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This is an unreviewed manuscript, primarily intended for informal exchange of information among NMC staff members.

Eight codes in the NMC subroutine library (W3LIB) which manipulate winds on stereographic grids, neglect to take proper account of wind values at the poles. This note discusses a solution to this problem and illustrates the methods used at NMC to represent winds at the poles and elsewhere. The procedures used in the aforementioned subroutines involve the conversions of true winds to grid oriented winds, the converse of this process, the conversion of vector wind $u, v$ components to true direction and speed, and finally the converse of this process. The results section summarizes and verifies the accuracy of the revised codes.

True direction and speed, in degrees and knots (or meters/second) and $u, v$ vector components are the most widely used methods of wind representation at NMC.

Wind direction, at all points excluding the pole point, is the true direction from which the wind is blowing (Figure 1a). Within one degree of the pole however, winds must be reported in accordance with the WMO convention (Figure 1b). A simple definition of this standard for wind direction at the poles is:

Facing the wind at or within one degree of the north pole, the wind direction is the west longitude meridian from which the wind is blowing. Facing the wind at or within one degree of the south pole, the wind direction is the east longitude meridian from which the wind is blowing.

A second convention for specifying winds at the north pole, the ATA (Air Transport Association) convention, orients the direction of the wind opposite from that of the WMO convention. The ATA convention is used primarilly for aircraft flight planning and therefore will not be dealt with in this note.

Two ways in which wind vectors (i.e. $u, v$ component form) can be graphically represented on NMC grids are: 1) with respect to a latitude/longitude (1ola) coordinate system and 2 ) with respect to a grid-oriented coordinate system. At all points except the pole points on the northern hemisphere lola coordinate system, a west wind (west to east) is defined as a positive u component while a south wind (south to north) is defined as a positive $v$ component. The grid-type system orients a positive $u$ component as a wind blowing from left to right along the horizontal axis of the NMC $65 \times 65$ polar stereographic grid and a positive $v$ component as a wind blowing from bottom to top along the vertical axis of the polar stereographic grid. Figure 2 illustrates positive $u$ and $v$ components on a northern and southern hemisphere grid-type system.

## PROCEDURE

The following variables are used throughout the W3FC codes dealing with winds (see table 2) and are defined as follows:

```
FFID = I grid-unit distance from the pole
                [ I(pole)-I(point) ]
FFJD = J grid-unit distance from the pole
            [ J(pole)-J(point) ]
FU = true u component of the wind
FV = true v component of the wind
FGU = grid-oriented u component of the wind
FGV = grid-oriented v component of the wind
DFP = grid unit distance from the pole SQRT [FFID 2 + FFJD' ]
SIN 0 = FFJD / DFP
COS 0= FFID / DFP
SPEED = wind speed SQRT [FU' 
DIR = true direction of the wind
```

In the following basic equations for a conversion from one two-dimensional coordinate system to a second two dimensional coordinate system, ( $x, y$ ) are units in the old system of cordinates and ( $x^{\prime}, y^{\prime}$ ) are units in the new system of coordinates (Figure 3). The new origin is coincident with the old origin and the new axes make an angle $\theta$ (defined as positive when rotating from old to new axis counterclockwise) with the old axes. (Burington, 1957)

$$
\begin{aligned}
& x^{\prime}=x * \cos \theta+y * \sin \theta \\
& y^{\prime}=-x * \sin \theta+y * \cos \theta
\end{aligned}
$$

From these equations follow:

$$
\begin{aligned}
& x=x^{\prime} * \cos \theta-y^{\prime} * \sin \theta \\
& y=x^{\prime} * \sin \theta+y^{\prime} * \cos \theta
\end{aligned}
$$

When converting from a lola system to a grid-oriented system at the north pole, a coordinate rotation of $100^{\circ}$ ( 1.745 radians) is made, making the positive v-axis coincident with $80^{\circ} \mathrm{W}-100^{\circ} \mathrm{E}$. At the south pole a coordinate rotation of $260^{\circ}$ ( 4.538 radians) is made, making the positive v-axis coincident with $100^{\circ} \mathrm{E}-80^{\circ} \mathrm{W}$. The following equations (derived from the basic rotation equations) are used in this transformation:

AT THE POLE
ELSEWHERE
(NORTHERN HEMISPHERE)

```
FGU = FU * COS }+\textrm{FV}*\operatorname{SIN}0\quadFGU=FU*(FFJD/DFP) + FV * (FFID/DFP
FGV = -FU*SIN\Theta + FV * COS 弶 FGV = FV * (FFJD/DFP) - FU* (FFID/DFP)
(SOUTHERN HEMISPHERE)
```






