AA Technical Memorandum ERL RFC-3

INTERPRETATION OF TEST RESULTS FOR PITOT STATIC PROBES

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noaa NATIONAL OCEANIC AND ATMOSPHERIC ADMINISTRATION / Environmental Research Laboratories

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Interpretation of Test Results for Pitot Static Probes

(1) Introduction

Three Pitot Static Probes (Rosemount 855 EB-1) were tested by Rosemount, Inc. in their wind tunnel. The results were summarized in Rosemount Report 27722 of 19 February 1977.

Because of spatial restrictions the instruments could not be tested for angle of attack and angle of sideslip influences directly but only for an inclination angle and a rotation angle. Thus a conversion from measured angles to usage angles must be performed.

The measurement results are given as the ratio of the difference between the measured pressure and a reference pressure divided by the impact pressure. Conversions will be presented to relate the given values to error in static pressure and airspeed.

(2) Errors in Static Pressure

Static pressure is defined as the absolute air pressure that would have existed at the point of the aircraft in the atmosphere if there were no pressure disturbances caused by the aircraft. Since even a very long boom ahead of the wing causes a pressure disturbance, it is not possible to calibrate a pressure measuring device in the lab exactly. On the basis of general knowledge of the disturbance caused by the Pitot Static Tube and its mounting Rosemount,has designed the probe so that it compensates for this anticipated value. Figure 3 of the Rosemount Report shows that this compensation is

$$\frac{P_m - P_1}{q_{c1}} = -0.00535 \begin{array}{l} +0.0005 \\ -0.0015 \end{array}$$

for all three instruments in the Mach range from M = 0.3 to M = 0.7. For a single instrument the compensation ratio changes by less than 0.001 in the above Mach range and differences between individual instruments are less than 0.0015. Only a flight test can provide the final static pressure correction. P is the real static pressure in flight, Pm the pressure measured in flight or in test, and P_1 the static pressure in the wind tunnel. The final static pressure correction A is then

$$\underbrace{P - P_m}_{A} = \underbrace{P - P_1}_{B} + \underbrace{P_1 - P_m}_{C}$$

where $B = P - P_1$ is the deviation between the actual static pressures in flight and in the wind tunnel that give the same reading of the instrument, and $C = P_1 - Pm$ is the compensation that Rosemount built into the design and tested by comparing the readout value of the instrument with the actual static pressure of the undisturbed flow in the wind tunnel.

For an airspeed of 150 m/sec (292 knots) the ratio q_c/P is about 0.15. Thus the compensation $\Delta P/q_c = -0.0535$ amounts to a compensation in static pressure of

$$\frac{\Delta P}{P} = \frac{\Delta p}{q_c} \cdot \frac{q_c}{P} = -0.0535 \cdot 0.15 = -0.008.$$

At a pressure of 1000 mb this is 8 mb. The compensation is proportional to the pressure; if it decreased to 500 mb the compensation is 4 mb.

For an error in the static pressure correction of $\partial P/q_c = \begin{matrix} +0.0005 \\ -0.0015 \end{matrix}$ we get

$$\frac{\partial P}{P} = \frac{\partial P}{q_c}\,\frac{q_c}{P} = \begin{matrix} +0.0005 \cdot 0.15 = +\ 0.000075 \\ -0.0015 \cdot 0.15 = -\ 0.000225. \end{matrix}$$

Thus the error is for

1000 mb	$\begin{matrix} +\ 0.075 \\ -\ 0.225 \end{matrix}$ mb
200 mb	$\begin{matrix} +\ 0.015 \\ -\ 0.045 \end{matrix}$ mb.

(3) Errors in Airspeed

The airspeed, AS, is computed from the dynamic pressure, q_c, by the equation

$$AS = v_s \sqrt{\frac{2}{\gamma-1}\left[\left(1 + \frac{q_c}{P}\right)^{\frac{\gamma-1}{\lambda}} - 1\right]} \qquad (1)$$

where v_s is the velocity of sound, p is the static pressure, and γ is the Poisson constant. For dry air

$$\frac{\gamma-1}{\gamma} = 0.287.$$

The velocity of sound, v_s, is

$$v_s = \sqrt{g\ \gamma RT} \tag{2}$$

with g the acceleration of gravity ($g = 9.81\ m/sec^2$)

γ the Poisson constant ($\gamma = 1.403$ for dry air)

R the gas constant ($R = 29.28\ m/°K$).

T the absolute Temperature (^{o}K).

In figure 1 the relation of airspeed to the pressure ratio q_c/p is drawn, and figure 2 shows the airspeed as a function of the Mach number, and figure 3 presents the pressure ratio q_c/p as the function of the Mach number.

Equation (1) contains 2 terms, q_c and p, that are influenced by Pitot static tube errors. Actually

$$\frac{q_c}{p} = \frac{p_p - p}{p} \tag{3}$$

where p_p is the Pitot pressure.

Figure 7 of the Rosemount report shows the Pitot pressure error. If we assume a base value of M = 0.45, the Pitot pressure error can be written as

$$p_{pm} - p_p = (-0.005 \pm 0.001)\ q_c$$

for values between M = 0.4 and M = 0.6 and for angles of attack up to 12 degrees.

If the bias error is included in the evaluation of the Pitot pressure the measuring error caused by the Pitot pressure error will be the one computed in the following discussion.

The airspeed error for an impact pressure error is given by

$$\frac{\partial AS}{\partial p_p} = \frac{v_s^2}{AS}\ \frac{1}{p}\frac{1}{\lambda}\left(\frac{p_p}{p}\right)^{-\frac{1}{\gamma}} . \tag{4}$$

Assuming an airspeed of AS = 150 m/sec, the velocity of sound v_s = 331.8 m/sec, the Poisson constant λ = 1.403, and the ratio of q_c/p = 0.15, we obtain

$$\frac{\partial AS}{\partial q_c} = \frac{331.8^2}{150} \, \frac{1}{p} \, \frac{1}{1.403} \, (1.15)^{-\frac{1}{1.403}}$$

$$\frac{\partial AS}{\partial q_c} = \frac{473.5}{p}.$$

For a Pitot error of $\partial q_c = \pm\, 0.001\, q_c$ we obtain

$$\partial AS = 473.5 \cdot 0.001 \, \frac{q_c}{p} \, .$$

If we assume q_c/p = 0.15

$$\partial AS = \pm\; 0.0.071 \text{ m/sec.}$$

In case the basic deviation of $-0.005\, q_c$ had not been taken into account, the airspeed error would be

$$\partial AS = \begin{matrix} -0.28 \\ -0.43 \end{matrix} \text{ m/sec.}$$

The influence of an error in static pressure on the airspeed is given by

$$\frac{\partial AS}{\partial p} = - \frac{v_s^{\,2}}{AS} \, \frac{q_c}{p^2} \, \frac{1}{\gamma} \, (1 + \frac{q_c}{p})^{-\frac{1}{\gamma}} . \qquad (5)$$

