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Introduction to Solar Terrestrial Activity for Geomagnetic Studies Part II: The Earth's Main Field

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INTRODUCTION TO SOLAR TERRESTRIAL ACTIVITY FOR GEOMAGNETIC STUDIES

PART II: THE EARTH'S MAIN FIELD

W. H. Campbell

This review concerns the main magnetic field as determined from the earth's surface observations. Starting from a simple magnetic dipole representation, the reader is led to the more complete spherical harmonic expansion of the field in normalized associated Legendre functions. Expansion coefficients for the International Geomagnetic Reference Field (IGRF) and recent satellite models are discussed. Sources of the earth's main field and secular change are briefly explained. Position of the geomagnetic, dip, and eccentric axis poles are given together with definitions of geomagnetic coordinates, conjugate positions, and time. This review then concludes with examples of chart representations of the field.

1. INTRODUCTION

The objective for this series of reviews is to discuss the magnetic field changes at the earth's surface resulting from solar-terrestrial activity. To establish a position for interpreting these changes it is necessary to review first the basic properties of the earth's main field. The influence of this giant magnetic dipole is confined by the solar wind within an extended cavity called the magnetosphere. Inside this region charged particle motion is largely controlled by the earth's steady field and the field contortions in reaction to the variable solar wind. The final chapter in this magnetic study series will be a presentation of various worldwide disturbance effects that are measured in conjunction with the magnetic perturbations. However, at this stage in our review we will confine our interest to the main geomagnetic field as seen at the earth's surface observatories, and we shall consider what that field behavior should be without the solar wind interaction.

2. MAGNETIC COMPONENTS

Since the earliest years of the scientific study of the earth's magnetism it has been apparent that the most imposing feature of the field is its resemblance to that of a dipole. Such a magnetic field configuration is associated with a current loop or a bar magnet having a concentration of field at each of its ends. A typical inexpensive compass which is formed by a small needle magnet of this type freely balanced or suspended at its middle will align itself in a general north-south direction. The north pointing end of this magnet is called the north pole. Opposite poles of magnets or compasses are found to attract each other. According to this convention in naming, therefore, the earth's dipole field, attracting the north pole of a magnet to the northern arctic region, should be called a south pole. Fortunately, to avoid such confusion, the convention is ignored for the earth's magnetic behavior and the Arctic magnetic pole is called a north magnetic pole. Other adjectives sometimes given are Boreal for the northern and Austral for the southern pole.

By universal agreement a special set of names and symbols are used to describe the earth's field. Figure 1 illustrates this nomenclature for a location in the northern hemisphere where the total field vector points into the earth. The coordinate system is right-handed with X, Y, Z taken as geographic directions, north, east, and down, respectively. The horizontal field vector pointing magnetically northward is called H; the angle this makes with geographic north is called the declination, D; and the angle the total field, F, makes with the horizontal plane is called the inclination, I, or dip angle. When the incremental changes in declination, ΔD, are studied, the small angle approximation, $\sin \Delta D \approx \Delta D$, applies, so the magnetic eastward component of the small change, ΔE, can be obtained from

$$\Delta E = H\Delta D, \tag{1}$$

where ΔD has been converted from degree to radian measure. The DHZ system is used at most world observatories because of its direct application, in olden times, to navigation and land survey. In the DHZ system, the data from different stations have different component orientation with respect to the earth's axis and equatorial plane. The XYZ coordinate system has been selected for the field recordings by some high latitude observatories because of the great disparity in the direction of magnetic north at many polar region sites.

Bibliography References: 7,18.

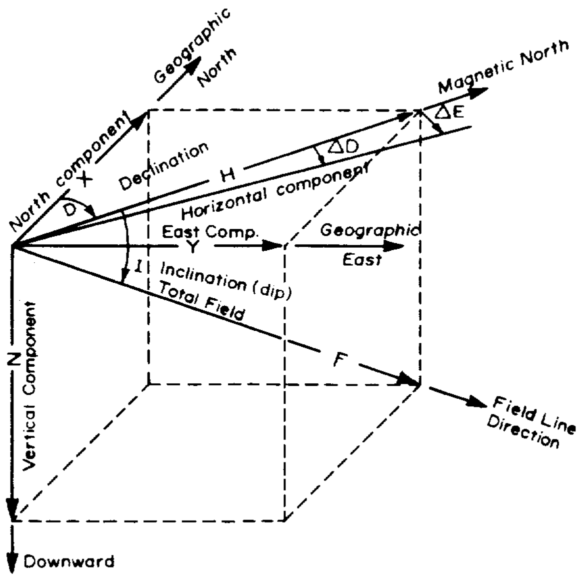

Figure 1. Components for geomagnetic field measurements for an example northern hemisphere total field vector F inclined into the earth.

3. THE SIMPLE DIPOLE FIELD

To most students, exposure to the term dipole first occurred in learning about the electric field of two point charges of opposite sign placed rather close together. The potential of a point charge in air is given by

$$\Omega = \frac{1}{4\pi\varepsilon_o} \frac{q}{r} , \tag{2}$$

where Ω is the electric scalar potential (volts), q is the charge (coulombs), ε_o is the inductive capacity in free space, and r (meters) is the distance to the observation point, P, in a right hand x, y, z orthogonal coordinate system. For the figure 2 dipole configuration of point charges separated along the z-axis by a distance, d, the electric scalar potential is given as

$$\Omega = \frac{1}{4\pi\varepsilon_o}\left[\frac{q}{\sqrt{[z-(d/2)]^2+x^2+y^2}} + \frac{-q}{\sqrt{[z+(d/2)]^2+x^2+y^2}}\right]. \tag{3}$$

When d<<r this becomes

$$\Omega = \frac{q}{4\pi\varepsilon_o r}\left[\left(1+\frac{1}{2}\frac{zd}{r^2}\right) - \left(1-\frac{1}{2}\frac{zd}{r^2}\right)\right]. \tag{4}$$

Thus we find that

$$\Omega = \frac{q\ d\ \cos\ \theta}{4\pi\varepsilon_o\ r^2} + \Delta A, \tag{5}$$

where ΔA represents higher order terms which become negligible when d<<r.

The product of the charge and the separation is called the dipole moment p = q d. The field of this dipole is then obtained from the negative gradient of the potential, $E = -\nabla\Omega$. Recognizing that the gradient of Ω in spherical coordinates is given by

$$\nabla\Omega = \underline{e}_r \frac{\partial\Omega}{\partial r} + \underline{e}_\theta \frac{1}{r}\frac{\partial\Omega}{\partial\theta} + \underline{e}_\phi \frac{1}{r\ \sin\ \theta}\frac{\partial\Omega}{\partial\phi} , \tag{6}$$

where the $\underline{e}_{r,\theta,\phi}$ are the unit vectors in the three coordinate directions, we can determine the field components at any point on a spherical surface. Symmetry about the dipole axis means that Ω doesn't change with changing ϕ; thus the $\underline{e}_\phi$ component is zero. We only need to concern ourselves with the field components E_r and E_θ,

$$\underline{E}_r = \frac{-\partial\Omega}{\partial r} = \frac{p}{2\pi\varepsilon_o}\frac{\cos\ \theta}{r^3}\underline{e}_r ,$$

and

$$\underline{E}_\theta = \frac{1}{r}\frac{\partial\Omega}{\partial\theta} = \frac{p}{4\pi\varepsilon_o}\frac{\sin\ \theta}{r^3}\underline{e}_\theta . \tag{7}$$

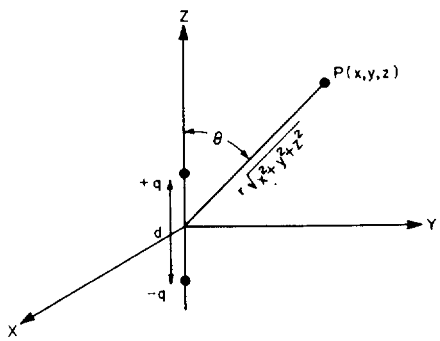

Figure 2. Electric dipole coordinates.

