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# Pseudo-Precipitation: A Continuous Precipitation Variable

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## Abstract

The discontinuous and highly nonlinear nature of precipitation makes its statistical manipulation, like the reduction of bias in quantitative precipitation forecasts (QPF) or probabilistic QPF (PQPF) problematic. To ease the statistical post-processing of precipitation, a new continuous variable, pseudo-precipitation (PP) is introduced. Pseudo-precipitation is equal to precipitation when precipitation is nonzero and is proportional to the vertically integrated water vapor deficit (with a negative sign) otherwise. The concept of PP and its practical application is illustrated with analyzed and forecast data samples.

**Keywords:** precipitation, statistical post-processing, calibration, bias correction, dry bias

## 1. Introduction

Precipitation forecasts have become more skillful over recent decades due to improvements in Numerical Weather Prediction (NWP) e.g., (Olson et al. 1995; Reynolds 2003.) However, due to the fine scale and complex thermodynamic and microphysical processes that lead to the formation of precipitation, these improvements have been relatively slower

than improvements in other variables such as temperature and pressure (Sanders 1986). Compounding the problems posed by its multiscale character, many of the processes contributing to the formation of precipitation are also highly non-linear. This results in the spatially and temporally highly variable and discontinuous character of precipitation. Additional improvements in QPF and PQPF skill are possible by statistically post-processing ensemble forecasts (e.g., Fritsch and Carbone 2004; Hamill and Whitaker 2006; Scheuerer and Hamill 2015).

The discontinuous nature of precipitation, however, poses special challenges: the statistical post-processing of ensemble forecasts; the formulation of tangent linear and adjoint models used in data assimilation such as for the Weather Research and Forecast model, (Huang et al. 2009); dynamical sensitivity and other applications; and forecast verification. With these difficulties in mind, we introduce a new statistically continuous variable called pseudo-precipitation (PP) that is related to precipitation. Section 2 offers further motivation for the development of this variable, while section 3 provides a formal definition. The application of PP is illustrated with observed/analyzed precipitation and ensemble forecast data and with a conceptual de-biasing example in section 4. Section 5 outlines a working definition for a transform function that ensures the continuity and differentiability of PP across and around the zero- precipitation threshold. The paper concludes with a brief summary and a discussion of possible future work (section 6).

## **2. Motivation**

While forecasts of continuous variables lend themselves to straightforward statistical post-processing techniques such as model output statistics (MOS, Glahn and Lowry 1972; Antolik 1995), PQPF usually involves a two-step process. Given an ensemble of forecasts and corresponding precipitation observations, probability of precipitation (POP) forecasts are usually calibrated first, followed by the statistical processing of the probability distribution of precipitation (PDP), conditioned on precipitation being greater than zero. Such a two-step statistical post-processing of precipitation is limited because it does not consider a number of moisture-related properties of nonprecipitating members of the ensemble (e.g., whether nonprecipitating members are associated with wet or very dry air conditions) that likely have discriminating information on the calibration of PDP forecasts. Likewise, the amount of precipitation in those members that do precipitate may have discriminating power in terms of POP forecast calibration.

Conceptually, the calibration of ensemble precipitation forecasts is simpler when the forecasts are *wet-biased*; one needs only to reduce the amount and/or area of precipitation coverage in each ensemble member to match ensemble forecast statistics with observed climatology. When a statistical post-processing scheme needs to decrease POP in order to ensure consistency with observations, precipitation for ensemble member(s) with the lowest nonzero precipitation at each grid point can simply be set to zero. The technique of Yuan et al. 2007 is an example of how ensemble numerical weather prediction (NWP) forecasts can be postprocessed to calibrate PQPF in case of mostly wet forecast bias. Correcting *dry-biased* ensemble members, on the other hand, requires assigning nonzero precipitation amounts to ensemble members that had zero precipitation in the raw forecast. In this case, the discontinuous nature of precipitation complicates the statistical adjustment of ensemble forecasts. Lacking any a priori information on which ensemble members with zero precipitation should be assigned nonzero precipitation amounts, a method must be devised to identify ensemble members that are closest to producing precipitation.

In nature, precipitation results from multiscale processes. Moist air may converge as part of the larger scale circulation. Under certain conditions, convection may be triggered on smaller scales. Through complex and even finer scale microphysical (MP) processes such as condensation and deposition, hydrometeors may form, fall, and eventually reach the Earth's surface. MP processes cannot be resolved, even in the finest resolution NWP models, and therefore must be parameterized. In coarser resolution models, even convective processes (CP) must be parameterized. Depending on the resolution of NWP models, precipitation is therefore (i) generated as a prognostic variable in MP parameterization schemes (i.e., falling hydrometeors reaching the ground, (Houze 2014)), and/or (ii) diagnosed from prognostic variables such as moisture, temperature, and wind as part of CP parameterization schemes (Arakawa 2004).

