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**U.S. DEPARTMENT OF COMMERCE**
NATIONAL OCEANIC AND ATMOSPHERIC ADMINISTRATION
Environmental Research Laboratories

## Statistical Analysis of EML 1 Multiple Cumulus Experiments in 1970, 1971, and 1972

JOANNE SIMPSON
WILLIAM L. WOODLEY
GERALD F. COTTON
JANE C. EDEN

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# STATISTICAL ANALYSIS OF EML[1] MULTIPLE CUMULUS EXPERIMENTS IN 1970, 1971, and 1972

Joanne Simpson
William L. Woodley
Gerald F. Cotton*
Jane C. Eden

[1] Experimental Meteorology Laboratory

*Meteorology Statistics Group, Air Resources Laboratory



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## ABSTRACT

In 1970 and 1971 EML conducted a dynamic multiple cumulus seeding program in a 4000 n $mi^2$ target area in south Florida. Randomization was by days, with an objective screening criterion aimed to restrict the sample to undisturbed conditions. The seeding decision was not known to project scientists, so that identical flights were conducted on seeded and control days. "Floating" targets were defined as those echoes "seeded" and all those merging with them so long as they remained in the fixed target. Owing to operational limitations, only 7 seeded and 5 control cases were obtained in the two year's effort.

Rain volumes were calculated by calibrated 10-cm radar, checked by rain gages. Unfortunately, comparison indicated that the 1971 rainfalls were underestimated compared to 1970. Adjusted and unadjusted data are tested by classical statistics, for both "floating" and total target. The mean "floating" target seeded rainfall exceeds the unseeded by a factor of 4-5 at the 0.1 significance level, with or without data adjustment. For the total target, mean seeded rainfall either very slightly exceeded or fell short of the unseeded, depending on the radar adjustment. Multi-variate non-parametric tests considering ratios of floating to total target variables together with the number of mergers provide rough bounds to significance levels of the seeding effects.

In July-August 1972, a "radar control" program was conducted using a light aircraft. The purpose was to specify natural rain distributions in "floating" and total targets and to calibrate two 10-cm radars against 5 clusters of 8 raingages. Four GO days were obtained with the aircraft, which are combined with the 1970-1971 cases for a Bayesian analysis. The

1 Experimental Meteorology Laboratory, NOAA

framework of the analysis was constructed earlier using the data from single cloud experiments. The analysis is expedited by the good fit of the gamma probability distribution to the rainfall from single clouds and multiple clouds and the finding with the single clouds that seeding alters only the scale parameter but not the shape parameter of the rainfall distribution.

Bayes equation is applied to obtain a probability distribution for the seeding factor, defined as the factor by which seeding multiplies the average rainfall (on seed vs. control cases). Several types of diffuse and non-diffuse prior probabilities are employed with both "floating", and total target data; results are shown to be adequately insensitive to the choice of priors.

With the reservation that natural distributions are not yet adequately specified, preliminary results are encouraging. For the "floating" target, Bayesian and classical statistics concur in evaluating a seeding effect on rain volume of about a factor of three; the posterior probability distribution is concentrated in the positive values and some confidence may be placed in the result. Although it is still not conclusive, the floating target seeding factor may be resolved in one additional experimental season. For the total target, the 1970-1972 expected value of seeding effect is about 1.7, but the standard deviation is too large (about 0.5) and the natural distributions not firmly enough specified either to confirm a positive effect or to delineate its magnitude.

Since dynamic seeding factors of 1.5 - 1.7 in the total target could mean benefit-cost ratios exceeding 60, investigation is conducted to outline the necessary conditions to resolve this experiment. Probability theory together with present data suggest about 50 pairs of cases or 5 - 10 years experimentation with the current experiment design and target. Alternative designs and/or targets are examined and reasons presented for their rejection.

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## 1. INTRODUCTION

The EML series of single cumulus seeding experiments, successfully concluded, was but the first step to assess the potential of dynamic seeding for practical rain augmentation and as a tool to understand convective processes. Despite the large and highly significant precipitation increases produced by seeding isolated convective clouds, even those reaching cumulonimbus stature make only a small contribution to the subtropical water budget. Dynamic seeding must be effective either in inducing cumulonimbus mergers or in increasing precipitation from those already in existence if it is to be an economically valuable tool for rain augmentation. While an isolated Florida cumulonimbus may produce 100-2000 acre-ft of rain, a merged system may produce up to 50,000 acre-ft (cf. Woodley et al, 1971).

In progressing from single cloud seeding to seeding in a 4000 n mi$^2$ area to promote cumulus mergers, EML has undertaken a convection problem

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[1] Experimental Meteorology Laboratory, NOAA

which is much more difficult to model and evaluate. It has been clear from the outset of this phase of the work that just raingage data analyzed by classical statistics would not be adequate to reach meaningful results in a reasonable time. We measure rainfall by means of calibrated 10-cm radar (Woodley, 1970) checked by gages (Woodley and Herndon, 1970; Herndon, Woodley, Miller and Samet, 1973) and are attempting to supplement classical statistics with Bayesian statistics, numerical modelling and detailed measurements, if possible, on every cloud in the target area (Fig. 1).

The design and first-year results of this experiment have been described in detail elsewhere (Woodley and Williamson, 1970; Simpson and Woodley, 1971). It is randomized by days in the area shown in Fig. l; the crossover design was precluded for lack of two comparable land areas within radar range and free of blind cones. An important design feature is the MSF or "meteorological suitability factor" which is used to screen out rainy, disturbed days, which we have shown are unsuitable for dynamic seeding (Simpson, Woodley, Miller and Cotton, 1971). The MSF is given by

$$MSF = S - N_e \qquad (1)$$

where S is seedability in km [defined as vertical growth potential from dynamic seeding (see Simpson, Brier and Simpson, 1967)] and $N_e$ is the number of hours between 9 a.m. local time and noon that the target area contains radar echoes. Suitable days for the experiment were those that satisfied the criterion MSF $\geq$ 1.5 in 1971 and 1.0 in 1970.

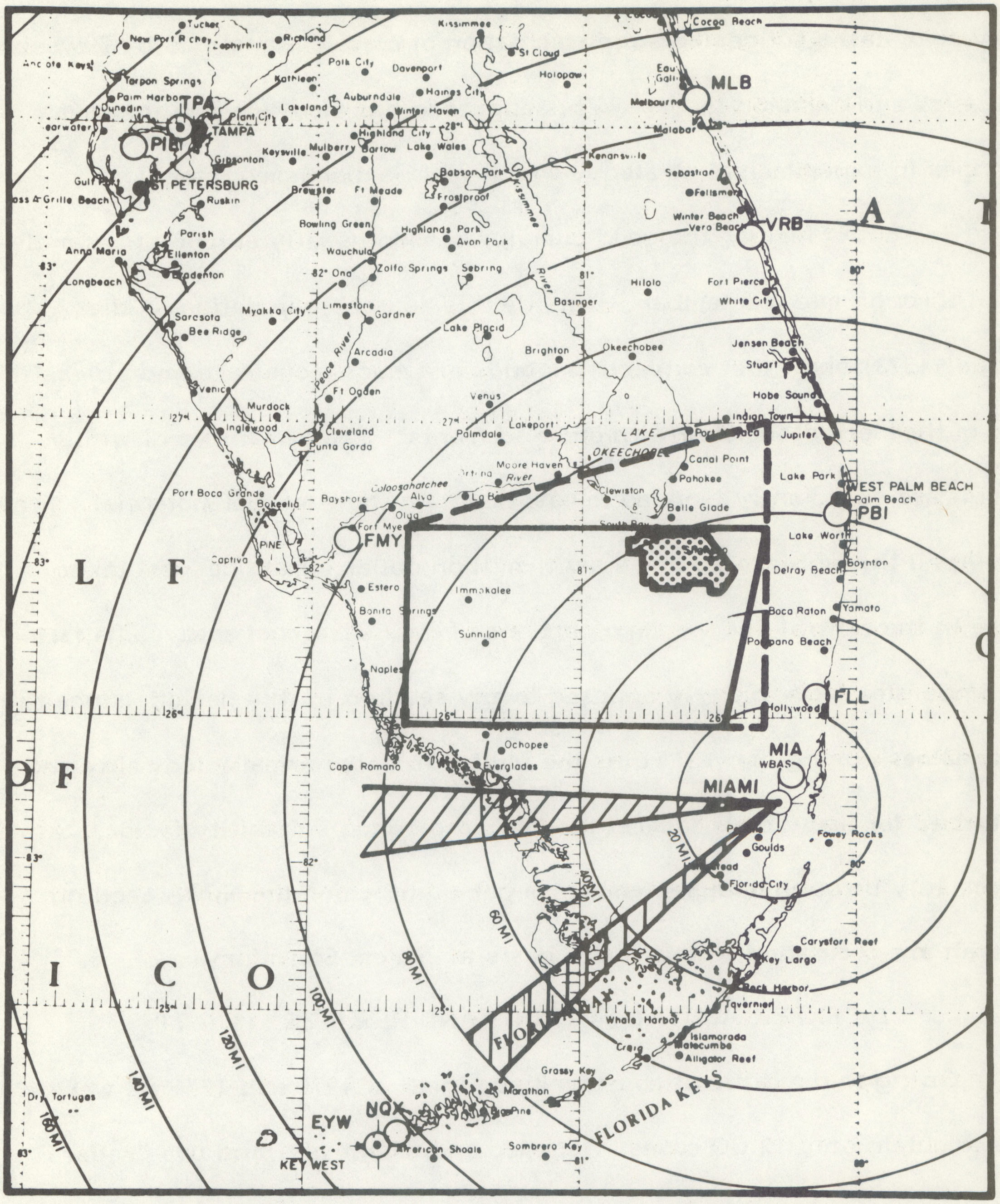

*Figure 1. EML area in south Florida for randomized multiple cumulus seeding experiment. Solid outline, 1970, expanded to dashed outline in 1971 and after. Dotted area shows location of dense raingage and surface measurement network. Cones are obstructed (blind) areas of University of Miami 10-cm radar.*

The difficulties we have been encountering in obtaining a substantial number of cases suggested supplementation of classical analysis by Bayesian statistics and decision theory. This approach has also resulted in some changes in experimental strategy, which will be discussed later.

In south Florida, the good cumulus season usually extends from mid-April through mid-September. Outside this season, population studies (Holle, 1973) show that convective clouds are much more rare and at least half of them are associated with old cold fronts. The remainder of winter cumuli are sufficiently isolated to cast doubt on their merger potential. From mid-April through early June a multi-million dollar tomato harvest takes place in the central and western portions of the EML target area. The farmers are understandably bitterly opposed to any seeding in this period, since rain on tomatoes during harvest ruins the fruit. June is normally too rainy and disturbed for many days to meet the "meteorological suitability factor", which leaves July through mid-September, a period in which the NOAA seeding aircraft are committed on a priority basis to Project Stormfury.

## 2. EXPERIMENTAL METHODS AND RESULTS, 1970-71

Owing to the difficulties described, in all of 1970 and 1971 we were able to obtain only 12 GO cases, 7 seeded and 5 controls, and two "radar control" cases. By a "GO" case, we mean one upon which a sealed envelope was opened to determine the "seed" or "no seed" instruction (known to the "randomizer" only), and a complete area experiment was executed. The "radar control" case in 1970 was obtained when the seeder aircraft was forced to abort due to a malfunction and target precipitation falling after a simulated seeding time was calculated. The radar control case in 1971 was intentional. On the day after the conclusion of the randomized operation, a special flight was made in a small twin-engine aircraft at 10,000 ft in conjunction with full operation of the University of Miami (UM) radars. A multiple cloud experiment was simulated from this altitude. The project scientist selected the clouds and the radar laboratory verified their top heights. If the clouds conformed to the selection criteria (described by Woodley and Williamson, 1970, loc. cit.) simulated seedings were made, without actual cloud penetration. The project scientist was very conscious of possible bias due to the non-randomized nature of this procedure. However, inspection of the rainfall on July 16, 1971 in Tables 2 and 3 will suggest that any bias may have worked in the reverse sense of that feared.

In carrying out the actual flights (identically on seeded and control days since the decision was unknown to project scientists) multiple seedings of individual clouds in close proximity were attempted to promote mergers using the preferred organization patterns evident in the unmodified convection as a guide. On days with adequately long cumulus lifetimes these attempts were apparently quite successful, especially when the B-57 jet worked in tandem with the DC-6 as a seeder. In most cases large, unmodified precipitating clouds in the target were avoided entirely because we currently believe that dynamic seeding will have little effect on such clouds. Their seeding would only have increased the uncontrolled background of the "floating targets". The concept of "floating targets" is important in many of our analyses. By "floating targets" we mean all clouds that are "seeded" and all those merging with them, so long as they remain in the target area.

An example of the way EML conducts its multiple cloud experiment is shown in Fig. 2 for a five-hour period on 8 July 1970. The smaller target area was in use at this time. The echo contours, from the minimum to the maximum, correspond to rain rates of .01, .10, .50 and 3.40 inches per hour respectively. The positions of seeding are numbered in the panels; the seedings were made in the half-hour period before the time that appears below the panel. Stippled echoes identify the floating target. Maximum radar measured cloud tops are included when available.

The experiment began in the west-central portion of the target at 1756 GMT on 8 July 1970 and continued there and in the north-center of the target until 1900 GMT. Several mergers of seeded clouds took place during this period. The large convective mass in the extreme western part of the target was avoided by the project scientist during the course of the experiment. Seeding it would have made little sense in view of its massive stature. Its seeding would have resulted in an undesirable increase in the natural precipitation noise of the floating target. It is systems like this one that produce the very high uncontrolled background of the total target.

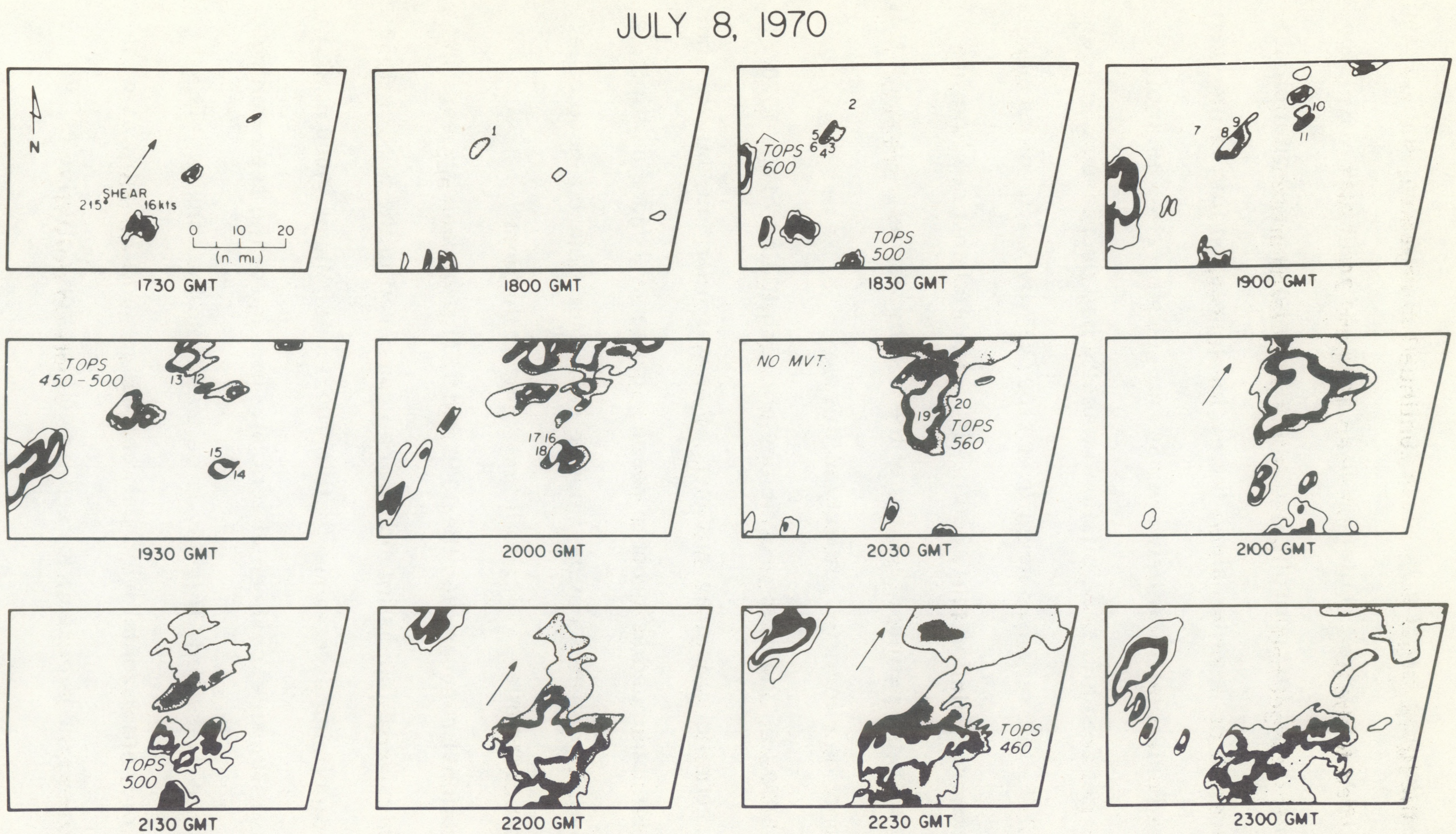


Figure 2. Example of multiple dynamic seeding strategy for 8 July 1970. Echo contours from maximum to minimum correspond to rain rates of 0.01, 0.10, 0.50, and 3.40 inches per hour. Seeding positions are numbered in the panels. Floating target echoes are stippled. Radar-measured cloud tops included when available. Vertical shear of the horizontal wind for the layer between 850 - 200 mb is represented by an arrow.

