1	Forecasting for recreational fisheries management: What's the catch?
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#### 23 Abstract

The re-authorized Magnuson-Stevens Fishery Conservation and Management Act 24 required regional fishery management councils to implement annual catch limits (ACLs) for 25 nearly all stocks under U.S. federal management. Since 2011, the number of stocks requiring 26 ACLs (and monitoring) has increased nearly tenfold, with strict accountability measures 27 requiring either in-season quota closures or shortening of subsequent seasons to avoid ACL 28 overages. Robust forecasts of catch can also provide a projected baseline for evaluation of 29 proposed management alternatives. We compared generalized linear models (GLM), generalized 30 31 additive models (GAM), and seasonal auto-regressive integrated moving average (SARIMA) models in terms of fit, accuracy, and f ability to forecast catches of four representative fish stocks 32 supporting recreational fisheries in the southeastern United States. All models were useful in 33 developing reliable forecasts to inform management. GAM models provided the best fit the 34 observed data; however, the SARIMA and GLM modeling approaches provided the best 35 36 forecasts for most scenarios. SARIMA and GLM also provided the best predictions of the seasonal trend in landings, a desirable feature for in-season quota monitoring. SARIMA was 37 more sensitive and GLM was less sensitive to recent trends, providing a useful bookend for 38 39 forecasts. The time span of input data affected forecast accuracy from all model types considered. This study suggests multiple forecasting models should be investigated, with 40 41 performance metrics carefully selected and evaluated, as no single model is likely to perform 42 best for all stocks of interest.

43

## 44 Introduction

The Magnuson-Stevens Fishery Conservation and Management (MSA) (U.S. Congress, 45 2006) requires regional fishery management councils to specify annual catch limits (ACLs) at a 46 level such that overfishing does not occur. Annual catch limits are required for all stocks under 47 U. S. federal management, except stocks with annual life cycles and those managed by 48 international agreement in which the U.S. participates. This provision was implemented in 2010 49 or earlier for stocks subject to overfishing, and in 2011 for all other federally-managed stocks. 50 This requirement results in a nearly tenfold increase in in the number of ACLs must be 51 52 monitored (from 2012 forward) relative to previous years (NMFS, 2014). To address this challenge, methods for forecasting fisheries catches and projecting season lengths to avoid ACL 53 overages are needed. Reliable forecasting methods are needed especially for recreational 54 fisheries in the southeastern region of the U.S. In this region, recreational landings comprise the 55 majority of total landings for many species (Coleman et al. 2004) yet, do have only limited in-56 season harvest information available (i.e., data available in two-month waves after 45 day delay 57 for each wave) that are often inadequate for current management needs. . 58 Forecasting fish landings is a critical element in the management of fisheries stocks 59 60 because it can inform strategy development and policy decisions (Thorson et al. 2014; Makridakis and others 2008; Hanson et al. 2006; Stergiou and Christou, 1996). Forecasts can be 61

62 used to apply in-season or post-season accountability measures and also to provide a baseline for

63 forecasting the impacts of proposed management actions. To date, forecasting applications in

64 fishery management applications are limited. Thorson and et al. (2014) evaluated a suite of

models across > 2,000 vertebrate taxa and provided some general guidance. In the U.S. South

66 Atlantic and Gulf of Mexico, Hanson et al. (2006) evaluated three models used to forecasts

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67 annual landings of Atlantic Menhaden and found that multiple regression and artificial neural networks could be used for this long-term commercial fishery. Forecasts of Brown Shrimp 68 growth and production are also forecasted in the Gulf of Mexico based on environmental 69 70 conditions in estuaries (Adamack et al. 2012). To be useful, appropriate methodologies need to be developed and evaluated, weighing the tradeoffs of model complexity, performance, and the 71 ability to inform management (Tsitsika and others 2007). Approaches to forecasting fish 72 landings are varied but generally fall into four broad categories: 1) using the previous year's 73 landings, 2) population dynamics models, 3) correlation-based regression models, and 4) time-74 75 series models.

Population dynamics models are advantageous because they attempt to characterize 76 factors affecting abundance, productivity, and growth potential of a stock (Hilborn and Walters, 77 78 1992; Buckland and others 2004; Newman and others 2006). Unfortunately, these models are data intensive, and require substantial time, effort, and resources to develop (Thorson et al. 79 2014). Due to these limitations, stock assessment models are only developed every 3-5 years for 80 economically important species in the Southeastern U.S. For many federally managed species, 81 adequate data are unavailable and resources are insufficient to develop population dynamics 82 assessment models (Berkson and Thorson, 2015; Carruthers et al. 2014). Moreover, when 83 forecasting is the primary objective, population dynamics models are not necessarily superior to 84 other less intensive methods as they require estimates of many parameters and have a tendency 85 to overfit, limiting their forecasting performance (Ward et al. 2014; Clark, 2004). 86