This study represents an initial attempt at creating a continuous precipitation related variable that can also be used to rank nonprecipitating forecast states to facilitate the calibration of ensemble forecasts. Instead of working with specific triggers that lead to precipitation in either of the two NWP precipitation generation algorithms, we consider slower and smoother aspects of precipitation formation that may be better represented in NWP forecasts. Moisture is an obvious consideration, expressed as total column-integrated water vapor (also known as precipitable water). Precipitable water in itself, however, is not sufficient because the same amount of vapor may produce precipitation on a cool day but not on a warm day. We are closer to the required attribute by computing the vapor deficit,

relative to saturation, in a column-integrated sense. The closer this value is to zero, in cold conditions or warm, the more susceptible a grid column is to producing precipitation.

### 3. Definition of Pseudo-Precipitation (PP)

Given the preceding discussion, our goal here is to define a new, continuous variable that is equal to precipitation ( $p$ ) when precipitation is nonzero, and is negative otherwise with an increasing absolute value as atmospheric conditions become less amenable to precipitation. Thus:

$$PP = p \text{ when } p > 0 \quad (1)$$

$$PP = f(J) \text{ otherwise,}$$

where  $J$  is a measure of conditions favorable to the creation of precipitation, and  $f$  is a suitable transform function that ensures the continuity of PP near zero. Further discussion on, and a working definition for  $f$  appears in section 5.

Ideally,  $J$  should reflect the dynamical and thermodynamical processes and variables contributing to the formation of precipitation, including moisture content, temperature profile, moisture convergence, and relevant microphysical processes. As a simple but suitable candidate, for grid point  $(i, j)$  with zero precipitation,  $J$  is chosen to be the vertically integrated water vapor deficit:

$$J(i, j) = \frac{1}{g} \int (q_v - q_{vsat}(T)) dp \quad (2)$$

where  $q_v$  is water vapor,  $q_{vsat}$  is saturated water vapor at temperature  $T$ , and  $g$  is the gravitational constant. This definition for  $J$  is based on the most basic ingredient (moisture) and process (saturation) needed for the formation of precipitation, effectively measuring how far conditions are from saturation. A favorable property of  $J$  in this definition of pseudo-precipitation is that it has the same physical unit ( $\text{kg/m}^2$ , mass per unit area) as precipitation, thus easing the constraints on  $f$  that ensure zero-crossing continuity. A more complex definition of  $J$  may also include moisture convergence and microphysical processes, which are important but secondary in the formation of precipitation.

For very short accumulation periods or for precipitation rate, fluctuations of  $J$  in time can be neglected and  $J$  is uniquely defined. When defining  $J$  for finite accumulation periods, however, temporal variations in  $J$  cannot be ignored. A possible approach in such cases is

to base  $J$  on the instantaneous atmospheric states during an accumulation period. Considering the discontinuous nature of precipitation (i.e., it can fall during short periods that are preceded and followed by unsaturated conditions),  $J$  could be defined to characterize the smallest absolute value of saturation deficit during the period over which precipitation is accumulated in a model. This definition may be more consistent with the natural processes governing precipitation formation compared to other plausible choices such as the mean or other functions of  $J$  over the accumulation period. Given the highly variable and nonlinear nature of processes leading to precipitation, a distance measure to saturation over an accumulation period (i.e., the easiest way to bring the atmosphere close to saturation over a period), however, cannot be uniquely defined. We note again that our goal is to find a variable that for nonprecipitating events measures how far the atmosphere is from precipitating. Considering that output from NWP forecast models from which  $J$  is calculated is typically available only at the start and end points of selected accumulation periods, in this study PP is determined using the average of  $J$  values from these two instances.

#### 4. Applications

Fig. 1 displays the empirical cumulative distribution function (CDF) of observed/analyzed pseudo-precipitation over an accumulation period of six hours, within a particular analysis grid box, and during a period of 121 days. Observed precipitation analyzed over a 4x4-km grid (the National Centers for Environmental Prediction - NCEP Stage IV analysis; <http://www.emc.ncep.noaa.gov/mmb/ylin/pcpanl/stage4/>) is aggregated onto a 1x1-degree latitude/longitude grid, while corresponding pseudo-precipitation values are calculated using the National Center for Atmospheric Research (NCAR)/NCEP Reanalysis data on the same 1x1-degree grid. For simplicity,  $J$  was determined using operational analysis variables with a simple identity matrix of  $f$ .