The conversion from grid-oriented coordinates to lola coordinates at the north pole uses a $260^{\circ}$ coordinate rotation, making the positive v-axis coincident with the prime meridian. At the south pole, a coordinate rotation of $100^{\circ}$ is made, making the positive v-axis coincident with $180^{\circ}$. The derived equations for these transformations are:

AT THE POLE
ELSEWHERE
(NORTHERN HEMISPHERE)

```
FU = FGU * COS }+\mathrm{ FGV * SIN }0\quadFU=FGU*(FFJD/DFP) - FGV * (FFID/DFP)
FV =-FGU * SIN }+\textrm{FGV}*\operatorname{COS}0\quad\textrm{FV}=\textrm{FGU}*(FFID/DFP) + FGV * (FFJD/DFP)
```

(SOUTHERN HEMISPHERE)

```
FU = -FGU * COS - FGV * SIN 
FV = FGU * SIN - FGV * COS 
```

The basic equations for converting from true direction and speed to $u, v$ components in either system at the north pole are:

```
u}=\operatorname{cos}0*\mathrm{ speed
v = sin}0*\mathrm{ speed (Figure 4).
```

At the north pole on the grid-oriented system, $\theta$ is the angle measured counterclockwise from $10^{\circ} \mathrm{E}$ to the west longitude meridian from which the wind is blowing. At the north pole on the lola system, $\theta$ is the angle measured counterclockwise from the prime meridian to the west longitude meridian from which the wind is blowing.

## AT THE NORTH POLE ELSEWHERE

1) 

$F G U=\operatorname{COS} \theta * \operatorname{SPEED} \quad \mathrm{FGU}=-\operatorname{SPEED} *[\operatorname{SIN}(D I R) *(F F J D / D F P)+\operatorname{COS}(D I R) *(F F I D / D F P)]$
$F G V=S I N \theta *$ SPEED $\quad F G V=S P E E D *[S I N(D I R) *(F F I D / D F P)-\operatorname{COS}(D I R) *(F F J D / D F P)]$
2)
$\mathrm{FU}=\cos \theta * \operatorname{SPEED} \quad \mathrm{FU}=-\operatorname{SPEED} * \operatorname{SIN}(\mathrm{DIR})$
$\mathrm{FV}=\operatorname{SIN} \theta * \operatorname{SPEED} \quad \mathrm{FV}=-$ SPEED $* \operatorname{COS}(D I R)$

The converse operations [ 3) grid-oriented components to direction and speed and 4) true components to direction and speed] use the following basic and derived equations:

$$
\theta=\arctan [u / v] \quad \text { (Figure 4) }
$$

## AT THE NORTH POLE

ELSEWHERE
3)
$\theta=\operatorname{ARCCOS}(F G U / \operatorname{SPEED}) \quad \operatorname{DIR}=\operatorname{ARCTAN}(F G U / F G V)+180$
4)

$$
\theta=\operatorname{ARCTAN}(F V / F U)
$$

$$
\mathrm{DIR}=\operatorname{ARCTAN}(\mathrm{FU} / \mathrm{FV})+180
$$

At the north pole, $\theta$ is defined the same as in the conversion from true direction and speed to $u, v$ components. From this angle (relative to either $10^{\circ} \mathrm{E}$ for grid-oriented winds or the prime meridian for lola oriented winds) the direction can be calculated in accordance with WMO standards.

## RESULTS

The aforementioned subroutines have been modified to calculate winds properly at the poles. Figures 5 and 6 show the conversions on a subset of the $65 \times 65$ grid surrounding the north pole [point (33,33)] from true $u, v$ components to grid-oriented $u, v$ components, and then back to true $u, v$ components. Figures 7 and 8 show conversions from grid-oriented $u, v$ components to true direction and speed and back to grid-oriented $u, v$ components. Figures 9 and 10 show the conversions from true $u, v$ components to true direction and speed and back to true $u, v$ components. Table 1 shows conversions using hypothetical winds at an arbitrary speed of 7.123 blowing from every five degrees across the north pole. This table confirms that the new code inserted into the subroutines indeed does work at the pole. Table 2 gives a list of the subroutines that have been modified, gives a brief synopsis of the conversions they do and gives their newly assigned names.


Figure 1a. Representation of a $140^{\circ}$, 45 knot wind at a point not at the pole.


Figure 1 b . Wind directions at either pole (indicated by direction of the arrows) are reported as the meridian (west longitudenorth pole, east longitude-south pole) from which the wind is blowing. eg: A wind blowing from $90^{\circ} \mathrm{E}\left(270^{\circ} \mathrm{W}\right)$ is reported as a $270^{\circ}$ wind at the north pole and as a $90^{\circ}$ wind at the south pole.


2a) Positive $U$ and $V$ components on a northern hemisphere grid-type coordinate system.