The map of the dipole field thus described is shown in figure 3. The field lines shown follow the relationship

$$r = r_e\ \sin^2\theta \tag{8}$$

in which r_e is the distance from the dipole center to the equatorial crossing of the field line.

From Maxwell's equations we know that in the region of no charge there is no divergence of the electric field, $\nabla \cdot E = 0$. Since E is the gradient of the scalar it follows that

$$\nabla \cdot E \equiv -\nabla \cdot \nabla \, \Omega = 0$$

or $$\nabla^2 \, \Omega = 0 \quad \text{(Laplace's equation)} \qquad (9)$$

of which Equation (5) is a solution for the dipole configuration of charges.

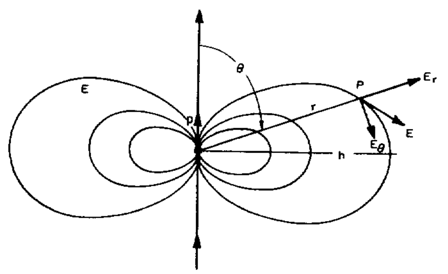

Figure 3. Electric dipole field lines.

We shall learn shortly that the earth's main field represents the case of a current loop. Let us now see how the typical field pattern of a current loop has a dipole form.

In figure 4 consider the current, i, flowing in the x-y plane about a loop enclosing area A (of radius b) to which z is in the normal direction. Let P be any point a distance, r, from the loop center and distance, R, from a current element ds. We wish to find the field components B_r and B at any point in the space about the loop under the simplifying condition that r>>b.

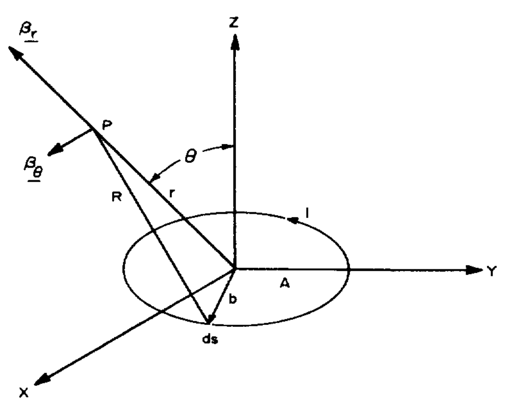

Figure 4. Current loop dipole coordinates.

From the vector relationship

$$\underline{R} = \underline{r} - \underline{b}, \qquad (10)$$

calling x' the projection of b upon the x axis, we may obtain

$$R^2 = r^2 + b^2 - 2rx' \sin \theta$$

and $$\frac{1}{R^3} = \frac{1}{r^3} \left\{1 + 3\frac{x'}{r} \sin \theta + . . .\right\}. \qquad (11)$$

Now, from the Biot and Savart law of electromagnetism, the field at a distance, $\underline{R}$, from a current element of length, ds, is given by

$$\underline{dB} = \frac{\mu i}{4\pi} \frac{\underline{ds} \times \underline{R}}{R^3} \text{ webers/meter}^2, \qquad (12)$$

where μ is the permeability of free space. Substituting (11) into this expression we may write

$$\underline{B} \doteq \frac{\mu i}{4\pi} \oint \left[\frac{\underline{ds} \times (\underline{r} - \underline{b})}{r^3} \left(1 + \frac{3x'}{r} \sin \theta\right)\right]. \qquad (13)$$

From (13) we can obtain

$$\underline{B} \doteq \frac{\mu i A}{4\pi r^3} \left[2\underline{k} + \frac{3 \sin \theta}{r} \underline{j} \times \underline{r}\right], \qquad (14)$$

where $\underline{j}$ and $\underline{k}$ are unit vectors in the y and z axis direction. Converting this to the $\underline{r}$, $\underline{\theta}$ coordinate directions (in which $\underline{e_r}$ and $\underline{e_\theta}$ are unit vectors) (14) becomes

$$\underline{B_r} \doteq \frac{\mu i A}{2\pi} \frac{\cos \theta}{r^3} \underline{e_r},$$

$$\qquad (15)$$

$$\underline{B_\theta} \doteq \frac{\mu i A}{4\pi} \frac{\sin \theta}{r^3} \underline{e_\theta}.$$

The form of these field variations is identical to (7), the electric dipole field form. Corresponding to p, the electric dipole moment, we can write for the magnetic dipole moment, M, of the current loop:

$$M = i \, A \qquad (16)$$

or in comparison to p = qd we can assume

$$M = m \, d, \qquad (17)$$

where d becomes the equivalent separation of magnetic poles of strength, m, which would give rise to the observed dipole field.

We saw in the parallel case of the electric dipole that E was obtained from a scalar potential in the charge free region. In a similar fashion we can write

$$\underline{F} = \underline{B} = -\nabla V \qquad (18)$$

where V is the magnetic scalar potential;

$$V = \frac{\mu M}{4\pi} \frac{\cos\theta}{r^2} . \qquad (19)$$

From Equation (18) and with Maxwell's relationship $\nabla \cdot B = 0$, in a current free region, we have $\nabla^2 V = 0$, for which (19) is a solution of the simple dipole (circular current loop) case. The form of this dipole field in space is the same as shown in figure 3. At the earth's surface we call r = a then

$$B_r = Z = 2\left(\frac{\mu_0 M}{4\pi a^3}\right) \cos\theta = Z_0 \cos\theta , \qquad (20)$$

$$B_\theta = H = \left(\frac{\mu_0 M}{4\pi a^3}\right) \sin\theta = H_0 \sin\theta , \qquad (21)$$

where $$H_0 = Z_0/2 \doteq 3.1 \times 10^4 \text{ gamma}, \qquad (22)$$

(1 gamma = 10^{-9} webers/m^2 = 1 nanotesla). Then for some of the other components of field in figure 1 we have

$$F = (H^2 + Z^2)^{\frac{1}{2}} \qquad (23)$$

$$\tan I = Z/H = 2 \cot\theta = 2 \tan X \qquad (24)$$

where the geomagnetic latitude, Φ, of a location is given by

$$\Phi = 90 - \theta \text{ (degrees)}. \qquad (25)$$

The trace along the earth's surface for which I = 0 is called the geomagnetic equator. This is to be differentiated from the dip equator - a line connecting the mean position of points with measured zero dip. Local crustal and core field anomalies cause the two equators to differ at many locations. Geomagnetic longitudes, Λ, are at equal angular increments about the dipole axis enumerated eastward from a zero (prime) geomagnetic meridian which passes through the geographic poles. Figure 5 illustrates these coordinate names.