With the above values this amounts to

$$\frac{\partial AS}{\partial p} = - \frac{331.8^2}{150} \frac{0.15}{p} \frac{1}{1.403} (1.15)^{-\frac{1}{1.403}}$$

$$\partial AS = 71.0 \frac{\partial p}{p} .$$

With an error of $\partial p/q_c$ of $\pm$ 0.0015

$$\frac{\partial p}{p} = \frac{\partial p}{q_c} \cdot \frac{q_c}{p} = 0.0015 \cdot 0.15 = \pm 0.000225,$$

and the error in airspeed becomes

$$\partial AS = \mp 0.016 \text{ m/sec.}$$

If no correction is entered for the angles of attack, and if sideslip $\partial p/q_c$ can be up to -0.14 for angles of attack up to 10^0 and sideslip angles up to 15^0, then

$$\partial AS = 1.49 \text{ m/sec.}$$

Actually the impact pressure q_c is measured directly whereas the wind tunnel measurements provide Pitot pressure and static pressure separately. After proper compensation for bias errors and influence of angle of attack and sideslip angles it is estimated that the quasi-random error in measuring the impact pressure q_c is

$$\frac{\partial q_c}{\gamma c} = \pm 0.0015.$$

The error in airspeed is then with the above values

$$\frac{\partial AS}{\partial q_c} = \frac{v_s^2}{AS} \frac{1}{p} \frac{1}{\gamma} (1 + \frac{q_c}{p})^{-\frac{1}{\gamma}} \qquad (6)$$

$$\partial AS - 473.5 \frac{\partial q_c}{p} = 473.5 \frac{\partial q_c}{q_c} \frac{q_c}{p}$$

$$\partial AS = 473.5 \cdot 0.0015 \cdot 0.15 = 0.11 \text{ m/sec.}$$

One should remember that these are the error contributions from the Pitot static probes alone. They must be combined with the other measuring errors before an estimate of the airspeed accuracy measurement can be given. In some cases the error of the ratio q_c/p can be assessed. Then the airspeed error can be computed from

$$\frac{\partial AS}{\partial (q_c/p)} = \frac{V_s}{\gamma} \frac{\left(1 + \frac{q_c}{p}\right)^{-\frac{1}{\gamma}}}{\sqrt{\frac{2}{\gamma-1}\left[\left(1 + \frac{q_c}{p}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]}} . \tag{7}$$

For dry air this is

$$\frac{\partial AS}{\partial (q_c/p)} = \frac{v_s}{1.403} \frac{\left(1 + \frac{q_c}{p}\right)^{-0.713}}{\sqrt{4.963\left[\left(1 + \frac{q_c}{p}\right)^{0.287} - 1\right]}} .$$

Or with $v_s = 331.8 \sqrt{1 + \frac{t}{273.16}} \left[\frac{m}{sec}\right]$

$$\frac{\partial AS}{\partial (q_c/p)} = 6.423 \sqrt{T} \frac{\left(1 + \frac{q_c}{p}\right)^{-0.713}}{\sqrt{\left(1 + \frac{q_c}{p}\right)^{0.287} - 1}} . \tag{8}$$

This relation is drawn in figure 4.

For $T = 273.16$ deg. K and $q_c/p = 0.15$,

this becomes

$$\frac{\partial AS}{\partial (q_c/p)} = 455.$$

With $\partial(q_c/p) = 0.000225$ the airspeed error becomes $\partial AS = 0.10$ m/sec.

Frequently the airspeed is expressed by the Mach number M. It is

$$M = \sqrt{\frac{2}{\gamma-1}\left[\left(\frac{p_p}{p}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]} \, . \qquad (9)$$

Then the errors are for the above values

$$\frac{\partial M}{\partial p_r} = \frac{1}{M}\,\frac{1}{p}\,\frac{1}{\gamma}\left(\frac{p_p}{p}\right)^{-\frac{1}{\gamma}}$$

$$= \frac{1}{0.45}\,\frac{1}{p}\,\frac{1}{1.403}\,(1.15)^{-\frac{1}{1.403}}$$

$$\partial M = 1.43\,\frac{\partial q_c}{p} = 1.43\,\frac{\partial q_c}{q_c}\,\frac{q_c}{p}$$

$$\partial M = 1.43 \cdot 0.001 \cdot 0.15 = 0.000215 \text{ for } q_c \text{ error}$$

$$\frac{\partial M}{\partial p} = 1.43\,\frac{q_c}{p}\,\frac{\partial p}{q_c}\,\frac{q_c}{p}$$

$$\partial M = -\,1.43 \cdot 0.15^2 \cdot 0.0015 = 0.000048 \text{ for p error.}$$

If the error in the ratio of q_c/p is known, the relative error in Mach number becomes

$$\frac{(\partial M)/M}{\partial(q_c/p)} = \frac{\gamma-1}{2\gamma}\;\frac{\left(1+\frac{q_c}{p}\right)^{-\frac{1}{\gamma}}}{\left(1+\frac{q_c}{p}\right)^{\frac{\gamma-1}{\gamma}} - 1}$$

$$= 0.14362\;\frac{\left(1+\frac{q_c}{p}\right)^{-\frac{1}{\gamma}}}{\left(1+\frac{q_c}{p}\right)^{0.287} - 1} \, .$$

For $\gamma = 1.403$ this gives for

q_c/p	M	$\frac{(\partial M)/M}{\partial(q_c/p)}$
0.05	0.265	9.829
0.10	0.371	4.835
0.15	0.451	3.174
0.20	0.517	2.346
0.25	0.573	1.851
0.30	0.623	1.522

For example, an error in the ratio of q_c/p of $\partial q_c/p = 0.000225$, which means a relative error of $\partial q_c/q_c$ of 0.15% at $q_c/p = 0.15$, would result in a relative Mach error of $\partial M/M = 3.174 * 0.000225 = 0.071\%$. The same relative error holds for the airspeed, so that at an airspeed of 150 m/sec the error would be 0.11 m/sec.

(4) Conversion from Tunnel Coordinates to Flight Coordinates

In figure 2 of the Rosemount report the geometry of the test arrangement is given. Here ϕ is the incidence angle and θ is the rotational angle in the test arrangement; α is the angle of attack, and β is the angle of sideslip. It is

$$\tan\alpha = \tan\phi\cos\theta$$
$$\tan\beta = \tan\phi\sin\theta. \tag{11}$$

The following table gives the conversion for some angles

	$\theta = 30^o$		$\theta = 60^o$	
ϕ	α	β	α	β
2°	1.73°	1.00°	1.00°	1.73°
4°	3.47°	2.00°	2.00°	3.47°
6°	5.20°	3.01°	3.01°	5.20°
8°	6.94°	4.02°	4.02°	6.94°
10°	8.68°	5.04°	5.04°	8.68°
12°	10.43°	6.07°	6.07°	10.43°
14°	12.18°	7.11°	7.11°	12.18°
16°	13.95°	8.16°	8.16°	13.95°

Since only some discrete values for α and β are available, a graphic interpolation is used to obtain the pressure error as a function of α and β.

In figure 5 first a relation between α and β for constant error is drawn. From these curves figure 5 is developed and give the error as a function of β for discrete values of α.

Then a fit to this family of curves was tried. A second order fit with the origin as constraint point was found to be adequate. Thus two coefficents have to be determined. They vary with the angle of attack α and the Mach number M. Thus the coefficients C1 and C2 were drawn as a function of α for 2 Mach numbers.

