weights ($1/\sigma^2$) multiplied by the output residuals squared (v^2). If all is perfect, $\Sigma \frac{v^2}{\sigma^2} = (n-u)$ for all observations. This summation, when divided by the degree of freedom (the number of observations minus the number of parameters) results in a value close to 1.0.

Table E.1 computes the unit variance for the solution of a typical three-photo block (fig. E.1) with case 1 considering the camera stations as unknowns and case 2 considering the camera station as constrained or known (character no. 12=1, record no. 2, COMMON file, sec. II.B.1).

Table E.1.--Computations of unit variance for a typical three-photo block. (See fig. E.1.)

		CASE 1	CASE 2
v w v = 1 for each plate coordinate			
	Photo 1	12	12
	Photo 2	20	20
	Photo 3	14	14
Camera Stations (if observed)		0	18
Ground Control (3 points, 7 components)		7	7
Σ v w v		53	71
m = 2 * number of plate measurements			
	Photo 1	12	12
	Photo 2	20	20
	Photo 3	14	14
c = 1 for each ground control component		7	7
s = 6 * number of camera stations		0	18
n = m + c + s		53	71
MINUS			
s		18	18
p		30	30
u = s + p		48	48
DOF (degrees of freedom) = n - u		5	23
σ_o^2 = Unit Variance sq. (Σvwv/DOF)		10.6	3.1
σ_o = Unit Variance		3.3	1.8

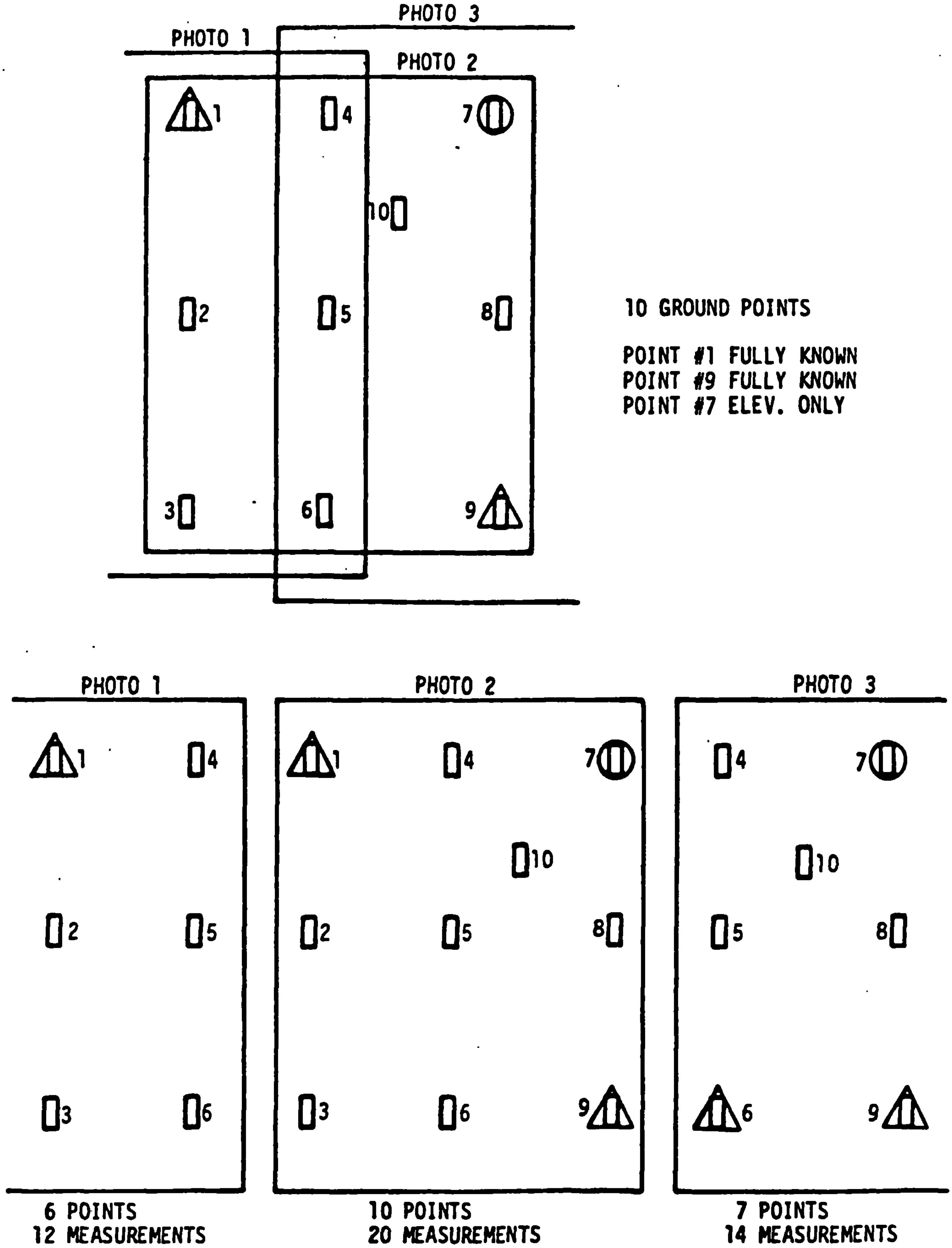


Figure E.1--A typical three-photo block

APPENDIX F.--ATMOSPHERIC REFRACTION

The GIANT program has an atmospheric refraction correction model applicable up to an altitude of 9,000 meters. The dynamic nature of this model makes it possible to execute a more accurate correction to the refraction effect. This correction is based not only on the altitude of the camera but also on its attitude. In the iterative adjustment process, the atmospheric refraction correction is carried out according to the updated state vector of the camera. The switch for applying an atmospheric refraction correction to the GIANT program (character 16, record no. 2, sec. II.B.1) is turned off if the correction has already been made in the "preprocessor." The application of this model slows down the convergence of the solution for only a slight improvement in results. This may discourage its use by production units as long as the results, obtained without its use, are good enough for their work. Also, the 9,000-meter limit on altitude will have to be overcome by replacing the model by another more universally applicable version.