While the floating target echo mass continued to organize, the seeding emphasis shifted to the target center between 1930 and 2030 GMT. By 2100 GMT the floating target had consolidated into a very large precipitation system that appeared to be oriented along the vector corresponding to the vertical shear of the horizontal wind between 850 and 200 mb (215°, 16 knots). The floating target area and precipitation intensity had decreased considerably by 2130 GMT, but regained new life at 2200 GMT by virtue of its merger with unmodified precipitation to its south. It is doubtful that seeding had anything to do with the reintensification of the floating target, although this possibility cannot be eliminated entirely at this time.

July 8, 1970 was the most prolific day of precipitation in the 1970-71 sample. The seeded clouds grew and merged as was intended, but this behavior cannot be construed as proof of seeding causality. More multiple cloud experimentation of this nature must be completed before a good case for causality can be made.

An operational summary for the 1970 and 1971 programs appears in Table I. All days were fair as intended when compared to the $S-N_e$ ($>1.50$) and echo coverage fair day criteria (within 100 n mi of Miami $<4000$ n mi$^2$) except 30 June 1970, an experimental control. In some of the seed-control comparisons to be described, this day is deleted from the sample. The vertical shear of the horizontal wind between 850 mb and 200 mb was weak or moderate during the program, exceeding 40 knots on only five of the fourteen days tabulated. The initial "seed" times for the sample are variable, but seeding durations (time of first seeding minus time of last seeding) are more comparable. The seed times on the radar control days were simulated. The amount of silver iodide expended per day of experimentation, real or simulated, was variable, ranging from 4 to 15 kgm. The latter figure is comparable to the amount of silver iodide used during a day of seeding in Project Whitetop (Braham, 1966).

The rainfall analyses for the multiple cloud experiments were completed in the manner as for the single clouds (Woodley, 1970). The radar representation of the rainfall was checked by comparison with raingages. Accuracy degraded in 1971 relative to that reported for 1968 (Woodley and Herndon, 1970) and 1970 (Herndon et al, 1971). An upward adjustment of the radar-derived rain volumes by 1.75 appeared to make the 1971 sample comparable to that obtained in earlier years, although it is not clear as yet that a single adjustment is valid for all days (see Herndon, Woodley, Miller and Samet, 1973, for details). The results are presented in Table 2 without adjustment and in Table 3 with the 1.75 upward adjustment of the 1971 rain values. All calculations are for the six hours after real or simulated seeding.

In Tables 2 and 3 the 1970 results are different from those presented earlier (cf. Simpson and Woodley, 1971) because they are now calculated for the larger target and for six hours after commencement of real or simulated seeding, instead of five hours as done earlier for the 1970 cases.

Table 1. *Multiple Cloud Seeding Experiment 1970 and 1971 Operational Summary*

| DATE 1970 | ACTION | S-$N_e$ † | ECHO COVERAGES ≠ (n. mi²) | PRED. NAT. GROWTH ø (km) | SHEAR 850-200mb (deg) | 1200 GMT (kts) | SEEDING TIMES (GMT) FIRST | LAST | No. SEEDING PASSES | No. FLARES | AMOUNT AgI (kgm) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| JUNE 29 | S | 3.00 | 1000 | 9.9 | 275 | 9 | 1743 | 2058 | 17 | 136 | 6.8 |
| JUNE 30 | NS | 1.10** | 4145 | 9.5 | 055 | 22 | 1714 | 1950 | 18 | (121) | (6.05) |
| JULY 2 | S | 5.00 | 710 | 7.7 | 085 | 40 | 2036 | 2202 | 13 | 96 | 4.8 |
| JULY 7 | NS | 3.20 | 550 | 7.4 | 235 | 22 | 1835 | 2113 | 16 | (126) | (6.3) |
| JULY 8 | S | 3.90 | 1275 | 9.1 | 215 | 16 | 1756 | 2005 | 20 | (151) | 7.55 |
| JULY 17 | RC | 2.80 | 775 | 10.2 | 145 | 17 | 1800 | —— | 0 | 0 | 0 |
| JULY 18 | S | 2.70 | 42 | 8.6 | 240 | 54 | 1851 | 2136 | 20 | 135 | 6.75 |
| DATE 1971 | | | | | | | | | | | |
| JUNE 16 | S | 2.40* | 61 | 8.8 | 234 | 13 | 1955 | 2330 | 22 | 307 | 15.35 |
| JULY 1 | NS | 1.70** | 711 | 7.9 | 094 | 8 | 1747 | 2146 | 20 | (208) | (10.40) |
| JULY 12 | NS | 2.50 | 365 | 10.3 | 070 | 20 | 1747 | 2200 | 15 | (182) | (9.1) |
| JULY 13 | S | 3.40 | 588 | 8.7 | 059 | 45 | 1738 | 2209 | 31 | 303 | 15.15 |
| JULY 14 | S | 2.60 | 365 | 8.6 | 058 | 53 | 1841 | 2211 | 26 | 256 | 12.80 |
| JULY 15 | NS | 2.40 | 974 | 9.1 | 092 | 41 | 1955 | 2235 | 28 | (305) | (15.25) |
| JULY 16 | RC | 3.40 | 162 | 4.5 | 316 | 29 | 1850 | 2213 | 27 | (285) | (14.25) |

* 1200GMT SOUNDING UNRELIABE – 1800GMT USED

** WSR-57 RADAR DOWN 1 HOUR – ECHO IN TARGET ASSUMED PRESENT

( ) AMOUNT THAT WOULD HAVE BEEN USED HAD IT BEEN A SEED DAY

† $1000m \le R \le 2500m$ — CLD BASE = 915m

ø R = 1000m — BASE = 915m

≠ WITHIN 100 n. mi. MIAMI

Table 2. *Multiple Cloud Seeding Experiment 1970 and 1971 Rainfall Calculations (Unadjusted)*

| DATE 1970 | ACTION | FLOATING TARGET RAINFALL acre-ft X $10^4$ | TOTAL TARGET RAINFALL acre-ft X $10^4$ | $W_{FT}/W_{TT}$ | DEPTH FT (in) | $D_{FT}/D_{TT}$* | No. MERGERS FT | SUM OF RANKS | SUCCESS |
|---|---|---|---|---|---|---|---|---|---|
| JUNE 29 | S | 0.16 (11) | 3.23 (7) | 0.05 (12) | 1.29 (5) | 0.81 (9) | 3 (12) | (56) | 11 |
| JUNE 30 | NS | 3.08 (3) | 6.95 (3) | 0.44 (7) | 2.72 (2) | 0.96 (4) | 9 (4) | (23) | 3 |
| JULY 2 | S | 1.11 (4) | 1.94 (10) | 0.57 (3) | 1.28 (6) | 0.96 (4) | 6 (7) | (34) | 5 |
| JULY 7 | NS | 0.78 (7) | 7.53 (4) | 0.10 (11) | 1.04 (8) | 0.47 (13) | 6 (7) | (50) | 8 |
| JULY 8 | S | 8.96 (1) | 11.90 (1) | 0.75 (2) | 2.49 (3) | 0.91 (6) | 18 (1) | (14) | 1 |
| JULY 17 | RC | —— (—) | 4.67 (—) | —— (—) | —— (—) | —— (—) | — (—) | (—) | — |
| JULY 18 | S | 4.56 (2) | 8.42 (2) | 0.54 (4) | 4.39 (1) | 0.91 (6) | 12 (2) | (17) | 2 |
| DATE 1971 | | | | | | | | | |
| JUNE 16 | S | 0.13 (13) | 0.14 (13) | 0.90 (1) | 0.95 (10) | 1.00 (3) | 1 (13) | (53) | 10 |
| JULY 1 | NS | 0.15 (12) | 0.90 (12) | 0.16 (9) | 0.58 (13) | 0.67 (12) | 6 (7) | (65) | 13 |
| JULY 12 | NS | 0.20 (10) | 4.31 (5) | 0.05 (12) | 0.75 (11) | 0.77 (10) | 4 (11) | (59) | 12 |
| JULY 13 | S | 0.90 (5) | 1.71 (9) | 0.53 (5) | 1.42 (4) | 1.04 (2) | 11 (3) | (28) | 4 |
| JULY 14 | S | 0.95 (6) | 2.80 (8) | 0.34 (8) | 0.70 (12) | 0.76 (11) | 5 (10) | (55) | 9 |
| JULY 15 | NS | 0.55 (9) | 1.10 (11) | 0.51 (6) | 0.96 (9) | 1.33 (1) | 8 (5) | (41) | 6 |
| JULY 16 | RC | 0.57 (8) | 4.00 (6) | 0.14 (10) | 1.18 (7) | 0.89 (8) | 8 (5) | (44) | 7 |

* $D_{FT}/D_{TT}$ FOR PERIOD WITH RAIN IN FLOATING TARGET
( ) RANK

Table 3. *Multiple Cloud Seeding Experiment 1970 and 1971 Rainfall Calculations (Adjusted)*

| DATE 1970 | ACTION | FLOATING TARGET RAINFALL acre-ft X $10^4$ | | TOTAL TARGET RAINFALL acre-ft X $10^4$ | | $W_{FT}/W_{TT}$ | | DEPTH FT (in) | | $D_{FT}/D_{TT}$* | | No. MERGERS FT | | SUM OF RANKS | SUCCESS |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| JUNE 29 | S | 0.16 | (13) | 3.23 | (8) | 0.05 | (12) | 1.29 | (9) | 0.81 | (9) | 3 | (12) | (63) | 12 |
| JUNE 30 | NS | 3.08 | (3) | 6.95 | (6) | 0.44 | (7) | 2.72 | (2) | 0.96 | (4) | 9 | (4) | (26) | 3 |
| JULY 2 | S | 1.11 | (6) | 1.94 | (10) | 0.57 | (3) | 1.28 | (10) | 0.96 | (4) | 6 | (8) | (41) | 7 |
| JULY 7 | NS | 0.78 | (9) | 7.53 | (4) | 0.10 | (11) | 1.04 | (12) | 0.47 | (13) | 6 | (8) | (57) | 11 |
| JULY 8 | S | 8.96 | (1) | 11.90 | (1) | 0.75 | (2) | 2.49 | (3) | 0.91 | (6) | 18 | (1) | (14) | 1 |
| JULY 17 | RC | — | (—) | 4.67 | (—) | — | (—) | — | (—) | — | (—) | — | (—) | (—) | — |
| JULY 18 | S | 4.56 | (2) | 8.42 | (2) | 0.54 | (4) | 4.39 | (1) | 0.91 | (6) | 12 | (2) | (17) | 2 |
| DATE 1971 | | | | | | | | | | | | | | | |
| JUNE 16 | S | 0.23 | (12) | 0.25 | (13) | 0.92 | (1) | 1.66 | (7) | 1.00 | (3) | 1 | (13) | (49) | 9 |
| JULY 1 | NS | 0.26 | (11) | 1.58 | (12) | 0.16 | (9) | 1.02 | (13) | 0.67 | (12) | 6 | (8) | (65) | 13 |
| JULY 12 | NS | 0.35 | (10) | 7.54 | (3) | 0.05 | (12) | 1.31 | (8) | 0.77 | (10) | 4 | (11) | (54) | 10 |
| JULY 13 | S | 1.58 | (5) | 2.99 | (9) | 0.53 | (5) | 2.49 | (3) | 1.04 | (2) | 11 | (3) | (27) | 4 |
| JULY 14 | S | 1.67 | (4) | 4.90 | (7) | 0.34 | (8) | 1.23 | (11) | 0.76 | (11) | 7 | (7) | (48) | 8 |
| JULY 15 | NS | 0.96 | (8) | 1.88 | (11) | 0.50 | (6) | 1.68 | (6) | 1.33 | (1) | 8 | (5) | (37) | 5 |
| JULY 16 | RC | 1.00 | (7) | 7.00 | (5) | 0.14 | (10) | 2.07 | (5) | 0.89 | (8) | 8 | (5) | (40) | 6 |

* $D_{FT}/D_{TT}$ *FOR PERIOD WITH RAIN IN FLOATING TARGET*

( ) *RANK*

Total target rainfall is the most straightforward and easily understood measure of the efficacy of dynamic seeding in augmenting rainfall over an area. However, at present it is very misleading. On several days large non-experimental precipitating clouds either formed or moved into the target area. They were in no way the result of the seeding operations, but they were included in the rainfall analyses by virtue of their presence in the target.

A more sensitive analysis of the effect of seeding is the "floating target" rainfall analysis that is limited to the experimental clouds and to those clouds with which they merge. The analysis area floats or moves with the clouds, but in all cases, the floating target is bounded by the fixed target. The floating target is one of the components of the total target and the two should be positively correlated.

If seeding can affect convective developments on the mesoscale, it should eventually be detectable in the EML series of experiments in Florida because of the seeding technique and Lagrangian nature of the analysis that is employed here. Seeding supercooled convective clouds near their tops is an ideal method of assuring that the seeding agent reaches the right spot at the right time for maximum efficiency. In addition, the high altitude provides the project scientists with the over view that is necessary for the interpretation of convective patterning and for mapping the seeding strategy. The seeding of individual clouds also permits their identification on the radar scope that is necessary for the definition of the floating target, a Lagrangian concept that minimizes the noise introduced by nonexperimental precipitation. These design advantages, coupled with the fact that dynamic seeding has a pronounced effect on isolated convective clouds in Florida, should permit detection of a seeding effect on the mesoscale in this region.

Examination of Tables 2 and 3 reveals that there is great variability in floating and total target rainfalls. The maximum floating target rainfall is 50 times more than the minimum value. The maximum of nearly $9 \times 10^4$ acre ft is 45 times greater than the maximum rain volume that has been computed for large isolated thunderstorms in Florida (Woodley, Norwood and Sancho, 1971). The ratios of floating to total target rainfalls (column headed $W_{FT}/W_{TT}$ in the Tables) are also quite variable, ranging from a minimum of 0.05 to 0.92.

Stratifications were formulated in an attempt to delineate the effect of seeding. Besides floating and total target rainfalls and their ratios, the depth of rainfall in the floating target (columns labelled DEPTH FT in Tables 2 and 3) and the ratio of floating target to total target depth (columns labelled $D_{FT}/D_{TT}$) were examined for each day of the experiment. These depths were computed in ten-minute intervals by dividing floating and total target rain volumes by the mean areal coverage of echoes in the respective targets and by then summing for the length of time that rain fell in the floating target. Although this is only a crude estimate of the actual water depth, it is clear that some of the rainfall amounts are not small. On three seeded days adjusted amounts exceed two inches and on one of them the depth exceeded four inches.

The rationale for the depth stratification is that seeded clouds by virtue of the larger size might produce heavier rain per unit area and have a greater floating target depth than unseeded clouds. If this hypothesis has validity, the ratio of floating to total target rain depth should be closer to one on seed days than on control days. However, the hypothesized depth effect might be rather small because our single cloud results suggest that the main effect of seeding is to increase cloud size and duration and not rain intensity (Simpson and Woodley, 1971). For the restricted multiple cloud sample, the mean ratios of floating to total target rain depth on seed and no seed days are 0.91 and 0.83, respectively. The difference is quite small, but in the postulated sense. No conclusions are warranted at this time.

All of the stratification parameters in Tables 2 and 3 were ranked, the largest number in each category receiving the number one rank. These stratification ranks were then summed and success (Tables 2 and 3) was defined as inversely proportional to the magnitude of the summed ranks. By this criterion, four of the first five most successful days in the unadjusted sample were seed days and three of the five most successful in the adjusted sample were seed days. Otherwise, there is a sprinkling of seed and no seed days. There is the suggestion, but no proof, that seed days were more successful than the controls.

