NOAA Technical Memorandum NOS NGS-25

REVISIONS OF THE HOACOS HEIGHT-CONTROLLED NETWORK ADJUSTMENT PROGRAM

National Geodetic Survey
Rockville, Md.
May 1980

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Classification, Standards of Accuracy, and General Specifications of Geodetic Control Surveys. Federal Geodetic Control Committee, John O. Phillips (Chairman), Department of Commerce, NOAA, NOS, 1974 reprinted annually, 12 pp (PB265442). National specifications and tables show the closures required and tolerances permitted for first-, second-, and third-order geodetic control surveys. (A single free copy can be obtained, upon request, from the National Geodetic Survey, OA/C18x2, NOS/NOAA, Rockville, MD 20852.)

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NOS NGS-1 Use of climatological and meteorological data in the planning and execution of National Geodetic Survey field operations. Robert J. Leffler, December 1975, 30 pp (PB249677). Availability, pertinence, uses, and procedures for using climatological and meteorological data are discussed as applicable to NGS field operations.

NOS NGS-2 Final report on responses to geodetic data questionnaire. John F. Spencer, Jr., March 1976, 39 pp (PB254641). Responses (20%) to a geodetic data questionnaire, mailed to 36,000 U.S. land surveyors, are analyzed for projecting future geodetic data needs.

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ABSTRACT: The HOACOS height-controlled adjustment program has been revised in several places. Its new features include the adjustment in the geodetic horizon system, a change of units for compatibility with other programs, a revised method of treating the direction of gravity at stations with no astronomic data, and the handling of observations when some heights are not given.

1. INTRODUCTION

HOACOS is a computer program for adjusting horizontal networks in space with heights held fixed and without reducing observations to the surface of an ellipsoid. Experience with this program disclosed the need for certain revisions to make it more general in application. These revisions are described below, using the same notation as in the previous report (Vincenty 1979) unless otherwise shown.

2. ADJUSTMENT IN THE GEODETIC HORIZON SYSTEM

In the original version of HOACOS the coordinate unknowns were linear shifts in the plane of the astronomic horizon at each station and the shifted points were reduced to the constant height surface after the adjustment. This is the most efficient method, making it unnecessary to compute numerous trigonometric functions of geodetic parameters. However, this does not provide compatibility with programs of other agencies which express the unknowns in angular units such as seconds. This problem arises when reduced normal equations of two countries are to be combined in the Helmert-Wolf block adjustment method, in which case the unknowns of all partial systems must be expressed in the same units. Moreover, the linear shifts dx', dy' in the astronomic horizon plane are not the same as the corresponding shifts dx, dy in the geodetic horizon; therefore, in order to ensure the desired compatibility, the shifts must first be expressed in the geodetic horizon system.

The azimuth and distance observation equations in the astronomic horizon system have the form

$$v = a_1 dx'_1 + a_2 dy'_1 + a_3 dx'_2 + a_4 dy'_2 + K. \qquad (1)$$

For use with the geodetic horizon they are rewritten as

$$v = b_1 dx_1 + b_2 dy_1 + b_3 dx_2 + b_4 dy_2 + K. \tag{2}$$

To derive the coefficients of (2), the following rotation matrices are first defined:

$$R_a = \begin{bmatrix} -\sin\phi\cos\lambda & -\sin\phi\sin\lambda & \cos\phi \\ -\sin\lambda & \cos\lambda & 0 \\ \cos\phi\cos\lambda & \cos\phi\sin\lambda & \sin\phi \end{bmatrix} \tag{3}$$

$$R_g = \begin{bmatrix} -\sin B\cos L & -\sin B\sin L & \cos B \\ -\sin L & \cos L & 0 \\ \cos B\cos L & \cos B\sin L & \sin B \end{bmatrix}. \tag{4}$$

The shifts in the astronomic and geodetic horizon systems are related by

$$[dx,\ dy,\ dH]^T = R_g R_a^T [dx',\ dy',\ dH']^T. \tag{5}$$

An evaluation of (5) with the usual second-order approximations gives

$$\begin{bmatrix} dx \\ dy \\ dH \end{bmatrix} = \begin{bmatrix} 1 & \sin\phi(\lambda - L) & (\phi - B) \\ -\sin\phi(\lambda - L) & 1 & \cos\phi(\lambda - L) \\ -(\phi - B) & -\cos\phi(\lambda - L) & 1 \end{bmatrix} \begin{bmatrix} dx' \\ dy' \\ dH' \end{bmatrix}. \tag{6}$$

