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ABSTRACT. The quantity and quality of atmospheric data pertaining to the global carbon cycle have improved to an extent that more realistic error estimates can now be attempted for regional sources and sinks of CO_2 derived from such data. *Enting et al.* [1995] describe a Bayesian synthesis methodology for a 3-D atmospheric transport model. Recently, *Ciais et al.* [1995] deconvoluted the ocean and land uptake using CO_2 mixing ratio and $\delta^{13}C$ observations in the atmosphere in a 2-D inverse model of atmospheric transport. In their work, more attention was given to the description of the method than to a precise estimate of errors. A coarse estimate was provided by sensitivity tests of the model using drastic alterations of the important parameters. We present here a detailed error analysis of the land and ocean fluxes inferred by *Ciais et al.* [1995]. Because the analytic expression for the fluxes is known explicitly, it is possible to use an analytical propagation of errors. In this manner, we quantify the main uncertainties associated with the three principle parameters of the model: the discrimination against ^{13}C by plant photosynthesis and the ocean and land isotopic disequilibria. This requires a physical, and thus partly subjective, estimate of the uncertainty of each of the three parameters. We also re-evaluate the "bootstrap" error associated with the longitudinal structure of data, which consist of flask samples from the Climate Monitoring and Diagnostics Laboratory (CMDL) global air sampling network, supplemented at high southern latitudes with Commonwealth Scientific and Industrial Research Organization/Global Atmospheric Sampling Laboratory (CSIRO/GASLAB) measurements [*Francey et al.,* 1994]. This is done by propagating the errors associated with the CO_2 and $^{13}CO_2$ sources separately, which have been inferred by a bootstrap analysis. The error propagation includes their covariance, accounting for the very strong correlation between the sources of CO_2 and $^{13}CO_2$.

Propagation of Errors

The analytical solution of the deconvolution of land and ocean fluxes by *Ciais et al.* [1995] is given in equation (1) and (2).

$$S_b = \frac{\alpha_{ao} R_a \cdot S + (R_f - \alpha_{ao} R_a) \cdot S_f - {}^{13}S + {}^{13}S_{bdis} + {}^{13}S_{odis} + {}^{13}S_{defdis}}{(\alpha_{ao} - \alpha_{ph}) \cdot R_a} \qquad (1)$$

$$S_o = \frac{\alpha_{ph} R_a \cdot S + \left(R_f - \alpha_{ph} R_a\right) \cdot S_f - {}^{13}S + {}^{13}S_{bdis} + {}^{13}S_{odis} + {}^{13}S_{defdis}}{\left(\alpha_{ao} - \alpha_{ph}\right) \cdot R_a} \tag{2}$$

where the terms are defined as follows:

R_a	Atmospheric $^{13}C/^{12}C$ isotopic ratio
R_f	Fossil fuel $^{13}C/^{12}C$ isotopic ratio
α_{ph}	Isotopic fractionation factor for plant photosynthesis
α_{ao}	Kinetic fractionation factor for the dissolution of gaseous CO_2 into seawater
^{13}S	Total sources of $^{13}CO_2$ inferred by the 2-D model
S	Total sources of CO_2 inferred by the 2-D model
S_f	Source of CO_2 from the burning of fossil fuels
$^{13}S_{bdis}$	Isotopic disequilibrium flux of $^{13}CO_2$ due to exchange between the land biosphere and the atmosphere
$^{13}S_{odis}$	Isotopic disequilibrium flux of $^{13}CO_2$ due to exchange between the ocean and the atmosphere
$^{13}S_{defdis}$	Isotopic disequilibrium flux of $^{13}CO_2$ produced by biospheric destruction