Mathematical Model

In its simplest form, atmospheric refraction can be expressed by (American Society of Photogrammetry 1980):

$$\Delta_\alpha = K_\alpha \tan \alpha$$

where

- Δ_α is the angle of displacement due to atmospheric refraction;
- α is the angle the ray makes with the "true vertical;"
- K_α is a constant related to the atmospheric conditions. (Refer to fig. F.1.)

The constant K_α can be varied as the amount of displacement (angular) attributable to a ray at 45 degrees from the vertical. K_α is a constant related to the atmospheric conditions. For flying heights (H) up to 9,000 meters, K_α can be given by:

$$K_\alpha = 13(H-h)[1-0.02(2H + h)] \text{ microradians}$$

where

H is the flying height (kilometers)
h is the ground point elevation (kilometers)

In a vertical photograph, the correction (Δ_r) for the effect of atmospheric refraction can be shown as:

$$\Delta_r = K_\alpha \left(r + \frac{r^3}{f^2}\right) \text{ or } K_\alpha r \left(\frac{R^2}{f^2}\right)$$

where

$$R^2 = (r^2 + f^2)$$

r is the radial distance of a point image from the photo nadir;
f is the focal length of the camera.

Obviously, the approach will work for near vertical photographs only.

However, tilts of the photograph must be considered, and computation of the correction made during the iterative solution. This can be accomplished quite simply by a change in the approximate Z (in a local system) coordinate. The change in the Z ground coordinate which will produce the same effect at the plate as the atmospheric refraction is given by (fig. F.2):

$$dZ = \frac{(Z_o - Z_G) \tan \alpha}{\sin^2 \alpha} \quad (\Delta_\alpha)$$

The angle α is the difference in the direction between true vertical and the point in question. The correction then can be written as:

$$dZ = \frac{\Delta X^2 + \Delta Y^2 + \Delta Z^2}{\Delta Z} K_\alpha$$

where

$$\Delta X = (X_o - X_G)$$

$$\Delta Y = (Y_o - Y_G)$$

$$\Delta Z = (Z_o - Z_G)$$

This correction is easily applied in each iteration of the triangulation solution for every point.