As expected, the CDF for PP defined with an identity matrix  $f$  in Fig. 1 displays a non-differentiable behavior at and just below zero PP. To avoid discontinuity and to ensure differentiability of pseudo-precipitation (particularly at PP=0),  $f$  must be chosen carefully. In particular, one must consider that (a)  $J$  cannot be measured directly and therefore has to be derived from other measured or calculated NWP model variables; (b)  $J$  has to be determined for both NWP analyses and forecasts; (c) different NWP models have unique ways of representing natural processes affecting precipitation, especially the initiation of precipitation, imparting a model's potential peculiarities into PP; (d) in nature, precipitation

starts before the column of air becomes completely saturated; and (e) precipitation is a monotonically increasing function of the length of an accumulation period, whereas  $J$  (as discussed in the previous section) is a different function of that time period.

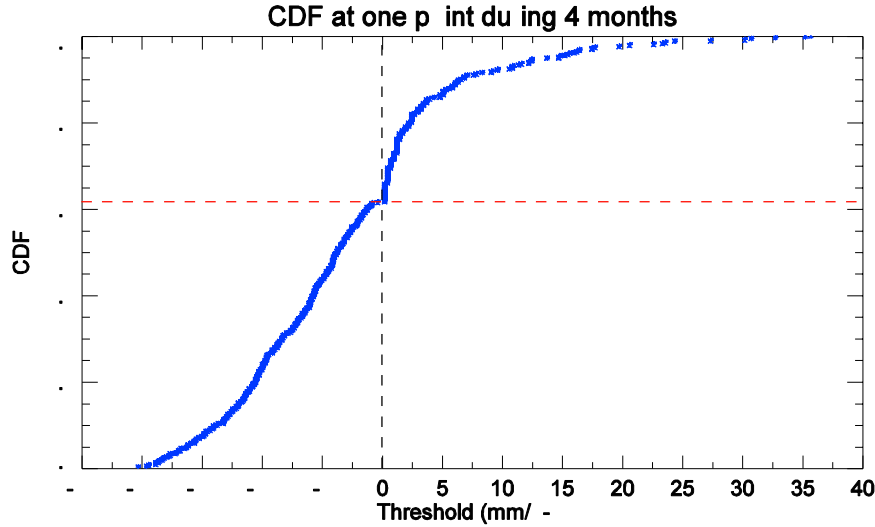


Fig. 1. Empirical cumulative distribution function (CDF) of observed/analyzed pseudo-precipitation for a 6-hour period for a 1x1-lat/lon grid box centered at 40°N and 121°W for the period December 2005–March 2006, based on Stage-IV precipitation and NCEP-NCAR saturation deficit reanalysis data. The red dashed horizontal line marks the observed/analyzed probability of having no precipitation.

These factors contribute to a narrow range of PP values just below zero over which no PP values occur. Another manifestation of discontinuity evident in Fig. 1 is the difference in the slope of the curves below and above zero PP. Note that CDF slopes for positive PP are also different for different accumulation periods (not shown). These problems can be attributed to issues related to differences in the sensitivity of precipitation and  $J$  to various accumulation periods (issue mentioned above) and will be considered in section 5.

In Fig. 2, we compare pseudo-precipitation CDF's for observed/analyzed and forecast conditions for a month-long period in the summer of 2006, at a grid point over Denver, CO. The black dotted curve displays the observed/analyzed PP as in Fig. 1. Nonzero precipitation is based on NCEP Stage IV precipitation analyses aggregated to a 1x1-latitude/longitude grid, while the negative PP values are computed at the same resolution, using the NCEP Global Forecast System (GFS) analysis data when the Stage IV data are equal to zero. The forecast CDF (blue dashed curve) is based on 1x1-degree resolution data



from the North American Ensemble Forecast System (NAEFS) (Toth et al. 2006) that includes ensemble members from the NCEP (four times per day) and the Canadian Meteorological Center (CMC), twice per day. Positive values represent 0–6h precipitation accumulation forecasts, while the negative PP values, where zero precipitation were forecast, have been computed using raw (uncalibrated) ensemble forecast data. This is because the NAEFS ensemble archive does not contain bias-corrected humidity fields. In future applications, however, PP will be derived using bias-corrected model variables.