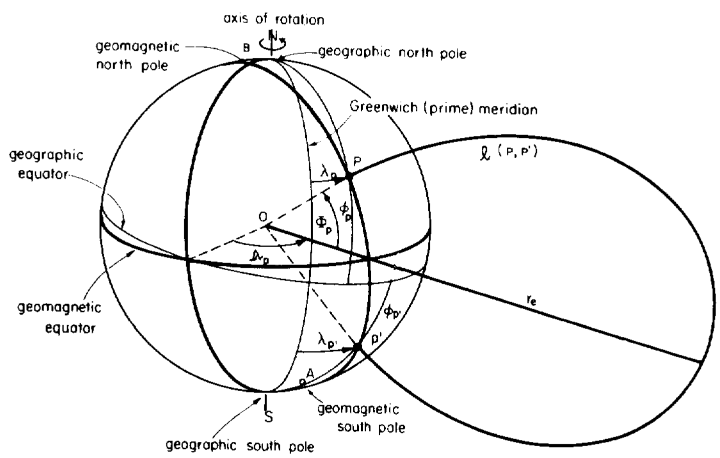

Figure 5. Geomagnetic coordinates of a location.

A field line originating in one hemisphere at a point, P, with geomagnetic latitude, Φ, and continuing to the surface in the opposite hemisphere (to a conjugate point, P') will extend out in the equatorial plane a distance, r_e, from the earth's center shown by figure 5. With the help of figure 6 a dipole field line length in space, ℓ, may be approximated for locations up to 65° in latitude by the relationship

$$\ell = 0.038\ \Phi_0 r_e \qquad (26)$$

where Φ_0 is in degrees latitude at which the field line leaves the earth's surface and ℓ has the same units as r_e.

The axis line of the dipole field through the earth's center is inclined about 11° to the geographic axis of rotation. We call the two intersections of this line with the earth's surface the geomagnetic poles. They are located about 78.8°N, 70.0°W (not far from Thule, Greenland) and about 78.8°S, 110.0°E (not far from Vostok, Antarctica). We may obtain the geomagnetic coordinates of a station from the geographic latitude, ϕ, and longitude, λ, from the equations

$$\sin\Phi = \sin\phi \cos 11.2^\circ + \cos\phi \sin 11.2^\circ \cos(\lambda - 290.0^\circ)$$

$$\sin\Lambda = \cos\phi \cdot \sin(\lambda - 290.0^\circ)/\cos\Phi. \qquad (27)$$

Near the equator the earth's field is about 3×10^4 gamma; in polar regions it reaches about 7×10^4 gamma. Equations (20, (21), and (22) of the centered dipole model with its axis aligned for the pole positions will describe the earth's field at most locations to about 90% accuracy on the average. Figure 5 illustrates the location of these poles, the geomagnetic equator and the differences of geomagnetic and geographic coordinates. In a later section we will show how the earth's field is derived more exactly from the surface measurements and how the locations of the magnetic poles are determined.

Bibliography References: 14, 16.

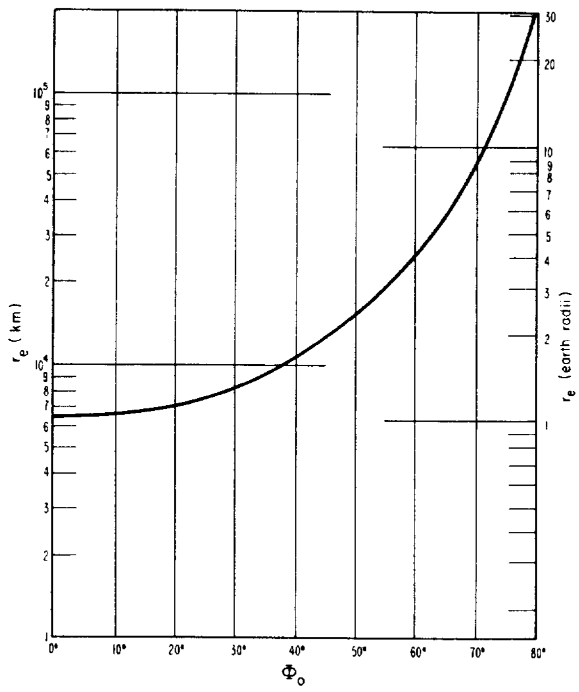

Figure 6. Equatorial distance r_e of a dipole field line starting from latitude Φ_0 at the earth's surface [from Sugiura and Chapman, 1966].

4. SOURCE OF THE EARTH'S MAIN FIELD

As we shall see in the following section, it is possible to prove from observatory records that the source of the principal geomagnetic field is not above the earth's surface but rather below it. The questions that arise are whether permanently magnetized materials or currents are the source, and where these are situated.

A uniformly magnetized spherical shell could produce a dipole field. However, the specific magnetization of the earth's average crustal material is several orders of magnitude too small to be a significant cause of the earth's main field. We also know that temperature and pressure within the earth increase with depth. The Curie temperature of iron (the temperature above which ferromagnetic characteristics disappear), which is about 750°C at sea level, decreases with pressure. Thus the Curie temperature is reached a short distance (less than 10 miles) into the earth. For this reason the existence of a permanently magnetized material at the earth's core is not likely.

There exist regular long term changes in the field. Paleomagnetic researchers report that the main field has reversed its N-S polarity on the average of once every 2×10^5 years. The location of the magnetic poles change gradually, wandering about the geographic axis in such a way that the two axes (geographic and geomagnetic) correspond when positions are averaged over 10^4 years or more. There is an excellent correlation between the earth's rotational changes and variations in westward motion of the dipole field. Unique nondipole features seen on maps of the main field exhibit a westward drift of 0.1 to 0.2°/year which is many times faster than any drift of the earth's geological features. All these field motions can be better satisfied by an electric current model of the field than by a permanent magnetized material model.

Where should the current flow? The crust of the earth is too poor a conductor for the flow of 10^9 amperes necessary to generate the main field. The earth's outer core of liquid metal, extending to over 1/2 the earth's diameter, seems to be a more suitable site. A self-exciting dynamo theory of the main field is presently popular. In this viewpoint the fluid motions within the earth's core arise from the earth's axial rotation and radial thermal convection. The resulting differential flows within the molten liquid of nickel-iron are thought to act as a self-exciting dynamo whose self-reinforced currents generate the observed dipole field. For this model the polarity is determined by the initial excitation field direction; fluctuations and westward drifts of the field patterns can occur and reversal of the main field are allowed.

Bibliography References: 12, 17, 19.

5. DETAILED EXPANSION OF EARTH'S MAGNETIC FIELD

As far as our problem of the earth's main field is concerned only negligible electric field changes occur and insignificant currents flow in the region immediately above the earth's surface. We can assume, therefore, that all our observatory measurements are external to the source and so we may write from Maxwell's equations

$$\nabla \times \underline{B} = 0. \tag{28}$$

This equation means that the field is obtainable from a gradient of a scalar potential

$$\underline{B} = -\nabla V. \tag{29}$$

Equation (29) together with the rigorously obeyed Maxwell's relationship,

$$\nabla \cdot B = 0, \tag{30}$$

gives the Laplace expression

$$\nabla \cdot \nabla V \equiv \nabla^2 V = 0 \ . \tag{31}$$

Written in spherical coordinates Equation (31) becomes

$$\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta \frac{\partial V}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2 V}{\partial\lambda^2} = 0 \tag{32}$$

in which r, θ, λ are the geographic earth centered coordinates of radial distance, colatitude, and longitude. Assuming a solution separable into the three coordinates with R a function of r alone, S a function of the angles, T a function of θ only, and L a function of λ only; we can write

$$V(r,\theta,\lambda) = R(r)S(\theta,\lambda) \tag{33}$$

with $$S(\theta,\lambda) = T(\theta)\ L(\lambda).$$

Using standard methods it is possible to show that there exists a converging series solution of the potential for Laplace's equation in which

$$\begin{aligned} V &= a \sum_{n=1}^{\infty} \left[\left(\frac{r}{a}\right)^n S_n^e + \left(\frac{a}{r}\right)^{n+1} S_n^i\right] \\ &= V^e + V^i , \end{aligned} \tag{34}$$

where a is the earth's radius. The e and i refer to external and internal field parts of S_n and V. This e and i separation means that if all sources are away from the coordinate origin less than the distance, a, the field must go to zero at infinity; the series $(r/a)^n$ must vanish, and then S_n^e is zero. Conversely for all sources of field outside the sphere of radius, a, S_n^i must be zero. The potential is thus divided into the internal and external parts.