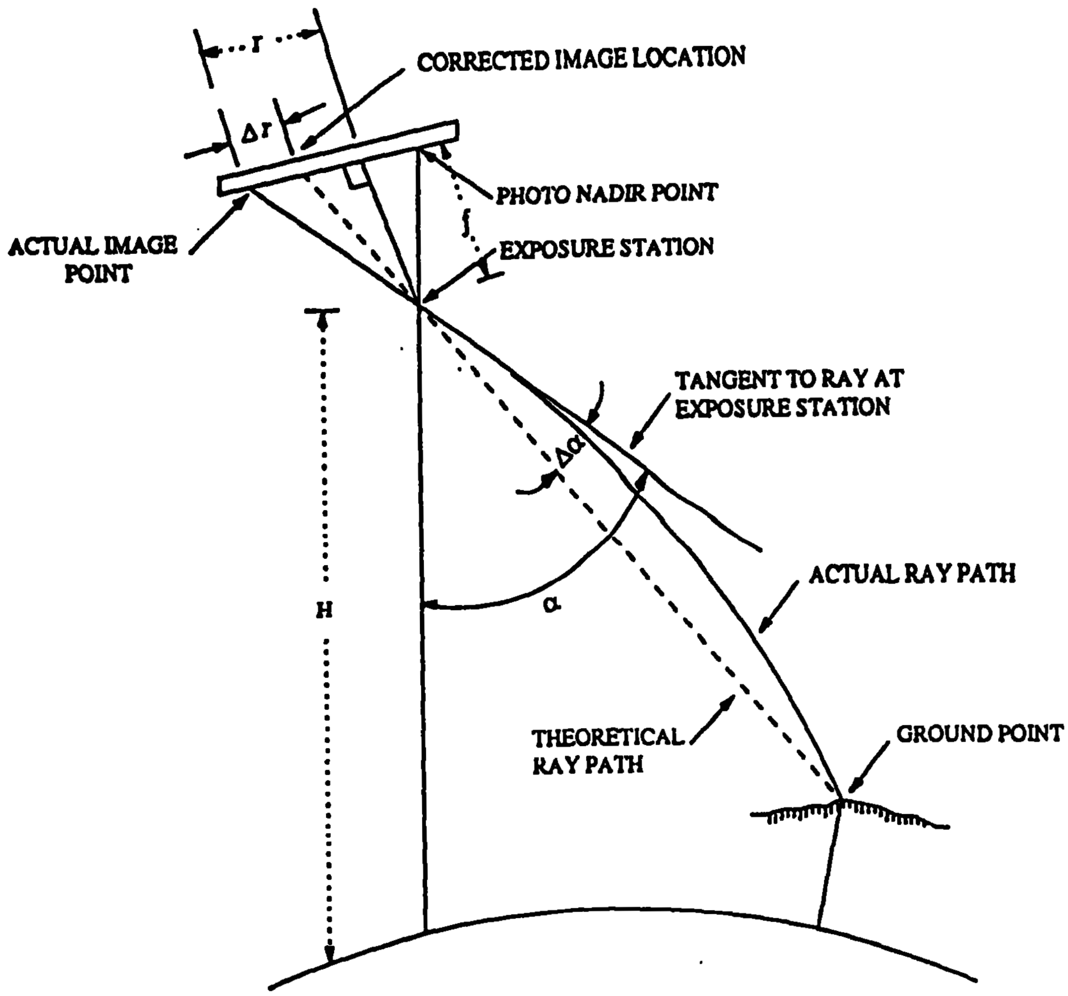


Figure F.1.--Atmospheric refraction.

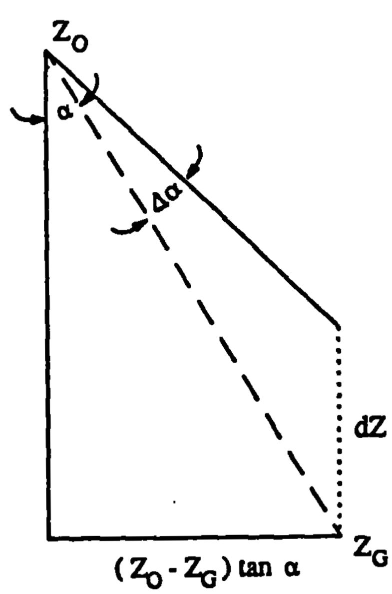


Figure F.2.--Correction for atmospheric refraction.

APPENDIX G.--PHOTOBATHYMETRY

To account for errors caused by water refraction of the light rays from underwater features, corrections are applied to each and every underwater point during each iteration of the triangulation solution. The switch for applying water refraction correction is character 17, record no. 2 of COMMON file. To apply the correction to underwater points, water level (meters) with respect to the reference ellipsoid at the time of photography is entered in characters 31-40, record no. 2 of COMMON file.

The basic approach for applying water refraction correction is similar to applying atmospheric refraction correction. The water refraction model may be expressed by (fig. G.1):

$$\Delta_\alpha = K_\omega \tan \alpha$$

where

Δ_α is the angle of displacement
α is the angle the ray makes with the true vertical at the camera station
K_ω is the constant related to water refraction for an underwater feature

It can be shown that the value of K_ω is given by (fig. G.1):

$$K_\omega = [\ \{\tan (\alpha + \Delta_\alpha)/ \tan \alpha\} - 1]/[1 + \tan (\alpha + \Delta_\alpha) \tan \alpha]$$

or, $$K_\omega = [\ (H + d)/(H + \bar{d}) - 1]\ /\ [1 + \tan^2\alpha]$$

and the correction (Δ_r) for water refraction (fig. G.2):

$$\Delta_r = K_\omega(r + \frac{r^3}{f^2}) = K_\omega r(\frac{R^2}{f^2})$$

where

$$R^2 = r^2 + f^2$$

r = radial distance of the image point from the photo nadir
f = focal length of the camera

The equation for water refraction correction (Δ_r) is similar to the air refraction correction (appendix F) except that the constant K_ω replaces K_α in the expression.

In the GIANT program the air and water refraction corrections are applied as a change (dz) in the Z coordinate (fig. F.2) given by:

$$dZ = \frac{(Z_o - Z_G) \tan \alpha\ (\Delta_\alpha)}{\sin^2\alpha}$$

The angle α is the difference in the directions between true vertical and the point in question. The correction dZ can be expressed as

$$dZ = \frac{\Delta X^2 + \Delta Y^2 + \Delta Z^2}{\Delta Z} (K_\omega)$$

where

$$\Delta X = X_o - X_G$$

$$\Delta Y = Y_o - Y_G$$

$$\Delta Z = Z_o - Z_G$$

This correction is easily applied to every underwater point (with negative elevation) and in every iteration of the solution. The expression is similar to the expression for air refraction correction. Both the air and water refraction corrections to the Z coordinate of a point can be applied by the formula

$$dZ = \frac{\Delta X^2 + \Delta Y^2 + \Delta Z^2}{\Delta Z} (K_\omega + K_\alpha)$$

where $(K_\omega + K_\alpha)$ replaces K_ω or K_α in the expression for the correction. For points at or above water level $K_\omega = 0$ in the algorithm, such that the correction is only for the air refraction.