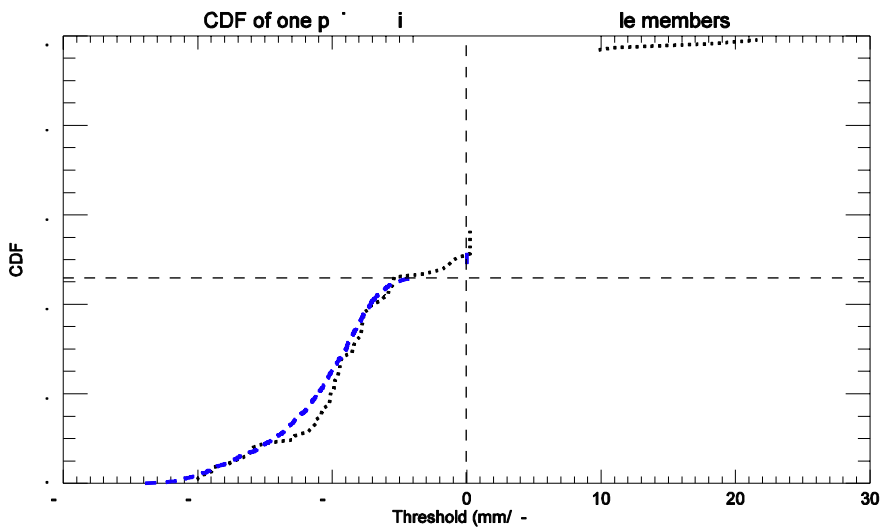


Fig. 2. Empirical CDF of 6-hr accumulated observed/analyzed (based on NCEP Stage IV precipitation and GFS analysis data, black dotted) and 0-6-hour forecast (based on NAEFS ensemble data, blue dashed) pseudo-precipitation at a grid point covering Denver, CO, over a 30-day period in the summer of 2006. The dashed horizontal line marks the forecast probability of having no precipitation.

Due to a larger sample that includes all ensemble members, the blue CDF curve based on the NAEFS forecasts is much smoother than the black CDF based on observed/analyzed PP. The probability of having no precipitation in Fig. 2 is the CDF value where the analyzed and forecast curves reach the zero pseudo-precipitation value. For this period, the ensemble forecasts tend to underestimate the probability of no precipitation and hence overestimate POP relative to observations as the forecast CDF lies below that of the analyzed/observed CDF curve around zero PP. NAEFS also underestimates extreme precipitation, with the highest forecast values being around 9 mm, compared to 20 mm in the observed/analyzed

data. Interestingly, the distribution of analyzed and forecast PP is rather similar over their negative range. This is because statistically, the distribution of 6-hr forecast and analyzed variables that affect PP are rather similar since the analysis is partly derived from a short-range forecast (Kalnay 2003).

A hypothetical application of PP for the bias correction of ensemble precipitation forecast is illustrated in the schematic of Fig. 3. In this example, only 20% of the raw ensemble members produced precipitation (see empirical cumulative histogram in left panel). Based on the assessment of systematic errors in similar forecast cases from the past, a calibration of the ensemble forecasts in this hypothetical example assigns positive precipitation amounts to members with closest to zero PP values. The resulting empirical cumulative histogram for the bias-corrected ensemble (right panel in Fig. 3) shows 83% of the members with precipitation, which is the expected percentage conditioned on a 20% forecast probability.