The experimental design was predicated on the production of mergers. The desirability of this design goal was checked by correlating the number of cloud mergers in the floating target with the precipitation that fell there. The correlation is an impressive 0.90, suggesting that merger productions should indeed be the prime design goal. However, at present the promotion of merger by seeding is more of an art than a science. This experiment cannot be brought to a definitive conclusion until this situation is reversed. Formulation of a sound physical hypothesis as to how to proceed to organize a field of clouds with a given set of initial conditions is a main goal of EML scientists.

Comparison of mean floating and total target rainfall (without 30 June and 17 July 1970) is shown in Table 4, which presents a dilemma of interpretation.

*Table 4. Mean Target (acre-feet) of Seven Seeded Cases and Five Controls, 1970-1971.*

| | Floating Target Rainfall X $10^4$ | | | Total Target Rainfall X $10^4$ | | |
|---|---|---|---|---|---|---|
| | S | NS | Diff. | S | NS | Diff. |
| Unadj. | 2.40 | 0.45 | 1.95 | 4.31 | 3.57 | 0.74 |
| Adj. | 2.61 | 0.67 | 1.94 | 4.80 | 5.11 | -0.31 |

* 1 acre-foot = $1.23 \times 10^9$ gm = $1.23 \times 10^3$ $m^3$

The floating target figures, with or without adjustment, show that the mean seeded rainfall is 4 or 5 times the unseeded, respectively, and hence suggests that seeding was increasing the rainfall, while the total target figures appear to suggest little if any effect of seeding. There are at least four possible interpretations of the results in Table 4, namely:

1) The sample is too small; the results are due to chance and no conclusions are warranted.

2) Seeding increased precipitation in the floating target, but nonexperimental precipitation masked it in the total target (uncontrolled background).

3) Seeding increased rain in the floating target, but decreased it in the total target.

4) The floating target is biased; seeding had no effect.

Interpretation (4) is a possibility with a floating target design if the experimenters are able to guess the seed decision. However, although the first two authors were on the flights and made systematic attempts to guess the decision at the end of each GO day, their performance was worse than chance (Table 5).

*Table 5. Guessing the Treatment Decision - 1971 Multiple Cloud Seeding Experiment*

| Date | Simpson | Woodley | Decision |
|---|---|---|---|
| 16 June | * | Seed | Seed |
| 1 July | Seed | Seed | No Seed |
| 12 July | Seed | Seed | No Seed |
| 13 July | Seed | Seed | Seed |
| 14 July | No Seed | No Seed | Seed |
| 15 July | Seed | Seed | No Seed |
| Score | 3 correct / 11 guesses | = 27% | |

* Not on aircraft

## 3. CLASSICAL STATISTICAL ANALYSIS OF 1970-1971 DATA

Firstly, we find a large and significant correlation between floating and total target rainfalls; the correlation coefficients between the two variables are 0.84 and 0.74, for unadjusted and adjusted data respectively.

A primary goal is to establish whether the apparently large floating target seeding effect is real, i.e., whether the seeded and control populations differ significantly. We first used the Wilcoxon-Mann-Whitney test, which involves ranking the rainfalls.

For the unadjusted data, the significance level is slightly better than 10%, while for the adjusted data, it is just slightly worse than 10%. This means that there is about one chance in ten that the two data samples are from the same population. Most meteorologists accept a 5% significance level as a satisfactory demonstration of causality; with a 10% significance level, one is encouraged but feels the need for further work. It is noteworthy that if the next twelve GO cases duplicate the rainfall results of the existing twelve, the 5% significance level would be exceeded with or without radar adjustment. An experimental period covering a normal south Florida July and August should be adequate to obtain about 20 additional cases, particularly if a back-up seeder aircraft were available. It is interesting to note that if the two very dry seed days (June 29, 1970 and June 16, 1971) were not in the sample, the remaining existing floating target data would show a seed-control difference significant to better than 0.5%.

In the statistical analysis of the single cloud experiments (Simpson, Woodley, Miller and Cotton, 1971) we often used fourth roots of the rainfall data, which we call transformed data. The transformed data have a more nearly normal distribution and the standard deviations of the two populations are more nearly equal, making some statistical tests applicable which could not be used with the raw data. In Section 5, we will show that the same advantages apply to the transformation of the area data. Hence we may use the student t test with the transformed data. Applying the one-sided t test to the 1970-1971 transformed adjusted data, we find for the floating target, that the seeded and control populations differ at a significance level just slightly worse than 10%, which does not permit rejection of the null hypothesis.

Next, we attempt a more sophisticated statistical approach to these data, namely multivariate nonparametric tests.

The EML multiple cloud seeding experiments provide, in addition to precipitation amounts, a large number of measured variables, which hopefully will provide data both for necessary scientific knowledge for further understanding of the physical processes involved as well as for statistical measures of success in precipitation enhancement. Certain of these variables can be examined in various ways to yield realistic measures of the magnitude of this success. Such analyses fall into the general category of multivariate statistical analysis. Here we specifically are interested in the two sample (seeded vs. unseeded) nonparametric ranking procedures. These procedures are extensions of the univariate ranking tests (Siegel, 1956) such as the Wilcoxon-Mann-Whitney (W-M-W) test which is used earlier in this report.

As with the W-M-W test these are used as tests of shift in the means (location shift) of the variables of the seeded sample relative to those of the nonseeded sample.

The usual multivariate test of location shift is the Hotelling $T^2$ test which requires that the variables have, at least approximately, a multivariate normal distribution. Since the variables considered jointly in this section are floating target rainfall, total target rainfall, $W_{FT}/W_{TT}$, depth, $D_{FT}/D_{TT}$ and the number of mergers, this assumption is unrealistic. Thus possible alternatives for establishing possible changes in the mean at this stage of analysis include certain nonparametric tests which are illustrated in the following. The advantage of nonparametric procedures is that we need not make assumptions about the joint distribution of the variables. The problem of obtaining realistic multivariate distributions and developing useful test statistics are increased substantially over that for the univariate case because one must be concerned with the joint distributions of all of the variables. There are, however, two important disadvantages with the tests used below. The first is that one is forced into the hypothesis testing situation which is less satisfactory than being able to obtain quantitative estimates of the mean shifts along with confidence limits about these estimates. Hypothesis tests provide only a probability statement about accepting or rejecting the null hypothesis, i.e., the statement that there is no difference between the seeded and non-seeded population. The second disadvantage of two of the tests given below is that the power of the test to distinguish between "populations" when in truth there is a difference, is unknown. This could lead to the unpleasant situation of being unable to reach a decision about real differences without a very large sample of seeded and unseeded cases. For the other two tests, this power is known only asymptotically.

The four rank tests described below that have been adapted for testing location shift of the seeded samples from the unseeded samples use jointly the six variables listed above. Simply, if F $(\underline{x})$ is the distribution function of these variables for the unseeded cases, where $\underline{x}$ is a vector of dimension six corresponding to the six variables, then F $(\underline{x} + \underline{\delta})$ is the distribution function of the seeded cases, where $\underline{\delta}$ is a vector of constants. Thus, the test of the null hypothesis, i.e. the hypothesis that there is no difference between the two samples, is that $\underline{\delta} = \underline{0}$, the zero vector. The only assumption necessary is that the distribution function F exists and that the treatments, seed and nonseed, have been randomly applied to the experimental subjects, in this case the collection of clouds in a specified target on a given experimental day. These tests are in the class of randomized permutation tests, i.e., conditional type tests that depend only on the given outcome of the experiment. Briefly, if one has $n_1$ seeded and $n_2$ unseeded cases, the test statistic is computed for each of the possible $\frac{(n_1 + n_2)!}{n_1!\, n_2!}$ permutations of the observed data between the two samples providing the distribution of the possible numerical values of the statistic. This can be done since under the null hypothesis all of the observations can be considered as coming from the same population. It is then possible to compare the statistic computed from the actual outcome with the distribution of all possible outcomes. If the value of this statistic lies in a region of small probability of this distribution, we can reject the hypothesis that the seeded and unseeded populations are identical.

<u>Test Statistics</u>

To facilitate the description of the tests let,

$Z_{ij}$ have the rank value of the jth variable for the ith observation,

$\sum_s$ represent summation over the set of subscripts of the seeded observation,

$\sum_u$ represent summation over the set of subscripts of the unseeded observations,

$n_1$ the number of seeded observations,

$n_2$ the number of unseeded observations, and $I_{ij}$ = 1 if observations i and j are both from the seeded set or both from the unseeded set

$I_{ij}$ = 0 if observations i and j are from different sets.

The first test is an adaptation of the rank sum test presented by Chung and Fraser (1958). As noted previously, the power of this test is unknown and, in addition, the statistic does not utilize the sample covariance function. The latter is important in this instance for adjusting for the correlations between the variables. The statistic which is simply the sum of the ranks in the seeded sample is easily computed from

$$R_1 = \sum_{S} \sum_{j=1}^{6} (Z_{ij} - \frac{n_1 + n_2 + 1}{Z})$$

The variables in Table 2 and Table 3 have been ranked so that the lowest ranks numerically are assigned to the maximum desired response of each of the variables. Thus, large negative values of $R_1$ measure optimum outcomes for this experiment. In the context of the discussion of hypothesis testing, this test has $\underline{\delta} < \underline{o}$ as the alternative to the null hypothesis.

The second test is a normal scores test (Chung and Fraser, 1958, loc. sit.) which has the same disadvantages as the rank sum test. The only difference between these two tests is that the ranks of the rank sum test are replaced by the expected values of the order statistic for a sample of size $n_1 + n_2$ from a normal distribution. The test can be written,

$$R_2 = \sum_{s} \sum_{j=1}^{6} \xi(U_{ij})$$

Where $\xi(U_{ij})$ is the expected value of the ith order statistic for the jth variable. The hypothesis test is the same as that described for the rank sum test. It should be noted that the normal scores test have been shown to perform extremely well in univariate cases studied by Monte Carlo simulations (Neave and Granger, 1968).

The third test is the rank sum test studied by Puri and Sen (1966) which has certain desirable properties. First, the test statistic uses the sample covariance function and second, the statistic can be approximated asymptotically by the chi-square variate when $n_1$ and $n_2$ are sufficiently large. In addition, Monte Carlo simulations have shown that the bivariate version compares very well with the $T^2$ statistic under normal alternatives (Battacharyva, Johnson and Neave, 1971). The test statistic is:

$$R_3 = n_1 \underline{T}_1^1 V^{-1} \underline{T}_1 + n_2 \underline{T}_2^1 V^{-1} \underline{T_2}$$

where

$$\underline{T}_R^1 = (T_R^1, T_n^2, \ldots, T_R^6) \quad , k = 1,2$$

$$T_1^j = \frac{1}{n_1(n_1+n_2+1)} \sum_{s} \left(Z_{ij} - \frac{n_1+n_2+1}{2}\right)$$

$$T_2^j = \frac{1}{n_2(n_1+n_2+1)} \sum_{u} \left(Z_{ij} - \frac{n_1+n_2+1}{2}\right) \qquad j = 1,2,\ldots,6$$

and V is the variance covariance matrix formed in the usual way except that the ranks replace the actual observations.

The alternate hypothesis consists of the set $\underline{\delta} \neq \underline{o}$ so that this test is equivalent to a two-sided test whereas the first two tests are one-sided equivalents. This test can equally well be performed as a normal scores type by replacing the individual ranks with the corresponding expected value of the order statistic from the normal distribution. The results were essentially the same as those for the rank sums and are omitted from further discussion as the comparison is adequately demonstrated by test #1 and test #2.

The principal drawback of this test in the present context is its two-sided nature, in other words, the test measures the magnitude of mean shifts at the expense of direction information. More explicitly, the desired direction of mean shift for each variable is toward low ranks in successful seeded cases and toward high ranks in unseeded cases yielding large values of $R_3$. However, similar large values of $R_3$ can be obtained by high ranks for one or more variables in the seeded cases and corresponding low ranks in the unseeded cases. Thus, given that the rank means of each variable have all shifted in the desired direction as has occurred in the present experiment to date, the test will provide a conservative estimate of significance level.

The last test presented here has properties similar to test #3 except that there is not explicit account taken of the inter-correlations between the variables. These properties and other variations of the test are discussed by Mantel and Valand (Mantel and Valand, 1970). The statistic used is the following:

$$R_4 = \sum_{i<k} I_{ik} \sum_{j=1}^{6} | Z_{ij} - Z_{kj} |$$

which is in the form of a U statistic having an asymptotic normal distribution. Small values of $R_4$ indicate that the seeded and unseeded observations tend to cluster into two separate groups, hence the test has the two-sided characteristic.

## Results

The outcomes of the four tests described above are presented in Table 6. These are not intended to be definitive at this time but rather indicative of the current status of the experiment and suggestive of required changes in the statistics to improve one or several for subsequent performance. This analysis should be considered as supportive and complementary to the on-going parametric analysis currently in the Bayesian framework presented in Sections 5 and 6 of this report. As can be seen in the table, these tests were applied to the adjusted and unadjusted data both for all 13 cases ($n_1$ = 7 and $n_2$ = 6) and with 11 cases ($n_1$ = 7 and $n_2$ = 4) where the two no seed days, i. e., day number 3, a wet day, and day number 13, a radar

*Table 6. Probability Levels for Significance for the Four Nonparametric Tests*

| | TEST #1 Chung-Fraser Rank Sum Test | TEST #2 Chung-Fraser Normal Scores Test | TEST #3 Puri-Sen Rank Sum Test | TEST #4 Mantel-Valand Cluster Test |
|---|---|---|---|---|
| **UNADJUSTED DATA** | | | | |
| ALL DATA (7 Seed / 6 No Seed) | 0.14 | 0.15 | 0.26 | 0.25 |
| SELECTED DATA* (7 Seed / 4 No Seed) | 0.06 | 0.08 | 0.27 | 0.21 |
| **ADJUSTED DATA** | | | | |
| ALL DATA (7 Seed / 6 No Seed) | 0.16 | 0.17 | 0.68 | 0.32 |
| SELECTED DATA* (7 Seed / 4 No Seed) | 0.08 | 0.08 | 0.72 | 0.26 |

* Day Number Three, a wet day, and Day Number Thirteen a radar control, were eliminated.

control, were removed from consideration.

To illustrate how the probability levels given in the table are obtained, consider the application of test #1 to the 11 cases of the unadjusted data. For the seven seeded and four unseeded cases, $R_1 = -33.5$. The distribution of the statistic computed for all possible combinations of these eleven cases (note that $\frac{(n_1+n_2)!}{n_1!n_2!} = \frac{11!}{7!\,4!} = 330$) is given in Table 7. These are 21 values as low as or lower than the observed value of $R_1$. Thus, the probability of obtaining a value this small with this set of data is 21/330 = 0.064. That is, one would reject the null hypothesis that seeding had no effect at significance level 0.064.

While none of the tests are entirely satisfactory for different reasons in the present experimental context, it is reasonable to conclude that the areal seeding effect has yet to be clearly established. The probability levels for tests #1 and #2 are very likely too low while those for the other tests are conservative hence providing rough bounds for the true levels.

Table 7. Distribution of the Sample Statistic for Unadjusted Data. All possible combinations of seven seed and four no seed cases.

| | CLASS INTERVALS OF VALUES OF STATISTIC | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | < -50 | -50 to -45 | -45 to -40 | -40 to -35 | -35 to -30 | -30 to -25 | -25 to -20 | -20 to -15 | -15 to -10 | -10 to -5 | -5 to 0 | 0 to 5 | 5 to 10 | 10 to 15 | 15 to 20 | 20 to 25 | 25 to 30 | 30 to 35 | 35 to 40 | 40 to 45 | 45 to 50 | > 50 | TOTAL |
| NUMBER OF TIMES VALUE OF STATISTIC FELL IN INTERVAL | 2 | 5 | 4 | 9 | 11** | 15 | 18 | 22 | 25 | 28 | 31 | 28 | 24 | 27 | 19 | 19 | 15 | 7 | 7 | 7 | 4 | 3 | 330 |

**The observed statistic was the lowest value in this class.

## 4. THE "RADAR CONTROL" PROGRAM OF 1972

In July and August 1972, an extensive rain measurement program was conducted in the EML target, with the purpose of comparing two 10-cm radars with each other, with results from 5 clusters of 8 recording raingages each, and with satellite evaluations of cloud and rainfall parameters. During days which met the meteorological suitability factor, a light twin-engined aircraft was flown at 10,000 ft to execute a simulated seeding program. Seedable clouds were selected in as nearly identical manner to a real experiment as possible and simulated "flare releases" were counted. The entire program and its results are discussed in detail by Herndon, Woodley, Miller and Samet (1973).

It would be far cheaper and less time-consuming if natural distributions could be documented without the necessity of flying, but we presently see no way that cumulus clusters suitable for dynamic seeding can be selected by radar alone or by any other currently available means of remote sensing. After the radar has been digitized[2], the total target distribution can be calculated for fair days meeting the meteorological suitability factor, with a radar-guided selection of "first seeding" time. If the sample is adequately large, inclusion of a few extra days that flying might have eliminated should not seriously affect the results.