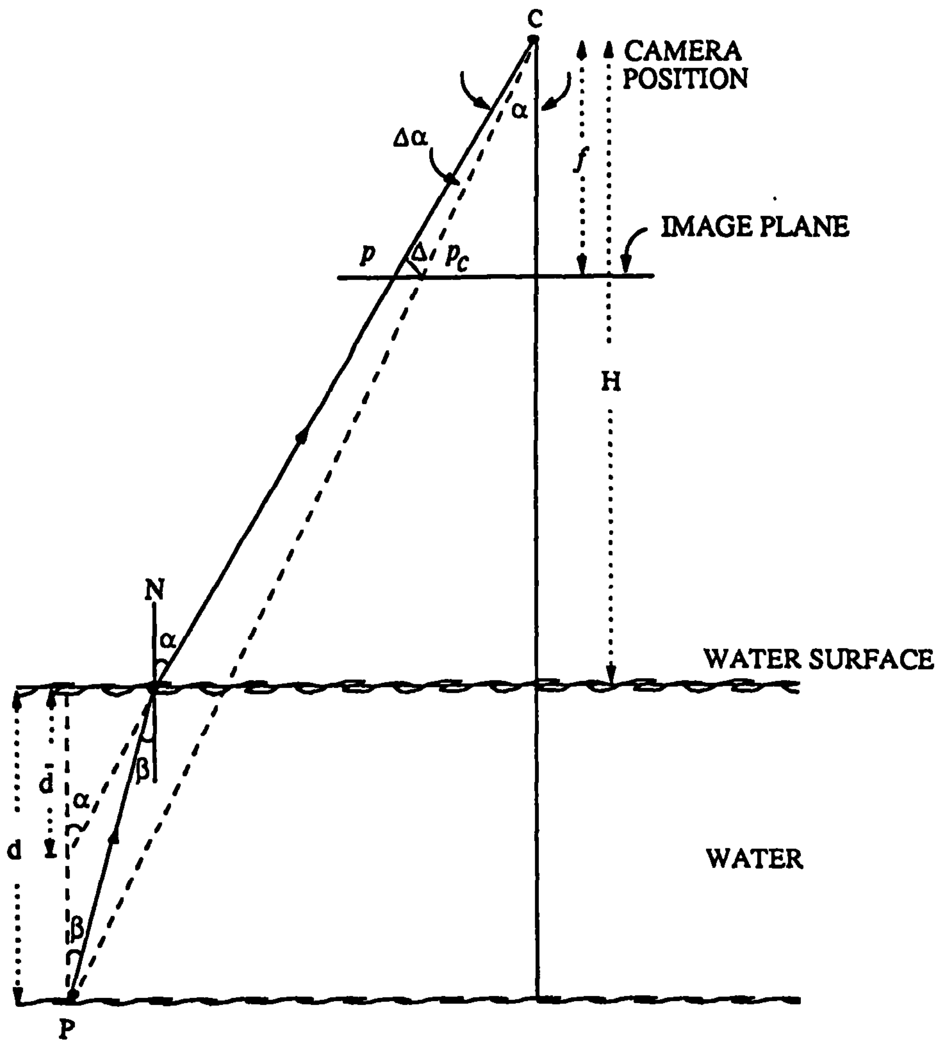


Figure G.1.--Water refraction of underwater target (P).

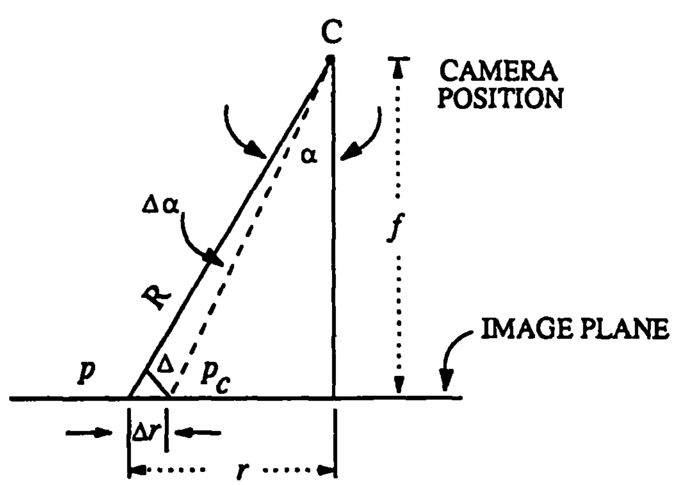


Figure G.2.--Image displacement due to water refraction.

APPENDIX H.--RUN STRATEGIES AND DATA EDITING

INTRODUCTION

The flexibility which makes GIANT useful also makes it difficult to establish a unique procedure for data editing and evaluation. The circumstances of any given job may necessitate a change in the procedure. However, the following discussion will help in establishing a logical procedure.

In general, aerotriangulation tasks performed by GIANT have the following characteristics:

1. The objective is to establish a sufficiently dense net of control to enable stereomodel setup for compilation.

2. Photography is flown with flight height of less than 9,000 meters above mean sea level with a calibrated mapping camera and near vertical orientation. (The atmospheric refraction model is valid up to the altitude of 9,000 meters. For higher altitudes, a more suitable model would be necessary).

3. External information for the camera parameters are enforceable. In any case, a close approximation of the camera parameters is desirable.

DATA EDITING

In a measured data set, such as plate coordinates, there are three general types of errors:

1. Accidental or fortuitous errors, which the least square technique minimizes;

2. Systematic error, which is not amenable to solution and has hopefully been removed by preprocessing;

3. Blunders or mistakes, which are the result of incorrect observations or recording.

It is this last type of error, the blunders, which must be recognized and removed. The first two types of errors must be recognized and accounted for, but the third type must be removed for a valid solution. It occasionally becomes difficult to differentiate between large accidental errors and blunders. The "rule of thumb" is that errors exceeding the 3 sigma (standard deviations) level may be considered blunders.

Editing Plate Coordinate Data and Other Input Data by Using Intersection-Only Run

An intersection-only computer run allows identification and removal of the following gross errors:

1. Very large errors in plate coordinate or misidentification.

2. Incorrect combinations of photograph numbers and associated points which appear on the photograph.

3. Consistently bad photographs which either have a blunder in one or more of the camera station parameters or in the preprocessing of the plate coordinate data.