We do not treat uncertainties associated with R_f, α_{ao} and $^{13}S_{defdis}$ because the sensitivity tests carried out by *Ciais et al.* [1995] show that they are negligible. The land and ocean fluxes (S_b and S_o) are functions of the three model parameters α_{ph}, $^{13}S_{odis}$ and $^{13}S_{bdis}$, which are assumed to be independent, and of the sources inferred by the inverse model (S and ^{13}S). The fossil fuel term is considered known, as well as its isotopic ratio. Because we know explicitly the expressions for S_b and S_o, it is possible to apply a straightforward propagation-of-errors scheme to estimate the 1σ errors associated with these fluxes. For the uncertainty in the uptake on land, we obtain the following expression.

$$\begin{aligned}\sigma_{S_b}^2 = {} & \left(\partial_{\alpha_{ph}} S_b \cdot \sigma_{\alpha_{ph}}\right)^2 + \left(\partial_{S_{odis}} S_b \cdot \sigma_{S_{odis}}\right)^2 + \left(\partial_{S_{bdis}} S_b \cdot \sigma_{S_{bdis}}\right)^2 + \\ & \left(\partial_S S_b \cdot \sigma_S\right)^2 + \left(\partial_{13S} S_b \cdot \sigma_{13S}\right)^2 + 2 \cdot \partial_S S_b \cdot \partial_{13S} S_b \cdot \mathrm{cov}\left(S, {}^{13}S\right)\end{aligned} \tag{3}$$

where

∂_j is the partial derivative of S_b with respect to parameter j.

The covariance term persists because S and ^{13}S are not independent. Indeed, even if CO_2 and $\delta^{13}C$ are measured independently, their surface sources S and ^{13}S are strongly correlated. This is because the relative variations of the $^{13}C/^{12}C$ ratio of any carbon containing compound on the surface of the earth are not more than a few percent. Furthermore, in the inverse transport model, the inferred source of a tracer depends essentially on atmospheric transport and the same transport is applied to CO_2 and $\delta^{13}C$. Using equation (1), we obtain the following expressions for the set of partial derivatives in (3).

$$\partial_{\alpha_{ph}} S_b = \frac{S_b}{\left(\alpha_{ao} - \alpha_{ph}\right)} \tag{4}$$

$$\partial_{13_S} S_b = \frac{-1}{\left(\alpha_{ao} - \alpha_{ph}\right) \cdot R_a} \tag{5}$$

$$\partial_{S_{bdis}} S_b = \frac{1}{\left(\alpha_{ao} - \alpha_{ph}\right) \cdot R_a} \tag{6}$$

$$\partial_S S_b = \frac{\alpha_{ao}}{\left(\alpha_{ao} - \alpha_{ph}\right)} \tag{7}$$

We now need to provide reasonable estimates of the errors associated with each of the three model parameters, and with the sources. The errors of the disequilibria and the discrimination will be estimated from an assessment of the representation of the data used to calculate these terms, which in turn is based on knowledge of the physical mechanisms influencing them. The errors of the sources will be estimated from the bootstrap analysis.

Parameter α_{ph} -- Discrimination by plants

In Table 1 we calculate the standard deviation of a large set of measurements of $\delta^{13}C$ in C-3 plants at low altitudes [*Körner et al.*, 1991] to get an estimate of the 1σ error of α_{ph}.

Table 1. 1σ error of the zonally-averaged photosynthetic discrimination (in units of ‰) by C-3 plants in each model grid box, numbered 1 through 20 from south to north [*Tans et al.*, 1989].

Box	Sigma	Box	Sigma	Box	Sigma	Box	Sigma
1	0.7	6	0.8	11	0.9	16	0.9
2	0.7	7	0.8	12	0.9	17	0.9
3	0.7	8	0.9	13	0.9	18	0.7
4	0.9	9	0.9	14	0.8	19	0.7
5	0.9	10	0.9	15	0.8	20	0.7

The data of *Troughton* [1972], although they do not separate plants growing at high altitudes from lowland plants, would have yielded similar errors. In the tropics, we must account for the uncertainty of the proportion of C-4 versus C-3 plants in a given latitude band. Note that the word "proportion" does not mean the true percentage of C-4 plants but rather the percentage weighted by gross primary productivity (GPP) of C-4 plants in a given latitude band. The proportion of C-4 plants is given by the SiB model but, because of large discrepancy with the study of *Lloyd and Farquhar* [1994], we arbitrarily enhance by a factor of 2 the uncertainty on α_{ph} in tropical areas where C-4 plants are encountered (i.e., box numbers 5-7 and 12-13 of our 2-D model). Including these values in equations (3) and (4), we obtain the following errors for the ocean and land partitioning due to parameter α_{ph} (Table 2).