Then a fit to these coefficients was made. C2 can be fitted by a straight line with the coefficient B1 and B2 while for C1 a second order fit with the coefficients A1, A2 and A3 had to be found. Since only two values of M were tested and no further information was available on the influence of the Mach number on the pressure error, a linear relationship of the A and B coefficients with M was assumed.

The result of these fittings is summarized by the following data where C is the error in static pressure for given values of an angle of attack α, an angle of sideslip β, and a Mach number M.

B1 0.5993 E-3 - (M - 0.4) * 0.3300 E-3
B2 -0.1208 E-4 + (M - 0.4) * 0.1110 E-4

A1 0.1154 E-3 + (M - 0.4) * 0.5950 E-3
A2 0.2919 E-4 - (M - 0.4) * 0.5620 E-4
A3 -0.1074 E-5 + (M - 0.4) * 0.0500 E-5

$$C_1 = A1 + A2 * \alpha + A3 * \alpha^2$$
$$C_2 = B1 + B2 * \alpha$$

$$C = C_1 * \beta + C_2 * \beta^2$$

Example:

M = 0.6 $\alpha = 10^\circ$ $\beta = 8^\circ$

A1 = 0.2345 E-3
A2 = 0.1795 E-4
A3 = -0.9740 E-6

B1 = 0.5333 E-3
B2 = -0.9860 E-5

C1 = 0.3166 E-3
C2 = 0.4347 E-3

C = 0.0340

The fits were computed on an interactive remote TELEX terminal so that iterations could easily be tested. The standard least squares fit program was expanded so that constraint points could be inserted and each input value was weighted. The program is copied in the Appendix. TESTCLS is the program to pick up the data, compute the fit, and print out the data. This program requires the subroutine CSTLSTS which in turn uses the subroutine SIMQ.

Another program POLYNOM was used to compute the values from given coefficients and determine their deviation from the fixed values.

The same data were used for both programs. In the first column are the degree of fit, the number of data points, the number of constraint points, and the coefficients for POLYNOM. The next lines contain the coordinate values, alternating x and y. Then comes a line with the x and y values of the constraint points. The remaining lines contain the weighting of each point. After some frustrating attempts to obtain usable curves for C1 and C2, we found it is not sufficient merely to compute fits to existing data. The data had to be improved until a very good fit was obtained. Finally the whole set of data for the whole family of curves was tabulated and higher order differences improved until the whole picture was smooth.

Figure 8 shows the data and the differences as they were finally used.

```
      PROGRAM TESTCLS (TAPE1, INPUT=TAPE1, OUTPUT)
      DIMENSION C(10),W(50),X(50),Y(50),XO(9),YO(9)
    1 FORMAT (1H1)
    2 FORMAT (///10E13.4//)
    3 FORMAT (//13X*X*10X*Y*7X*COMPUTED*
       6X*ERROR*5X*REL.ERROR*5X*WEIGHTING*//)
    4 FORMAT (4X,5F12.5,F13.0)
    5 FORMAT (5X,I1,1X,I3,1X,I1)
    6 FORMAT (5X,8F8.4)
      READ 5, M,N,MC
      READ 6, ((X(I),Y(I)),I=1,N)
      READ 6, ((XO(I),YO(I)),I=1,MC)
      READ 6, (W(I),I=1,N)
      CALL CSTLSTS (M,N,X,Y,W,MC,XO,YO,C,IERR)
      MP1=M+1
      PRINT 1
      PRINT 2, (C(I),I=1,MP1)
      PRINT 3
      DO 900 I=1,N
      YC=C(1)
      DO 850 J=2,MP1
      YC=YC+C(J)*X(I)**(J-1)
  850 CONTINUE
      YD=YC-Y(I)
      YP=YD/Y(I)
      PRINT 4, X(I),Y(I),YC,YD,YP,W(I)
  900 CONTINUE
      END
```

FITTING PROGRAM

```
      SUBROUTINE CSTLSTS (M,N,X,Y,W,MC,XO,YO,C,IERR)
C
C --- N E E D S   S U B R O U T I N E   S I M Q ---
C     LEAST-SQUARES APPROXIMATION TO FIT A POLYNOMIAL TO A GIVEN SET
C     OF DATA POINTS WITH CONSTRAINT POINTS
C     PARAMETERS ARE IN SINGLE PRECISION, CALCULATIONS ARE IN DOUBLE
C     M       DEGREE OF POLYNOMIAL (MIN=1, MAX=9)
C     N       NUMBER OF POINTS (MIN=M+1, MAX=5000)
C     X       X COORDINATES OF POINTS (NOT NECESSARILY EVENLY SPACED)
C     Y       Y COORDINATES OF POINTS
C     MC      NUMBER OF CONSTRAINT POINTS
C     XO,YO CONSTRAINT POINTS  (MIN=0)  (MAX=M)
C     W       1/SIGMA SQUARED OF EACH POINT
C             FOR EQUAL WEIGHTING ALL W(I)=1.0
C     C       COEFFICIENTS OF GENERATED EQUATION
C             Y = C(1) + C(2)*X + C(3)*X**2  + C(4)*X**3 + .....
C     IE      NORMALLY IE=0     IF AN ERROR OCCURRED  IE=1
C ---- MAKE SURE THAT N IS GREATER THAN M
C
      DIMENSION C(11),CI(11),E(10,11),S(18),W(N),X(N),Y(N),XO(MC),YO(MC)
      TYPE DOUBLE CI,E,S
    1 FORMAT (5X,I4,1X*POINTS INSUFFICIENT TO DETERMINE A *I1* DEGREE *
     +*POLYNOMIAL*//)
      IERR=0
      IF  (N  .GT .  M)  GOTO 100
      PRINT 1, N,M
      IERR=1
      RETURN
  100 MP1=M+1
      MP2=M+2
      MS=M+M
      IF  (MC  .EQ.  0)  GOTO 250
      DO 200  I=1,MC
      E(I,1)=1.0
      E(I,MP2)=YO(I)
      DO 200  J=2,MP1
      E(I,J)=XO(I)**(J-1)
  200 CONTINUE
  250 MD=MC+1
  280 E(MD,1)=0.0
      DO 290 L=1,N
      E(MD,1)=E(MD,1)+W(L)
  290 CONTINUE
      DO 300  K=1,MS
      S(K)=0.0
      DO 300  L=1,N
      S(K)=S(K)+(X(L)**K)*W(L)
  300 CONTINUE
      IF  (K.EQ.1)  E(MD,1)=E(MD,1)+W(L)
      DO 400  I=MD,MP1
      DO 400  J=1,MP1
      IF ((I.EQ.MD).AND.(J.EQ.1))  GOTO 400
      E(I,J)=S(I+J-MC-2)
```

```
  400 CONTINUE
      DO 500  I=MD,MP1
      E(I,MP2)=0.0
      DO 500  L=1,N
      XE=1.0
      IF (I  .NE.  MD)  XE=X(L)**(I-MD)
      E(I,MP2)=E(I,MP2)+XE*Y(L)*W(L)
  500 CONTINUE
      CALL SIMQ (MP1,E,CI,IERR)
      DO 600  I=1,MP1
      C(I)=CI(I)
  600 CONTINUE
  700 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE SIMQ(NN,E,AN,IERR)
C
C          THE LAST COLUMN OF THE AUGMENTED MATRIX, E CONTAINS THE
C          RIGHT HAND SIDE OF THE SIMULTANEOUS SYSTEM OF EQUATIONS
C
      TYPE DOUBLE E,AN
      DIMENSION E(10,11),M(10),AN(10)
      N=NN
      L=N+1
      IERR=0
      DO  1  J=1,N
    1 M(J)  =  J
      DO 11 I=1,N
      MP=M(I)
      IF(E(MP,I))3,4,3
    4 JD=I+1
      DO 21 J=JD,N
      MN=M(J)
   23 IF(E(MN,I))20,21,20
   21 CONTINUE
   24 FORMAT (38H NO SOLUTION TO SIMULTANEOUS EQUATIONS)
      IERR=1
      RETURN
   20 MP=M(J)
      M(J)=M(I)
      M(I)=MP
    3 K=L
    5 E(MP,K)=E(MP,K)/E(MP,I)
      K=K-1
      IF(K-1)6,5,5
    6 DO 11 J=1,N
    9 IF(J-I)7,11,7
    7 K=L
      MQ=M(J)
   10 E(MQ,K)=E(MQ,K)-E(MQ,I)*E(MP,K)
      K=K-1
      IF (K-I)11,10,10
   11 CONTINUE
      DO 2 J=1,N
      MP=M(J)
    2 AN(J)=E(MP,L)
      RETURN
      END
```

```
      PROGRAM POLYNOM (TAPE1,INPUT=TAPE1,OUTPUT)
      DIMENSION C(10),X(50),Y(50)
    1 FORMAT (1H1)
    2 FORMAT (///10E13.4//)
    3 FORMAT (//13X*X*10X*Y*7X*COMPUTED*
     6X*ERROR*5X*REL.ERROR*//)
    4 FORMAT (4X,5F12.5,F13.0)
    5 FORMAT (5X,I1,1X,I3,1X,I1,5(4X,E8.4))
    6 FORMAT (5X,8F8.4)
      READ 5, M,N,MC,(C(I),I=1,5)
      READ 6, ((X(I),Y(I)),I=1,N)
      MP1=M+1
      PRINT 1
      PRINT 2, (C(I),I=1,MP1)
      PRINT 3
      DO 900 I=1,N
      YC=C(1)
      DO 850 J=2,MP1
      YC=YC+C(J)*X(I)**(J-1)
  850 CONTINUE
      YD=YC-Y(I)
      YP=YD/Y(I)
      PRINT 4, X(I),Y(I),YC,YD,YP
  900 CONTINUE
      END
```