In eq. (6), $\lambda - L$ is an approximation of $\sin(\lambda - L)$ which can be written as

$$\sin(\lambda - L) = (X\sin\lambda - Y\cos\lambda)/p \tag{7}$$

where $p = \sqrt{X^2 + Y^2}$. This leads to the following coefficients for use with eq. (2):

$$\begin{aligned} b_1 &= a_1 + m_1 a_2 & \qquad b_3 &= a_3 + m_2 a_4 \\ b_2 &= a_2 - m_1 a_1 & \qquad b_4 &= a_4 - m_2 a_3 \end{aligned} \tag{8}$$

in which

$$m_1 = \sin\phi_1(X_1\sin\lambda_1 - Y_1\cos\lambda_1)/p_1 \tag{9}$$

$$m_2 = \sin\phi_2(X_2\sin\lambda_2 - Y_2\cos\lambda_2)/p_2\ . \tag{10}$$

Let $M' = M + H$ and $N' = (N + H) \cos B = p$. Then the observation equation for an astronomic azimuth, with the shifts dB, dL and the constant term expressed in seconds, becomes

$$v = b_1 M'_1 dB_1 + b_2 N'_1 dL_1 + b_3 M'_2 dB_2 + b_4 N'_2 dL_2 + K \tag{11}$$

and the observation equation for a distance, with K in linear units, becomes

$$v = \text{arc } 1''(b_1 M'_1 dB_1 + b_2 N'_1 dL_1 + b_3 M'_2 dB_2 + b_4 N'_2 dL_2) + K \,. \tag{12}$$

The radii of curvature M and N are needed to five or six figures but they are computed in the program to much better precision. For this purpose a rapid formula is used to obtain tan B:

$$\tan B = \frac{Z(a + H)^2}{p(b + H)^2} \tag{13}$$

where a and b are major and minor semiaxes of the ellipsoid. Eq. (13) is an approximation of

$$\tan B = \frac{Z(a + H)^2}{p(b + H)^2} [1 + \frac{1}{4} e^4 H(Z^2 - p^2)/a^3] \tag{14}$$

(Vincenty 1980), in which the elliptic term amounts to at most 0.0001" per 1000 meters of H in all terrestrial applications.

3. STATIONS WITHOUT ASTRONOMIC COORDINATES

Theory requires that the direction of gravity be known at all stations from which directions are measured. This is seldom the case, therefore some approximate values must be used for astronomic coordinates. In the classical method of adjusting on the ellipsoid the implied assumption in such cases is that astronomic coordinates are the same as the corresponding most recent geodetic values. The original version of HOACOS assumed that they are the same as the first preliminary geodetic values. Neither approach is theoretically correct. In the present HOACOS version the sines and the cosines of such pseudo-astronomic coordinates are updated between iterations to correspond to geodetic values. This is accomplished by

$$d(\sin \phi) = \cos \phi \; dB \tag{15a}$$

$$d(\cos \phi) = - \sin \phi \; dB \tag{15b}$$

and similarly for $\sin \lambda$ and $\cos \lambda$. This approach does not improve the results of an adjustment (though it may change them slightly), but in the case when the astronomic coordinates have been derived from gravimetric deflections and from preliminary geodetic positions it automatically corrects the assumed direction of gravity for the change in geodetic position, as is necessary.

4. STATIONS WITHOUT HEIGHTS

In the practice employed by the National Geodetic Survey the heights of unoccupied intersection stations are not used in the adjustment. In the classical method of adjustment on an ellipsoid by the TRAV10 program (Schwarz 1978) the directions to such points do not receive deflection corrections nor skew normals corrections. In the height-controlled method on which HOACOS is based, these corrections are implicit. There may be cases when the standpoint is at an elevation of some thousands of meters. The forepoint, with no elevation given, is assumed to be at zero height. If, in addition, astro-geodetic deflections at the standpoint are large, the computed astronomic azimuth (and consequently also the adjusted direction) over this line could be wrong by tens of seconds because of the fictitious steepness of the line as defined by ΔX, ΔY, ΔZ values between these points. Some method is needed to nullify the errors caused by the assumption of zero height at the forepoint.

In such cases the elevated point is projected on the ellipsoid by

$$\delta X = -(X/p)\ H \cos B \tag{16a}$$

$$\delta Y = -(Y/p)\ H \cos B \tag{16b}$$

$$\delta Z = -\ H \sin B \tag{16c}$$

and the observation equation for this line is formed as if both points were at zero height. The coefficients of dx_1 and dy_1 are multiplied by

$$F = a/(a + H)\ . \tag{17}$$

A similar situation occurs when a direction and a sea level distance are given over a short line, normally within 100 meters, and the height of one of the stations is missing. Then the same procedure is used as in the case of a direction to an intersection station.

5. ELIMINATION OF STATION ORIENTATION UNKNOWNS

A separate version of HOACOS was created in which station orientation unknowns are eliminated by the well-known Schreiber method. It has not yet been determined whether the elimination of these unknowns is profitable. One disadvantage of this method is that it does not give directly the standard errors of the eliminated unknowns which are needed for the computation of the tau statistics to flag observations for possible rejections (Pope 1976).

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