Table 2. 1σ errors of the calculated land uptake in broad latitude bands due to the discrimination by plants (units Gigaton (10^{15} g) C, or GTC yr^{-1}). The errors of the ocean uptake are very close to these values and correlate with the errors on land, so that the sum of ocean and land fluxes always equals the net flux of total CO_2.

Latitude band	90-30°S	30°S-Eq	Eq-30°N	30-90°N	Global
Partitioning sigma	0.07	0.06	0.09	0.19	0.22

Parameter $^{13}S_{odis}$ -- Ocean disequilibrium.

The ocean isotopic disequilibrium term is expressed by equation (8).

$$^{13}S_{odis} = F_{oa}\,(\alpha_{oa}(T)\,R_o - \alpha_{ao}(T)\,R_a) \qquad (8)$$

where

- F_{oa} Gross flux of CO_2 from ocean to atmosphere
- R_o $\delta^{13}C$ of dissolved inorganic carbon in surface ocean
- α_{oa} Kinetic fractionation factor for the transfer of dissolved inorganic carbon to CO_2 in the atmosphere
- α_{ao} Kinetic fractionation factor for the transfer of gaseous CO_2 to dissolved inorganic carbon

To estimate the isotopic disequilibrium between the surface oceans and the atmosphere, we use observations of $\delta^{13}C$ of dissolved inorganic carbon in surface waters (R_o). To take into account the wind speed dependence of the gas exchange coefficient, *Ciais et al.* [1995] initially used a climatology of the gas exchange coefficient based on winds obtained from a general circulation model (GCM) [*Erickson*, 1989]. We now use a determination of the gross flux (F_{oa}) calculated from monthly satellite data using the Liss and Merlivat formulation of the gas exchange coefficient [*Etcheto et al.*, 1991; *Liss and Merlivat*, 1986]. We apply a scaling factor of 1.6 to make the gas exchange coefficient consistent with the bomb ^{14}C ocean inventory [*Broecker et al.*, 1985]. The ocean isotopic disequilibrium is also a function of the sea surface temperature (SST) through α_{oa}, but we neglect the experimental uncertainty of the fractionation factors compared to the large uncertainties of R_o and F_{oa}. We make the conservative approximation that R_o is independent of F_{oa}. Considering monthly fields, this is justified by the fact that R_o is controlled mostly by the biological activity and the advection of nutrient-rich, $\delta^{13}C$ depleted water from the deep ocean rather than by the air-sea exchange. The 1σ error of $^{13}S_{odis}$ is obtained by propagating errors in equation (8).

$$\sigma_{S_{odis}}{}^2 = [\alpha_{oa} \cdot F_{oa}]^2 \cdot \sigma_{R_o}{}^2 + [\alpha_{oa} R_o - \alpha_{ao} R_a]^2 \cdot \sigma_{F_{oa}}{}^2 \tag{9}$$

We now have to estimate 1σ errors for R_o and F_{oa}. The error of R_o is essentially due to the lack of data, about one cruise per ocean basin [*Bentaleb*, 1994; *Francois et al.*, 1993; *GEOSECS*, 1987; *Quay et al.*, 1992]. In our approach, we average the observations of R_o over 10° latitude bands [*Tans et al.*, 1993]. Therefore, we estimate σ_{R_o} to be the standard error of the mean of the observations, when grouped in 10° latitude bands (Table 3). The bands with fewer measurements of R_o yield larger values of σ_{R_o}. In Table 4 the error estimates have been converted to areas corresponding to the 2-D atmospheric model of *Tans et al.* [1989].