The four GO days flown and analyzed for 1972 are summarized in Table 8.

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[2] Hopefully by the summer of 1973

*Table 8. "Radar Control" Multiple Cloud Experiment 1972 (Light Aircraft)*

| Date 1972 | S-Ne | Echo Cov. ($n\ mi^2$) | Pred. Nat. Growth (KM) | "Seeding" Times (GMT) First | "Seeding" Times (GMT) Last | No. "Seeding" Passes | No. "Flares" | Rain Volume Floating ($10^4$ acre-ft) | Rain Volume Total ($10^4$ acre-ft) |
|---|---|---|---|---|---|---|---|---|---|
| July 21 | 2.00 | 147 | 7.1 | 1815 | 2150 | 25 | 200 | .0958 | .2196 |
| Aug. 4 | 1.35 | 0 | 10.9 | 1813 | 2106 | 24 | 191 | .22891 | .25718 |
| Aug. 9 | 1.70 | 84 | 5.4 | 1820 | 2200 | 20 | 189 | 2.14696 | 3.03738 |
| Aug. 18 | 1.30 | 126 | 10.1 | 1730 | 1943 | 27 | 217 | 1.14945 | 2.71023 |

Effects on conclusions of radar errors and variations in calibration, etc. are discussed in the extensive comparison by Herndon, Woodley, Miller and Samet (loc.cit.) 1973). Here the values in the tables will be taken as correct, and only the sampling problem will be examined. Two points resulting from the comparisons must especially be kept in mind throughout the following: Firstly, randomization by days makes these experimental results more vulnerable to radar calibration variations than were the single cloud data, in which several seeded and control cases normally occurred on the same day. Secondly, if we are treating seeding factors on rainfall greater than about two, the seeded-control differences are probably outside the common measurement errors, but when seeding factors are less than two, the adequate measurement of convective rainfall by any currently available method becomes a serious problem.

With these reservations, we have combined the four control cases in Table 8 with the control cases in Table 3. Results for 1970, 1971 and 1972 combined are presented in Table 9.

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3 The study by Herndon et al. (1973, loc. cit.) shows, for example, that to determine daily convective rainfall in Florida by raingages to within a factor of two 90% of the time, gages are required to be spaced one every 10 square miles or closer.

*Table 9. EML Area Experiment - 1970, 1971, and 1972 Combined.*

| Random Cases<br>7 Seed; 4 Control | Floating Target<br>($x10^4$ acre-ft) | Non-Random Cases<br>5 Control<br><br>Total Target<br>($x10^4$ acre-ft) |
|---|---|---|
| A. Control | | |
| Sample Mean | .7746 | 3.528 |
| Sample Std. Deviation | .6434 | 3.026 |
| Sample Coeff. of Variation | .8307 | 0.858 |
| B. Seed | | |
| Sample Mean | 2.61 | 4.80 |
| Sample Std. Deviation | 3.16 | 4.04 |
| Sample Coeff. of Variation | 1.21 | 0.84 |

With the floating target, the average seed-control ratio is above three. With the Wilcoxon-Mann-Whitney test, the populations differ at a significance level of 10%. Repeating the same data sample once more would raise the significance level to better than 5%.

With the one-tailed student t-test applied to the transformed 1970-71-72 data, the floating target seeded and control population means also differ at the 10% significance level. We must recall, however, the three main assumptions upon which use of the t-test is based, namely normal populations, equal standard deviations and random sampling. In Section 5, we will show that the first two assumptions are justified with the transformed data.

However, the entire sample was not selected on a strictly randomized basis. Therefore, we cannot yet firmly conclude the floating target seeding factor although we can be hopeful that one more experimental period may lead to this conclusion.

In the case of the total target, the average seed-control ratio is 1.36 and the difference between the populations is not significant using either the Wilcoxon-Mann-Whitney test with the raw data or the t test with transformed data.

## 5. EXPERIMENT DESIGN AND BACKGROUND FOR THE BAYESIAN APPROACH

After the execution of the first EML area experiment in 1970, the senior author attended a course in Decision Analysis at the Thayer School of Engineering, Dartmouth College, in order to learn to adapt the techniques of Bayesian analysis to the modification experiments. The early steps in constructing this framework were reported in a Technical Memorandum by Simpson and Pezier (1971), hereinafter referred to as I. In the course of this work it was found that the transformed (fourth root) single cloud rain volumes were well fitted by a gamma distribution. In the EML randomized single cloud seeding series, it was found (Simpson, 1972) that seeding altered only the scale parameter of the rainfall distribution and not the shape parameter. In I, (prior to the 1971 area experiment) these properties were carried over by assumption to the area experiment. Physical conclusions from the work at that stage, however, were precluded owing to the existence of only one undisturbed control day in 1970. Bayes equation was applied with the seeded data to obtain a probability distribution for the scale parameter of the seeded gamma function. Only transformed (fourth root) rainfall data were used.

Subsequently it was found that the raw single cloud rainfall data were also well fitted by a gamma distribution. The Bayesian framework was evolved considerably further with the single cloud data in a Technical Memorandum by Simpson, Eden, Olsen and Pezier (1973), hereinafter referred to as II. In this work, both raw and transformed data were used and Bayes equation was applied not only to the gamma scale parameter but to the seeding factor itself, namely

$$F \equiv \frac{\langle R \rangle \text{ seed}}{\langle R \rangle \text{ control}} \tag{1}$$

where $\langle R \rangle$ seed is the expected value of the seeded rainfall distribution and $\langle R \rangle$ control is the expected value of the control rainfall distribution.

Also

$$\hat{F} = \frac{\overline{R} \text{ seed}}{\overline{R} \text{ control}} \tag{2}$$

where $\hat{F}$ is often used as an estimator for F, just as the sample averages, $\overline{R}_{seed}$ and $\overline{R}_{control}$ are often used as estimators for the expected value of rainfall distributions.

Monte Carlo methods, together with probability theory, were applied in II to specify the number of cases necessary to resolve seeding factors of various sizes. With the single cloud distributions, seeding factors of about 3 are adequately resolved with 25 - 50 cases, a result found earlier with classical statistics (Simpson, Woodley, Miller and Cotton, 1971).

The Bayesian approach to these problems offers both attractions and difficulties. One attraction is that use of Bayes equation permits us to obtain a probability distribution for the seeding factor F and hence an upper and lower bound on its magnitude for any specified range of integrated probability. This type of result contains more information than does the mere rejection of the null hypothesis (that the means of the two populations are not different) at a specific significance level, which is all that can be done with many of the classical non-parametric tests that we have used.

Another attraction, accompanied by a pitfall, is that with the Bayesian approach the experiments can be designed to obtain the natural distributions more rapidly and cheaply than with a fully randomized seeding program. This involves use of a light aircraft or could possibly be done even without flying, after the radar is digitized. The pitfall is that the Bayesian results do not yet explicitly state the degree of uncertainty with which we have obtained the natural distribution. The probability statement is confined to the seeded distribution, given that the natural distribution is, in fact, as assumed.

Consequently, caution is necessary and we have adopted a compromise position. Namely, we continue the fully randomized experiment on roughly one-to-one randomization for as many cases as possible and <u>in addition</u>, obtain "radar control" cases. By the autumn of 1974, it is reasonable to hope for 15 - 20 pairs of random GO cases <u>and</u> 30 "radar control" cases. Thus we should have a total of 50 control (radar plus random) cases to determine the natural distribution parameters. It should then be possible to resolve fairly well by statistical tests whether there has been any bias in radar versus random controls, i.e. whether the cases could have been selected at random from the same population or not. Whether or not 15 - 20 seeded cases on top of this will be adequate to resolve the seeding factor F depends then upon the value of F itself, with a better prognosis the larger that F is. The problems of sample size and the cases required to resolve various postulated values of F are considered further in Section 7.

The gamma probability density function may be written

$$P(R) = \frac{\beta^{\alpha}}{\Gamma(\alpha)} R^{\alpha-1} e^{-\beta R} \tag{3}$$

where P(R) in this paper is the probability density of a rainfall amount (usually in acre-ft) in either the floating or total target. The scale of the distribution is determined by the parameter $\beta$ and the shape by the parameter $\alpha$. $\Gamma$ is the gamma function (cf. Pearson et al., 1957). The first two moments of the gamma function are well known to be (see, for example, Tribus and Pezier, 1970, or Kendall and Stuart, 1963)

$$\mu_1 = \langle R \rangle = \alpha/\beta \tag{4}$$

and

$$\mu_2 = \sigma^2 = \alpha/\beta^2 \tag{5}$$

where $< R >$ is the expected value and $\sigma^2$ is the variance. Therefore, the coefficient of variation, V, is

$$V = \frac{\sigma}{<R>} = \frac{1}{\sqrt{\alpha}} \tag{6}$$

In I and II we presented and used a computer program (DAMAX) based on the principle of maximum entropy (Tribus, 1969, p. 197) to find the best fit distribution (out of seven selected distributions) for any set of data. The program also calculates the best-fit parameters for each distribution[4] and a value of chi-squared.

With this program, we found for the single clouds that the shape parameter $\alpha \simeq 0.6$ for the raw data and $\alpha \simeq 7$ for the transformed, with negligible difference between seeded and control populations. In I, we attempted to treat the four then extant area seeded cases by assuming that the natural transformed rainfall distribution obeyed a gamma function with $\alpha \simeq 7$ and $< R >$ obtained from the one extant control case that occurred on an undisturbed day (July 7, 1970). Our skepticism regarding these assumptions precluded conclusions and/or formal publication at that time.

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4 Identical to those obtained by the method of maximum likelihood.

With only 9 control (4 random and 5 radar) cases now, we are still unable to specify the natural distribution adequately. However, we have a fighting chance to estimate $\alpha$, the shape parameter, approximately. This estimate is enabled by combining all the seed and control cases currently available and applying DAMAX. Monte Carlo experiments made in connection with II showed that when observations were drawn from two different gamma populations with the same $\alpha$ (i.e. differing in scale parameter by the amount of the single seeded versus control clouds) and DAMAX was applied to the combined data set, $\alpha$ was recovered to a reasonably accurate degree (usually better than 10%).

Table 10 shows the results of applying DAMAX to all available area observations for 1970, 1971 and 1972 combined. Table 11 explains the distributions. In the case of the floating target, June 30, 1970 (disturbed) has been added to the 7 seeded cases, 4 random controls and 5 radar controls, making 17 cases. In the case of the total target, June 30 and July 17, 1970 (radar control with no floating target) have been added, making a total of 18 cases. Tables 15 in II showed that with n = 20 cases, for the raw data ($\alpha \sim 0.6$) $\alpha$ should nearly always be recoverable to better than a factor of 2, with a standard deviation of about 0.27 of its value. However, Table 16 in II showed that the situation is much worse for the larger $\alpha$'s associated with transformed data; hence results in Part B of Table 10 are viewed with greater reservations.

*Table 10. Application of DAMAX to Combined Seed and Control Data 1970, 1971, and 1972.*

A. Raw Data

I. Floating Target (17 cases)

| Dist. | -Log (A) | B | C | PROB | $\chi^2$ (4 D.F.) |
|---|---|---|---|---|---|
| 1 | 0.51039 | -0.60026 | 0 | 0.108 | 4.17 |
| 2 | 1.12422 | -1.11926 | 0.33867 | 0.372 | 3.26 |
| 3 | 0.50886 | -0.08800 | 0.65919 | 0.144 | 3.76 |
| 4 | 0.67043 | -0.14310 | 0.51436 | 0.124 | 3.76 |
| 5 | 1.37254 | -0.41221 | 0.03969 | 0.026 | 4.30 |
| 6 | 0.93858 | -1.87808 | 0.37760 | 0.163 | 8.32 |
| 7 | 1.51559 | -1.62671 | 0.02478 | 0.063 | 6.09 |

II. Total Target (18 cases)

| Dist. | -Log (A) | B | C | PROB | $\chi^2$ (4 D.F.) |
|---|---|---|---|---|---|
| 1 | 1.91909 | 0.00764 | 0.01804 | 0.322 | 2.43 |
| 2 | 1.43184 | -0.32877 | 0.34463 | 0.014 | 0.34 |
| 3 | 1.55533 | 0.12200 | 0.18816 | 0.175 | 2.43 |
| 4 | 1.44043 | 0.18124 | 0.27612 | 0.150 | 2.43 |
| 5 | 1.85996 | -0.08414 | 0.01600 | 0.339 | 2.43 |
| 6 | 0.49758 | -1.67837 | 0.73415 | 0 | 17.62 |
| 7 | 1.25260 | -1.47238 | 0.07963 | 0 | 19.82 |

*Table 10. Application of DAMAX to Combined Seed and Control Data 1970, 1971 and 1972—Continued.*

B. Transformed Data

I. Floating Target (17 cases)

| Dist. | -Log (A) | B | C | PROB. | $\chi^2$ (4 D.F.) |
|---|---|---|---|---|---|
| 1 | 5.05997 | 10.63167 | 5.30883 | 0.082 | 3.76 |
| 2 | - 0.26206 | - 1.47707 | 5.41872 | 0.194 | 3.26 |
| 3 | - 0.87743 | 2.64800 | 0.65919 | 0.075 | 3.76 |
| 4 | -11.30244 | 10.04891 | 11.02778 | 0.177 | 3.76 |
| 5 | - 2.92834 | 4.83674 | 2.65930 | 0.139 | 3.76 |
| 6 | -10.36777 | -12.08690 | 10.13473 | 0.182 | 3.76 |
| 7 | - 2.44145 | - 6.88573 | 2.25224 | 0.150 | 3.76 |

II. Total Target (18 cases)

| Dist. | -Log (A) | B | C | PROB. | $\chi^2$ (4 D.F.) |
|---|---|---|---|---|---|
| 1 | 7.30513 | 11.21867 | 4.22177 | 0.256 | 2.43 |
| 2 | 0.04555 | 1.68496 | 5.51413 | 0.035 | 0.34 |
| 3 | 0.16858 | 3.48600 | 0.18833 | 0.454 | 2.43 |
| 4 | - 9.22856 | 11.42633 | 9.35190 | 0.081 | 0.34 |
| 5 | - 1.69763 | 6.14573 | 1.89652 | 0.157 | 2.43 |
| 6 | -12.41106 | -11.20141 | 12.38114 | 0.013 | 6.25 |
| 7 | - 3.37070 | - 5.98486 | 3.27514 | 0.004 | 6.25 |

Table 11 shows the distributions used in DAMAX and the equation associated with each, thereby explaining what the A, B and C in Table 10 refer to.

*Table 11. Probability Density Functions Compared in DAMAX.*

| Distribution | Equation for P(R) |
|---|---|
| 1. Normal family | $A \, EXP \, (BR - CR^2)$ |
| 2. Log Normal | $A \, R^B \, EXP\left[-C\,(LOGR)^2\right]$ |
| 3. Weibull | $A \, R^B \, EXP \, (-CR^{\beta+1})$ |
| 4. Gamma | $A \, R^B \, EXP \, (-CR)$ |
| 5. Rayleigh | $A \, R^B \, EXP \, (-CR^2)$ |
| 6. Inv. Gamma | $A \, R^B \, EXP \, (-C/R)$ |
| 7. Inv. Rayleigh | $A \, R^B \, EXP \, (-C/R^2)$ |

As indicated by Table 17 in II, the application of DAMAX to a randomly selected sample of 20 observations from a gamma function frequently results in the relative probability of the gamma distribution ranking 3 or 4 out of the seven distributions compared, as is the case in Table 10. In Table 10, the values for $\chi^2$ for the gamma distribution are satisfactory since they do not indicate rejection of the null-hypothesis for that distribution. Fig. 3 shows histograms of the data compared with the best-fit gamma distribution and some of the other distributions that fit the data as well or better than the gamma distribution.