87 Correlation-based regression models (e.g., linear models) have been used successfully to
88 predict menhaden landings in the U.S. Atlantic and Gulf of Mexico since at least 1975 (Schaaf
89 and others 1975) and were used for more than three decades to produce annual forecasts of

landings (Hanson and others 2006). However, landings for many species follow non-linear 90 trajectories where the response variable may be more appropriately modeled using non-Gaussian 91 error distributions (Ward et al. 2014). Generalized linear models (GLMs: Nelder and 92 93 Wedderburn, 1972) are extensions of linear models that can accommodate response variables following exponential family distributions (e.g., Poisson, negative binomial) and may be superior 94 to linear models for modeling fish landings data. Generalized additive models (GAMs; Wood, 95 2006) extend the GLM by allowing non-parametric relationships between the response and 96 explanatory variables (Wood, 2003). Rigorous routines for model selection and validation may 97 prevent overfitting than occur with these models (Zuur and others 2010). Most correlation-based 98 methods do not account for time explicitly in the model, although some methods may provide 99 this capability (e.g., generalized estimating equations). If covariates are used, a determination of 100 101 future values of covariates is required to develop a forecast. In some cases, this can be quite realistic (e.g., landings restriction due to closed season); however, in other cases it may be 102 difficult or impossible to predict (e.g. environmental conditions). 103 104 Time series models are conceptually simple and popular tools for forecasting. Seasonal auto-regressive integrated moving average (SARIMA) models can be constructed using only the 105 information contained in the series (Dennis and others 1991; Holmes, 2001; Ives and others 106 2010) and aim to describe the autocorrelation in these data (Hyndman and Athanasopoulos 2014; 107 Ward et. al 2014). More simply, this can be thought of as a multiple regression model with 108 lagged values as covariates. These models are flexible and assume that future conditions are 109 110 similar to the past conditions that generated these observed data. SARIMA models assume that the time series is stationary with stable variance throughout the time period. Unfortunately, these 111

112	assumptions are frequently violated with fisheries data, although this can often be resolved
113	through differencing and/or transformation (Box and others 2013).
114	The purpose of this study was to evaluate a suite of approaches to produce short-term
115	forecasts at two-month intervals (i.e., 'waves') necessary to inform fisheries management
116	decisions for U.S. federally managed species in the Gulf of Mexico and Atlantic Ocean.
117	Specifically, we considered approaches that could be fit with minimal data (e.g., landings data)
118	and applied to a range of species with varied life histories and fisheries characteristics. We used
119	four representative fish stocks/stock complexes supporting recreational fisheries that are
120	currently managed by the South Atlantic or Gulf of Mexico Fishery Management Councils and
121	compared the performance of GLMs, GAMs, and SARIMA in terms of model fit, accuracy, and
122	forecasting ability. The goal of these approaches was to develop reliable methods for predicting
123	timing of in-season closures to avoid exceeding an ACL and predicting total annual landings in
124	the absence of a quota closure.

125

#### 126 Materials and Methods

#### 127 *Recreational Fisheries Catch Data*

Recreational landings data were obtained from the NMFS Southeast Fisheries Science Center (SEFSC) ACL Dataset (accessed May 2013), which provided aggregated landings data from 1986-2012 from the Marine Recreational Fisheries Statistics Survey (MRFSS), the Southeast Headboat Survey (HBS), and the Texas Parks and Wildlife Department (TPWD) Creel Survey. Landings data from the various surveys are provided in both numbers and pounds. The ACL dataset provides improved quality assurance and quality control on the raw data generated by each of these surveys; for example, the ACL dataset implements a hierarchical procedure to

- backfill missing weight estimates from MRFSS (now MRIP;
- 136 <u>http://www.st.nmfs.noaa.gov/recreational-fisheries/index</u>). In short, samples are aggregated
- upward (i.e., wave, mode) to ensure adequate sample size (i.e., >= 30).

The MRFSS intercepts collect data on port agent observed landings ('A' catch) and 138 angler reported landings ('B1' catch) and discards ('B2' catch) in numbers by species, two-139 month 'wave' (e.g., Wave 1 = Jan/Feb, ..., Wave 6 = Nov/Dec), area fished (inland, state, and 140 federal waters), mode of fishing (charter, private/rental, shore), and state (North Carolina to 141 Louisiana). These dockside intercepts are expanded using effort data collected via telephone 142 surveys (private/rental: random digital dial during each wave; for-hire: weekly 10% random 143 sample). In 2012, MRFSS was nominally replaced by the Marine Recreational Information 144 Program (MRIP). In 2013, the MRFSS survey methodology was modified by MRIP, resulting in 145 146 some changes that are still being calibrated by SEFSC. Thus, MRIP values from 2013 forward were not considered for this modeling exercise. 147 Landings of headboats (i.e., recreational vessels where customers pay "by the head") are 148 149 calculated using a combination of logbook reports and dockside sampling, and adjustments to landings are made based on underreporting and misreporting determined through dockside 150 validation by port agents. Southeast Headboat Survey 151 (http://www.sefsc.noaa.gov/labs/beaufort/sustainable/headboat/) fishing records contain trip-152 level information on number of anglers, trip duration, date, area fished, landings (number of fish) 153