Table 3. Standard error of the mean of the observations of R_o (units ‰) grouped into 10° latitude bands (centered around the latitudes listed) for each ocean basin and for the world ocean (area units 10^6 km^2).

Latitude	Value	-60°	-50°	-40°	-30°	-20°	-10°	0°
Pacific	Area	8.0	11.0	13.1	13.9	15.3	17.5	18.9
	1σ	0.2	0.1	0.1	0.2	0.1	0.2	0.2
Indian	Area	4.5	8.4	10.2	11.2	8.9	9.9	8.3
	1σ	0.2	0.2	0.3	0.2	0.1	0.1	0.1
Atlantic	Area	4.6	6.1	7.2	7.5	6.7	6.0	6.4
	1σ	0.3	0.4	0.3	0.3	0.3	0.3	0.3
Global	Area	12.5	24.0	29.4	32.3	31.7	34.1	33.2
	1σ	0.2	0.2	0.2	0.2	0.2	0.2	0.2
Latitude		**10**	**20**	**30**	**40**	**50**	**60**	**70**
Pacific	Area	21.1	18.5	15.7	12.3	9.2	5.8	1.4
	1σ	0.2	—	0.1	0.1	0.1	0.1	0.1
Indian	Area	6.1	4.8	0.0	0.0	0.0	0.0	0.0
	1σ	0.1	0.1	—	—	—	—	—
Atlantic	Area	6.7	8.2	9.4	8.5	5.8	5.1	7.0
	1σ	0.2	0.2	0.2	0.2	0.2	0.2	0.2
Global	Area	33.6	30.0	23.9	21.7	17.7	11.6	6.5
	1σ	0.2	0.1	0.1	0.1	0.1	0.1	0.2

Table 4. Standard error of the mean of the observations of R_o for each model grid box (units ‰).

Box	Sigma	Box	Sigma	Box	Sigma	Box	Sigma
1	0.2	6	0.2	11	0.2	16	0.1
2	0.2	7	0.2	12	0.1	17	0.1
3	0.2	8	0.2	13	0.1	18	0.1
4	0.2	9	0.2	14	0.1	19	0.1
5	0.2	10	0.2	15	0.1	20	0.2

The error of F_{oa} is two-fold. First, the uncertainty of the scaling factor used to make F_{oa} consistent with bomb ^{14}C is 20% [*Broecker et al.*, 1985]. Note that a recent budget of bomb ^{14}C in the atmosphere would even suggest a 25% error [*Hesshaimer et al.,* 1994]. Secondly, the uncertainty of the satellite 10 m wind speeds used to determine F_{oa} is about 5% when considering zonal averages on the 2-D model grid [Etcheto, personal communication]. In reality, this latter uncertainty should be larger at high wind speed values. Accounting for uncertainties both in the ^{14}C inventory and the gas exchange coefficient formulation, we believe that a 30% error for the value of F_{oa} is reasonable.

$$\frac{\sigma_{F_{oa}}}{F_{oa}} = 0.30 \qquad (10)$$

Substituting equations (9) and (10) into (3), and multiplying by 2, in order to be conservative for this important error, Table 5 gives the error of the ocean and land partitioning associated with the uncertainty in the isotopic disequilibrium between the surface oceans and the atmosphere.

Table 5. Estimated errors of the land uptake in broad latitude bands due to the uncertainty of the ocean disequilibrium (units GTC yr^{-1}). The errors of the ocean uptake are very close to these values and correlate with the error on land such that the sum of ocean and land fluxes always equals the net flux of total CO_2.