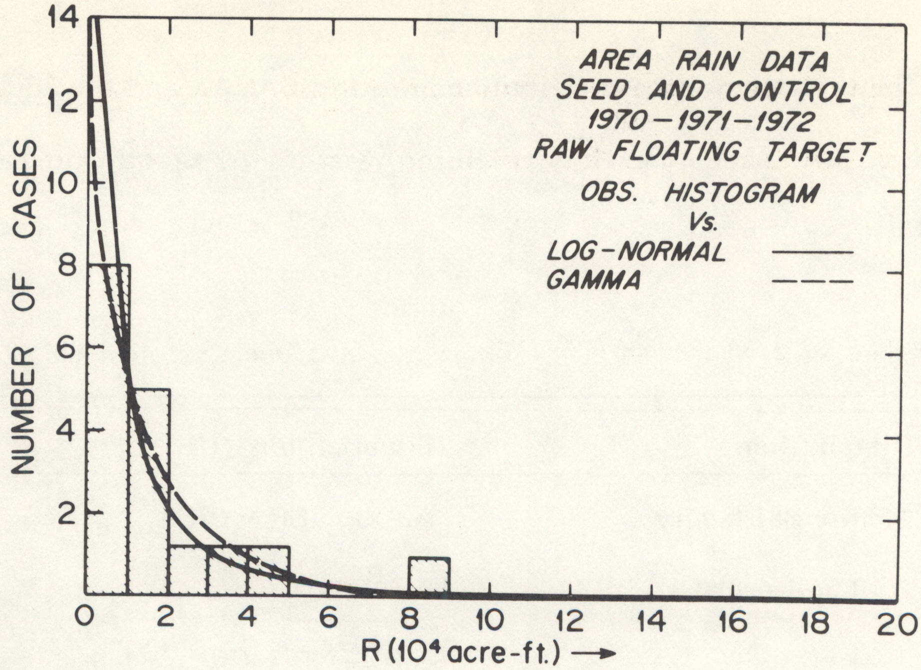


*Figure 3. Histograms of 1970-1972 area data compared with best fit and gamma distribution. a. Raw floating target.*

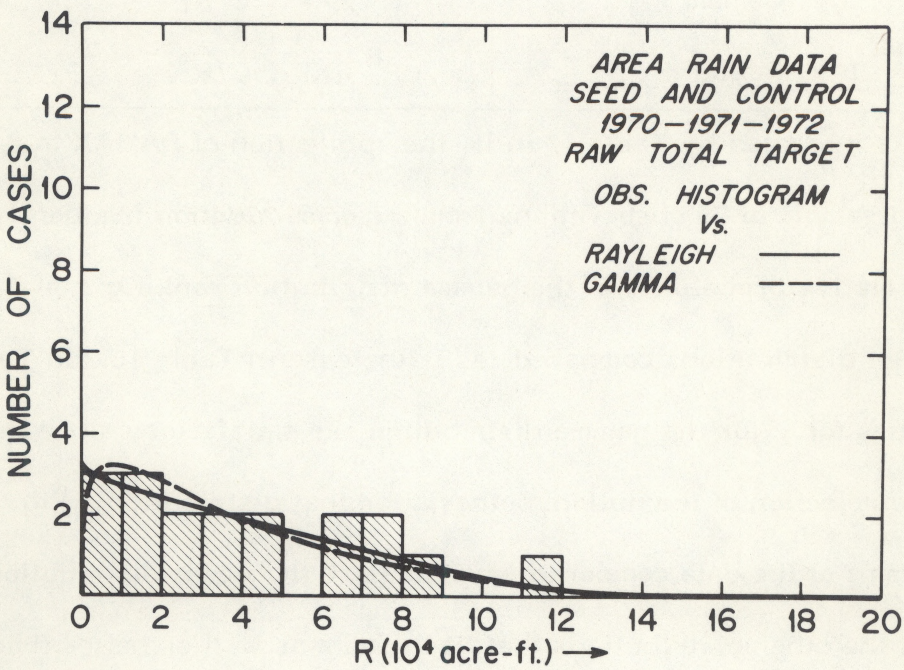


*Figure 3b. Raw total target.*

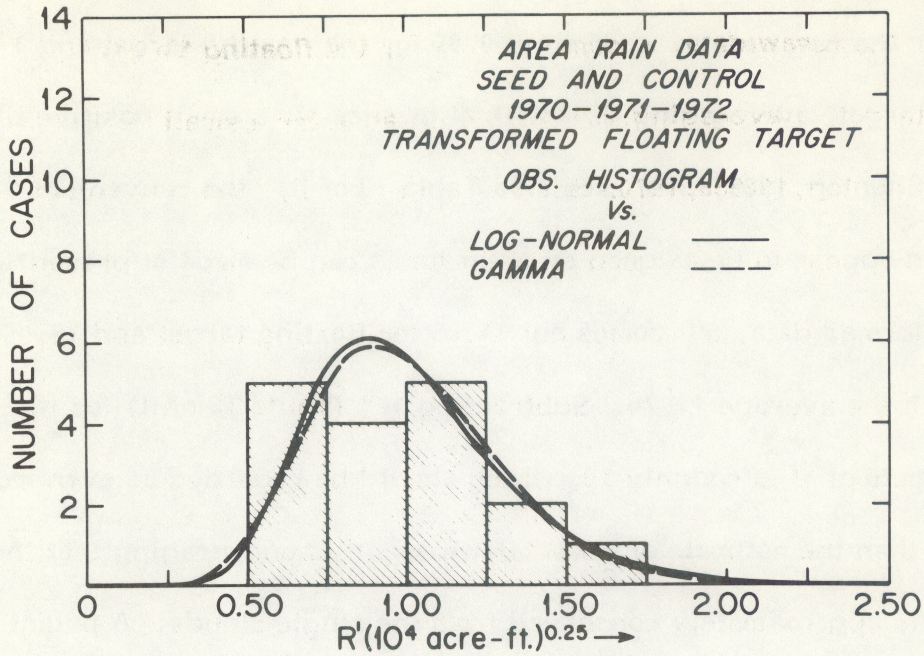


*Figure 3c. Transformed floating target.*

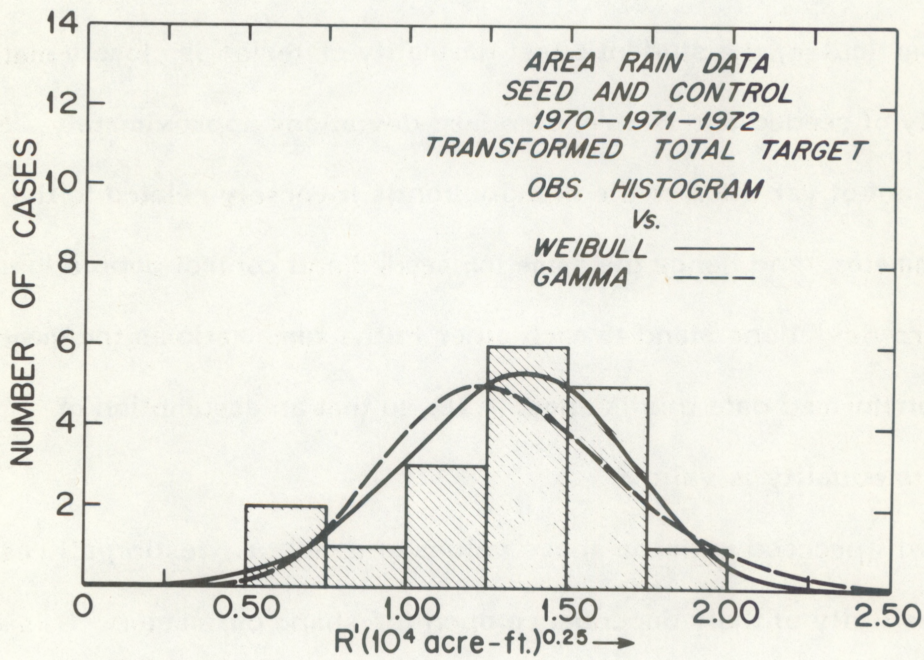


*Figure 3d. Transformed total target.*

For the raw data, $\alpha$ comes out 0.86 for the floating target and 1.18 for the total target, averaging 1.02. With allowance for a small positive bias (Bowman and Shenton, 1968, 1970; see also Table 18 of II), the convenient value of 1 would appear to be as good an estimate as can be made at present. For the transformed data, $\alpha'$ comes out 11.05 for floating target and 12.43 for total, with the average 11.74. Subtracting 17% (Table 18 of II) for bias, the best estimate of $\alpha'$ is roughly 10, which should be regarded as even more tentative than the estimate of $\alpha \sim 1$. However, it is encouraging that the ratio of $\alpha'$ to $\alpha$ is approximately conserved from the single clouds. A potentially important result, if $\alpha' \sim 10$ is confirmed by further measurements, is that statistical tests requiring normal distributions may be applicable to the transformed area data.

In particular, the student t-test normality criterion is closely met and the equality of seeded and control standard deviations approximately. Since the coefficient of variation of the distribution is inversely related to the shape parameter, and hence the same for seeded and control populations, the standard deviations stand to each other in the same ratio as the means. For the transformed data this is about 1.26, so that an assumption of approximate equality is valid.

We will proceed with the above values of $\alpha$ and $\alpha'$, testing all results for the sensitivity of their dependence upon the shape parameter. It is reasonable to believe fairly firmly that for the raw data, $\alpha$ must be between

0.6 and about 1. It stands to reason as we go up the precipitation space and time scale the near zero values will decrease in frequency and the coefficient of variation of the distribution function $(1/\sqrt{\alpha})$ should not increase. If it decreases more than the estimate above, then $\alpha > 1$. The latter would imply a population with natural variability somewhat less than our current model population, and hence a readier resolution of seeding factor, or a smaller requirement on sample size as illustrated in Fig II 4 of Appendix II of II. There is an indication, as expected, that the shape parameters for the total target distributions are slightly larger than those for the floating target distributions. As more data become available, it may become desirable to take this into account.

The assumptions in the Bayesian work to follow here in the area cases, both floating and total target, are:

1) Control and seeded rainfalls obey a gamma distribution with $\alpha = 1$ and $\alpha' = 10$.

2) The control populations are then completely specified by using the sample averages as estimators of $< R >$.

The consequences of and sensitivity to these assumptions will be tested in the course of the work.

## 6. BAYESIAN RESULTS FOR AREA SEEDING FACTOR

In II we found that Bayesian techniques can be applied to the area experiment in several alternative ways. We assigned prior probabilities and applied Bayes equation to obtain a posterior probability distribution after seeing the area data. Two main variables were considered, namely the scale parameter $\beta$ in equation (3) and the seeding factor F. We did the calculations for both floating and total target, using both raw and transformed data in each case. In all cases we strove to do at least a few examples with a diffuse prior probability on the variable whose distribution was being specified. In the course of this work it was shown that a diffuse prior probability on scale parameter $\beta$ was anything but a diffuse prior on seeding factor F. In fact, when prior $\beta$ is uniform, the corresponding prior F has most of its probability in low values and hence is a negatively-prejudiced prior. From the viewpoint of Bayesian logic, it seems more reasonable to assign a prior probability to the variable we are most interested in, and for which we may have some physical understanding. Hence we will work directly here with the seeding factor F.

For this purpose we eliminate $\beta$ from the rainfall probability density distribution and write instead of equation (3)

$$p(R) = \frac{\left(\frac{\alpha}{<R>_{NS}\ F}\right)^{\alpha}}{\Gamma(\alpha)} R^{\alpha-1}\ e^{-\left(\frac{\alpha}{<R>_{NS}\ F}\right)R} \tag{7}$$

where $<R>_{NS}$ is equated to $\bar{R}$control, the sample average of the control population. F is the seeding factor defined by equation (1) which has been used together with (4) to eliminate $\beta$.

As found in II, the classes of prior probability on F which are tractable include the inverse gamma distribution and the uniform distribution over some specified range. Here we treat only the latter, namely

$$p(F) = \frac{1}{b-a} \text{ for } a \leq F \leq b \tag{8}$$

Applying Bayes equation, we write

$$P(F|D) = P(F)\ \frac{P(D|F)}{P(D)} \tag{9}$$

which becomes

$$P(F|D) = C_1\ F^{-n\alpha} \exp\left[-\frac{\alpha \sum_{i=1}^{n} R_i}{<R>_{NS}\ F}\right] \text{ for } a \leq F \leq b \tag{10}$$

$= 0$ elsewhere

The normalizing constant $C_1$ is found in II, viz:

$$C_1 = \left( \frac{\alpha \sum_{i=1}^{n} R_i}{\langle R_{NS} \rangle} \right)^{n\alpha-1} \Bigg/ \left[ \gamma\left( n\alpha-1, \frac{\alpha \sum_{i=1}^{n} R_i}{\langle R_{NS} \rangle a} \right) - \gamma\left( n\alpha-1, \frac{\alpha \sum_{i=1}^{n} R_i}{\langle R_{NS} \rangle b} \right) \right] \quad (11)$$

To simplify, let $\mu = \frac{\alpha \sum_{i=1}^{n} R_i}{\langle R_{NS} \rangle}$ then

$$C_1 = \mu^{n\alpha-1} \Big/ [\gamma(n\alpha-1,\mu/a) - \gamma(n\alpha-1,\mu/b)]$$

$$\langle F \rangle = \mu \frac{\gamma(n\alpha-2,\mu/a) - \gamma(n\alpha-2,\mu/b)}{\gamma(n\alpha-1,\mu/a) - \gamma(n\alpha-1,\mu/b)} \quad (12)$$

$$\langle F^2 \rangle = \mu^2 \frac{\gamma(n\alpha-3,\mu/a) - \gamma(n\alpha-3,\mu/b)}{\gamma(n\alpha-1,\mu/a) - \gamma(n\alpha-1,\mu/b)}$$

Here $\gamma$ is the incomplete gamma function[5] as in II

---

[5] When using the normalized incomplete gamma function it is necessary to multiply $\langle F \rangle$ by $1/(n\alpha-2)$ and $\langle F^2 \rangle$ by $1/[(n\alpha-2)(n\alpha-3)]$.

Two ranges of uniform prior probability on seeding factor F were first considered, namely from F = 0.8 to 5 and from F = 0.5 to 10. Results for both floating and total targets are shown in Table 12 and Fig. 4.

*Table 12. Bayesian Results for Seeding Factor - 1970 - 72 Area Experiment*

| | Floating Target | Total Target |
|---|---|---|
| Prior 0.8 - 5: | | |
| After Data: | $\langle F\|D\rangle = 3.48$ | $\langle F\|D\rangle = 1.87$ |
| | $\sigma(F\|D\rangle = 0.83$ | $\sigma(F\|D) = 0.77$ |
| Prior 0.5 - 10: | | |
| After Data: | $\langle F\|D\rangle = 4.44$ | $\langle F\|D\rangle = 1.90$ |
| | $\sigma(F\|D) = 1.72$ | $\sigma(F\|D) = 0.92$ |

In the case of the floating target, results are encouraging. Even if two standard deviations are subtracted from the expected value, F would be one or greater, indicating only a small probability of a negative effect.

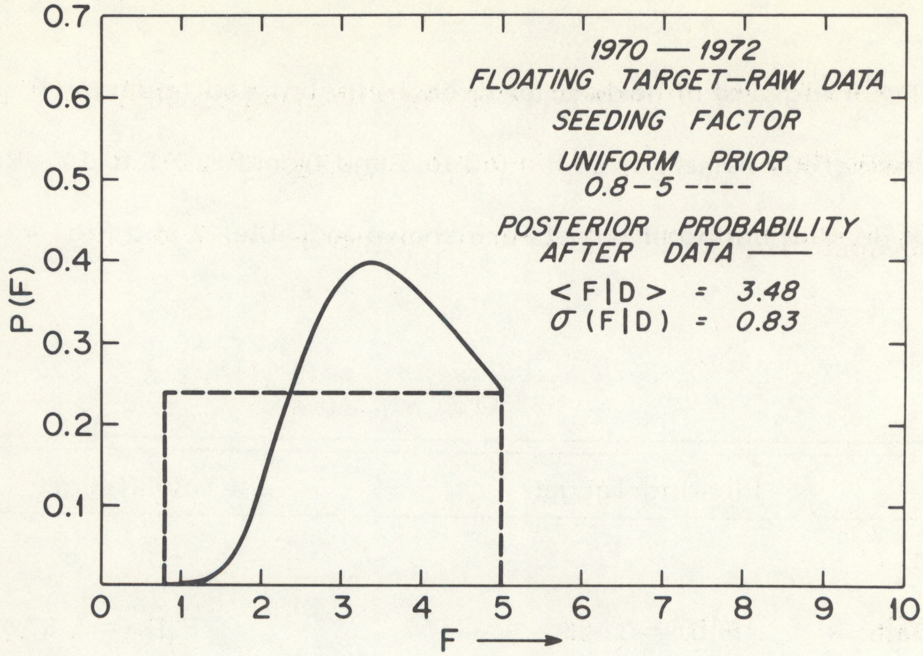


*Figure 4. Prior and posterior probability distributions for seeding factor F. Raw data, area experiment 1970 - 1972.*
*a. Floating target. Uniform prior F=0.8-5.*