- and releases (number fish) by species.
- The TPWD Creel Survey (<u>https://tpwd.texas.gov/fishboat/fish/didyouknow/creel.phtml</u>)
  generates estimates of landings for private/rental boats and charter vessels fishing off Texas.
  TPWD conducts a stratified random angler-intercept survey at specified boat-access sites

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158	throughout the year. TPWD landings are reported in numbers by 'high-use' (May 15-November
159	20) and 'low-use' time periods (November 21-May 14), area fished (state and federal waters),
160	and mode (charter, private/rental). TPWD high and low use landings estimates are re-estimated
161	by NMFS personnel to correspond to MRFSS two-month waves.
162	Landings time series for three recreationally-important stocks and one incidentally-
163	caught stock complex with relatively simple management histories were assembled. Landings
164	for vermilion snapper (Rhomboplites aurorubens), and Gray Snapper (Lutjanus griseus)
165	managed by the GMFMC as well as Red Porgy (Pagrus pagrus) and the 'Grunts' complex
166	managed by the SAFMC were computed as the sum of MRFSS, HBS, and TPWD landings by
167	year and wave. The SAFMC 'Grunts' complex contains White Grunt (Haemulon plumierii),
168	Margate (Haemulon album), Sailor's Choice (Haemulon parra), and Tomtate (Haemulon
169	aurolineatum); most of these stocks are incidentally caught on trips targeting other species.
170	During the years considered for this analysis, none of these stocks were subject to quota closures.
171	Management histories were reconstructed for all four species to account for the timing of
172	federal recreational quota closures and closed seasons. For projection purposes, all recreational
173	landings were assigned to two-month waves. Model inputs for each species were expressed as
174	catch (in pounds whole weight) per open day (catch was assumed equal for all open federal days
175	within a wave). Expressing catch as a daily rate was important for determining the date a catch
176	limit might be exceeded and also for handling any closures in the management history of the
177	stock. As states adopted compatible seasonal regulations for the species of concern, all landings
178	were assumed to occur within the federal season; thus, the federal open days by wave were used
179	as the divisor for computing wave-specific catch-per-day. To reduce prediction bias associated
180	with reductions in catches due to fisheries closures in the Gulf of Mexico following the

181	Deepwater Horizon/BP oil spill in April 2010, values for April-December 2010 in the Gulf of
182	Mexico were recomputed as the average of 2009 and 2011 values for the same time period.
183	Shorter duration events such as hurricanes and red tides were not considered.
184	Modeling approach
185	Time series of recreational harvest for each species were fitted using GLMs (Hardin and
186	Hilbe, 2007), GAMs (Wood, 2006), and seasonal autoregressive integrated moving average
187	models (SARIMA;(Box et al., 2013)). Projected catch per day by wave was projected in pounds
188	instead of numbers because the ACLs for these stocks are specified in pounds.
189	Generalized Linear Model
190	Long-term and seasonal trends in the catch-per-day time series were captured using a
191	GLM, fit with Proc GENMOD in SAS v9.2 software (SAS Institute, Inc., 2000). Mean catch-
192	per-day (lbs) was dependent upon a linear predictor of year and a quadratic predictor of wave,
193	which were linked via a log link function with a negative binomial response error distribution
194	(Nelder and Wedderburn, 1972). Residual diagnostics and Akaike's Information Criterion
195	(AIC; Akaike, 1974) values were used to select the final model configurations.
196	Generalized Additive Model
197	Generalized additive models were also fit to each time series. Mean catch per day (lbs)
198	was predicted using a cubic-spline smoother (s) for the main effects (year) and a tensor product
199	spline (te) (De Boor and others 1978) for the interaction term (wave, year) (Gasper and others
200	2013). GAMs were fit using the mgcv library (Wood, 2006) in R v.3.0.2 (R Development Core
201	Team, 2013). Backward selection was used to determine if predictors or interactive effects could
202	be removed without compromising model performance. AIC and a log-likelihood ratio test were
203	used to determine whether more complex models were warranted (Froeschke and others 2012).

## 204 Seasonal ARIMA Model

Time series exhibiting a long-term trend and a seasonal trend may be well-suited to a 205 SARIMA model (Box et al., 2013). In a SARIMA  $(p,d,q)^*(P,D,Q)$  model, the auto-regressive 206 207 component (p) represents the lingering effects of previous observations, the integrated component (d) represents temporal trends, and the moving average component (q) represents 208 lingering effects of previous random shocks (or error). SARIMA models were implemented 209 using Proc ARIMA in SAS v9.2 (SAS Institute, Inc., Cary, NC). All possible combinations of 210 single-difference SARIMA models for catch-per-day by wave were considered (Table A1). A 211 single-difference SARIMA model only considers a maximum of one differencing term in the 212 annual and one differencing term in the seasonal component. All SARIMA models were fit 213 using conditional least squares. Stationarity tests were used to guide differencing selection. 214 215 Final SARIMA model selection was guided by examination of autocorrelations, inverse autocorrelations, partial autocorrelations, cross-correlations, residual diagnostics, and AIC. 216 *Model evaluation and performance* 217 Time series of three different lengths (i.e., 1999-2011, 2004-2011, and 2007-2011) were 218