TEST PROGRAM

2	16	1	0000E+00		2520E-03		5950E-03	
0.0		0.0	1.0	0.0007	2.0	0.0026	3.0	0.0058
4.0		0.0101	5.0	0.0156	6.0	0.0223	7.0	0.0303
8.0		0.0394	9.0	0.0497	10.0	0.0613	11.0	0.0740
12.0		0.0879	13.0	0.1031	14.0	0.1194	15.0	0.1370
0.0		0.0						
1.0		1.0	1.0	1.0	1.0	1.0	1.0	1.0
1.0		1.0	1.0	1.0	1.0	1.0	1.0	1.0

DATA FILE

0. .1154E-03 .6011E-03

X	Y	COMPUTED	ERROR	REL.ERROR	WEIGHTING
0.00000	0.00000	0.00000	0.00000	I	1.
1.00000	.00070	.00072	.00002	.02353	1.
2.00000	.00260	.00264	.00004	.01351	1.
3.00000	.00580	.00576	-.00004	-.00759	1.
4.00000	.01010	.01008	-.00002	-.00208	1.
5.00000	.01560	.01560	.00000	.00027	1.
6.00000	.02230	.02233	.00003	.00142	1.
7.00000	.03030	.03026	-.00004	-.00128	1.
8.00000	.03940	.03939	-.00001	-.00018	1.
9.00000	.04970	.04973	.00003	.00054	1.
10.00000	.06130	.06126	-.00004	-.00060	1.
11.00000	.07400	.07400	.00000	.00002	1.
12.00000	.08790	.08794	.00004	.00048	1.
13.00000	.10310	.10308	-.00002	-.00015	1.
14.00000	.11940	.11943	.00003	.00025	1.
15.00000	.13700	.13698	-.00002	-.00017	1.

$\alpha = 0^0$ M = 0.4

0. .2345E-03 .5352E-03

X	Y	COMPUTED	ERROR	REL.ERROR	WEIGHTING
0.00000	0.00000	0.00000	0.00000	I	1.
1.00000	.00070	.00077	.00007	.09958	1.
2.00000	.00260	.00261	.00001	.00378	1.
3.00000	.00550	.00552	.00002	.00371	1.
4.00000	.00950	.00950	.00000	.00014	1.
5.00000	.01460	.01455	-.00005	-.00324	1.
6.00000	.02070	.02067	-.00003	-.00123	1.
7.00000	.02790	.02787	-.00003	-.00119	1.
8.00000	.03610	.03613	.00003	.00081	1.
9.00000	.04550	.04546	-.00004	-.00082	1.
10.00000	.05590	.05587	-.00003	-.00061	1.
11.00000	.06740	.06734	-.00006	-.00089	1.
12.00000	.07990	.07988	-.00002	-.00020	1.
13.00000	.09350	.09350	-.00000	-.00001	1.
14.00000	.10810	.10818	.00008	.00078	1.
15.00000	.12390	.12394	.00004	.00032	1.

$\alpha = 0^0$ M = 0.6

$\alpha = 5^0$ M = 0.4

0. .2999E-03 .4803E-03

X	Y	COMPUTED	ERROR	REL.ERROR	WEIGHTING
0.00000	0.00000	0.00000	0.00000	I	1.
1.00000	.00070	.00078	.00008	.11450	1.
2.00000	.00260	.00252	-.00008	-.03046	1.
3.00000	.00520	.00522	.00002	.00422	1.
4.00000	.00890	.00888	-.00002	-.00184	1.
5.00000	.01360	.01351	-.00009	-.00693	1.
6.00000	.01910	.01909	-.00001	-.00060	1.
7.00000	.02560	.02563	.00003	.00124	1.
8.00000	.03310	.03314	.00004	.00107	1.
9.00000	.04160	.04160	-.00000	-.00001	1.
10.00000	.05100	.05102	.00002	.00047	1.
11.00000	.06140	.06141	.00001	.00015	1.
12.00000	.07270	.07276	.00006	.00076	1.
13.00000	.08500	.08506	.00006	.00072	1.
14.00000	.09840	.09833	-.00007	-.00073	1.
15.00000	.11260	.11256	-.00004	-.00040	1.