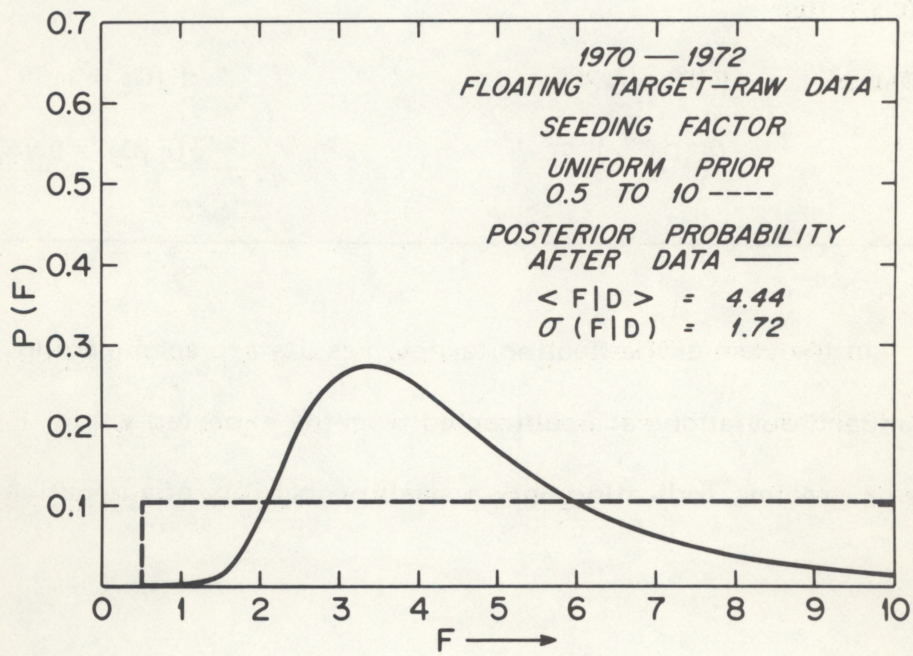


*Figure 4b. Floating target. Uniform prior prior F=0.5-10.*

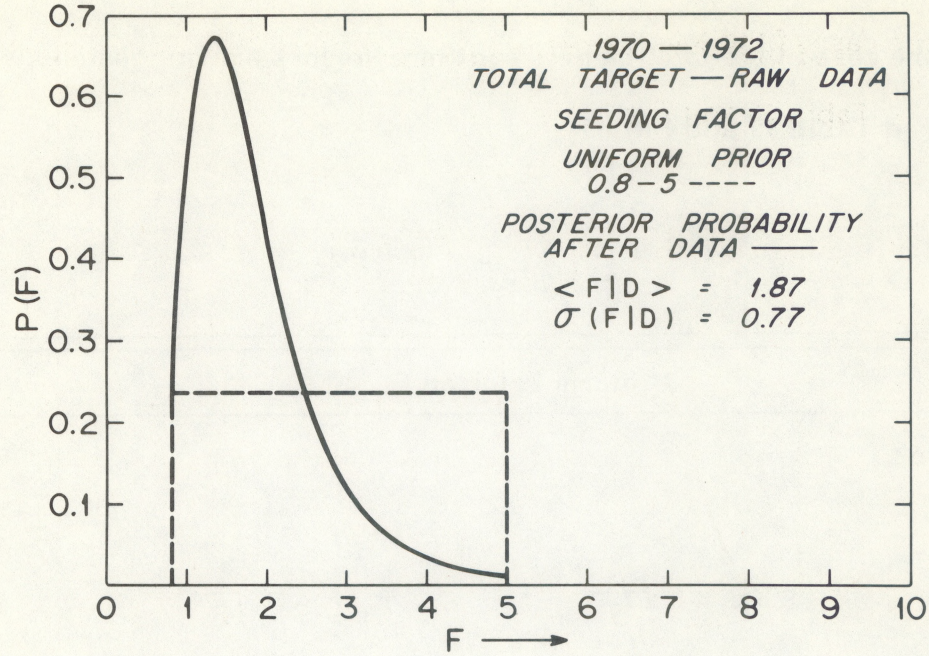


*Figure 4c. Total target. Uniform prior F=0.8-5.*

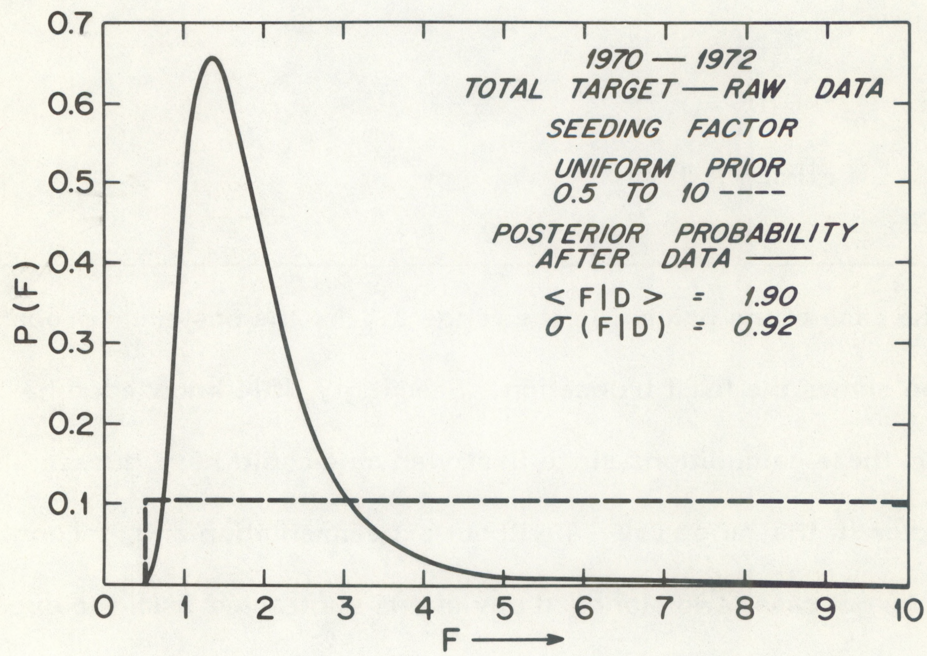


*Figure 4d. Total target. Uniform prior F=0.5-10.*

In the case of the total target, additional priors are suggested, which are tested in Table 13 and Fig. 5.

*Table 13. Bayesian Results for Seeding Factor - Total Total Target 1970 - 72*

| Uniform Prior on F | |
|---|---|
| Range Prior | |
| 0.2 - 3 | |
| After Data: | $\langle F \mid D \rangle = 1.67$ |
| | $\sigma(F \mid D) = 0.54$ |
| Range Prior | |
| 0.4 - 4 | |
| After Data: | $\langle F \mid D \rangle = 1.79$ |
| | $\sigma(F \mid D) = 0.69$ |

In the case of the prior F in the range 0.4 - 4 the posterior probability distribution shows the least truncation. Seemingly little knowledge has been gained from these calculations since firstly anyone could have guessed a seeding factor in the range 0.4 - 4 without experimentation and secondly, in all total target cases two standard deviations subtracted from the expectation takes us into the range $F < 1$, or seeding decreasing rainfall.

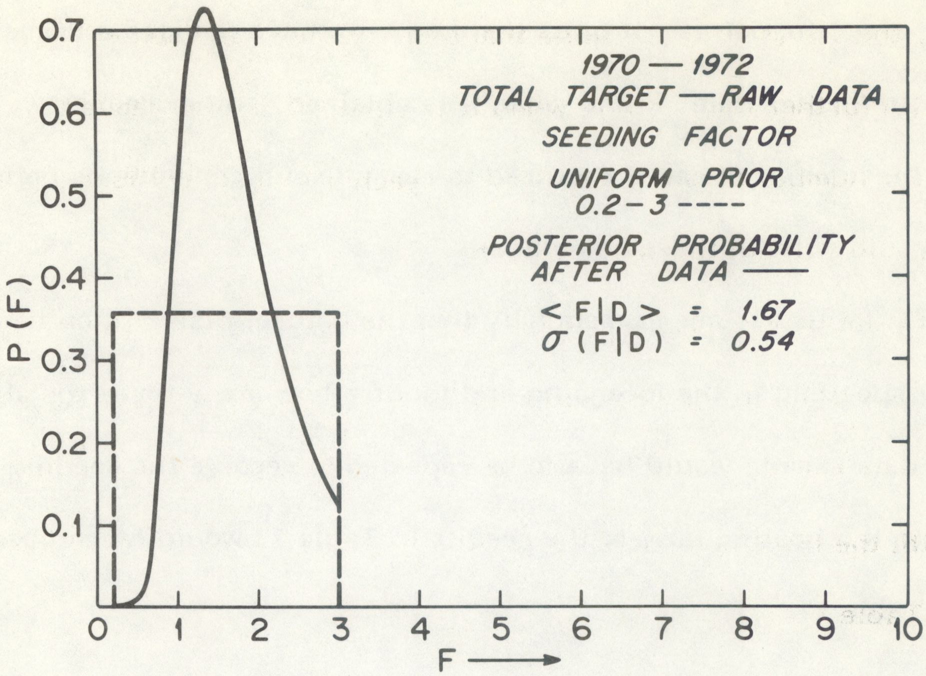


*Figure 5. Prior and posterior probability distributions for seeding factor F. Total target, raw data, 1970 - 1972.*
*a. Uniform prior F-0.2-3.*

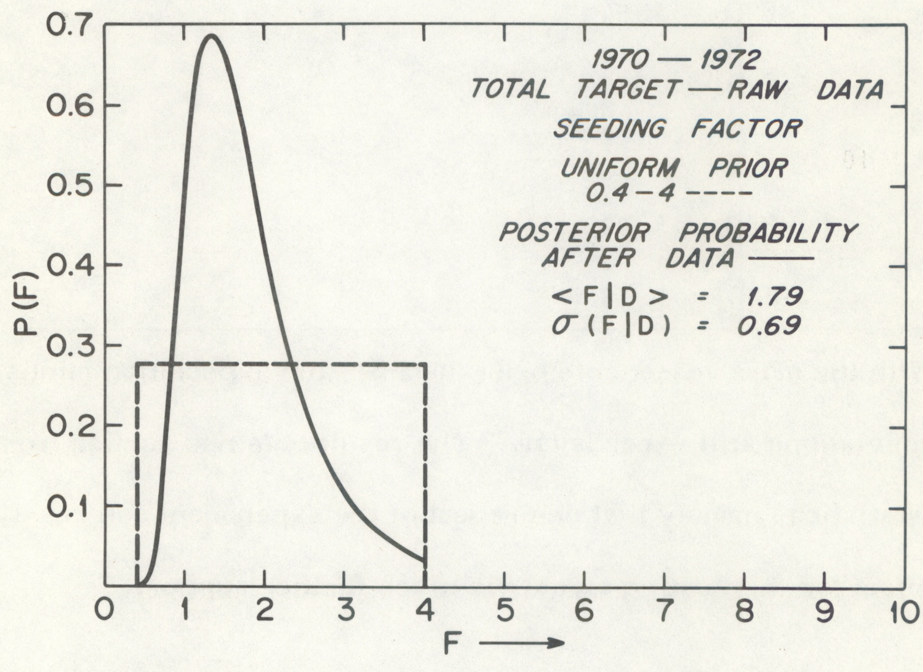


*Figure 5b. Uniform prior F=0.4-4.*

However, the situation is not quite that bad. We have devised a framework to deal with further data, if and when it is obtained, and moreover we can estimate the number of cases required to reach useful conclusions better than we could without these calculations.

First let us assume momentarily that the natural distribution has been correctly specified in the foregoing and inquire how many times the already obtained data sample would have to be repeated to resolve the seeding factor.

With the floating target, the results in Table 12 would be replaced by those in Table 14.

*Table 14. Area Experiment - Floating Target - Doubled Data Sample (14 Cases).*

| | |
|---|---|
| Prior 0.8 - 5 | |
| After Data: | $\langle F \mid D \rangle = 3.54$ |
| | $\sigma(F \mid D) = 0.72$ |
| Prior 0.5 - 10 | |
| After Data: | $\langle F \mid D \rangle = 3.92$ |
| | $\sigma(F \mid D) = 1.16$ |

With the more reasonable prior (0.8-5), the expectation minus 3 standard deviations still exceeds one. The result obtained earlier from classical statistics, namely that one repeat of the experiment can resolve a positive effect for the floating target, is given further support.

With the total target, Table 15 suggests the likelihood of having to increase the data sample to 56 cases, a utopian but perhaps unavoidable goal!

*Table 15. Area Experiment - Total Target - Augmented Data Sample.*

Uniform prior F in range 0.8 - 5

A. Double sample - 14 cases

$\langle F|D \rangle = 1.59$

$\sigma(F|D) = 0.47$

B. Quadruple sample - 28 cases

$\langle F|D \rangle = 1.47$

$\sigma(F|D) = 0.29$

C. Eight-fold sample - 56 cases

$\langle F|D \rangle = 1.41$

$\sigma(F|D) = 0.19$

Next we investigate the important subject of the sensitivity of results to uncertainty in our specification of natural distributions. As seen in equation (10), the natural distribution enters our results via $\alpha$ and $\langle R \rangle_{NS}$ (evaluated by $\bar{R}$ control). Therefore we examine the dependence of results upon variations in each of these. Table 16 shows that neither floating nor total target results are highly sensitive to small variations in shape parameter $\alpha$.

*Table 16. Sensitivity of Area Results to Shape Parameter Errors.*

A. Uniform prior on F in range 0.8 - 5

| | Floating Target | | Total Target | |
|---|---|---|---|---|
| | $\alpha = 0.9$ | $\alpha = 1.1$ | $\alpha = 0.9$ | $\alpha = 1.1$ |
| $\langle F \mid D \rangle$ | 3.47 | 3.50 | 1.93 | 1.82 |
| $\sigma(F \mid D)$ | 0.85 | 0.82 | 0.82 | 0.73 |

B. Uniform prior on F in range 0.5 - 10

| | $\alpha = 0.9$ | $\alpha = 1.1$ | $\alpha = 0.9$ | $\alpha = 1.1$ |
|---|---|---|---|---|
| $\langle F \mid D \rangle$ | 4.52 | 4.36 | 1.98 | 1.84 |
| $\sigma(F \mid D)$ | 1.80 | 1.65 | 1.04 | 0.84 |

C. Uniform prior on F in range 0.4 - 4 - Total target only

| | | $\alpha = 0.9$ | $\alpha = 1.1$ |
|---|---|---|---|
| $\langle F \mid D \rangle$ | - | 1.82 | 1.76 |
| $\sigma(F \mid D)$ | - | 0.72 | 0.66 |

Table 17 shows that dependence upon control sample average is possibly more critical. Variations in the sample average of 10%, 20% and a factor of two are examined.

*Table 17. Sensitivity of Area Results to Errors in Control Sample Average.*

| | Most reasonable* uniform prior on F | | | |
|---|---|---|---|---|
| | Floating Target | | Total Target | |
| $\langle R \rangle_{NS}$ Change | Expectation | Std. Dev. | Expectation | Std. Dev. |
| +10% | 3.32 | 0.86 | 1.65 | 0.67 |
| -10% | 3.65 | 0.79 | 1.95 | 0.71 |
| +20% | 3.17 | 0.88 | 1.53 | 0.64 |
| -20% | 3.81 | 0.74 | 2.12 | 0.72 |
| 2 factor | 2.22 | 0.86 | 0.96 | 0.44 |
| 0.5 factor | 4.32 | 0.52 | 2.80 | 0.66 |

* Range 0.8 - 5 for floating target; 0.4 - 4 for total target

In II (Table 13 and Appendix II) we examined departures in sample average from the true $\langle R \rangle$ when observations were taken at random from a gamma distribution with $\alpha = 0.6$. Bowman and Shenton (1970) showed that when $\alpha = 1$ the situation is a little less favorable. With 10 observations per sample, we have less than a 10% chance to be off by a factor of two, but we have an 80% chance of being worse off than 10% and a 63% chance of being worse off than 20%. With the floating target, these problems are less serious than with the total target. However, with the latter, the standard deviations are unacceptably large and will remain so until we have approximately 50 pairs of cases. This conclusion is further developed and supported in the next section.

## 7. ON THE NUMBER OF OBSERVATIONS REQUIRED

The most serious obstacle against progress in weather modification is high natural variability. Natural variability becomes an increasingly serious problem with smaller anticipated seeding factors. In the EML experiments, we have been fortunate with the work on isolated clouds and probably with floating targets as well, since seeding factors of about 3 are indicated. With the variabilities encountered, we found that a sample of 20-25 pairs of cases should be adequate to resolve these seeding factors. A caveat with the floating targets is the effect of measurement uncertainties, still under analysis, since randomization was by days. Unlike the single clouds, we do not have seed and control cases on the same day.

With the total targets, present data suggest a seeding factor in the neighborhood 1.5 - 1.7. While a rainfall increase of this size could be of immense practical value, it will be a long, difficult effort to verify. Here we will assume the measurements are correct and investigate the sampling problem only; a later report will combine the effects of sampling with those of measurement uncertainties.

In II, we undertook a series of Monte Carlo experiments to investigate the sampling problem and then (in Appendix II) solved the same problem using probability theory, with virtually identical results. The basic idea is that we obtain estimates of F, namely $\hat{F}$, from simulated seeded and control rainfall populations which are gamma distributions (here with $\alpha = 1$) with the true F (and corresponding scale parameters) specified in advance. We draw a sample of n cases each from both populations, average them, and take the average ratio as $\hat{F}$. The experiment is repeated in m times with each n to see what percentage of the time, for example, we will get $\hat{F} > 1.50$ when the true F is equal to one. We can then decide from the results how much risk we are prepared to take that our experimental $\hat{F}$ is in error by any specified amount due to the sampling problem.