4. Differences in ground control coordinates which are proofread for differences.

In this computer run, one must only look for gross errors (blunders). Gross errors could be due to misidentification of points, recording errors, etc. Since this run is made with initial approximations only, large patterned residuals should be expected, especially in plate residuals. What one looks for then is a <u>break</u> in the pattern.

If examination of the run shows the computed elevation of a ground control point to be higher than the camera station position, one of the two most probable blunders occurred.

<u>Sign of f</u>. The most probable blunder is the sign of the focal length being incorrect. If the plate coordinate data, as preprocessed, should be reconstructing a photo positive, the sign of the focal length should be negative; if a photo negative, the sign should be positive.

<u>Yaw</u>. The other probable cause is that the yaw angle (κ) is incorrect. This may be checked by plotting several points from a photo positive on a map and rechecking the relationship of image space 'y' and north.

Editing by the Study of Plate Residuals in a Photogrammetric Adjustment Run

Major blunders are easy to identify and rectify in plate coordinate data. Difficulty occurs when gross errors are eliminated and a judgment must be made on eliminating points such that the large residuals are removed. There is a human tendency to start eliminating plate coordinates with large residuals until a run is produced with all small residuals. This procedure may be carried over several runs. In this procedure, the user may inadvertently eliminate readings in an area until all readings connecting adjoining plates have been dropped. This leads to weak solutions and results in poor coordinate determinations.

Listed below are some phenomena which should occur as one approaches the best solution and which will not be obvious to the casual user. When editing, the following must be kept in mind:

Residuals will be grouped by the number of photos (rays) "seeing" a point. The residuals will appear larger for those points seen by more rays.

Residuals in the direction of flight will tend to become zero. The error resolves itself in the vertical component. This is especially true for two-ray points. The elevation of the computed ground position should be watched along with the plate residuals.

Ground control will tend to show a different residual grouping than for uncontrolled points. This tendency is directly related to the weighting of the plate coordinates and the control coordinates.

The residuals should balance for each point, i.e., the positive and negative residuals should add to zero. This will be approximate, but generally true for a well-adjusted run.

There should not be any undesirable pattern of errors, i.e., no systematic component. The residuals should conform with the laws of normal distribution: small errors are more likely than large errors, the error zero is most probable, and positive and negative errors are equally likely.

Editing by the Study of Camera Parameters

When editing photogrammetric computer runs with unconstrained camera stations, the camera stations are reflecting only the influence of the other data, the plate coordinate, and the ground control. The rule of thumb is twofold: mapping photography is flown straight and level as far as possible; and an aircraft never flies exactly straight and level.

On each run, the user should examine the camera positions and orientations, and ensure they are following a consistent path. Any deviation should be explained.

Editing by the Study of Ground Control

Three possibilities may cause errors: misidentification of control; poor point transfer; and bad coordinates of the point.

Remedial action is determined by the cause. Options for the remedial action are:

- o downgrading the "type" of point; e.g., from fully known to horizontal, only if the elevation component is bad;
- o increasing the associated standard deviations to reflect the point is not as well known as others;
- o changing the type from control to a passpoint; and
- o removing or rereading bad plate coordinates;

All coordinates of ground control points are treated as weighted unknowns. Furthermore, the program provides initial estimates of the values and their weights for the unknown components of ground points, referred to as UNHELD components. In determining the degree of freedom for the solution, direct weighted equations for UNHELD components of ground control are not counted. On the other hand, the HELD coordinate components of ground control are held in the solution to the extent of their assigned weights. The program uses its supplied values as best estimates and counts the direct weighted equations in determining the degree of freedom for the adjustment.

The following "key numbers" should be watched carefully:

A Posteriori Estimate of Variance of Unit Weight

This is an important single number by which to judge a run. For a normal case, this number should approach one (1.0). The number starts out very large and as data editing and bootstrapping improve the data, it comes down to a reasonable value. Remember, this number only reflects the balance between the input standard deviations and the output residuals. If for some reason the weighting is not realistic, this number may not approach one (1.0). Watch it carefully as an indicator of overall performance along with the contributing components of the number.

Weighted Sum of the Squares

This number, along with the changes in camera station parameters, is printed for each iteration. It can be used to judge how much each iteration is changing the solution and, to some extent, where the change is occurring. This number, which most often is huge at the beginning, is an estimate of the sum of the squares of the plate residuals. It is used as a convergence test, i.e., when this number changes less than a predetermined percentage the solution is stable and iterations stop. If the number increases between iterations, the run has "diverged," usually because of bad data or weak geometry, and is incapable of reaching a good solution. Edit the data or submit with fewer iterations and then edit.

Number of Iterations

The default value for this input number is <u>four</u> which is sufficient for most runs. This represents the maximum number of times it can compute a correction and update the solvable parameters to improve a solution. A run may cut off before reaching the maximum number of iterations because of other established criteria. The reason for having this number variable is that in some cases one may want to perform a less amount of iterations to isolate blunders from the solution. In other cases, one may want to extend the number of iterations to reach the best solution, regardless of computer time involved.

Weighting

The weight matrix is the inverse of the input variance-covariance matrix, which is composed of the input standard deviations for all measurements. It is possible to "warp" the solution in any manner desired by manipulating the weights. The best available guide is to assign realistic values to weights. If control is scaled from a 1:24,000 map, make the weights appropriate to the accuracy of the map and scaling error. Plate coordinates, likewise, should reflect the care and accuracy of the equipment and personnel and requirements of the job; 5 to 15 micrometers (microns) is the normal range of standard deviations assigned in the input data.