It has been demonstrated that a knowledge of field potential, V, as well as its radial gradient over the surface of the earth enables a determination of the proportional division of V^e and V^i. Studies have proved that it is sufficient for the main field studies to consider only the internal contributions. With this assumption the full solution of (32) becomes

$$V = a \sum_{n=1}^{\infty} \left(\frac{a}{r}\right)^{n+1} \sum_{m=0}^{n} (g_n^m \cos m\lambda + h_n^m \sin m\lambda)\ P_n^m (\cos\theta) \tag{35}$$

for which the g_n^m and h_n^m are called Gauss coefficients and P_n^m (cos θ) are the Schmidt normalized, associated Legendre functions. The S (θ,λ) function part of (35) represents a surface spherical harmonic of degree n, order m and is analogous to the various possible standing wave patterns that may develop upon a constrained spherical surface. The procedure of matching the observed surface potential to this series of spherical functions is mathematically similar to the fitting of a Fourier series to an irregular two-dimensional curve. The Gauss coefficients in the Schmidt normalization show the ordered relative importance of the corresponding terms in the series. An increasing number of terms are required for increasing accuracy of the fit of the expansion to the observations.

The special spherical harmonic factors used in (35) may be determined from the following relationships:

The first three Legendre polynomials P_n are

$$\begin{aligned} P_0(u) &= 1 \\ P_1(u) &= u \\ P_2(u) &= 1/2(3u^2 - 1). \end{aligned} \tag{36}$$

The recurrence relationship for obtaining Legendre polynomial $P_{n+1}(u)$ from the two preceding polynomials is

$$n \cdot P_{n-1}(u) + (n+1) \cdot P_{n+1}(u) = (2n+1) \cdot u \cdot P_n(u) \ . \tag{37}$$

With (36) and (37) the Schmidt normalized associated Legendre functions can be obtained from

$$P_n^m(u) = (1-u^2)^{m/2} \left[(2-\delta_{om})\frac{(n-m)!}{(n+m)!}\right]^{1/2} \frac{\partial^m P_n(u)}{\partial u^m} \tag{38}$$

where u = cos θ and the Kronecker delta $\delta_{om} = 0$ if $m \neq 0$ or $\delta = 1$ if $m = 0$.

The earth's orthogonal field components (F, X, Y, Z in figure 1) are obtained from the negative gradient of the potential in spherical coordinates (Equation 6):

$$Z = -B_r = \frac{\partial V}{\partial r}$$

$$X = -B_\theta = \frac{1}{r}\frac{\partial V}{\partial \theta} \tag{39}$$

$$Y = B_\lambda = -\frac{1}{r \sin\theta}\frac{\partial V}{\partial \lambda} .$$

From Equation (35) the above resolved to

$$Z = -\sum_{n=1}^{\infty} (n+1)\left(\frac{a}{r}\right)^{n+2} \sum_{m=0}^{n} (g_n^m \cos m\lambda + h_n^m \sin m\lambda)\, P_n^m(\theta)$$

$$X = \sum_{n=1}^{\infty} \left(\frac{a}{r}\right)^{n+2} \sum_{m=0}^{n} (g_n^m \cos m\lambda + h_n^m \sin m\lambda)\, \frac{\partial P_n^m(\theta)}{\partial\theta}$$

$$Y = \sum_{n=1}^{\infty} \left(\frac{a}{r}\right)^{n+2} \sum_{m=0}^{n} \frac{m}{\sin\theta} (g_n^m \sin m\lambda - h_n^m \cos m\lambda)\, P_n^m(\theta) \tag{40}$$

and $$F = \sqrt{X^2+Y^2+Z^2} .$$

These Equations (40) will give us a spheric harmonic representation of the earth's field components. All we need is to find the appropriate coefficients g_n^m and h_n^m.

Bibliography References: 2, 4, 5, 7, 10, 13, 14, 18.

6. MAGNETIC SCALAR POTENTIAL TABLES

For typical computations a is taken to be the mean earth radius value, 6371.2 km, and r = a + h where h is the altitude of the observation point. The Gauss coefficients, g_n^m and h_n^m, can be obtained for a fixed number of polynomial P_n^m terms since the higher orders provide only successively smaller corrections to the field that are eventually less than the useful accuracy of the harmonic representation. The coefficients can be obtained from the Equations (40) by a converging adjustment of possible values in such a manner that the sum of the squares of the differences between all the measured and computed magnetic fields for the observing sites are minimized. Since the earth's field is gradually changing with time the coefficients are identified with a particular year called the determination epoch. Occasionally station values from years differing from that epoch are included in the calculation by estimating the observatory secular change (slow regular drift of the main field) and extrapolating the field values to the desired year. In most cases the insufficient information on this secular change is responsible for the main errors in the harmonic representation. Calculations for two or more epochs provide the annual rate of change of the Gauss coefficients $\dot{g}_n^m$ and $\dot{h}_n^m$. For increasing accuracy in the earth's field determination allowance may be made for the oblateness of the earth's shape by appropriate identification of the station's position in geocentric (position from the center) rather than geodetic (position on an assumed spherical surface) coordinates. For a single epoch, slightly different coefficients have been obtained by different research groups, largely due to a variance in data sources and methods of fitting the observations to Equations (40). A compromise field for the period 1960 to 1969 has been internationally agreed upon (International Geomagnetic Reference Field, IGRF, epoch 1965.0) as a working standard (table 1). The IGRF (1965.0)

Table 1. Gauss coefficients IGRF epoch 1965.0 for expansion in geographic, rotational axis coordinates [from Cain and Cain, 1968].

		Main Field (gamma)		Secular Change (gamma/year)	
n	m	g	h	$\dot{g}$	$\dot{h}$
1	0	-30339	0	15.3	0.0
1	1	-2123	5758	8.7	-2.3
2	0	-1654	0	-24.4	0.0
2	1	2994	-2006	0.3	-11.8
2	2	1567	130	-1.6	-16.7
3	0	1297	0	0.2	0.0
3	1	-2036	-403	-10.8	4.2
3	2	1289	242	0.7	0.7
3	3	843	-176	-3.8	-7.7
4	0	958	0	-0.7	0.0
4	1	805	149	0.2	-0.1
4	2	492	-280	-3.0	1.6
4	3	-392	8	-0.1	2.9
4	4	256	-265	-2.1	-4.2
5	0	-223	0	1.9	0.0
5	1	357	16	1.1	2.3
5	2	246	125	2.9	1.7
5	3	-26	-123	0.6	-2.4
5	4	-161	-107	0.0	0.8
5	5	-51	77	1.3	-0.3
6	0	47	0	-0.1	0.0
6	1	60	-14	-0.3	-0.9
6	2	4	106	1.1	-0.4
6	3	-229	68	1.9	2.0
6	4	3	-32	-0.4	-1.1
6	5	-4	-10	-0.4	0.1
6	6	-112	-13	-0.2	0.9
7	0	71	0	-0.5	0.0
7	1	-54	-57	-0.3	-1.1
7	2	0	-27	-0.7	0.3
7	3	12	-8	-0.5	0.4
7	4	-25	9	0.3	0.2
7	5	-9	23	-0.0	0.4
7	6	13	-19	-0.2	0.2
7	7	-2	-17	-0.6	0.3
8	0	10	0	0.1	0.0
8	1	9	3	0.4	0.1
8	2	-3	-13	0.6	-0.2
8	3	-12	5	0.0	-0.3
8	4	-4	-17	-0.0	-0.2
8	5	7	4	-0.1	-0.3
8	6	-5	22	0.3	-0.4
8	7	12	-3	-0.3	-0.3
8	8	6	-16	-0.5	-0.3

