Synthesis of Vortex Rossby Waves. Part I: Episodically Forced Waves in the Inner Waveguide

AMARYLLIS COTTO

National Weather Service, WFO, San Juan, Puerto Rico

ISRAEL GONZALEZ III AND HUGH E. WILLOUGHBY

Department of Earth and Environment, Florida International University, Miami, Florida

(Manuscript received 22 December 2014, in final form 19 June 2015)

ABSTRACT

Spiral cloud bands dominate tropical cyclones' appearance in satellite and radar images. It is generally accepted that at least some of them are vortex Rossby waves that propagate on the radial gradient of mean-flow-relative vorticity. This study models these features in Fourier and time domains as linear, barotropic, nondivergent waves on a maintained mean vortex scaled to resemble tropical cyclones. This formulation is the simplest one imaginable that encompasses the essential rotational dynamics.

The modeled waves are episodically forced by a rotating annular train of sinusoidal vorticity sources and sinks that crudely represents eyewall convection. Substantial quiescent time intervals separate forced intervals. The waves propagate wave energy predominantly outward and converge angular momentum inward. Waves' energy is absorbed as their perturbation vorticity becomes filamented near the outer critical radii where their Doppler-shifted frequencies and radial group velocities approach zero. The waves can propagate spatially only in narrow annular waveguides because of their slow tangential phase velocity and the restricted Rossby wave frequency domain. Although radial shear of the mean flow distorts their velocity field into tightly wound spirals, their streamfunction and geopotential fields assume the form of elliptical gyres or broad trailing spirals that do not resemble observed hurricane rainbands.

1. Introduction

Vortex Rossby waves (VRWs; e.g., MacDonald 1968; Montgomery and Kallenbach 1997) have been plausibly advanced as an explanation for a subset of spiral bands in tropical cyclones (TCs)—with some observational and modeling support. Here we undertake a reexamination of their essential rotational dynamics in using a barotropic, nondivergent (BND) model. Our purpose is to articulate a computationally and conceptually simple perspective on VRW propagation, responses to forcing, and interactions with the mean vortex. A companion paper (Gonzalez et al. 2015) reexamines vortex motion on a beta plane from a VRW perspective.

These modeled VRWs have a common, specified tangential wavenumber and a spectrum of frequencies.

DOI: 10.1175/JAS-D-15-0004.1

Their intermittent forcing is synthesized using a Fourier series that represents a sinusoidal vorticity source that rotates around the storm twice as it ramps up and then back down to zero. Subsequently, the forcing is zero for the equivalent of six more rotation periods. The resulting VRW wave train is a superposition of steady, rotating components each forced by its Fourier component of the vorticity source. The evolution of the wave field thus reflects constructive and destructive interference among the components. The Fourier forcing is purely rotational with minimal Gibbs phenomenon and no net induced axially symmetric vorticity.

Around the eye, observed spiral rainbands rotate cyclonically at speeds generally slower than that of the axially symmetric tangential wind. They are elongated strands of precipitating clouds and convection that tend to wrap around the vortex. These bands can extend hundreds of kilometers from the eyewall (Romine and Wilhelmson 2006). Geometrically they can be represented as trailing equiangular spirals (Senn and Hiser 1959). Their phases appear to propagate outward as

Corresponding author address: Dr. Hugh E. Willoughby, Department of Earth and Environment, Florida International University, 11200 SW 8th Street, AHC-5 360, Miami, FL 33199. E-mail: hugh.willoughby@fiu.edu

they are advected cyclonically downwind by the mean swirling flow.

MacDonald's (1968) interpretation that spiral rainbands are Rossby waves propagating upstream upon the negative radial gradient of mean-vortex relative vorticity was analogous to midlatitude Rossby wave propagation on the meridional gradient of planetary vorticity. The hypothesis was tenable because the spiral bands tilt upstream, they move more slowly than the mean wind, and convective cells advect through them. If one continues the analogy with midlatitude cyclones, they should transport angular momentum inward and wave energy outward.