Invariably, an occasion may arise when insufficient information is available. When things are not working right and the user does not know why, it may be helpful to change the weights: lock the control, loosen the camera positions, tighten the orientations. Any combination can be manipulated until the cause of the problem can be identified. In the absence of sufficient information, these are legitimate data editing techniques done deliberately. If possible, the weights are changed back to realistic values.

RUN STRATEGY

From the above discussions, run strategy should suit the job. Multiple runs will be required to produce the best solution. The objectives of these runs are listed below. A run may achieve one or more objective from a single run, or multiple runs may be required for each objective, depending on the amount of data and the number of blunders.

Blunder Edit

The first run should be with the INTERSECTION ONLY option exercised. This option overrides the normal solution and allows the program to go through all the motions. Taking only the initial approximations, it computes plate residuals and ground coordinates. No adjustment is performed. This is extremely useful in the first run to eliminate large blunders before they mess up an adjustment beyond recognition.

Clean-up Plate Coordinates

The second objective should be to tie the plates together; i.e., to achieve consistent and low-plate coordinate residuals. This can be achieved with little or no control and the camera station parameters relatively "loose;" i.e., free to adjust. If there are many problems, it may be necessary to constrain the camera station parameters more tightly to prevent them from over-reacting to the errors. It may also be desirable to cut the number of interactions to one or two.

Fit the Control

The objective now is the twofold requirement of improving camera station position and orientation, and fitting them to the ground control. A run is made with only those ground control points which seem to fit or show systematic discrepancies from the preliminary runs. If the solution shows marked improvement, it may be desirable to save the camera station parameters and change the initial approximations to those of the last run. This is often called "bootstrapping." DO NOT tighten the weights (lower the values of the input standard deviations) because no new information has been received, nor are the parameters known any better.

Error Propagation

The last run should include the error propagation option. This is an expensive option to exercise and does not yield much information in the initial phases. The output shows the spread of accuracies of the points and camera station parameters.

Bandwidth Errors

Bandwidth errors are the result of a point appearing on photographs beyond the program limit. The first and last appearances of a point may not exceed a certain number (program limit) of photographs. The most common cause of this error is the duplication of a point number, inadvertently, or separate sets of photographs.

If this message appears during a run, or other very bad unexplained results occur, rerun the job with the INTERSECTION only option exercised. An error message identifying the point in question will be listed. If this is not the cause, the user will probably be able to isolate the problem.

Adding Points After Job Completion

A common occurrence is to finish a triangulation project and then later receive a request for additional point coordinates from the same project. This may be accomplished easily by saving the last set of camera station parameters and setting up an INTERSECTION only run. The new points are marked, measured, and preprocessed as were the points used in the previous adjustment. They are then either added to the appropriate frames or run in the program by themselves, preferably with some other points previously determined for checks. If this is done, the solution will not be affected and a consistent set of coordinates will be produced.

APPENDIX I.-- SELF CALIBRATION OF PRINCIPAL DISTANCE AND PRINCIPAL POINT LOCATION OF A CAMERA

(Model No. 1 in GIANT V3.0)

Precise location of the principal point in the image plane and the principal distance of a camera are important in the reconstruction of photogrammetric stereomodels, and are the fundamental parameters determined in camera calibration. These parameters can be calibrated by laboratory methods, or in-flight, during a mission for aerotriangulation. A note of caution in calibrating the pricipal distance is that it is highly correlated with flying height. Therefore, the camera station position must be known precisely in order to be able to calibrate principal distance. However, the in-flight method may be used to determine principal point location without the above constraint.

In the generalized photogrammetric solution, two parameters of the principal point coordinates, x_o, and y_o, and one for the principal distance, f, are included in the data reduction for aerotriangulation. Using collinearity condition equations, model no. 1 in GIANT V3.0 enables the determination of these three parameters in addition to the usual aerotriangulation parameters.

The collinearity condition (fig. K.1) gives the following relationship (eqn. 1, appendix K):

$$\begin{bmatrix} x - x_o \\ y - y_o \\ - f \end{bmatrix} = \lambda \begin{bmatrix} M \\ (\omega, \phi, \kappa) \end{bmatrix} \begin{bmatrix} X_p - X_c \\ Y_p - Y_c \\ Z_p - Z_c \end{bmatrix} \qquad (1)$$

where,

x , y - plate coordinates of an image point p

x_o, y_o - principal point coordinates in the plate coordinate system

f - principal distance of the camera (sensor)

M - rotation matrix in terms of rotations ω, ϕ, κ

The equations can be reduced to two linear observation equations in which x_o, y_o and f are carried into the least squares adjustment. The aerotriangulation solution, using model no. 1 in GIANT V3.0 program, gives the calibrated coordinates of the principal point location and the calibrated principal distance.

APPENDIX J.--COMPENSATION OF UNMODELLED SYMMETRIC RADIAL ERRORS
(Model No. 2 in GIANT V3.0)

INTRODUCTION

Symmetric radial errors introduced in the data acquisition system could be the ones which may not lend themselves easily to mathematical modelling (e.g., due to the dynamics of the situation) or the ones whose models may not have been incorporated in the data reduction scheme. A good example of such an error is the one caused by the glass plate in front of the camera lens at the time of photography. In GIANT V3.0 a generalized mathematical model based on an odd order polynomial has been selected to simulate such errors. The coefficients of the polymomial are introduced as the additional parameters in the generalized photogrammetric least squares solution for higher precision in a photogrammetric project.