Latitude band	90-30°S	30°S-Eq	Eq-30°N	30-90°N	Global
Partitioning Error	0.34	0.32	0.26	0.14	0.54

Parameter $^{13}S_{bdis}$ -- Land Biosphere Disequilibrium

The land biosphere disequilibrium (or soil carbon disequilibrium) is given by equation (11)

$$^{13}S_{bdis} = \alpha_{ph}\, S_{resp} \sum_i x_i \cdot (R_a\,(t - \tau_i\,) - R_a\,) \qquad (11)$$

where

S_{resp} Flux of CO_2 respired by soils
x_i Fraction of S_{resp} respired by pool number i of soil carbon (the four pools considered are detrital, microbial, slow and passive carbon).
τ_i Turnover time of soil carbon in pool i

Ciais et al. [1995] have shown, using the turnover calculations of *Schimel et al.* [1994], that the "slow" carbon pool is by far the largest contributor to the isotopic disequilibrium. Therefore, we will limit the error estimate to this pool of carbon in soils, neglecting passive carbon and fast carbon pools. The dependence of S_{bdis} on α_{ph} causes negligible error (α_{ph} is very close to 1). Again, the 1s error of S_{bdis} is obtained by propagating errors in equation (11).

$$\sigma^2_{S_{bdis}} = \left[\alpha_{ph} S_{resp}\, x_{slow} \cdot \left[R_a(t-\tau_{slow}) - R_a\right]\right]^2 \cdot \sigma^2_{S_{resp}}$$

$$+\left[\alpha_{ph} S_{resp} \cdot \left[R_a(t-\tau_{slow}) - R_a\right]\right]^2 \cdot \sigma^2_{x_i} \qquad (12)$$

$$+\left[\alpha_{ph} S_{resp}\, x_{slow} \frac{\partial R_a}{\partial_t}\bigg|_{t-\tau_{slow}}\right]^2 \cdot \sigma^2_{\tau_{slow}}$$

An explicit error estimate is not straightforward because we do not know the analytical expression of the atmospheric decrease $R_a(t-\tau_i)$. We simplify the problem by fitting a cubic polynomial to a time series of ice core ^{13}C data [*Leuenberger et al.*, 1992; Francey, personal communication] to obtain an approximation of the atmospheric decrease since the last century. From the range of uncertainty of the characteristics of the slow carbon pool given by *Schimel et al.* [1994], we estimate the following errors of τ_{slow} and x_{slow}

$$\frac{\sigma_{\tau_{slow}}}{\tau_{slow}} = 20\% \qquad (13)$$

$$\frac{\sigma_{x_{slow}}}{x_{slow}} = 15\% \qquad (14)$$

We consider that the 1σ error of the flux of CO_2 respired by soils is

$$\frac{\sigma_{S_{resp}}}{S_{resp}} = 20\% \qquad (15)$$

Substituting (13)-(15) into (12) yields the total error of S_{bdis} reported in Table 6.

Table 6. Relative error of the land biosphere-atmosphere disequilibrium flux defined as the ratio $\sigma S_{bdis}/^{13}S_{bdis}$.

Box	Sigma	Box	Sigma	Box	Sigma	Box	Sigma
1	0.36	6	0.19	11	0.29	16	0.26
2	0.35	7	0.20	12	0.33	17	0.26
3	0.26	8	0.20	13	0.23	18	0.19
4	0.40	9	0.18	14	0.25	19	0.18
5	0.24	10	0.19	15	0.26	20	0.16

The resulting error of the land and ocean partitioning given in Table 7 is obtained by replacing the estimates in Table 6 in equation (3).

Table 7. 1σ errors of the land uptake in broad latitude bands due to the biosphere disequilibrium(units GTC yr^{-1}). The errors of the ocean uptake are very close to these values and correlate with the error on land such that the sum of ocean and land always equals the net flux of total CO_2.