compared in terms of model fit and forecasting performance. Exploring time series of varying 219 lengths is important as stocks vary in the period for which reliable catch data exists and this 220 approach permits a mechanism to examine trade-offs with model complexity across time series 221 of different lengths that are not confounded by indidual species effects. Although data were 222 available prior to 1999, preliminary projections suggested model performance was occasionally 223 improved by truncating the time series but not by extending it to prior to 1999. To evaluate 224 forecast utility, we evaluated the proportion of variation explained by the covariates  $(R^2)$ , and the 225 226 mean error (i.e., observed - fitted values) for the final year of data. For Atlantic stocks, we also

removed the terminal year from the time series (i.e., 'drop-one'), re-fit the model to 2004-2010 227 data and predicted landings for 2011 to provide a more robust evaluation of forecast performance 228 by using the fitted model to forecast beyond the data that were used to build the model and more 229 230 closely simulate how these models would be used in practice by resource managers. The deviance between the forecast and the actual landings in the final year provided an additional 231 estimate of accuracy. This 'drop-one' approach was only applied to Atlantic stocks due to the 232 confounding impact of having up to 36.6% of the Gulf of Mexico EEZ closed to fishing in 2010 233 due to the Deepwater Horizon/BP Oil Spill. Finally, a variation on the 'drop-one' approach was 234 applied to all four stocks by plotting cumulative landings time series to evaluate model fits from 235 1999-2011, 2004-2011, and 2007-2011 data relative to observed values in 2011 and model 236 forecasts relative to observed values in 2012. A simple approach of using the previous year's 237 238 landings as a forecast was also explored for all scenarios. As SARIMA uses a Gaussian error 239 structure and permits negative forecast values, all SARIMA-based predictions of negative catches within a wave were converted to zeroes for these comparisons. 240

241 **Results** 

Most stocks exhibited long-term trends as well as seasonal periodicity in landings. Catch was typically lowest during winter (i.e., Waves 1 and 6) and peaked during summer (i.e., Waves 3-4). Model statistics are provided in Table 1. For the longer time series (i.e., 1999-2011, 2004-2011), a SARIMA (0,1,1)x(0,1,1) structure fit the data best of the different SARIMA models considered; meaning the data were differenced at the previous time step and the seasonal timestep, and a moving average term was used on both to fit the data. In the shortest time series evaluated (i.e., 2007-2011), a SARIMA (1,1,0)x(0,1,1) structure fit the data best of the different

SARIMA models considered, indicating an autoregressive term did better capturing the trendwith a limited set of data than a moving average.

251 *Gulf of Mexico* 

From 1999 to 2011, vermilion snapper landings peaked during summer each year and 252 total annual landings increased during the time series (Figures 1, 2). All modeling approaches 253 captured this pattern after appropriate model fitting and selection routines. For vermilion 254 snapper, R<sup>2</sup> increased with shorter time series for all models and the GAM model provided the 255 256 best fit to the observed data (Table 1). Examining the mean error during the final year of data indicated the SARIMA model fits were much closer to the observed values, with the lowest 257 mean error in the final year provided by the shortest time series (2007-2011; SARIMA: 162.66, 258 GAM: 735.01, GLM: 607.86 lb/day). This time series was non-stationary with increasing catch 259 260 rates toward the end of the period. Of the models considered, SARIMA most closely captured this pattern in the observed data, and only under-estimated landings by 5% for the 2007-2011 261 input time series. The other modeling approaches resulted in much higher under-estimation of 262 263 total landings (19-53%), with GLM showing the greatest fluctuation in accuracy dependent upon input time series. 264

From 1999 to 2011, Gray Snapper landings peaked during summer each year and total annual landings increased and then decreased during the time series (Figures 1, 3). As with vermilion snapper, all three models captured this pattern and R<sup>2</sup> increased with shorter time series for all three model approaches; the GAM was the best fit to the observed data (Table 1). In terms of explained variance, the GLM and SARIMA models were comparable although SARIMA had lower mean error in the final year of the times series. The lowest mean error in the final year was provided by the shortest time series (2007-2011; SARIMA: 59.31, GAM:

650.30, GLM: 463.01 lb/day). The time series was non-stationary; however, the greatest annual
landings of Gray Snapper occurred in the middle third of the time series. Fits of regression
models to the final year in the time series were highly dependent on the input time series
selected, and in most cases were outperformed by the previous year's landings. In the shortest
time series considered (i.e., 2007-2011), the SARIMA provided the best fit, overestimating
cumulative landings by only 2%. The SARIMA model produced negative catch predictions for
some waves (Figure 3).