2b) Positive $U$ and $V$ components on a southern hemisphere grid-type coordinate system.


Figure 3. Coordinate transformation or rotation.


Figure 4. $\operatorname{COS} \theta=\mathrm{U} / \mathrm{SPEED}$
$\mathrm{U}=\operatorname{COS} \theta \times$ SPEED
SIN $\theta=\mathrm{V} / \mathrm{SPEED}$
$V=\operatorname{SIN} \theta \times$ SPEED
TAN $\theta=\mathrm{V} / \mathrm{U}$
$\theta=\operatorname{ARCTAN}(V / U)$


B


Figure 5. Gridplot of true and grid-oriented $u, v$ components.
a) true u components, pole value $=13.8773$
b) true v components, pole value $=3.1438$
c) grid-oriented $u$ components, pole value $=.6863$
d) grid-oriented $v$ components, pole value $=-14.2124$

| +38 |  |
| :---: | :---: |
| $+37$ | AAAAAAAAAAAAAAA |
|  | AAA HHHHHFHHHHHHHHHM AS HHHHYHHHHHHHHHH HHA |
|  |  |
| +36 | + U001-0007-0U0G-U010-0011-0005-0002-0001 $+0001+0000$ |
|  | AH A |
|  | AAA AAAAAA |
| +35 | +UOD4+UOU1+0U04+0003-0001+0 |
|  | ЛАЛAAAAAAAAAAAAAAAAAAAAAAAAAA -003-0 HHH |
|  | AAAANAAAASAAAAAAAAAAAAAAAAAAAA HHHHH |
|  | AAAAAAAAAAAAAAAAAAAAAAAAAAAAA HHHHH |
| $+34$ | AAAAAAAAAA $05=0000+U 005+0001-0009=0009+0001$ |
|  | AAAAAARA AA AAA |
| 433 | AAAAAAAA H AA AAAA AAA |
| 433 | OOL $2+0 \cup 07-0001-0014+0014+0010+0000-0002+0005$ |
|  | AAAAAAAS HH AAAAAAAAAAAAAAAARAAAA |
|  | AAAAAAAAA H AAAAAAAAAAAAAARAAAAA |
| +32 | $0018+0011+0007+0002-0007-0006+0005+0004+0004+0009$ <br> AAAAAAAAAAAAA AAAMAAAAAAAAAAAAA <br> AAAAAAAA AAAHAAMAAAAAMAAAAAAAAAAAAAAAAAA |
|  | AAAAMAAAAA AAAAAAAAAAMAAAAAAAAAAAAAA |
| +31 | $+0014+U 007+0008+0014+0014+0010+0004+0000+0006+0014$ AAAAAAAAAA AAAAAA AAAAAAA |
|  | AAAAAAAAAAA BBEBBBB AAAAA AAAAAA |
|  | AAAAAAAMAAA BBBHBBBBBH AAAAA AAAAA |
| $+30$ |  |
|  | AAAAA AAAAA BBBEBBRBBE AAAA AAAAA |
|  | AAAA A AAA EBBEBEBB AAAA AAAA |
| +29 | +U0UR - U0U3+0U01+U014+0024+0018+00 |

## B

|  |  |
| :---: | :---: |
| $+37$ |  |
| +36 | $\begin{aligned} & \text { AAAAAA } \\ & \text { +UU1OA } 1 \text { AOOQ+OUAAAAA } \\ & \text { AAAARAAAAAARAAAAA } \\ & \text { AAAAAAAAAAAAAAAAA } \\ & \text { AAAAAAAAAAAAAAA } \end{aligned}$ |
| $+3$ |  |
| + 3 | -0008-0012-0014-0017-0013-0002-0005-0011-0015 WHFIHHHMTIHHMHFIFHHH HHH HHRHHHOHHHHHFHHHH <br> HHHCHHH <br> HHHHHHH |
| +33 | $\begin{array}{r} 0009-0 \cup 10-0009-0009+0003+0004-0003-0009=0012 \\ \text { AAAAAAAAAAAAAAA } \\ \text { AAAA } \end{array}$ |
| $+32$ |  |
| +31 | $=0005+U 0 D 4+0 U 13+0015$ $+0011+0009+0009+0011+0011+0004$ <br> AAAAA AAAAAAAAAAAAAA <br> AAAA AAAAAAAAAAAAAAAA <br> AAAA AAAAAA |
| $+3$ | $-0006+0008+0020+0019+0008-0002-0006+0004+0025+0014$ <br> AAAA BBUBE AAAA AAAA |
|  | AAA BBBBBB AAA MHHHH ABBA |
| 29 | $003+0015+0025+0019+0003-0012=0018-0 \cap 06$ |