Most of the times in photogrammetric applications precise value of the altitude (Z coordinate) of photography is not needed such that symmetric radial errors are compensated for by the adjusted value of the camera altitude. However, when GPS is utilized in aerotriangulation, the camera position is determined precisely such that this compensation for the symmetric radial errors is not possible. Therefore, with the introduction of GPS in photogrammetric solutions it is necessary to compensate for such errors by some sort of a mathematical model. A polynomial (eqn. 1) in the generalized photogrammetric solution is used.

<u>An odd order polynomial to express x and y components of the symmetric radial errors (Δx and Δy) are derived from the radial error (Δr):</u>

$$\Delta r = K_1 r^3 + K_2 r^5 + K_3 r^7 + \qquad (1)$$

such that

$$\Delta x = ((\Delta r)/r) * (x - x_o) \qquad (2a)$$

and

$$\Delta y = ((\Delta r)/r) * (y - y_o) \qquad (2b)$$

where,

- x , y — image coordinates of a point p at a radial distance r from the point ofsymmetry (principal point).
- x_o, y_o — principal point coordinates
- Δr — symmetric radial error
- r — radial distance of the image point under consideration.

Substituting value of Δr from equation (1):

$$\Delta x = (K_1 * r^2 + K_2 * r^4 + K_3 * r^6) * (x - x_o)$$

$$\Delta x = C * (x - x_o) \qquad (3a)$$

similarly,

$$\Delta y = C * (y - y_o) \tag{3b}$$

where,

$$C = (K_1 * r^2 + K_2 * r^4 + K_3 * r^6)$$

The collinearity relationship (eqn.1, appendix K) can be further expressed as:

$$\frac{x-x_o+\Delta x}{f} = \frac{m11*(X_p - X_c) + m12*(Y_p - Y_c) + m13*(Z_p - Z_c)}{m31*(X_p - X_c) + m32*(Y_p - Y_c) + m33*(Z_p - Z_c)} = \frac{Mx}{Mz}$$

In functional form:

$$F(x) = (x - x_o + \Delta x) + f * (Mx/Mz) = 0 \tag{4a}$$

similarly,

$$F(y) = (y - y_o + \Delta y) + f * (My/Mz) = 0 \tag{4b}$$

where,

m11, m12, m13, ... , m32, m33 are the components of rotation matrix M:

$$M = \begin{bmatrix} m11 & m12 & m13 \\ m21 & m22 & m23 \\ m31 & m32 & m33 \end{bmatrix}$$

and

x, y measured plate coordinates

x_o, y_o principal point plate coordinates

f principal distance

$\Delta x, \Delta y$ symmetric radial errors in terms of K_1, K_2, K_3

In the solution of collinearity equations of the form above, the coefficients K_1, K_2, K_3 for the polynomial are solved for to obtain their adjusted values. This procedure, using model no. 3 in the GIANT V3.0 program, is suitable to compensate unmodelled symmetric errors in the data acquisition system.

APPENDIX K.--PRECISION KINEMATIC GPS IN AEROTRIANGULATION
(Model No. 3 in GIANT V3.0)

INTRODUCTION

NAVSTAR Global Positioning System (GPS) has been used in obtaining camera positions at the instant of photographic exposure. The GPS-derived camera positions were then used to compute the positions of the ground targets. It has been shown (Lucas 1987) that if the positions of a photogrammetric camera can be independently determined to an accuracy of 5 cm, then comparable accuracies may be obtained for points on the ground with little or no ground control required. Thus, the economic benefit of precision kinematic GPS in aerotriangulation is obvious. Also, this procedure can be effectively used with other imaging and nonimaging sensors.

DETERMINATION OF FIXED VECTOR: (ANTENNA TO CAMERA NODE)

To apply kinematic GPS in aerotriangulation, fixed antenna to camera-node vector must be determined precisely. The GPS measurements give the position of the receiver antenna located on the aircraft at the instant of photographic exposure. The antenna to camera-node vector then relates the antenna position to the camera position at exposure. Collinearity condition equations are then generated for a generalized photogrammetric aerotriangulation solution to obtain coordinates of ground points.

In the GPS experiments (Lucas 1987 and Lucas and Mader 1989) the antenna to camera-node offset vector was measured with the help of steel tape and a level. The offset components measured were fore/aft (x), starboard/port (y), and up/down (z). These components were then used to relate the camera position to the observed antenna position.