contains only Gauss coefficients up to n=m=8 so wavelengths under 45° in angular size are not represented. There is good reason to believe that the NASA models (GSFC (12/66)) for the periods 1900 to 1965 or POGO (10/68) for 1965 to the present are more accurate representations. Large time extrapolations from the given epoch are undesirable because the secular changes are not as linear as is assumed by most models.

The associated Legendre polynomial formulation of the field exhibits a particularly interesting geometric characteristic. When the potentials for dipole, quadrupole, octupole, and successive n^{th} order multipole configurations of charge (figure 7) are calculated, the resulting mathematical expressions are identical to the corresponding n^{th} order terms of Equation (35). For this reason we can refer to the dipole, quadrupole, etc. parts of the earth's main field.

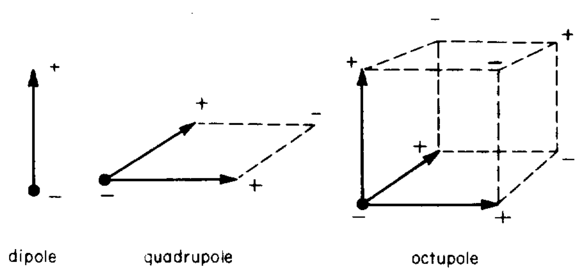

Figure 7. Multipole configurations.

From Equation (35) the earth's dipole potential (1st order polynominals) is written as

$$V_1 = \left(\frac{a^3}{r^2}\right) \left[g_1^0 \cos\theta + g_1^1 \sin\theta \cos\lambda + h_1^1 \sin\theta \sin\lambda\right] \quad (41)$$

If the coordinate system were aligned along the tilted geomagnetic dipole axis rather than the earth's rotation axis, V_1 would have the same form as Equation (19). For magnetospheric modeling studies a second series of Gauss coefficients are often used in which the polynomial expansions are from just such an earth centered dipole axis coordinate arrangement. The IGRF epoch 1965.0 field for such an expansion has coefficients $g_1^0 = -30953.5$, $g_1^1 = 0$, $h_1^0 = h_1^1 = 0$, and $g_1^0 = 16.01$. The northern location of the resulting pole is taken to be

$$\theta_0 = 11.435° \text{ geocentric colatitude}$$
$$\lambda_0 = -69.761 \text{ east longitude} \quad (42)$$

for which the secular change is

$$\dot{\theta}_0 = -3.70 \text{ X } 10^{-3} \text{ °/year}$$
$$\dot{\lambda}_0 = -6.936 \text{ X } 10^{-2} \text{ °/year} \quad (43)$$

for years from January 1, 1965.

The reader will sometimes come upon other than spherical polynomial representations of the earth's field such as a distribution of many magnetic dipoles whose strength and locations are arranged within the earth to approximate the field at the surface. There is no present indication that such exercises will improve upon the more usual method described above or will increase our understanding of the source region.

The relative importance of a polynomial term in Equation (40), for a computation of the earth's field, decreases with increasing order of the multipole and with increasing height above the earth's surface. Thus, for a finite number of Schmidt normalized associated Legendre polynomial terms, the matching to the earth's field should improve with altitude. This would be especially true were it not for the existence of currents within the magnetosphere and for some deformation of the magnetosphere by its encounter with the solar wind. We shall discuss these effects later.

A third series of Gauss coefficients may be obtained by allowing both the tilt of the axis for our polynomial expansion (the dipole axis) and its distance from the earth's center to be adjusted in such a way that the quadrupole term is made to vanish. This is about equivalent to placing the earth in a sack, then allowing the measured field to fix the location of a dipole axis for a polynomial description such that the dipole term is maximized. This configuration is called the eccentric dipole of the earth's field. The earth surface coordinates of this pole are, of course, not geodetically symmetric. Computations show that a displacement from the earth's center of only about 0.07 of the earth's radius is required.

Bibliography References: 3, 4, 10, 11, 13, 15, 18, 19.

7. GEOMAGNETIC LOCATIONS AND TIME

Geomagnetic pole positions are obtained from a spheric harmonic analysis in which all the station measurements about the earth contribute to the determination. The two pairs typically reported are the geomagnetic poles, named for a coordinate system about the dipole which passes through the earth's center, and the eccentric (axis) geomagnetic poles, named for the off-center dipole axis. In contrast, the dip poles (on early maps these were call the magnetic poles) are obtained from local observations of the maximum dip angle, and they thus reflect the joint effects of the earth core field and the local crustal geology. Table 2 gives the location of these three pole types.

Table 2

Pole Type (epoch)	Northern Latitude	Northern Longitude	Southern Latitude	Southern Longitude
Dip (1965)	75.3°N	100.5°W	66.5°S	140.0°E
Geomagnetic (1965)	78.8°N	70.0°W	78.8°S	110.0°E
Eccentric geomagnetic (1955)	81.0°N	84.7°W	75.0°S	120.4°E

For similar reasons locations of the equator obtained from polynomial expansions and dip measurements differ. Although the effects of crustal anomalies diminish rapidly with altitude, the problem we must consider is what field representation to use for various upper atmosphere studies. This question has not been adequately resolved to the satisfaction of all researchers. We presently recommend the following:

A. At all latitudes for general ionospheric studies of phenomena such as radiowave propagation and dynamo currents, use the projected (to altitude) values of a surface field in which anomalies of under about 50 km in horizontal size have been smoothed out.

B. From the auroral zone to the equator for ionospheric studies of the arrival of field line guided wave phenomena or of bombarding magnetospheric particles, use the full expansion of the spherical harmonic field or the eccentric dipole field or the dipole (geomagnetic coordinate) field, (in decreasing order of accuracy).

C. At altitudes above 2000 km but on field lines which extend to less than 5 R_e at their equatorial position, the first few terms of the spherical harmonic field or eccentric dipole field usually provide sufficient accuracy.

D. At the remaining locations, higher than the latitude limitation in B (above) and on longer field lines than in C (above), the interaction of the earth's dipole field with the solar wind, to be described in the next chapter, must be taken into consideration.