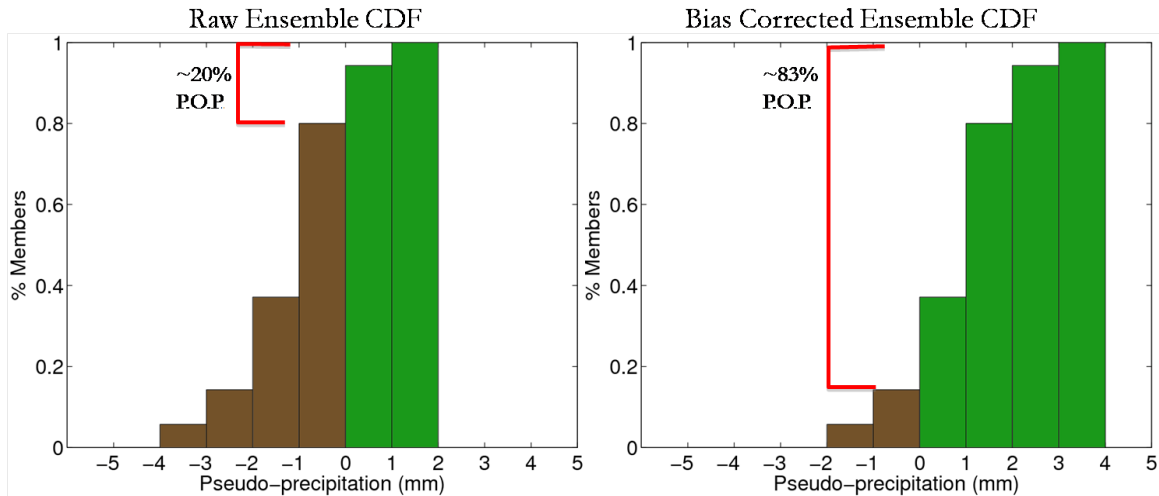


Fig. 3. A schematic example of cumulative histograms for raw (left) and bias-corrected (right) ensemble pseudo-precipitation forecasts. Dry forecast members closest to saturation from the left are adjusted so the calibrated forecasts on the right panel reflect expected precipitation conditioned on the raw forecast distribution. For further details, see text.

### 5. Choice of Transform Function

Following Eq. 1, when observed (for the real world) and forecast (for NWP forecasts), precipitation is nonzero, and pseudo-precipitation is set equal to that amount. When

precipitation is zero, PP is defined as a function of  $J$  (see Eq. 2). For consistency and comparability with PP for the real world, for PP of NWP forecasts,  $J$  will be calculated using bias-corrected NWP forecast variables that have the same mean and variance as the NWP analysis.

The typical discontinuous behavior of the CDF for observed/analyzed and forecast PP with  $f$  as an identity matrix is shown in the schematic of Fig. 4. Rather than attempting to tackle various underlying issues related to the use of a simple identity matrix  $f$  separately, a new  $f$  is proposed here as a simple operation that maps the CDF of negative PP so it seamlessly blends with the CDF of positive precipitation in a way that ensures the continuity of the full CDF. First, a particular accumulation period is chosen for PP (e.g., six hours). Second, a parametric distribution is fit over the observed/analyzed precipitation values (green curve), assuming that only values greater than zero can be observed and that values below zero are unobserved (i.e., censored) but account for a one-POP portion of the full distribution (dashed red curve). Third, another parametric distribution is fit over a climatic sample of  $J$  values with cases of zero precipitation, based on reanalysis data (brown curve).  $f$  is then defined as a mapping operation that transforms the reanalysis-based  $J$  into new values that fit onto the negative side of the parametric distribution (dashed red curve) that was created by the censored fitting of the observed precipitation values (green curve). This transformation is chosen such that the new  $J$  values correspond to the same percentile (i.e., percentile matching) over the negative range of the observed distribution (dashed red curve) as they do on the original distribution of the  $J$  values (brown curve).

Algorithms to fit distributions to a given sample of data as required by this method are described by Krzysztofowicz (2016) and tested by Wang et al. (2019). In practice, these fits are made for each grid point and day of the year, using a long climate record of observationally based precipitation analysis to produce a spatially and seasonally dependent definition of  $f$ . As long as all forecast variables used to define  $J$  for the calculation of forecast PP are bias-corrected (i.e., have the same mean values as the observed analysis<sup>1</sup>),  $f$  as defined above can be readily used for the calibration of forecast

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<sup>1</sup> Note that in an operational environment, analysis and NWP modeling tools are periodically updated and therefore are different from those frozen for use in the generation of reanalysis. The updates ensure that operational analyses at any time have the highest quality, and therefore are used for bias correction of the forecasts, while the reanalysis is used to determine climatological distribution characteristics. Systematic

PP. The use of  $f$ , as described above, effectively eliminates the problem discussed under issue (b) in section 4.