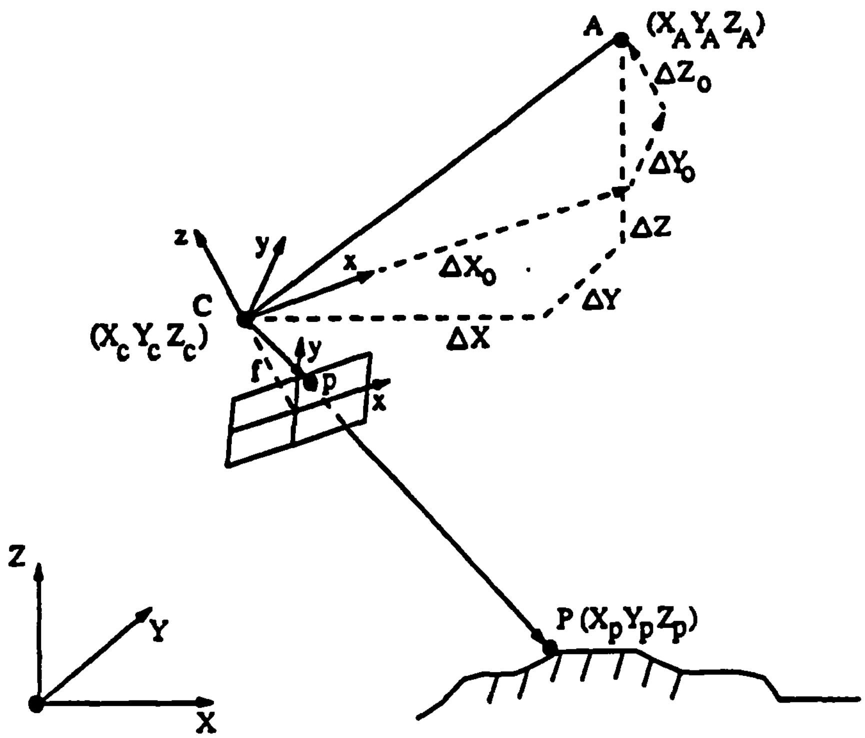
MATHEMATICAL MODEL

Model no. 3 of photogrammetric triangulation in GIANT V3.0 determines the three offset components of the antenna to camera-node vector in the image coordinate system, in addition to:

- o the adjusted values of all other conventional parameters solved for in the generalized photogrammetric solution, and

- o the adjusted coordinates of all ground (target) points.

The modified collinearity equations (eqn. 3) used in the model include the antenna to camera-node offsets. (See fig. K.1.)



LEGEND

P	–	Ground point
C	–	Camera node
A	–	Antenna
X, Y, Z	–	Geodetic coordinate system
x, y, z	–	Plate coordinate system
f	–	Principal distance of the camera
p	–	Image point corresponding to P
$\Delta X, \Delta Y, \Delta Z$	–	Offsets in geodetic coordinate system
$\Delta X_0, \Delta Y_0, \Delta Z_0$	–	Offsets in plate coordinate system

Figure K.1--Offsets: antenna to camera node.

Considering the ground point P, the image point p, and the camera-node C, we obtain the following collinearity condition relationship (eqn.1):

$$\begin{bmatrix} x \\ y \\ -f \end{bmatrix} = \lambda \begin{bmatrix} M \\ (\omega, \phi, \kappa) \end{bmatrix} \begin{bmatrix} X_p - X_c \\ Y_p - Y_c \\ Z_p - Z_c \end{bmatrix} \tag{1}$$

where,

x,y - measured and refined plate coordinates of point P

f - principal distance of the camera scale factor

M - rotation matrix, ground to photo coord.system conversion, implicit in ω, ϕ, κ

ω, ϕ, κ - rotations about x, y and z axes

X_p, Y_p, Z_p - Geodetic coordinates of a point P

X_c, Y_c, Z_c - Geodetic coordinates of camera node

Also,

$$\begin{bmatrix} X_c \\ Y_c \\ Z_c \end{bmatrix} = \begin{bmatrix} X_A - \Delta X \\ Y_A - \Delta Y \\ Z_A - \Delta Z \end{bmatrix} = \begin{bmatrix} X_A \\ Y_A \\ Z_A \end{bmatrix} - M^T \begin{bmatrix} \Delta X_o \\ \Delta Y_o \\ \Delta Z_o \end{bmatrix} \tag{2}$$

X_A, Y_A, Z_A - geodetic coordinates of antenna

M^T - rotation matrix, photo to ground

ΔX, ΔY, ΔZ - offsets, antenna to camera node, in geodetic coordinate system

ΔX_o, ΔY_o, ΔZ_o - offsets, antenna to camera node, in image (photo) coordinate system

Thus,

$$\begin{bmatrix} x \\ y \\ -f \end{bmatrix} = \lambda \begin{bmatrix} M \\ (\omega, \phi, \kappa) \end{bmatrix} \begin{bmatrix} X_p - X_A \\ Y_p - Y_A \\ Z_p - Z_A \end{bmatrix} + \begin{bmatrix} \Delta X_o \\ \Delta Y_o \\ \Delta Z_o \end{bmatrix} \tag{3}$$

Knowing X_A, Y_A, Z_A from GPS, and the measured and refined plate coordinates x,y of a target point, the unknown parameters:

ΔX_o, ΔY_o, ΔZ_o; ω, ϕ, κ; X_p, Y_p, Z_p

are solved for.

The complexity of the generalized collinearity equations is not brought out in this presentation. This only illustrates as to how the three components: ΔX_o, ΔY_o, ΔZ_o, of the fixed vector, antenna to camera node, are introduced into the formation of equations based on collinearity condition.

Observation equations of the form described above provide a formal solution to the problem of aerotriangulation without ground control, but additional conditions must be satisfied before an unambiguous solution is possible. It can be seen that some ground control is required for a single strip of GPS-controlled photography. Aerotriangulation using GPS without any ground control is possible only when multiple photo strips with side overlap are used. Ground points, where only the elevation is known, are sufficient constraints for a single strip of GPS-controlled photography, provided this elevation control is far enough away from the vertical projection of the line through the antenna positions (Lucas and Mader 1989).