A map of the geomagnetic coordinates (figure 8) representing the earth centered tilted dipole (Equation (41)) is convenient in many studies for rapid recognition of a station's location with respect to the main field in space (e.g. items B and C above). For ease in determining the geographic location of a site the chart is often displayed in equal latitude-longitude coordinates. The dipole coordinate system becomes of minor value for the equatorial region, where the dip equator (figure 9) defines the location of an intense ionospheric electrojet current, or for the polar regions where the magnetospheric distortions become important in the location of ionospheric phenomena. Some improved utility is obtained by use of an eccentric axis spherical coordinate system which is projected onto the off-centered earth. Nevertheless, some auroral zone researchers find the

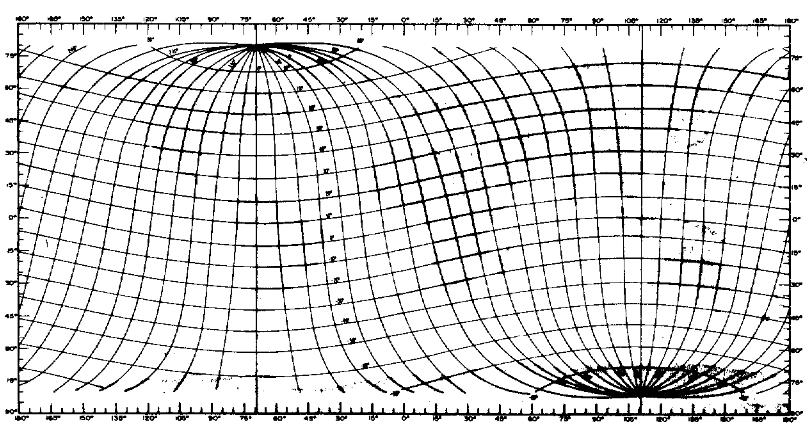

Figure 8. Geomagnetic coordinate map.

geomagnetic coordinate maps of convenience in plotting their data at the start of analysis before proceeding to more accurate representations.

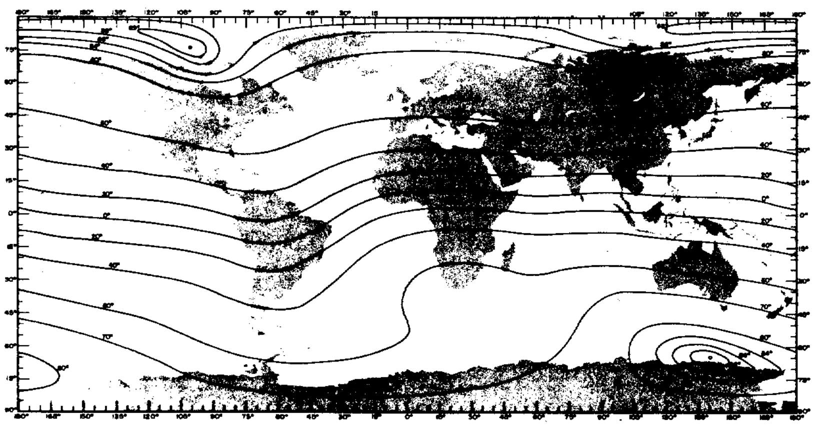

Figure 9. Magnetic dip contours.

For studies of phenomena which have a dependence upon the direction of the sun with respect to the earth's field, a system of <u>geomagnetic time</u> has been devised. Let us use the polar view of figure 10 to describe this time. To start with a simple example, assume the solar direction toward the bottom of the page and assume the day is the June solstice. In the figure geomagnetic meridians of longitude are shown, the two northern poles (geographic N and geomagnetic B), stations P_1, P_2, P_3, and the subsolar point S at the Tropic of Cancer are indicated. The geomagnetic time of a station is obtained from the locations of the geomagnetic longitude of the subsolar point and the geomagnetic longitude of the station. In figure 10 station P_1, on the same geomagnetic meridian as the subsolar point S, is at geomagnetic noon. Station P_2, 180° geomagnetic longitude from the subsolar point, is at geomagnetic midnight. Geomagnetic time is thus defined as the eastward hour angle between the geomagnetic meridian passing through the station and the geomagnetic meridian 180° from that passing through the sun. Station P_3 which is 75° geomagnetic longitude degrees from the subsolar meridian + 180° is thus 05h in geomagnetic time.

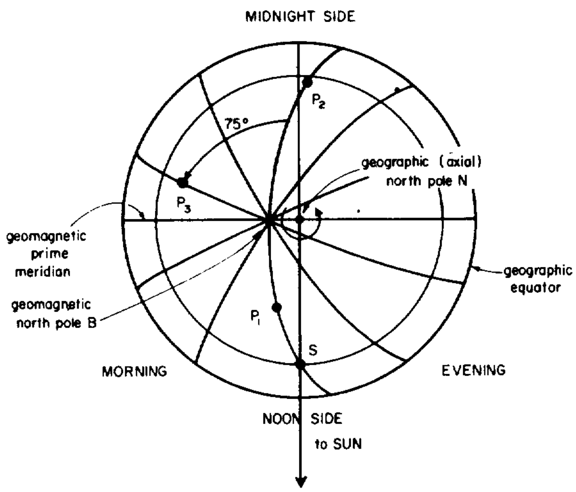

Figure 10. Geomagnetic time configuration.

Three particular features should be noted. Geomagnetic time intervals vary slightly throughout the day. The relationship of geomagnetic time to local solar time changes a little during the year. The concept of geomagnetic time, like local solar time, becomes useless near the coordinate poles; in those regions, positions a very short distance apart can be on quite different longitudes and thereby have as much as 12 hours difference in assigned "time."

Conjugate locations are those paired positions on the earth's surface which are connected by one magnetic line of force (figure 5). Conjugate positions at middle and low latitudes can be rather exactly mapped because the earth's field is easily described there. For more polar latitudes some allowance must be made for the distortion of the dipole field threading the outer magnetosphere. This distortion experiences solar-terrestrial-activity-level, seasonal, and diurnal changes. Figure 11 allows one to made a rapid estimation of the conjugate positions; the curved coordinates illustrate how the orthogonal geographic coordinates transform to the opposite (N/S) hemisphere along magnetospheric field lines. Above about 60° geomagnetic latitude such maps become less reliable and more dependent upon the particular variation of outer magnetospheric modeling.

Bibliography References: 1, 6, 7, 15, 16, 18.

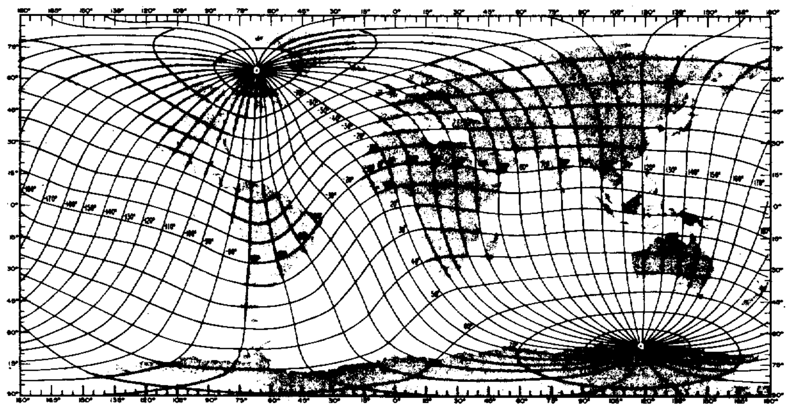

Figure 11. In reading this conjugate coordinate map note that the E-W directed, curved, solid lines indicate conjugate latitude: + for °N, - for °S; the N-S directed, curved, solid lines indicate conjugate longitude: + for °E, - for °W. For any geographic location these curved coordinates, read from the map, will give the approximate geographic latitude and longitude for the conjugate site. For example, the geographic position 55°N, 79°W near Great Whale River, Canada, occurs at the intersection of the -80° and -120° curved grid solid lines. The approximate conjugate location in geographic coordinates is therefore (80°S, 120°W) Byrd Station in Antarctica [from Campbell and Matsushita, 1967].