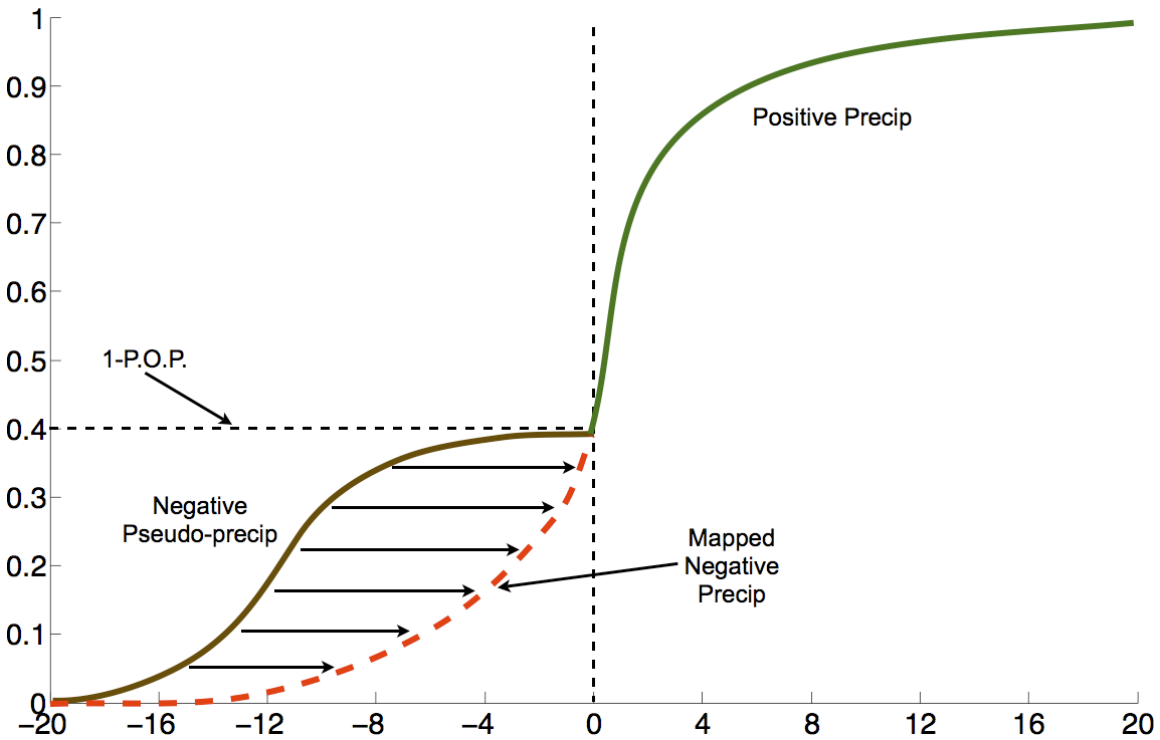


Fig. 4. Schematic CDF fitted to “censored” observed precipitation values (green curve), with a natural extension to the range of negative pseudo-precipitation (PP) values (red dashed), along with a CDF of negative PP values reflecting saturation deficit based on NWP analysis data (brown). Transform function  $f$  corresponds to frequency matching of the brown CDF with the red (“pseudo”) portion of the observed precipitation CDF, indicated by the horizontal arrows. For further details, see text.

## 6. Summary

To address difficulties that the discontinuous and nonlinear nature of precipitation poses for statistical post-processing and other NWP-related applications, a new, continuous variable called pseudo-precipitation (PP) was introduced. According to its definition, PP is equal to precipitation when there is precipitation, and is negative with an absolute magnitude proportional to the water vapor deficit over the atmospheric column when

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differences between operational and reanalysis values can be continuously monitored and appropriately applied to avoid related inconsistencies in calibrated forecasts.

precipitation is zero. Continuity of the distribution of negative PP values with that of observed/analyzed precipitation is achieved through the application of an empirically defined transform function.

Many statistical post-processing methods sensitive to continuity characteristics can then be applied on forecast PP, resulting in calibrated precipitation forecasts. PP thus facilitates a one-step post-processing of precipitation. Unlike the conventional approach of first calibrating POP and then producing a calibrated CDF *conditioned* on the presence of precipitation, a one-step post-processing approach allows for all (not only the wet or dry) members of an ensemble to influence the entire (not only the wet or dry part of the) posterior CDF, potentially increasing the statistical resolution of the post-processed forecast. In case of a dry bias, ensemble forecasts can also be conveniently calibrated by wettening members with the highest negative values of PP (i.e., least dry members).

The outlined bias correction algorithm has yet to be tested with real forecast and observed/analyzed precipitation data. Critical components in the concept of pseudo-precipitation involve the basic definition (presently using an *if* statement, see Eq. 1), and the working definition of transform function  $f$  presented in section 5. Other possible choices for Eq. 1 and  $f$  are being actively explored and will be reported separately. Future PP applications can utilize observed/analyzed datasets such as the climatology-calibrated precipitation analysis (Hou et al. 2014; CCPA), in place of Stage IV analyses used in this study. New reanalysis and reforecast datasets, as they become available, can also be utilized in PP-based calibration studies.

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