$+28+29+30+31+32+33+34+35+36+37$

Figure 6. Gridplot of true $u$, $v$ components.
a) true $u$ components, pole value $=13.8773$
b) true $v$ components, pole value 3.1438


B

$+28+29+30+31+32+33+34+35+36+37+24+24+3 u+31+32+33+34+35+36+37$

Figure 7. Gridplot of grid-oriented $u, v$ components and true direction and speed.
a) grid-oriented u components, pole value $=$. 6863
b) grid-oriented v components, pole value =-14.2124
c) true direction (tens of degrees), pole value $=257.2351$
d) wind speed (knots), pole value $=14.2290$
$+28+29+30+52+32+33+34+35+36+37$

|  | -0014-0006-0U02-0001+0001+0003-0001-0005-0010-0011 <br> H AASAAAAAAAAAAAAAAAAAAAAA <br> AAAAAAAAAAAAAAAAAAAAAAAAAAAA <br> HHHHHH <br> AANAAMAAA <br> AAA <br> HHHHHH |
| :---: | :---: |
| +37 | +0UO2+UOIO+OU14+UOAAAAAAAAAAAAAA HHHHHHH AAAA |
|  | AAAA $\quad \therefore$ AAAAAAAAA |
| + 3 |  |
|  | AAAAAAAAAAAAHAAAAAAAAAAAAAAAASAA |
|  | AAAAAAAAAAAAHAAAAAAAAAAAAAAAAAAAA |
| +35 | +U004+UUUO-DUO2-UDOJ-000J-0001+000 |
|  | AAA AAAAAAAAAAAAAAAAA |
|  | HHHIHHHHHHHHH |
| $+34$ |  |
| +33 | $\begin{aligned} & \text { HHM } \\ & =0 H \end{aligned}$ |
|  | AAAAAAAAA AAAAAAAAA |
| $\div 3$ | AAAAAAAAA |
|  | - |
|  | תAAAAAAA AAAAAA AAA |
| +3 |  |
|  | ДAAAAAA $\quad$ BUHEBBBB $+017+0010=0001=0008=0006+000$ |
|  | АAAAAAA В $\triangle$ ВВВВВВВВВВН AAAA |
|  |  |
| $+30$ | $+U 00 \cdots+U 007+0017+U 026+002 B+0022+0007-0004=0007-0002$ <br> AAAAAAA <br>  |
|  | AAAAAAA BHBUBBBBBBBG $\triangle A A A$ |
|  | AAAACAA ABCBBBBBBBB AAAAA |
| $+29$ | $+0003+0008+0015+0021+0023+0017+0008-0000-0004-$ |

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Figure 8. Gridplot of grid-oriented $u, v$ components.
a) grid-oriented $u$ components, pole value $=.6864$
b) grid-oriented v components, pole value $=\mathbf{- 1 4 . 2 1 2 4}$


B


```
+28+29+30+31+32+33+34+35+36+37+28+29+30+51+32+33+34+35+36+37
```

Figure 9. Gridplot of true $u, v$ components and true direction and speed.
a) true $u$ components, pole value $=13.8773$
b) true v components, pole value. $=3.1438$
c) true direction (tens of degrees), pole value $=257.2354$
d) wind speed (knots), pole value $=14.2290$


## B



Figure 10. Gridplot of true $u, v$ components.
a) true u components, pole value $=13.8773$
b) true $v$ components, pole value $=3.1438$







Table 1. Progressive transformation of arbitrary winds at the north pole from true (per WMO convention) u,v components (columns 1 and 2) to grid u,v components (columns 3 and 4) and true (per WMO convention) direction and speed (columns 5 and 6), then back to grid $u, v$ components (columns 7 and 8) and true (per WMO convnetion) au, v components (columns 9 and 10), true (per WMO convention) direction and speed (columns 11 and 12) and finally true (per WMO convention) u,v components (columns 13 and 14).

|  | NORTHERN HEMISPHERE |  |
| :---: | :---: | :---: |
| W3FC07 | grid-oriented components to true components | W3FC07 |
| W3FC08 | true components to gridoriented components | W3FC08 |
| W3FC02 | grid-oriented components to true direction and speed | W3FC02 |
| W3FC00 | true components to true direction and speed | W3FC05 |
| W3FC03 | true direction and speed to grid-oriented components | W3FC03 |
| W3FC01 | true direction and speed to true components | W3FC06 |
|  | SOUTHERN HEMISPHERE |  |
| W3FC11 | grid-oriented components to true components | W3FC11 |
| W3FC10 | true components to gridoriented components | W3FC10 |

Table 2.

## REFERENCE

1. Handbook of Math Tables and Formulas, Burington, 1957, pp. 27.