Latitude band	90-30°S	30°S-Eq	Eq-30°N	30-90°N	Global
Partitioning Error	0	0.04	0.04	0.08	0.1

Source Terms S and ^{13}S

1. Longitudinal structure of the observations

The longitudinal error is the uncertainty of the partitioning due to the longitudinal structure of the atmospheric $\delta^{13}C$ and CO_2 observations. *Ciais et al.* [1995] estimate this error in a bootstrap analysis, picking sets of sites randomly and using them as input to the 2-D inverse model. They used the standard deviation of the fluxes S_b and S_o inferred from different bootstrap runs as a proxy for the longitudinal error. We now estimate analytically the longitudinal error from the sources S and ^{13}S inferred in each bootstrap run. In addition, we include two more sites in the inverse calculation (Mawson, Antarctica and Macquarie Island, in the Southern Ocean, operated by CSIRO). We calculate the 1σ errors of the sources (σ_S and $\sigma_{^{13}S}$) as the standard deviation of the results of 20 bootstrap runs. For the covariance term in equation (3), we use an estimate of the covariance given by equation (16).

$$\mathrm{cov}\left(S,{}^{13}S\right)=\frac{\sum_{1}^{N_{boot}}\left(S_i-\overline{S}\right)\cdot\left({}^{13}S_i-\overline{{}^{13}S}\right)}{N_{boot}-1} \tag{16}$$

where

N_{boot}	Number of bootstrap runs (currently 20)
Overbars	The mean of S and ^{13}S from the N_{boot} bootstrap runs

We report in Table 8 the individual 1σ errors of S and ^{13}S, together with their correlation coefficient $\rho = cov(S,^{13}S)/(\sigma_S \sigma_{^{13}S})$. It is observed that the sources S and ^{13}S are very strongly correlated because the ^{13}C fraction of any source or sink of carbon is always close to 1.1% and because they are inferred by the same transport fields. Had we incorrectly assumed S and ^{13}S to be independent, the resulting error would have been much larger.

Table 8. 1σ errors of the sources S and ^{13}S as estimated by the bootstrap analysis. The bootstrap analyses the variability caused by using different observation sites, often at different longitudes, in the 2-D model of atmospheric transport.

Model box	1	2	3	4	5
Sigma S	0.053	0.14	0.18	0.11	0.27
Sigma $^{13}S \times 10^2$	0	0.13	0.15	0.12	0.24
Corr (S, ^{13}S)	1.00000000	0.99998990	0.99993970	0.99994944	0.99995384
Model box	6	7	8	9	10
Sigma S	0.22	0.14	0.24	0.35	0.26
Sigma $^{13}S \times 10^2$	0.21	0.16	0.29	0.35	0.26
Corr (S, ^{13}S)	0.99994623	0.99997790	0.99998271	0.99999085	0.99999536
Model box	11	12	13	14	15
Sigma S	0.19	0.27	0.27	0.3	0.15
Sigma $^{13}S \times 10^2$	0.20	0.28	0.28	0.29	0.24
Corr (S, ^{13}S)	0.99995813	0.99993794	0.99996140	0.99994417	0.99989728
Model box	16	17	18	19	20
Sigma S	0.4	0.38	0.18	0.17	0.2
Sigma $^{13}S \times 10^2$	0.37	0.37	0.21	0.18	0.2
Corr (S, ^{13}S)	0.99999032	0.99994998	0.99990901	0.99996454	0.99993876

Entering the numbers in Table 8 into equation (3) gives the error of the land and ocean partitioning as shown in Table 9.

Table 9. 1σ errors of the land uptake in broad latitude bands due to the longitudinal variation of the CO_2 and $\delta^{13}C$ observations (units GTC yr^{-1}). The errors of the ocean uptake are very close to these values and correlate with the error on land so as the sum of ocean and land always equals the net flux of total CO_2.

Latitude band	90-30°S	30°S-Eq	Eq-30°N	30-90°N	Global
Partitioning Error	0.26	0.39	0.57	0.56	0.93

2. Other sources of errors in the inferred fluxes

The bootstrap does not explore the uncertainty associated with applying the smoothing procedure to the time series of flask data at each site of the CMDL network. There is an uncertainty associated to such smoothing because, if we had more flasks during a certain period of the year, then the value of the smoothed time series would be slightly different from what we use. This is why we attribute larger weights in the latitudinal fit to sites with more data. A more rigorous approach would invoke a bootstrap selection of the individual flask values as we do for he sites used to constrain the latitudinal gradient. We tested the sensitivity of our model using different degrees of smoothing when constructing the smoothed time series at each site. This was done by changing the low-pass cut-off frequency, used in filtering the time series in the time domain, by approximately 50% about its standard value of 75 days for $\delta^{13}C$ and 40 days for CO_2. We observed no significant change in the corresponding annual mean partitioning by broad latitude bands.