279 *Atlantic* 

Red porgy annual landings were relatively stable from 1999-2012 with the exception of a 280 trough in 2000 and a peak in 2007 (Figure 1). Atlantic Red Porgy displayed a distinct seasonal 281 pattern with catch rates peaking during summer each year (Figure 4) and this pattern was 282 283 captured by all models. Model fits improved with shorter time series and the GAM model provided the best fit to the observed data (Table 1). Examining the mean error during the final 284 year of data indicated the SARIMA model fits were much closer to the observed values. The 285 286 lowest mean error in the final year was provided by the middle time series (2004-2011; SARIMA: 10.51, GAM: 43.33, GLM: 20.41 lb/day). Of the models considered, SARIMA most 287 closely captured the inter-annual pattern in the observed data and model fits to the 2004-2011 288 time series underestimated 2011 cumulative landings by only 1%. 289 From 1999 to 2011, 'Grunts' complex landings peaked during summer each year and 290 total annual landings were relatively stable over the study period (Figure 1, 5). Similar to the 291

other species examined, the explained variance in annual landings of 'grunts' for all models

increased with shorter time series and the GAM provided the best fit to the observed data (Table

1). In terms of explained variance, the GLM and SARIMA models were comparable although

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SARIMA was more accurate than GLM when comparing the fitted and observed values in the
final year of the times series. The lowest mean error in the final year was provided by GAM in
the middle time series (2004-2011; SARIMA: 368.10, GAM: 4.38, GLM: 119.34 lb/day). There
was a spike HBS and MRFSS landings in Wave 3, 2007 that was not captured by any models.

299 Forecast and Summary

The trend for the 'Grunts' complex was dynamic (see Figure 5). The drop-one scenario 300 model fits to the final year were excellent for SARIMA (only 7% error), but model predictions 301 from SARIMA were poor (a 67% underestimate). Both SARIMA and GAM overweighted the 302 long-term decline in landings (Figure 6). For the Atlantic 'Grunts' complex the most accurate 303 prediction was provided by the previous year's landings. For Red Porgy, SARIMA provided the 304 best model fit to the final year of data (a 6% overestimate) and the best forecast accuracy (a 4% 305 306 overestimate). Catch levels in 2011 for both stocks were within the long-term range of previous catch levels. 307

Examination of model fits to cumulative observed landings for 2011 indicated that in 9 of 308 309 12 scenarios, model fits from SARIMA were superior to GLM and GAM (closer to observed values; Figure 7). SARIMA confidence intervals were much larger than confidence intervals for 310 fitted GLM or GAM models (Figure 7). For SARIMA models, the confidence interval contained 311 the observed values in all twelve scenarios examined whereas the confidence intervals estimated 312 using GLM and GAM did not always contain the observed values. A comparison of the percent 313 deviation from the observed cumulative landings trend by wave, across stocks and time series, 314 indicated that GLM provided the best overall model fits  $(0.2\% \pm 36.2\% \text{ error; mean} \pm \text{SD})$ , 315 followed by SARIMA (5.7%  $\pm$  76.7%). GAM and the previous year's landings provided similar 316 overall predictive error (13.9%  $\pm$  31.1%, 13.2%  $\pm$  43.3%, respectively). One undesirable feature 317

of SARIMA is that declining trends in landings during a given wave may be forecast as zero or
negative landings, as observed with Gulf Gray Snapper (Figure 3). In this study, negative
forecasts were replaced with zeroes; however, it may be preferable to substitute the most recent
year's landings for that wave to avoid underestimating harvest. This approach reduced mean
error from SARIMA predictions by wave, across stocks and time series by nearly half (from
5.7% to 2.9%).

Total 2012 landings for Gulf vermilion snapper were 23% lower than 2011 landings; 324 whereas total 2012 landings for the other stocks evaluated were 35-42% higher than 2011 values. 325 Examination of model forecasts to cumulative observed landings for 2012 indicated that in 5 of 326 12 scenarios, mean forecast values of SARIMA were closest to observed values predictions 327 (Figure 8). In 5 of the remaining 7 scenarios, GLM provided the best predictions (Figure 8). For 328 329 Gulf Gray Snapper, SARIMA provided the best prediction using the 2007-2011 time series (8% error). For Gulf vermilion snapper, GLM provided the best prediction using the 2004-2011 time 330 series (-1% error). For the Atlantic 'Grunts' complex, the best predictions were obtained from 331 332 SARIMA and GLM using the 1999-2011 time series (-4% and +4% error in the cumulative landings prediction, respectively). For Atlantic Red Porgy, the best prediction was from GLM 333 using the 1999-2011 time series (-16% error). SARIMA confidence intervals were much larger 334 than confidence intervals for fitted GLM or GAM models, indicating greater uncertainty (Figure 335 8). SARIMA tended to be more responsive to short-term trends in catch that deviate from the 336 long-term average trend. For SARIMA models, the confidence interval contained the observed 337 values in all twelve scenarios examined whereas the confidence intervals estimated using GLM 338 and GAM did not always contain the observed values. Overall, SARIMA fits to seasonal 339 340 patterns were less biased but all model fits became more similar as the length of the input time