$\alpha = 5^0$ $M = 0.6$ $\alpha = 10^0$ $M = 0.4$

0. .3166E-03 .4366E-03

X	Y	COMPUTED	ERROR	REL.ERROR	WEIGHTING
0.00000	0.00000	0.00000	0.00000	I	1.
1.00000	.00070	.00075	.00005	.07605	1.
2.00000	.00240	.00238	-.00002	-.00846	1.
3.00000	.00490	.00488	-.00002	-.00420	1.
4.00000	.00830	.00825	-.00005	-.00574	1.
5.00000	.01250	.01250	-.00000	-.00012	1.
6.00000	.01760	.01762	.00002	.00102	1.
7.00000	.02360	.02361	.00001	.00045	1.
8.00000	.03050	.03048	-.00002	-.00077	1.
9.00000	.03820	.03822	.00002	.00041	1.
10.00000	.04680	.04683	.00003	.00060	1.
11.00000	.05630	.05631	.00001	.00024	1.
12.00000	.06670	.06667	-.00003	-.00041	1.
13.00000	.07790	.07790	.00000	.00006	1.
14.00000	.09000	.09001	.00001	.00011	1.
15.00000	.10300	.10299	-.00001	-.00011	1.

$\alpha = 10^0$ $M = 0.6$

```
2  11  0    1154E-3     2919E-4      -1074E-5
 0.0      1154E-3   1.0      1435E-3     2.0       1695E-3 3.0       1933E-3
 4.0      2150E-3   5.0      2345E-3     6.0       2519E-3 7.0       2671E-3
 8.0      2802E-3   9.0      2911E-3    10.0       2999E-3
 0.0      0.0
 1.0      1.0       1.0      1.0         1.0       1.0      1.0      1.0
 1.0      1.0       1.0      1.0         1.0       1.0      1.0      1.0
```

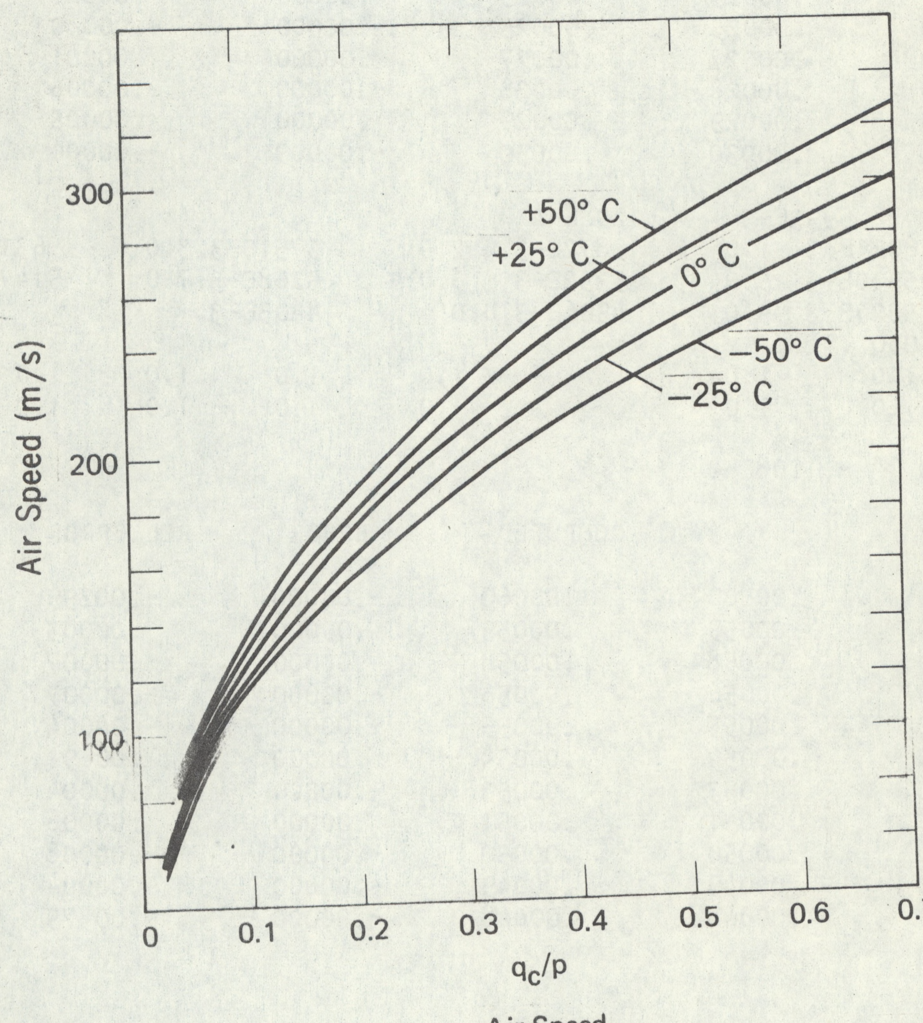
.1154E-03   .2919E-04   -.1074E-05

X	Y	COMPUTED	ERROR	REL.ERROR
0.00000	.00012	.00012	0.00000	0.00000
1.00000	.00014	.00014	.00000	.00011
2.00000	.00017	.00017	-.00000	-.00009
3.00000	.00019	.00019	.00000	.00002
4.00000	.00022	.00021	-.00000	-.00011
5.00000	.00023	.00023	-.00000	-.00000
6.00000	.00025	.00025	-.00000	-.00010
7.00000	.00027	.00027	.00000	.00001
8.00000	.00028	.00028	-.00000	-.00006
9.00000	.00029	.00029	.00000	.00005
10.00000	.00030	.00030	-.00000	-.00000

```
1  11  0    5993E-3     -1208E-4
 0.0      6011E-3   1.0      5872E-3     2.0       5751E-3 3.0       5631E-3
 4.0      5510E-3   5.0      5352E-3     6.0       5268E-3 7.0       5147E-3
 8.0      5027E-3   9.0      4906E-310.0           4803E-3
 0.0      0.0
 1.0      1.0       1.0                  1.0       1.0      1.0      1.0
 1.0      1.0       1.0                  1.0       1.0      1.0      1.0
```

.5993E-03   -.1208E-04

X	Y	COMPUTED	ERROR	REL.ERROR
0.00000	.00060	.00060	-.00000	-.00299
1.00000	.00059	.00059	.00000	.00003
2.00000	.00058	.00058	.00000	.00007
3.00000	.00056	.00056	-.00000	-.00007
4.00000	.00055	.00055	-.00000	-.00004
5.00000	.00054	.00054	.00000	.00691
6.00000	.00053	.00053	.00000	.00004
7.00000	.00051	.00051	.00000	.00008
8.00000	.00050	.00050	-.00000	-.00008
9.00000	.00049	.00049	-.00000	-.00004
10.00000	.00048	.00048	-.00000	-.00375



300
200
100
Air Speed (m/s)
+50° C
+25° C
0° C
−50° C
−25° C
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
q_c/p
Air Speed