With probability theory, the question is phrased: What is the per cent probability that the sample seeding factor from n observations lies within specified limits? In terms of a probability statement

$$\Pr \left[ a \leq \hat{F} \leq b \right] \tag{13}$$

gives the probability that F is within the interval a to b. Statements of the above form require only the use of incomplete beta tables to determine the probability, since F has a distribution which is a scalar multiple of the Snedecor F-distribution. Note that the distribution changes parameters as n changes. Tables 18 - 25 show the results pertinent to the EML area experiment.

*Table 18. Percent Probability that $\hat{F}$ (Sample Seeding Factor Estimate) Lies Within Specified Intervals When F=1.*

α = 1

| Interval | Sample Size n 10 | 20 | 50 |
|---|---|---|---|
| 0.0 - 0.5 | 6.49 | 1.56 | 0.03 |
| 0.5 - 0.8 | 24.64 | 22.64 | 13.38 |
| 0.8 - 1.2 | 34.50 | 47.43 | 68.32 |
| 1.2 - 1.5 | 15.76 | 18.16 | 16.08 |
| 1.5 - 2.0 | 12.12 | 8.65 | 2.16 |
| 2.0 - 2.5 | 4.15 | 1.32 | 0.03 |
| 2.5 - 3.0 | 1.45 | 0.20 | 0 |
| 3.0 - 4.0 | 0.73 | 0.04 | 0 |
| 4.0 - 5.0 | 0.13 | 0 | 0 |
| 5.0 - 6.0 | 0.02 | 0 | 0 |
| 6.0 - 7.0 | 0.01 | 0 | 0 |
| > 7.0 | 0 | 0 | 0 |
| Mean | 1.11 | 1.05 | 1.02 |
| Variance | 0.29 | 0.12 | 0.04 |
| Std. Dev. | 0.54 | 0.35 | 0.21 |

*Table 19. Percent Probability That $\hat{F}$ (Sample Seeding Factor Estimate) Lies Within Specified Intervals When F=1.*

$\alpha = 1$

| Interval | Sample Size n: 10 | 20 | 50 |
|---|---|---|---|
| 0.0 - 0.5 | 0.89 | 0.04 | 0 |
| 0.5 - 0.8 | 7.55 | 2.48 | 0.10 |
| 0.8 - 1.2 | 22.69 | 21.68 | 13.31 |
| 1.2 - 1.5 | 18.87 | 25.80 | 36.59 |
| 1.5 - 2.0 | 23.68 | 31.63 | 42.34 |
| 2.0 - 2.5 | 13.20 | 12.81 | 7.07 |
| 2.5 - 3.0 | 6.63 | 4.0 | 0.56 |
| 3.0 - 4.0 | 4.81 | 1.43 | 0.03 |
| 4.0 - 5.0 | 1.19 | 0.12 | 0 |
| 5.0 - 6.0 | 0.33 | 0.01 | 0 |
| 6.0 - 7.0 | 0.10 | 0 | 0 |
| 7.0 - 8.0 | 0.04 | 0 | 0 |
| 8.0 - 9.0 | 0.01 | 0 | 0 |
| 9.0 -10.0 | 0.01 | 0 | 0 |
| >10.0 | 0 | 0 | 0 |
| Mean | 1.67 | 1.58 | 1.53 |
| Variance | 0.66 | 0.27 | 0.097 |
| Std. Dev. | 0.81 | 0.52 | 0.31 |

*Table 20. Percent Probability That $\ddot{\hat{F}}$ (Sample Seeding Factor Estimate) Lies Within Specified Intervals When F=2.*

$\alpha = 1$

| Interval | Sample Size n | | |
|---|---|---|---|
| | 10 | 20 | 50 |
| 0.0 - 0.5 | 0.16 | 0.01 | 0 |
| 0.5 - 0.8 | 2.18 | 0.23 | 0 |
| 0.8 - 1.2 | 10.77 | 5.31 | 0.59 |
| 1.2 - 1.5 | 13.20 | 12.81 | 7.06 |
| 1.5 - 2.0 | 23.69 | 31.64 | 42.35 |
| 2.0 - 2.5 | 18.86 | 25.78 | 36.57 |
| 2.5 - 3.0 | 12.53 | 14.01 | 11.24 |
| 3.0 - 4.0 | 12.11 | 8.64 | 2.16 |
| 4.0 - 5.0 | 4.16 | 1.33 | 0.03 |
| 5.0 - 6.0 | 1.45 | 0.20 | 0 |
| 6.0 - 7.0 | 0.52 | 0.03 | 0 |
| 7.0 - 8.0 | 0.21 | 0.01 | 0 |
| 8.0 - 9.0 | 0.09 | 0 | 0 |
| 9.0 -10.0 | 0.03 | 0 | 0 |
| >10.0 | 0.04 | 0 | 0 |
| Mean | 2.22 | 2.11 | 2.04 |
| Variance | 1.18 | 0.48 | 0.17 |
| Std. Dev. | 1.09 | 0.69 | 0.41 |

*Table 21. Percent Probability That $\hat{F}$ (Sample Seeding Factor Estimate) Lies Within Specified Intervals When F=3.*

$\alpha$ = 1

| Interval | Sample Size n 10 | 20 | 50 |
|---|---|---|---|
| 0.0 - 0.5 | 0 | 0 | 0 |
| 0.5 - 0.8 | 0.24 | 0 | 0 |
| 0.8 - 1.2 | 2.10 | 0.24 | 0 |
| 1.2 - 1.5 | 4.15 | 1.32 | 0.03 |
| 1.5 - 2.0 | 12.12 | 8.65 | 2.16 |
| 2.0 - 2.5 | 15.76 | 18.15 | 16.08 |
| 2.5 - 3.0 | 15.63 | 21.64 | 31.73 |
| 3.0 - 4.0 | 23.68 | 31.63 | 42.34 |
| 4.0 - 5.0 | 13.20 | 12.81 | 7.07 |
| 5.0 - 6.0 | 6.63 | 4.00 | 0.56 |
| 6.0 - 7.0 | 3.23 | 1.13 | 0.03 |
| 7.0 - 8.0 | 1.58 | 0.30 | 0 |
| 8.0 - 9.0 | 0.79 | 0.09 | 0 |
| 9.0 -10.0 | 0.40 | 0.03 | 0 |
| >10.0 | 0.49 | 0.01 | 0 |
| Mean | 3.33 | 3.16 | 3.06 |
| Variance | 2.66 | 1.08 | 0.38 |
| Std. Dev. | 1.63 | 1.04 | 0.62 |

*Table 22. Percent Probability That $\hat{F}$ (Sample Seeding Factor Estimate) Will Exceed Fo When F=1.*

| $F_o$ | $x = \frac{1}{1 + F_o/F}$ | n= 10 | 20 | 50 |
|---|---|---|---|---|
| 0.0 | 1 | 100.00 | 100.00 | 100.00 |
| 0.5 | 0.6667 | 93.51 | 98.44 | 99.97 |
| 0.8 | 0.5556 | 68.87 | 75.80 | 86.59 |
| 1.0 | 0.5000 | 50.00 | 50.00 | 50.00 |
| 1.2 | 0.4545 | 34.37 | 28.37 | 18.27 |
| 1.5 | 0.4000 | 18.61 | 10.21 | 2.19 |
| 2.0 | 0.3333 | 6.49 | 1.59 | 0.03 |
| 2.5 | 0.2857 | 2.34 | 0.24 | 0.00 |
| 3.0 | 0.2500 | 0.89 | 0.04 | 0.00 |
| 4.0 | 0.2000 | 0.16 | 0.00 | 0.00 |
| 5.0 | 0.1667 | 0.03 | 0.00 | 0.00 |
| 6.0 | 0.1429 | 0.01 | 0.00 | 0.00 |
| 7.0 | 0.1250 | 0.00 | 0.00 | 0.00 |
| 8.0 | 0.1111 | 0.00 | 0.00 | 0.00 |
| 9.0 | 0.1000 | 0.00 | 0.00 | 0.00 |
| 10.0 | 0.0909 | 0.00 | 0.00 | 0.00 |

*Table 23. Percent Probability That $\hat{F}$ (Sample Seeding Factor Estimate) Will Exceed Fo When F=1.5.*

| Fo | $x = \frac{1}{1 + F_o/F}$ | n= 10 | 20 | 50 |
|---|---|---|---|---|
| 0.0 | 1 | 100.00 | 100.00 | 100.00 |
| 0.5 | 0.7500 | 99.11 | 99.96 | 100.00 |
| 0.8 | 0.6522 | 91.56 | 97.48 | 99.90 |
| 1.0 | 0.6000 | 81.39 | 89.79 | 97.81 |
| 1.2 | 0.5556 | 68.87 | 75.80 | 86.59 |
| 1.5 | 0.5000 | 50.00 | 50.00 | 50.00 |
| 2.0 | 0.4286 | 26.32 | 18.37 | 7.66 |
| 2.5 | 0.3750 | 13.12 | 5.56 | 0.59 |
| 3.0 | 0.3333 | 6.49 | 1.56 | 0.03 |
| 4.0 | 0.2727 | 1.68 | 0.13 | 0.00 |
| 5.0 | 0.2308 | 0.49 | 0.01 | 0.00 |
| 6.0 | 0.2000 | 0.16 | 0.00 | 0.00 |
| 7.0 | 0.1765 | 0.06 | 0.00 | 0.00 |
| 8.0 | 0.1579 | 0.02 | 0.00 | 0.00 |
| 9.0 | 0.1429 | 0.01 | 0.00 | 0.00 |
| 10.0 | 0.1304 | 0.00 | 0.00 | 0.00 |

*Table 24. Percent Probability That $\hat{F}$ (Sample Seeding Factor Estimate) Will Exceed Fo When F=2.*

| $F_o$ | $x = \frac{1}{1 + F_o/F}$ | n= 10 | 20 | 50 |
|---|---|---|---|---|
| 0.0 | 1 | 100.00 | 100.00 | 100.00 |
| 0.5 | 0.8000 | 99.84 | 99.99 | 100.00 |
| 0.8 | 0.7143 | 97.66 | 99.76 | 100.00 |
| 1.0 | 0.6667 | 93.54 | 98.45 | 99.96 |
| 1.2 | 0.6250 | 86.89 | 94.45 | 99.41 |
| 1.5 | 0.5714 | 73.69 | 81.64 | 92.35 |
| 2.0 | 0.5000 | 50.00 | 50.00 | 50.00 |
| 2.5 | 0.4444 | 31.14 | 24.22 | 13.43 |
| 3.0 | 0.4000 | 18.61 | 10.21 | 2.19 |
| 4.0 | 0.3333 | 6.50 | 1.57 | 0.03 |
| 5.0 | 0.2857 | 2.34 | 0.24 | 0.00 |
| 6.0 | 0.2500 | 0.89 | 0.04 | 0.00 |
| 7.0 | 0.2222 | 0.37 | 0.01 | 0.00 |
| 8.0 | 0.2000 | 0.16 | 0.00 | 0.00 |
| 9.0 | 0.1818 | 0.07 | 0.00 | 0.00 |
| 10.0 | 0.1667 | 0.04 | 0.00 | 0.00 |

*Table 25. Percent Probability That $\hat{F}$ (Sample Seeding Factor Estimate) Will Exceed Fo When F=3.*

| Fo | $x = \frac{1}{1 + F_o/F}$ | n= 10 | 20 | 50 |
|---|---|---|---|---|
| 0 | 1 | 100.00 | 100.00 | 100.00 |
| 0.5 | 0.8571 | 100.00 | 100.00 | 100.00 |
| 0.8 | 0.7895 | 99.76 | 100.00 | 100.00 |
| 1.0 | 0.7500 | 99.11 | 99.96 | 100.00 |
| 1.2 | 0.7143 | 97.66 | 99.76 | 100.00 |
| 1.5 | 0.6667 | 93.51 | 98.44 | 99.97 |
| 2.0 | 0.6000 | 81.39 | 89.79 | 97.81 |
| 2.5 | 0.5455 | 65.63 | 71.64 | 81.73 |
| 3.0 | 0.5000 | 50.00 | 50.00 | 50.00 |
| 4.0 | 0.4286 | 26.32 | 18.37 | 7.66 |
| 5.0 | 0.3750 | 13.12 | 5.56 | 0.59 |
| 6.0 | 0.3333 | 6.49 | 1.56 | 0.03 |
| 7.0 | 0.3000 | 3.26 | 0.43 | 0.00 |
| 8.0 | 0.2727 | 1.68 | 0.13 | 0.00 |
| 9.0 | 0.2500 | 0.89 | 0.04 | 0.00 |
| 10.0 | 0.2308 | 0.49 | 0.01 | 0.00 |

The calculations in Tables 18 - 21 are illustrated in Figs. 6 - 9. With F = 1 - 1.5 the conclusion is inevitable that 50 pairs of cases will be necessary for firm conclusions. This means with the current rate of progress, it may take 10 - 15 years to resolve F for the total target. In 1973, it is planned to extend the experimental period to all of July and August. If this can be done, a reasonable maximum expectation is 10 pairs of cases per experimental year. With an accelerated effort, then, the experiment could be concluded in 4 - 5 experimental years. The worthwhileness of this goal and alternative options are considered in the conclusions.

## 8. CONCLUSIONS

The conclusions which may be drawn from this work so far are as follows:

1) The floating target seeding factor on rainfall is probably, but not conclusively, positive and in the vicinity of 3. With one more experimental season, or about 7 more pairs of cases, there is reason to hope that this part of the experiment can be resolved at the 5% significance level.

2) Present indications are that a total of about 50 pairs of cases will be required to resolve the seeding factor for the total target, requiring 5 - 10 years of experimentation. This estimate is based on our current evaluation of natural fluctuations and seeding factor, both of which are subject to revision with further data. For the total target, the present expected value of $\hat{F} \sim 1.7$ is tentative. The standard deviation of $\hat{F}$ of $\sim 0.5$ is too large

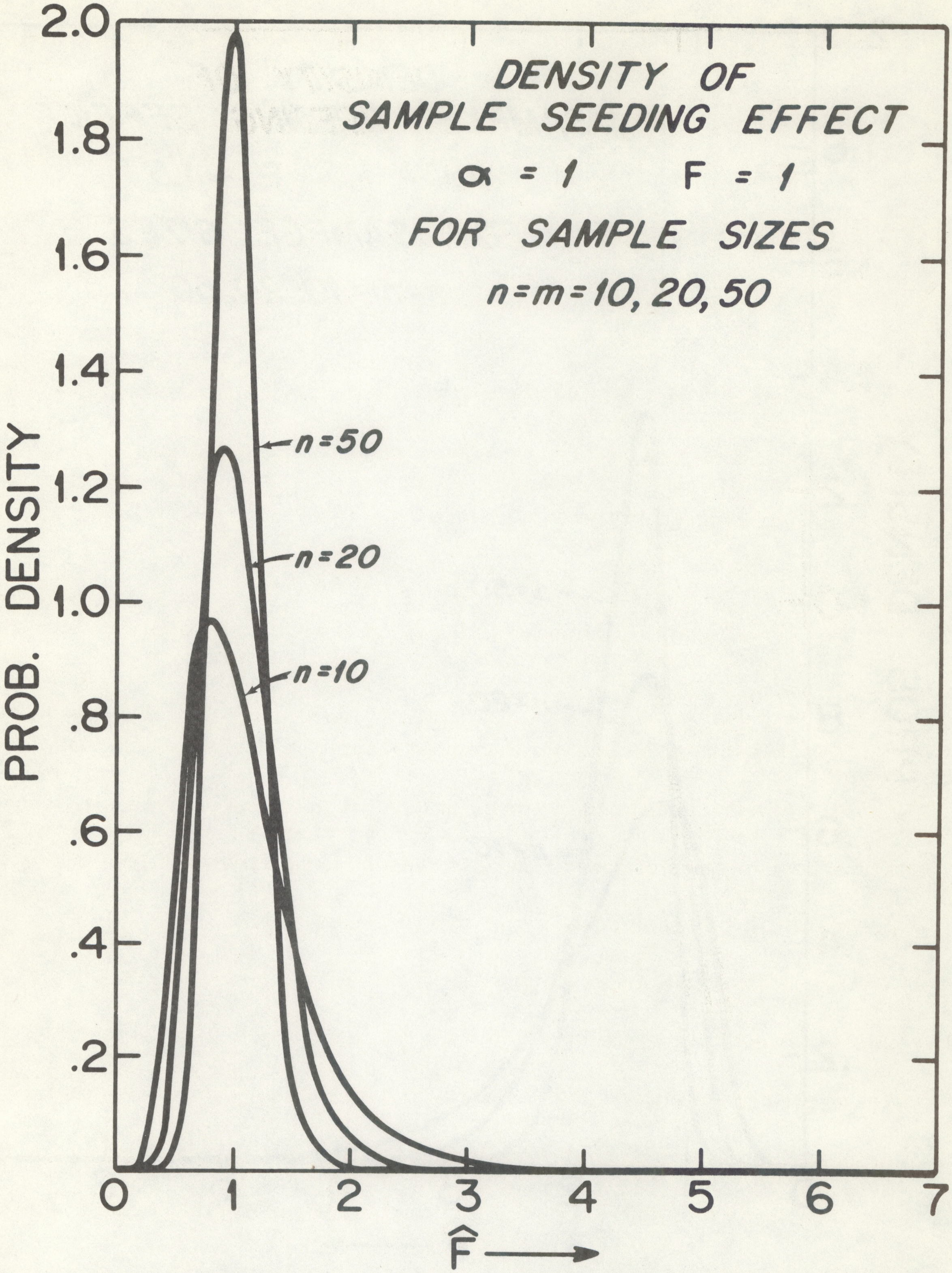


*Figure 6. Probability density of sample seeding factor $\hat{F}$ when F=1, α=1. Sample sizes n=10, 20, and 50.*