341	series was reduced. In the 12 scenarios explored, at least one regression-based approach
342	provided a superior prediction relative to using the previous year's landings.
343	A graphical comparison of the model fitting and forecasting performance of GLM, GAM,
344	and SARIMA models across the four stocks illustrates the tradeoffs in terms of model fit,
345	explained variance and forecasting performance (Figure 9). In terms of fitting the model to the
346	observed data, the flexibility of the GAM provided superior fits for each stock relative to
347	SARIMA and GLM. However, in terms of predictive performance, as indicated by fits to the
348	terminal year of the time series and accuracy of drop-one scenario forecasts, SARIMA and GLM
349	were generally superior to GAM.
350	
351	Discussion
352	Federal requirements implemented in the amended MSA (U.S. Congress, 2006) require
353	specification (and monitoring) of ACLs for most federally managed stocks. Resources are
354	insufficient to develop population dynamics-based landings projection models for most managed
355	stocks (Martell and Froese 2013; Hanson et al., 2006; Hilborn and Walters, 1992). Thus, other
356	methods must be identified to predict catch rates to ensure landings remain within prescribed
357	ACLs (Carruthers and others 2014). Given the large number of stocks that must be monitored
358	(Berkson and Thorson 2015), routines must be robust to widely varying temporal patterns that
359	characterize recreational landings patterns for most species (Ward et al. 2014). Similar to
360	previous efforts with Atlantic and Gulf menhaden, this study suggested statistical forecasting
361	could be a viable approach to predicting landings (Hanson et al., 2006; Ives et al., 2010). A
362	major goal of recreational fisheries management is to prevent catch limit overages - this can be
363	accomplished by in-season closures or post-season adjustments to the regulations or season

length in the following year. Accurate forecasts of recreational catches are critical to theapplication of both of these accountability measures.

Our study suggests semi-automated SARIMA or GLM model fitting and selection 366 routines could be used to develop short-term (i.e., one year) forecasts to inform management 367 decisions; however, the quality and time span of input data can affect the accuracy of model 368 forecasts. Longer time series tended to include up and down fluctuations in catch, whereas 369 370 cutting the regression input time series omitted these fluctuations. By fitting to a shorter time series, the short-term trend tended to be better-captured at the expense of long-term fluctuations 371 in catch. No single model or time series performed best across all stocks of interest; thus 372 performance metrics need to be carefully selected and evaluated across multiple models. Our 373 projections implicitly integrated the highly correlated terms of catch and effort by expressing 374 375 catch rates as catch per open day. Changes in management regulations, environmental conditions, or economic conditions that might lead to changes in catch per unit effort would lead 376 to increased uncertainty in forecasts; if these changes are anticipated, they can be incorporated as 377 378 covariates in the models.

In general, SARIMA models performed well across a range of time series and would 379 serve as an appropriate starting point for forecasting landings. The SARIMA model mean 380 forecasts were generally un-biased in fits to observed data although confidence limits were 381 consistently greater than those produced from GLM or GAM. SARIMA models can 382 accommodate but do not require additional covariates for either model building or forecast, a 383 distinct advantage over GLM and GAM. For in-season quota monitoring, the manager's goal is 384 to close the fishery before the landings exceed the quota, but without forgoing harvest up to the 385 386 quota. Thus, the predicted trajectory of cumulative landings is more important than the final

projected total. Comparisons of SARIMA, GAM, and GLM forecasts fit to the 2011 cumulative observed landings time series indicated the SARIMA approach best fit the cumulative landings time series for most scenarios. However, for some stocks, GLM performed better than SARIMA and was less sensitive than SARIMA or GAM to recent trends, providing a useful bookend for forecasts.

SARIMA forecasts should be treated with skepticism when they generate negative 392 landings values, as they are likely overfitting a recent trend. Negative forecast values from any 393 catch forecast model should minimally be replaced with zeroes, as negative catches are not 394 possible. In this study, substitution of landings values for the most recent year of fishing 395 improved forecast accuracy over replacement with zeroes in most cases. For model projections 396 to 2012, the SARIMA model forecast negative catch rates in 2012 for Atlantic Red Porgy in 397 398 wave 1 using all three time series, and in wave 6 using the 1999-2011 and 2007-2011 time series. Replacing these forecasts with the previous year's landings resulted in minor improvements in 399 cumulative total forecast accuracy (projected cumulative landings relative to observed 400 401 cumulative landings) as compared to replacement with zeroes (1999-2011: +6%, 2004-2011: +1%, 2007-2011: +4% more accurate). Replacement of the wave 6 landings for Gulf vermilion 402 snapper in the 2007-2011 forecast reduced forecast accuracy by 11% compared to substituting 403 zeroes. SARIMA forecast negative catch rates in 2012 for Gulf Gray Snapper in waves 1, 2, 5, 404 and 6 using the 1999-2011 and 2004-2011 time series. Replacing these forecasts with the 405 previous year's landings resulted in major improvements in cumulative total forecast accuracy as 406 compared to replacement with zeroes (1999-2011: +26%, 2004-2011: +42% more accurate). In 407 summary, post hoc replacement of negative SARIMA values with landings from the most recent 408 409 year of fishing is recommended.