Another source of error not accounted for in this study relates to how well air samples measured by the NOAA/CU and CSIRO programs represent the bottom layer of atmospheric transport models. In an inverse model where all the variability of observations is reflected in the inferred distribution of sources, local boundary layer biases in the observations may cause some errors. This source of errors is somewhat reduced by the in-situ sampling strategy per wind sector.

More importantly, there is an uncertainty associated with the advective transport fields used in the 2-D model [*Plumb and Mahlman*, 1987]. The model is calibrated by using CFCs and ^{85}Kr, which provide reasonable constraints on inter-hemispheric transport but not for the transport within one hemisphere, as for instance between high and mid-northern latitudes. Also, as is the case in several transport models, using either GCMs or observed winds, the transport at high southern latitudes is poorly constrained. A comparison of our 2-D analysis with high resolution 3-D modeling will bring useful information. A simple way to make such a comparison is to carry out a forward run in a 3-D model using the fluxes we obtain in the inverse 2-D model (equally distributed in longitude), then compare the concentrations simulated in the 3-D to the original observations. Also, we anticipate further calibration of models with tracers of the atmospheric circulation, for example SF_6 [*Maiss and Levin*, 1994].

Discussion

In Table 10, we compare the statistically-based errors estimated in this work with the previous estimate made by *Ciais et al.*, 1995 on the basis of sensitivity tests.

The first conclusion is that the errors estimated by both methods are in fair agreement. However, the analytical method yields, on average, smaller values. The largest source of errors is associated with the longitudinal variation of the data. We infer a smaller longitudinal error in the southern hemisphere than did *Ciais et al.* [1995], most likely because we use two additional CSIRO sites at high southern latitudes to constrain the inversion. The second conclusion is that errors in the global partitioning are smaller than those previously calculated by *Ciais et al.* [1995], mostly due to smaller error associated with the bootstrap. Global means are the sum of positive and negative contributions in latitude. In this work we treat sources in wide latitude bands as being independent statistical variables, but we consider that errors within each latitude band are not independent. In the real world, the isotopic discrimination and disequilibria can be treated as independent (the discrimination in one ecosystem does not depend on its neighbors). But errors pertaining to inferred sources at different latitudes (bootstrap estimates) cannot be considered as independent because the initial fit to the observations is not strictly local. This means that source patterns inferred in one band are correlated to the adjacent one. Only over wide latitude bands can we expect sources to have little correlation. On the other hand, fluxes averaged over the scale of large latitude bands generally exceed the errors, permitting the reliable use of $\delta^{13}C$ as a constraint on the land and ocean partitioning on such scales.

Table 10. Comparison of the error estimates of model parametersfrom this work (1) with the estimates made by *Ciais et al.* [1995] (2). Again, the units are GT. C yr^{-1}.

Model Parameter	Estimate Source	90-30°S	30°S-Eq	Eq-30°N	0-90°N	Global
Discrimination	1	0.07	0.06	0.09	0.19	0.22
	2	<0.1	<0.1	0.1	0.2	0.3
Ocean disequilibrium	1	0.34	0.32	0.26	0.14	0.54
	2	0.4	0.4	0.4	0.1	0.7
Land disequilibrium	1	0	0.04	0.04	0.08	0.1
	2	0.1	0.1	<0.1	0.2	0.3
Longitude variation	1	0.26	0.36	0.66	0.52	0.2
	2	0.4	0.7	0.5	0.4	1.0
Total error	1	0.43	0.49	0.72	0.58	0.62
	2	0.6	0.8	0.7	0.5	1.3

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