Figure 1

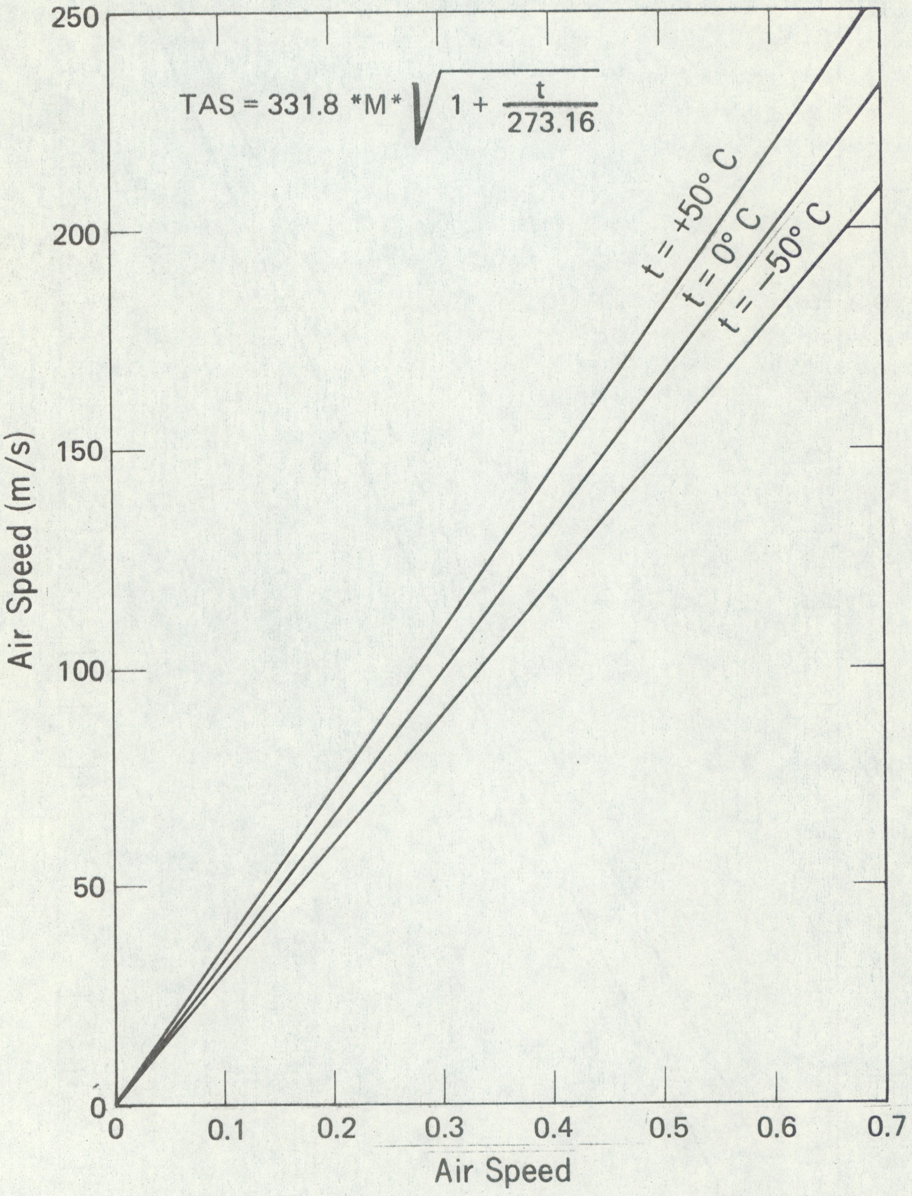
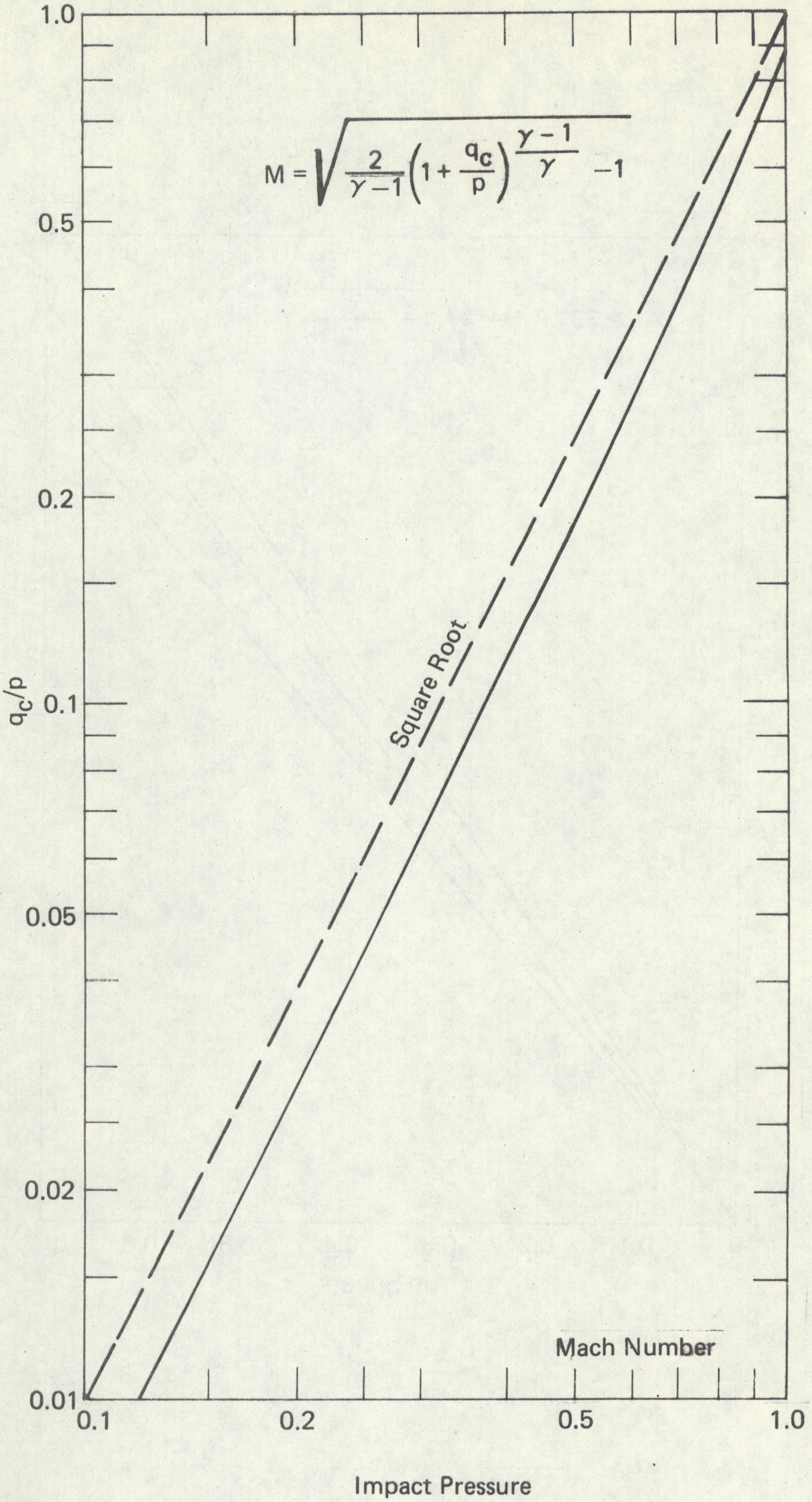

250
200
150
100
50
0
TAS = 331.8 *M* $\sqrt{1 + \frac{t}{273.16}}$
t = +50° C
t = 0° C
t = −50° C
Air Speed (m/s)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
Air Speed

Figure 2



$$M = \sqrt{\frac{2}{\gamma-1}\left(1+\frac{q_c}{p}\right)^{\frac{\gamma-1}{\gamma}} - 1}$$
1.0
0.5
0.2
0.1
0.05
0.02
0.01
q_c/p
Square Root
Mach Number
0.1
0.2
0.5
1.0
Impact Pressure