410	A strength of GAM models is the ability to fit noisy, non-linear data, however, this
411	flexibility can also permit overfitting of the model to these data if careful model selection and
412	validation routines are not employed (Wood 2006). GAM models provided the best fit to the
413	observed data in nearly all cases owing to the additional flexibility of this model to accommodate
414	noisy data. However, their tendency to overfit, despite model selection and validation resulted in
415	reduced forecasting performance in comparison to SARIMA and GLM models. While
416	overfitting can be addressed in GAMs (Zuur et al., 2010) by controlling the "wiggliness" of the
417	smoothing function, this can be quite arbitrary with small data sets. Alternatively, cross-
418	validation could be used, though the appropriateness of this approach in this present study is
419	doubtful given the moderate size of the input data sets.
420	As with any model, the reliability of our forecasts was dependent upon both the accuracy
421	and the consistency of the historical data. Recreational data in the southeastern U.S. is based
422	upon surveys (i.e., SE Headboat Survey, MRFSS, and the TPWD Creel Survey). Each of these
423	surveys contains uncertainty and spikes in landings estimates may occur when high catch rates
424	from a limited subsample are expanded out. Survey data based upon dockside intercepts
425	extrapolated to a fishing population comprising millions of people is subject to variability which
426	may reflect sampling issues rather than actual landings trends. Changes in survey methodologies
427	or management regulations may reduce the predictive utility of historical data. Future
428	forecasting modeling should attempt to incorporate uncertainty in wave-specific recreational
429	catch estimates to avoid model overweighting of outliers that may be an artifact of survey design.
430	Additionally, the utility of all of the methods explored in this study is contingent upon the ability
431	of historical trends to represent future landings. Angler behavior is notoriously difficult to
432	predict (Johnston and others 2010; Branch and others 2006), and changes in management

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433	regulations (i.e., closed seasons, bag limits, size limits) within or following the historical time
434	series make forecasting future recreational catches even more challenging. Future forecasting
435	modeling could explore the use management regulation time series as covariates, and also
436	evaluate the utility of economic predictors of recreational fishing effort such as United States
437	Gross Domestic Product or mean fuel prices. Finally, changes in stock size due to rebuilding
438	may also pose a problem, as increasing catch rates may result in higher-than-expected landings.
439	When a stock assessment is available, catchability may be combined with historical and
440	projected abundance-at-age to produce a time series of exploitable abundance. Exploitable
441	abundance may be a useful predictive covariate for catch forecasting models (N.A. Farmer,
442	unpublished data).
443	
444	Conclusions
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- 460

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544	TablesTable 1. Goodness of fit (R <sup>2</sup> ), mean error in terminal year across waves (ME; in pounds per day), total percent error in final
545	projected cumulative landings (TE), and mean error in projected year across waves (Drop1) for different stocks, modeling approaches,
546	and time series. Note 'Drop1' denotes forecasts where the terminal year of data is removed, the model refit, and the terminal year fit
547	is compared to the observed values.

STOCK	MODEL	R <sup>2</sup> (1992- 2011)	R <sup>2</sup> (1999- 2011)	R <sup>2</sup> (2004- 2011)	R <sup>2</sup> (2007- 2011)	ME (1992- 2011)	ME (1999-2011)	ME (2004- 2011)	ME (2007-2011)	TE (1992- 2011)	TE (1999- 2011)	TE (2004- 2011)	TE (2007- 2011)	ME_ Drop1 (2004-2010)	TE <sub>fit2010</sub> _Drop1 (2004-2010)	TE <sub>predict2011</sub> _Drop1 (2004-2010)
	SARIMA	0.66	0.69	0.73	0.86	895	518	513	163	29%	17%	17%	-5%	N/A	N/A	N/A
Gulf of Mexico	GAM	0.70	0.79	0.81	0.91	1140	1031	995	735	-36%	-33%	-32%	-23%	N/A	N/A	N/A
Vermilion	GLM	0.75	0.75	0.75	0.91	1660	1067	1070	608	-53%	-34%	-34%	-19%	N/A	N/A	N/A
Shapper	PrevYr	N/A	N/A	N/A	N/A	1097	1097	1097	1097	-34%	-34%	-34%	-34%	N/A	N/A	N/A
	SARIMA	0.64	0.60	0.66	0.85	1002	840	1307	59	-25%	-21%	-33%	2%	N/A	N/A	N/A
Gulf of Mexico	GAM	0.70	0.79	0.81	0.91	480	591	2475	650	18%	22%	93%	24%	N/A	N/A	N/A
Gray	GLM	0.50	0.52	0.63	0.65	2034	1892	900	463	76%	71%	34%	17%	N/A	N/A	N/A
Snapper	PrevYr	N/A	N/A	N/A	N/A	385	385	385	385	16%	16%	16%	16%	N/A	N/A	N/A
	SARIMA	0.51	0.42	0.37	0.31	212	51	368	169	-25%	-6%	-42%	-19%	735	7%	-67%
South Atlantic	GAM	0.70	0.79	0.81	0.91	68	80	4	84	7%	9%	-1%	-10%	218	17%	-26%
"Grunts"	GLM	0.45	0.45	0.50	0.64	290	332	119	66	33%	38%	13%	-8%	213	52%	24%
Complex	PrevYr	N/A	N/A	N/A	N/A	104	104	104	104	-12%	-12%	-12%	-12%	104	86%	-12%
	SARIMA	0.40	0.61	0.66	0.65	36	17	11	58	-12%	-5%	-1%	-25%	6	6%	4%
South	GAM	0.72	0.70	0.84	0.85	1	43	43	30	0%	-22%	-22%	-15%	129	-20%	-66%
Red Porgy	GLM	0.63	0.60	0.66	0.85	5	114	20	52	2%	58%	10%	-27%	67	51%	34%
	PrevYr	N/A	N/A	N/A	N/A	18	18	18	18	-9%	-9%	-9%	-9%	18	46%	-9%