8. MAIN FIELD CHARTS, ANOMALIES, AND SECULAR CHANGE

The charting of the changing earth's surface field has been a necessary and regular function of the major navigation nations of the world since the early days of world exploration. Typically contour-type lines of equal (isomagnetic) increments of field measurement are plotted. Because mercator projections preserve azimuthal relationships, the magnetic field declination is best represented on such maps. For consistency, the other field components are similarly displayed. Through the years several special adjectives have arisen to specify types of isomagnetic charts:

- isoclinic - equal inclination (dip), I
- isodynamic - equal intensity of field components F, H, X, Y, or Z
- isogonic - equal declincation, D. A line of zero declination is called "agonic."
- isoporic - equal annual time derivative (secular change) of the field component.

Usually the published charts are a representation of some harmonic analysis of the field. For these charts, one should take note of the type of analysis, order and degree of the special harmonic cofficients, epoch, and source of the original data. Figures 12, 13, 14, and 15 show the principal components of the earth's field according to the IGRF (1965.0) spherical harmonic coefficient expansion of order and degree 8, obtained from an internationally accepted mixture of models and data. Accurate charts of the various magnetic contours for 12th order and degree of spherical harmonic coefficients are prepared for general use by the US Navy Oceanographic Office and the Coast and Geodetic Survey in conjunction with the Royal Greenwich Observatory of Great Britain.

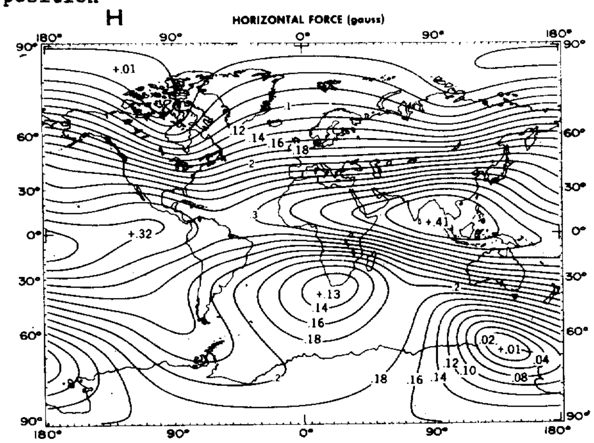

Figure 12. H calculated from IGRF (1965.0) order and degree 8 [from Cain and Cain, 1968].

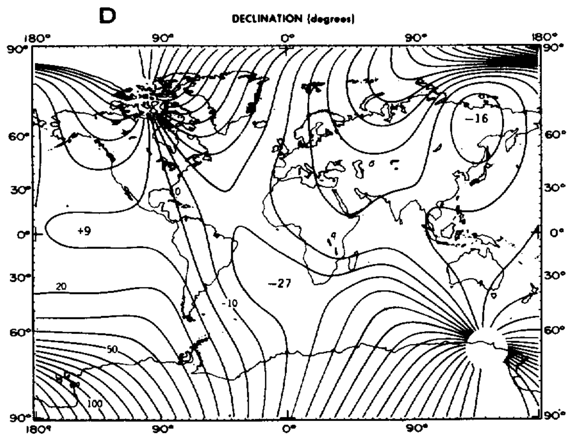

Figure 13. D calculated from IGRF (1965.0) order and degree 8 [from Cain and Cain, 1968].

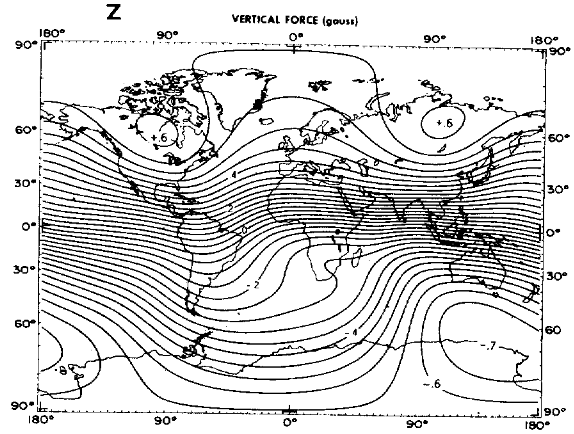

Figure 14. Z calculated from IGRF (1965.0) order and degree 8 [from Cain and Cain, 1968].

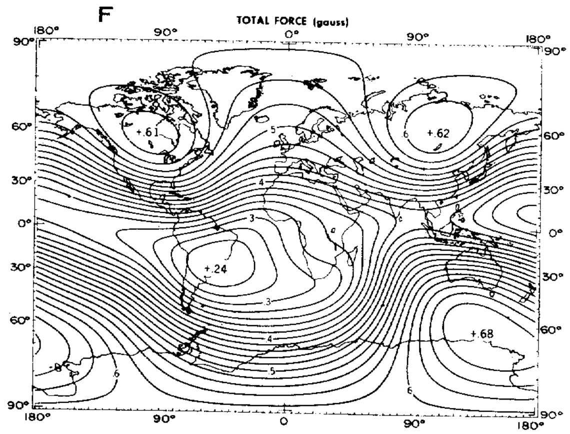

Figure 15. F calculated from IGRF (1965.0 order and degree 8 [from Cain and Cain, 1968].

An isoclinic map was shown in figure 9. Occasionally surface observatories are identified by their dip latitude, ϕ, which is obtained from the field inclination, I, using $\tan I = 2 \tan \phi$.

In all these charts we see patterns that are quite irregular in form with respect to the dipole field coordinates (figure 8). These irregularities may be emphasized by mapping the residuals after subtracting the dipole component of the earth's field. Such nondipole departures are called field anomalies and are often divided by size into local and regional types. The local anomalies, usually under 100 km in extent, seem to be correlated with the geological features of the earth's immediate surface. The field effect of these local magnetic anomalies disappear rapidly with altitude and make very little contribution to the principal magnetosphere. Intense fields exceeding that of the earth's dipole may occur close to deposits of ferromagnetic materials. Special geological surveys of local field anomalies are used in prospecting for natural resources (figure 16). Examples of ore discovery by magnetic survey alone are rare. Rather it is the magnetic identification of lines of discontinuity in rock conductivity which join with other geological characteristics in aiding the mineral exploration.

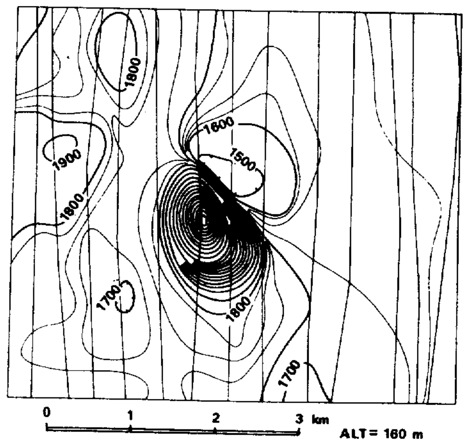

Figure 16. Total field intensity (gammas) measured at 160 meters altitude above Marmore iron-ore deposit about 50 km north of Lake Ontario, Canada (from Serson in Zmuda, 1971).

Regional anomalies, usually more than several thousand kilometers in size, are responsible for the predominant nondipole features on magnetic maps. Because of their great size and gradual change over the years these anomalies are most likely due to the source-current characteristics within the fluid core of the earth. The major regional anomalies reach a maximum of about 4×10^4 γ in Asia and the minimum of about 2×10^4 γ in the South Atlantic.