Figure 3

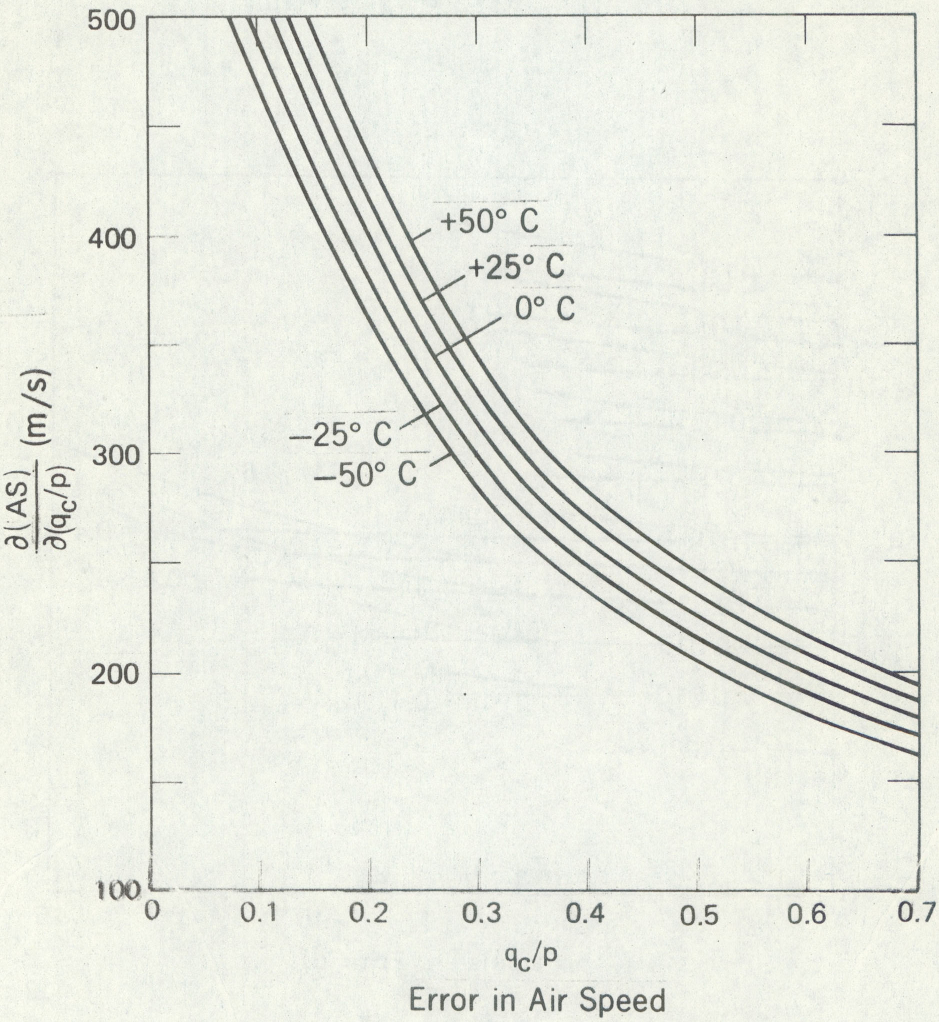
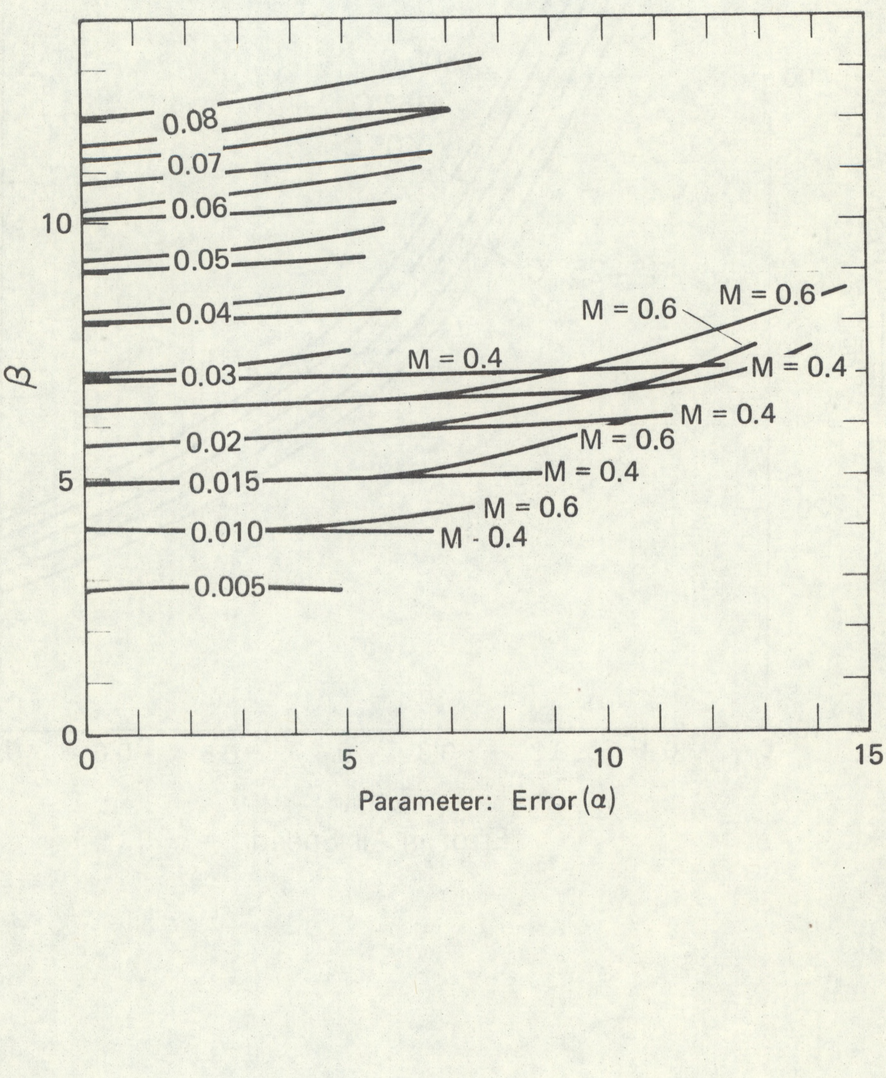

500
400
300
200
100
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
+50° C
+25° C
0° C
−25° C
−50° C
$\frac{\partial(AS)}{\partial(q_c/p)}$ (m/s)
q_c/p
Error in Air Speed

Figure 4



0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.015
0.010
0.005
M = 0.6
M = 0.6
M = 0.4
M = 0.4
M = 0.4
M = 0.6
M = 0.4
M = 0.6
M - 0.4
β
0
5
10
0
5
10
15
Parameter: Error (α)