#### 548 **Figure Captions**

549 Figure 1. Time series of recreational landings data, in millions of pounds whole weight, for Gulf

of Mexico vermilion snapper and Gray Snapper, and Atlantic 'grunts' complex and Red Porgy,

551 by data source (Texas Parks and Wildlife Department Creel Survey: TPWD, Marine

552 Recreational Fisheries Statistics Survey: MRFSS, and Southeast Headboat Survey: HBS).

**Figure 2.** Three statistical models (solid gray line) and their 95% confidence limits (dashed gray

line) were fit to landings data of Gulf of Mexico vermilion snapper from 1999 to 2011 (open

555 circles), to evaluate model fits across model types and times series.

**Figure 3.** Three statistical models (solid gray line) and their 95% confidence limits (dashed gray

line) were fit to landings data of Gulf of Mexico Gray Snapper from 1999 to 2011 (open circles),

to evaluate model fits across model types and times series.

**Figure 4.** Three statistical models (solid gray line) and their 95% confidence limits (dashed gray

line) were fit to landings data of Atlantic Red Porgy from 1999 to 2011 (open circles), to

561 evaluate model fits across model types and times series.

**Figure 5.** Three statistical models (solid gray line) and their 95% confidence limits (dashed gray

line) were fit to landings data of the Atlantic 'grunts' complex from 1999 to 2011 (open circles),

to evaluate model fits across model types and times series.

**Figure 6.** Three statistical models (solid gray line) and their 95% confidence limits (dashed gray

line) were fit to landings data of Atlantic Red Porgy and the 'grunts' complex from 1999 to 2010

567 (open circles), withholding 2011 landings data (open squares) from the model, to evaluate

568 forecast accuracy across model types and times series.

**Figure 7.** Cumulative landings plots showing SARIMA (red), GAM (blue), and GLM (green)

570 model fits and 95% confidence limits (shaded areas) relative to observed cumulative landings for

571 2011, based on 1999-2011, 2004-2011, and 2007-2011 time series data for Atlantic Red Porgy,

572 Atlantic 'grunts' complex, Gulf of Mexico Gray Snapper, and Gulf of Mexico vermilion snapper.

573 Figure 8. Cumulative landings plots showing relative model performance between SARIMA

(red), GAM (blue), and GLM (green) forecasts with 95% confidence limits (shaded areas)

- relative to observed cumulative landings for 2012, based on model fits to 1999-2011, 2004-2011,
- and 2007-2011 time series data for Atlantic Red Porgy, Atlantic 'grunts' complex, Gulf of
- 577 Mexico Gray Snapper, and Gulf of Mexico vermilion snapper.
- 578 **Figure 9.** Radar plots showing relative model performance between SARIMA (solid line), GAM
- 579 (dashed line), and GLM (dotted line) forecast models with regards to model fitting  $(R^2)$  to

580 different time series lengths, mean error in model in the final year for model fits, and mean

accuracy of model forecasts under 'drop-one' fit scenarios for four recreationally exploited

582 stocks.

## 584 APPENDIX

- **Table A1.** Seasonal (s) autoregressive integrated moving average (SARIMA)  $(p,d,q)^*(P,D,Q)s$
- 586 model combinations evaluated, where the auto-regressive component (*p*) represents the lingering
- 587 effects of previous observations, the integrated component (*d*) represents temporal trends, the
- 588 moving average component (q) represents lingering effects of previous random shocks (or error),
- and s denotes the seasonal time step. As recreational landings are primarily collected in two-
- 590 month waves, s was set equal to 6. A'1' denotes an active component in the model.

ARIMA( <i>p</i> , <i>d</i> , <i>q</i> )X( <i>P</i> , <i>D</i> , <i>Q</i> )s Model
ARIMA(0,1,1)X(0,1,1)s
ARIMA(1,0,0)X(0,1,1)s
ARIMA(0,0,1)X(0,1,1)s
ARIMA(0,1,1)X(1,1,0)s
ARIMA(1,0,0)X(1,1,0)s
ARIMA(0,0,1)X(1,1,0)s
ARIMA(1,1,0)X(0,1,1)s
ARIMA(1,1,0)X(1,1,0)s

591