When we ask how the earth's field is changing with time we must consider that accurate registration of the field of the whole earth has only been possible for the last 100 years. A few reliable station records are available back about 400 years. To investigate the earth's field as long as 2000 years ago, we rely upon the study of archeomagnetism, which concerns dated archeological remains, such as bricks and pottery into which the contemporary field had been baked. For the prehistory of our earth's field we must rely upon the evidence of paleomagnetism which tells us how the earth's field, back to about 4.5 million years, has left its magnetic imprint in the formation of rocks.

Paleomagnetic researchers find that the earth's field has been essentially dipolar and aligned close to the earth's rotational axis. Yet this field direction has reversed by 180° many times. Some studies report that the reversal period takes about 4.6×10^3 years. There is evidence that the present earth dipole field moment has been continually decreasing in strength since the early 19th century. This change is now at such a rate that the dipole field would essentially disappear in a few thousand years. At present, the dipole energy seems to be transferring to the multipole forms. In 1970 the dipole changes amounted to about 35 gammas/year at the equator.

Several additional features of the earth's main field contribute to the changing field vector at a station. Accurate measurements from recent years indicate a gradually slowing, westward precessional drift of the geomagnetic pole location, 0.04°/year; although paleomagnetic results show that the earth's rotational and dipole axes coincide when averaged over about 10^4 year or more.

Identifiable patterns of the non-dipole part of the earth's field seem to drift westward at a rate of approximately 0.10°/year. This would cause these anomalies to circle the globe in about 3,000 years were it not for the fact that their form may be noticeably altered in less than 100 years. On the average, the nondipole components are changing by about 50 gamma/year. For some spherical harmonic field expansions, both the first and second time derivatives of the Gauss coefficients are determined to allow some prediction of the field pattern variation with change in time from the original analysis epoch. Westward drift of the field may be explained by a differential rotation speed between the earth's mantle and its liquid core. The eccentric axis dipole location is particularly sensitive to the multipole parts of the earth centered harmonic expansion. The "non-dipole westward drift" can also be described by the westward motion of the eccentric axis location, about 0.11 degrees/year in 1966/67. A correlation of 0.93 has been established between the change in this westward drift and the change in the earth's rotation speed seven years earlier. This effect is explained to be consequence of the fluid-core dipole current source behavior and of the time for changes to reach the earth's surface.

At most stations about the world a secular change of the field appears quite clearly in a graph of the monthly average of local field variations (figure 17). In such examples the secular change for any component, w, at a station is described by the best fitting of constants, a, in the power series,

$$w = a_0 + a_1 t + a_2 t^2$$

where t is the time in years from the desired epoch. For the global picture isophoric (isovariation) charts are prepared. These show the secular change for a magnetic field component as established for particular epochs, analysis techniques, and data sources. Figure 18 gives an example of the secular variation in total field, F, for the IGRF (1965). Such charts are obtained from the first time derivative of the Gauss coefficients (table 2). A curious unexplained feature seen on all isotropic charts over the years is the small secular changes in the Pacific hemisphere contrasting with the larger values throughout the Atlantic hemisphere.

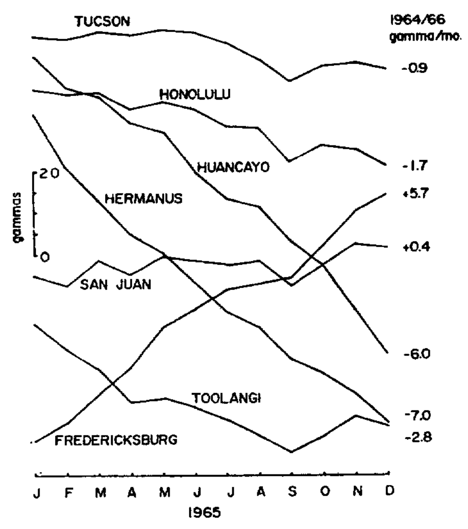

Figure 17. Secular change shown in values of monthly average of H field components at the six stations indicated for 1965. The mean changes in gammas/month for the quiet sun years 1964 through 1966 are indicated to the right of the diagram.

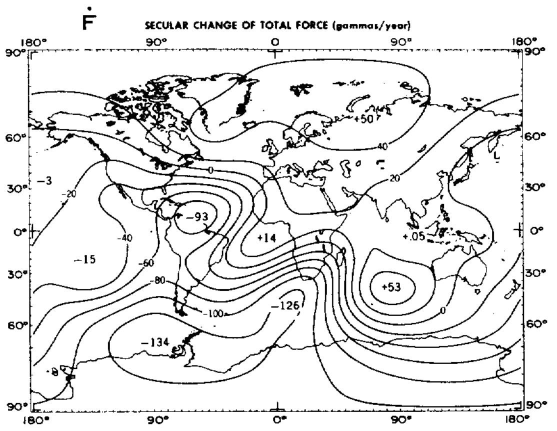

Figure 18. Secular change in F calculated from IGRF (1965.0) order and degree 8 [from Cain and Cain, 1968].

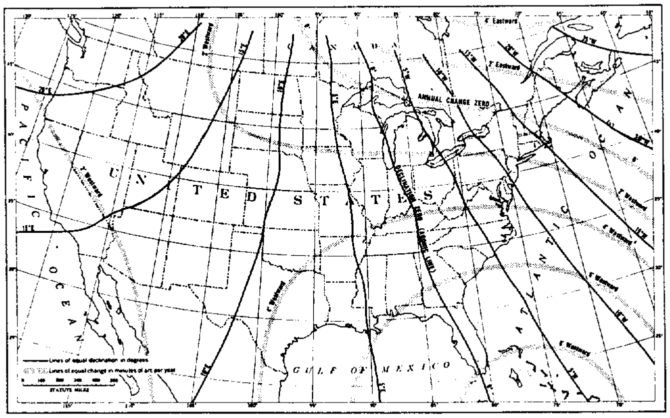

Figure 19. Contours of declination and its secular change for portion of USA epoch 1970 (from C&GS chart 3077 courtesy of E. Fabiano).

Field changes are so large in some areas that without secular information new component charts would otherwise need to be made at least yearly to provide necessary navigational accuracy. The declination and its secular change are also of considerable importance for land surveys. Figure 19 illustrates the manner in which such charts are often presented.

Bibliography References: 3, 5, 7, 8, 9, 10, 12, 18, 19.

9. ACKNOWLEDGEMENTS

The original manuscript for this paper was prepared by the author at the High Altitude Observatory of the National Center for Atmospheric Research. In revised form it is intended to become a part of an introductory textbook "Magnetism of the Earth in Space" which will be released as one of the Space Science Text Series of John Wiley and Sons, Inc. There exists a general interest and need of local observers presently engaged in geomagnetic measurements to have an introductory document on Solar-Terrestrial activity. Permission was therefore obtained from the book publisher to allow the early release of this partial chapter for a NOAA Technical Report.

As a review article the presentation relied heavily upon published material listed in the accompanying bibliography. The interested reader is encouraged to peruse these primary sources for a more detailed discussion of the subjects.

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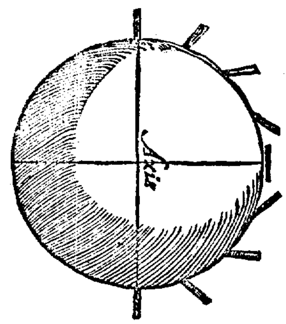

Illustration from William Gilbert, De Magnete, Book II, Chapter VI, 1600 AD, "How Magnetized Iron and Smaller Loadstones Conform to the Terrella and to the Earth Itself, and are Governed Thereby."

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