Figure 5

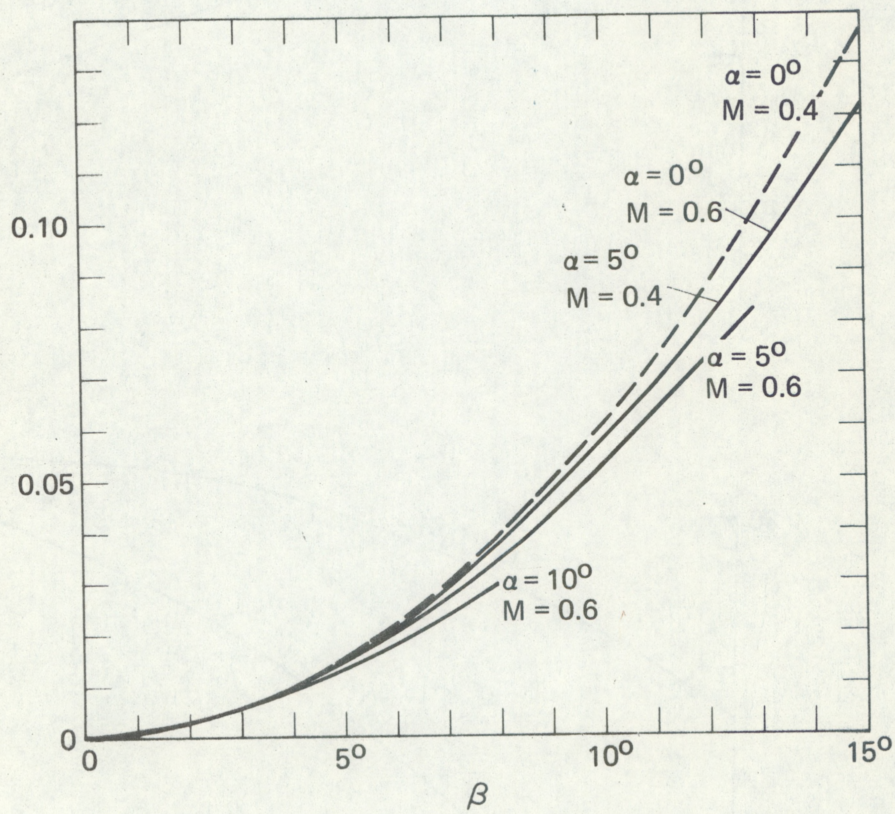


Figure 6

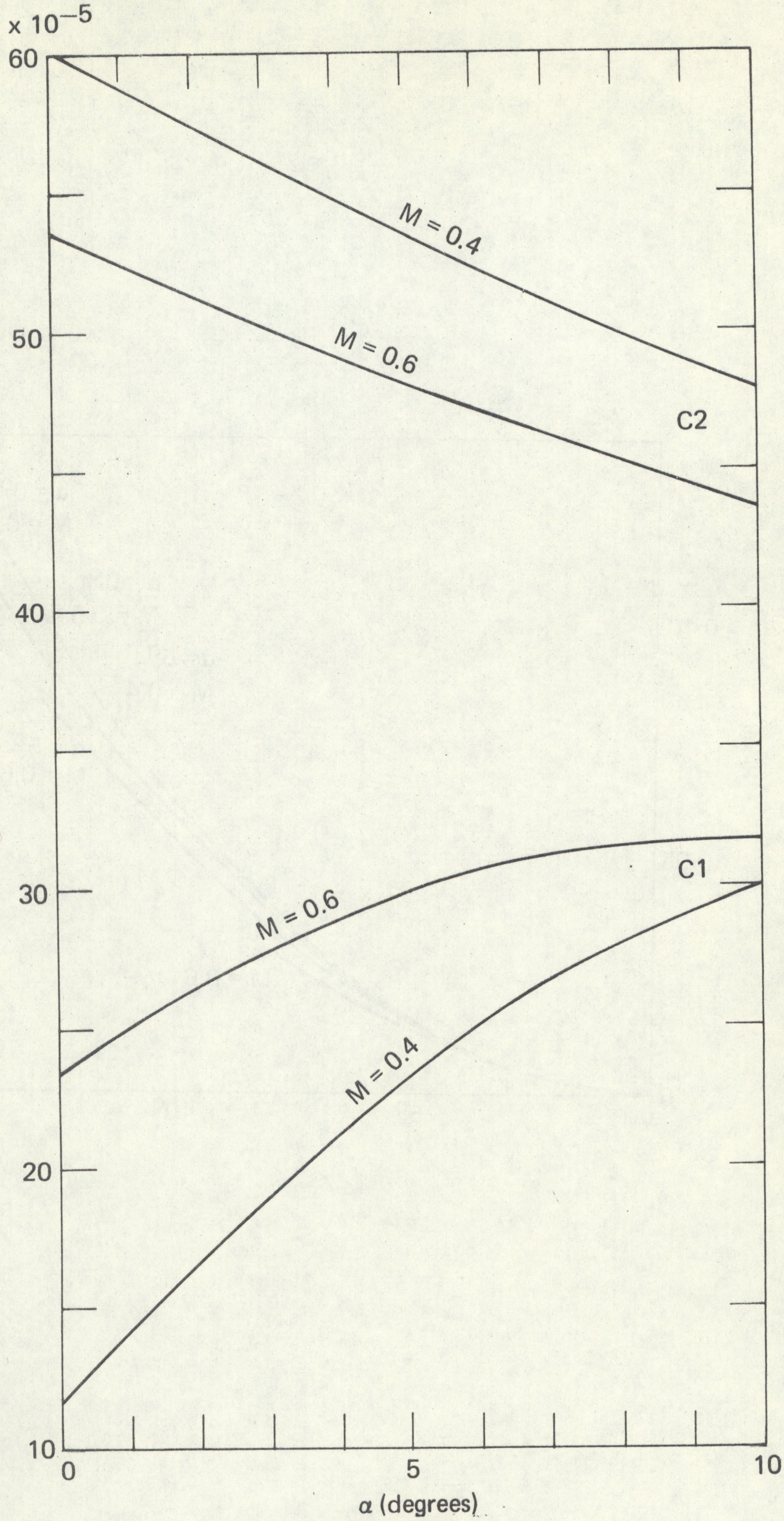


Figure 7 Coefficients

	FIT DATA	DATA	DAT	DA
1	7	7	7	7
2	26<	26<	26<	24<
	32<	29<	26<	25<
3	58< 9	55< 11	52< 11	49< 9
	43<	40<	37<	34<
4	101< 12	95< 11	89< 10	83< 8
	55<	51<	47<	42<
5	156< 12	146< 10	136< 8	125< 9
	67<	61<	55<	51<
6	223< 13	207< 11	191< 10	176< 9
	80<	72<	65<	60<
7	303< 11	279< 10	256< 10	236< 9
	91<	82<	75<	69<
8	394< 12	361< 12	331< 10	305< 8
	103<	94<	85<	77<
9	497< 13	455< 10	416< 9	382< 9
	116<	104<	94<	86<
10	613< 11	559< 11	510<10	468< 9
	127<	115<	104<	95<
11	740< 12	674< 10	614< 9	563< 9
	139<	125<	113<	104<
12	879< 13	799< 11	727< 10	667< 8
	152<	136<	123<	112<
13	1031< 11	935< 10	850< 11	779< 9
	163<	146<	134<	121<
14	1194< 13	1081< 11	984< 9	900< 9
	176	157	143	130
15	1370<	1239<	1127<	1030<

Figure 8