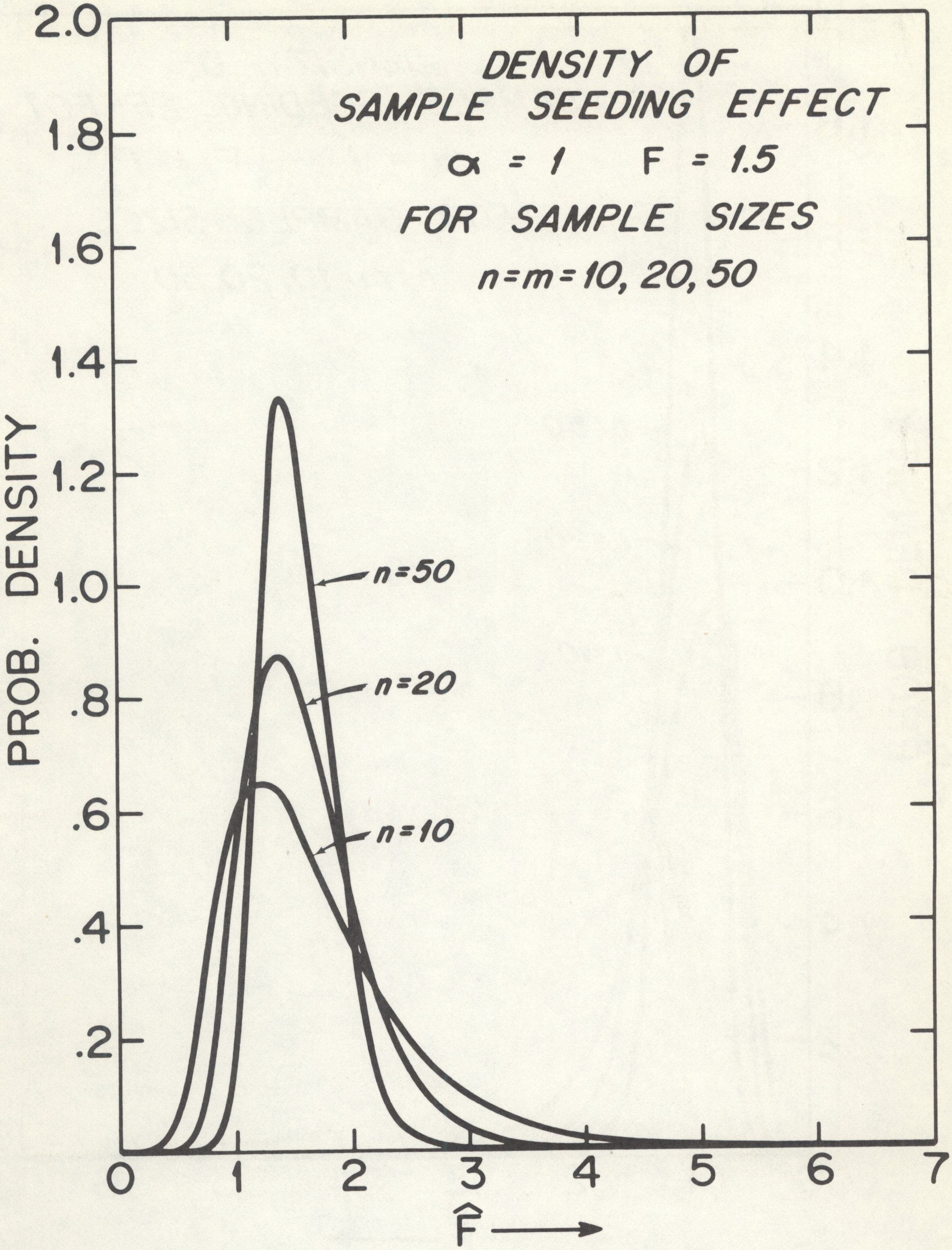


*Figure 7. Probability density of sample seeding factor $\hat{F}$ when F-1.5, α=1. Sample sizes n=10, 20, and 50.*

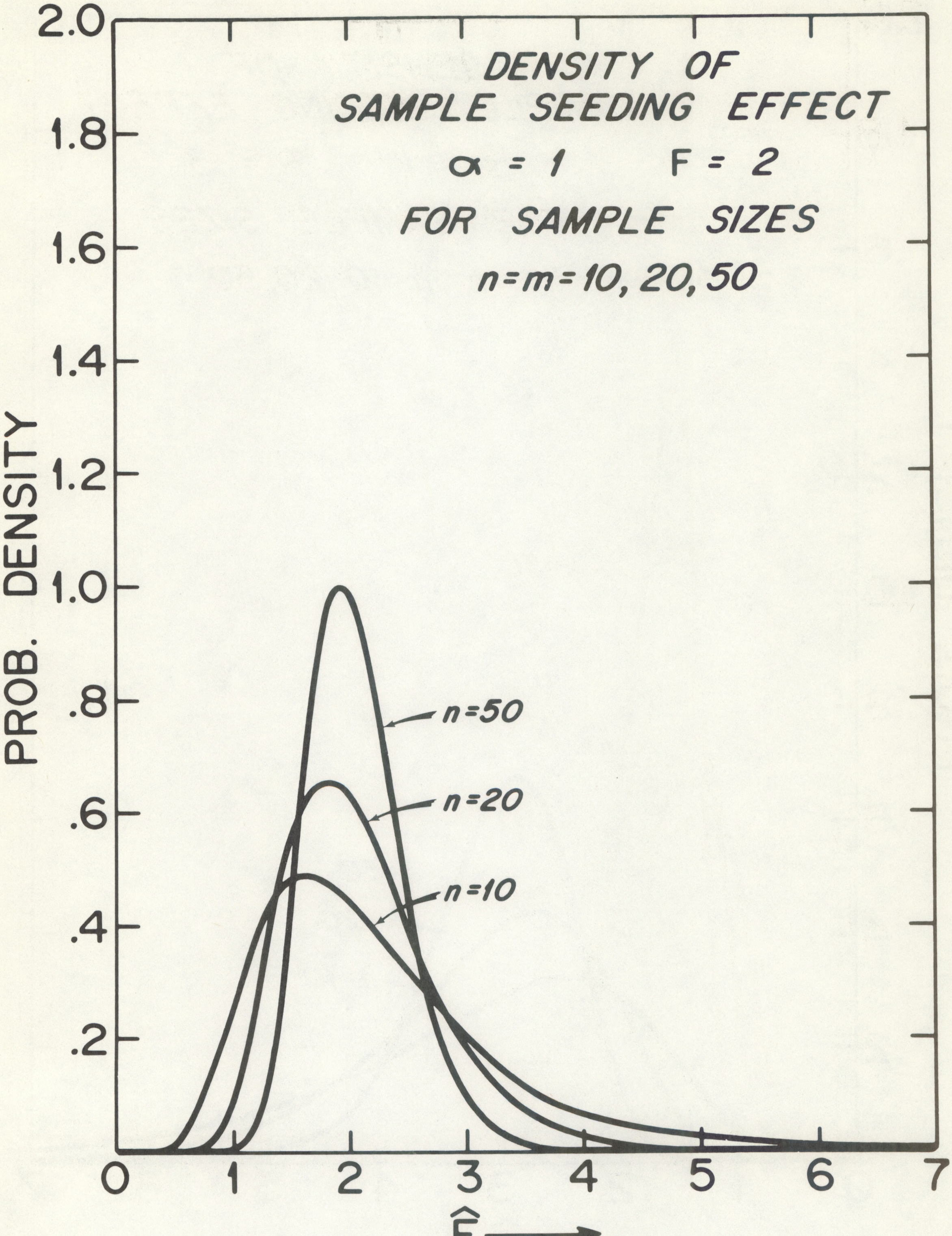


*Figure 8. Probability density of sample seeding factor $\hat{F}$ when F=2, α=1. Sample sizes n=10, 20, and 50.*

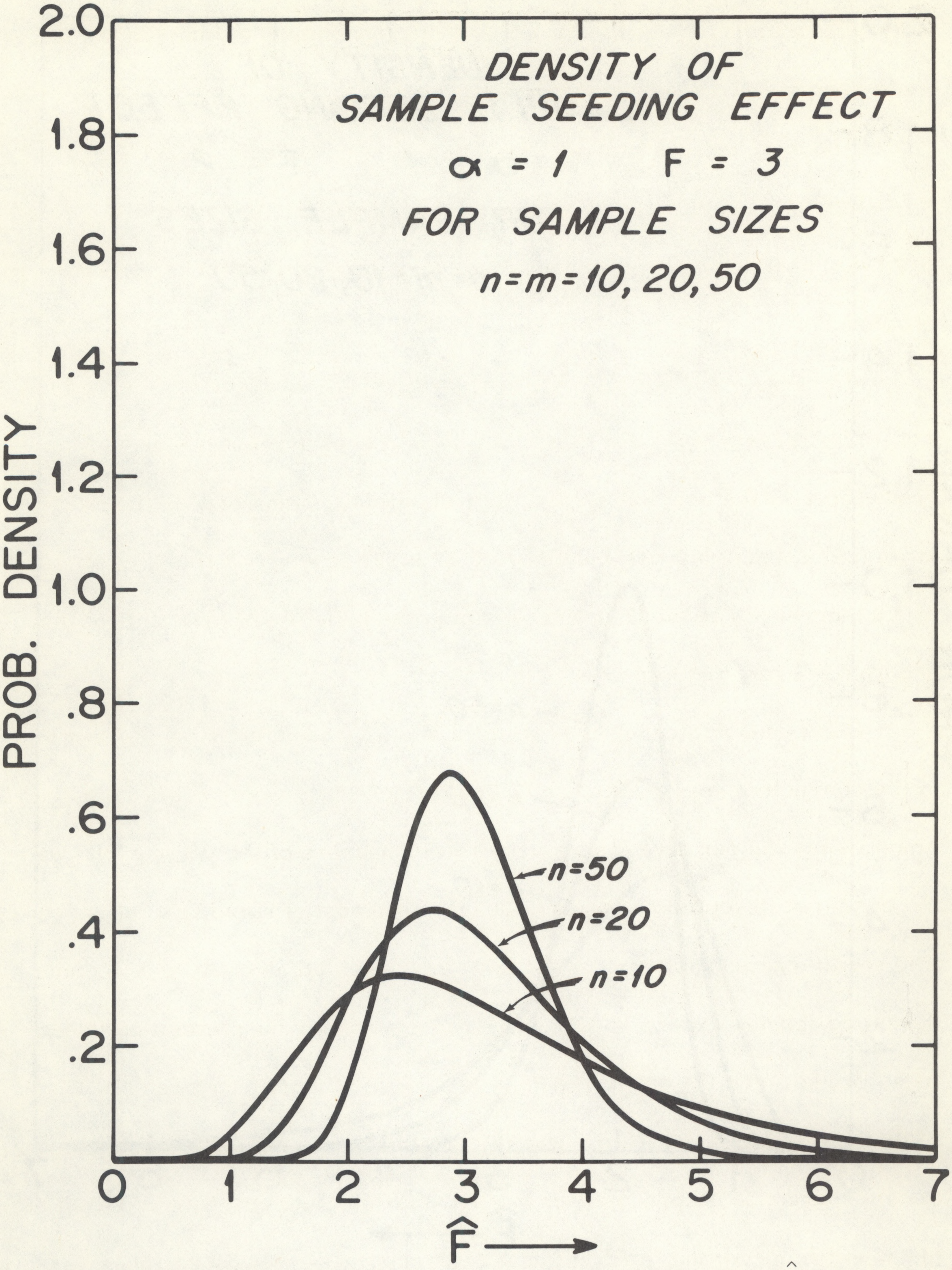


*Figure 9. Probability density of sample seeding factor $\hat{F}$ when F=3, α=1. Sample sizes n=10, 20, and 50.*

to draw conclusions, even if the natural distributions of rainfall in the target were more firmly known than they are.

The long and costly road which probably must be travelled to resolve this experiment raises two critical questions, firstly is it worthwhile? And secondly, if it is worthwhile, may there not be a better way to tackle the problem?

Practically speaking, if the total target seeding factor is anything like 1.5 - 1.7, there is no doubt that continued experimentation is worthwhile. From Table 9, the average seeded-control rainfall difference for the total target per seeded day is indicated as $1.27 \times 10^4$ acre-ft. If this amount can, by further work, be attributed to seeding, the benefit-to-cost ratios of the program are large indeed.

Our climatological studies suggest about 10 seedable days per month in the convective season, so that $12.7 \times 10^4$ acre-ft of water might be gained. Again using \$50 per acre-ft as a very rough measure of the value of the water, the benefits are roughly 6 million dollars. The cost of running a full scientific seeding program, with two heavily instrumented aircraft, pyrotechnic flares, calibrated radars, gages and a highly trained complement of scientists, would be on the order of \$100,000, giving a benefit-cost ratio of 60. After the seeding factors were soundly established, a minimal-cost operational program could probably be run for one-fourth to one-third the cost of the scientific program, raising the benefit-cost ratio well above 100.

Even if the direct practical benefits prove much smaller than the above, the potential scientific benefits from solving this problem in themselves justify its pursuit. So far, no area seeding experiment on fair weather cumuli in tropical air has been brought to a conclusive positive result. Moreover, the merger process, at the focus of the EML area experiment, is a key problem in atmospheric dynamics and severe weather prediction.

Granting then, that carrying dynamic seeding of cumuli over an area to a conclusive result is worthwhile, is there a better way to achieve the goal? Possible options are first, improved experiment design in south Florida and/or second, execution of the experiment elsewhere. To consider the first question, successive meetings have been held with prominent statisticians. In particular, the randomized crossover design has been suggested and carefully examined. Unfortunately, with a fine weather cumulus regime, there are no two areas in south Florida (in the area ranges of 2000 - 4000 sq miles) where a correlation in precipitation is as high as 0.5; we suspect this would be true with convective situations in most other places. Worse yet, even if two nearby highly correlated areas could be found, the spectre of dynamic contamination is too large to live with. Anyone who has seen a huge cumulonimbus, with its mushrooming anvil, literally wipe out convection over one thousand or more square miles, is not tempted to design a randomized crossover experiment for dynamic seeding.

The only remaining option might then be to move the experiment elsewhere, since about 6 weeks of the early convective season in south Florida is prohibited from seeding owing to the tomato harvest. Consequently, we have investigated other areas, especially in the midwest. In particular, the Illinois State Water Survey has collected a priceless set of natural rainfall records in several potential target areas in central Illinois. Five years of data are published for the 550 square-mile Little Egypt network and ten years of data for the 400 sq mile East Central Illinois network (Changnon and Huff, 1967). The data are stratified so that the 115 and 161 cases, respectively, of convective precipitation can be examined separately. Applying the same types of analyses to these data as in the foregoing, we find that the natural fluctuations are larger than in south Florida; for example, the standard deviation of the rainfall is about twice the average value (cf. Table 9) and the distributions are such that longer experimentation would probably be necessary to resolve the same size seeding factors in the two regions. Furthermore, all indications are that seedabilities are considerably lower in the midwest and high plains regions than they are in tropical regions such as Florida (Simpson and Dennis, 1972).

At present, moving to a different tropical location is contra-indicated, in view of the ready accessibility of the present target to the necessary facilities, its flatness, and the years of effort and knowledge already invested. A major conclusion from this work, and most other work in weather modification, is that there are few short-cuts to significant results. Planning should therefore be undertaken with a 5 - 10 year effort in mind.

## 9. ACKNOWLEDGMENTS

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