Alternatively, Willoughby (1977, 1978) proposed that spiral bands are inward-propagating inertia-buoyancy (IB) waves. Simulated bands in this model exhibited transport of energy toward the center of the vortex and outward transport of angular momentum. Waves excited at the vortex periphery propagated upstream, against the mean flow (like Rossby waves), and were advected slowly downstream. If the cyclone was strong enough (maximum velocity $> 50 \,\mathrm{m \, s^{-1}}$), IB waves could be Doppler shifted to the Brunt–Väisälä (buoyancy) frequency N. At this IB critical radius, which more or less corresponded to the radius of maximum wind (RMW), the radial wavenumber became locally infinite and the waves were absorbed. The work in the 1970s initially focused on simulation of VRWs but then shifted to IB waves when the VRW model was unable to simulate narrow, trailing geopotential spirals that extended over substantial radial intervals because (we now know) of VRW's slow intrinsic phase propagation.

In addition to slower speed, VRW propagation differs significantly from IB waves because VRWs are confined to a narrow waveguide enclosed by loci where their frequencies are Doppler shifted to the local propagation frequency of a one-dimensional VRW (where their local radial wavenumber is zero) or to zero frequency, which defines the VRW critical radius (where their local radial wavenumber becomes large). For VRWs in the TC core where vorticity decreases outward, the radius at which the frequency is Doppler shifted to the one-dimensional VRW frequency is nearer the center and the critical radius farther from it, opposite to the arrangement for IB waves.

Guinn and Schubert (1993) analyzed Rossby wave characteristics and the relationship between spiral bands and the potential vorticity (PV) field, neglecting friction and mass sources or sinks in their shallow water, *f*-plane PV model. Their waves propagated on a circular, piecewise-continuous distribution of mean-vortex PV, such that PV perturbations appeared as undulations of the boundaries. By analogy with the general circulation, PV contours assumed sinusoidal wave patterns; centers of positive and negative PV propagated upwind. In an outer "surf zone," where the PV gradients were relatively weak, the radial group propagation slowed, resulting in accumulation of wave energy and transfer their PV to the mean flow.

Shapiro and Montgomery (1993) advanced the asymmetric balance (AB) approximation as a way to represent asymmetries in TCs. It is the high-Rossby-number (Ro) analog to the synoptic-scale quasigeostrophic formulation. The AB approximation, in which the second time derivative was assumed to be slow compared with the square of inertia frequency, allowed for balancedwind calculations in high-Ro flows. It also allowed for divergent perturbations, whose radial wavenumber increased with time as wave packets propagated across the radially shearing mean flow and energy transfers from the asymmetric flow to the mean vortex.

The Eliassen–Palm theorem (EPT; Eliassen and Palm 1960) was originally developed in a quasigeostrophic context to describe synoptic-scale flows in geostrophic and hydrostatic balance. As rederived for the TC case, the EPT describes variations of radial eddy fluxes of wave energy and angular momentum that interact with the mean flow only where the waves are forced or where they experience critical-surface absorption (Andrews and McIntyre 1976a,b; Boyd 1976).

Montgomery and Kallenbach (1997) used AB to compute Wentzel-Kramers-Brillouin (WKB) solutions for spiral, vorticity-wave structures in both Rankine-like and continuous vortices. The waves exhibited the same characteristics that MacDonald (1968) described. The AB formulation filtered out IB waves. With zero heating or friction, wave propagation depended entirely upon conservation of PV. Energy and momentum transferred from asymmetric PV anomalies to the symmetric mean flow could force mean-vortex intensity changes over time (e.g., Montgomery and Enagonio 1998). In the continuous-vortex wavenumber-1 version of the problem, the vortex center was displaced and the circulation intensified as an initial wavenumber-1 PV anomaly wrapped around the vortex in tightly wound spiral filaments.

Möller and Montgomery (1999, 2000) confirmed intensification through incorporation of initial PV anomalies into axially symmetric shallow-water and three-dimensional baroclinic vortices. A key common factor in the work of Guinn and Schubert (1993) and Montgomery and his coauthors was formulation as an initial-value problem in which a preexisting perturbation with net cyclonic PV evolved dynamically. In Guinn and Schubert (1993), outward diffusion from the high-PV core became filamented into spiral bands. In Montgomery et al.'s work, the net cyclonic part of initial PV distribution became incorporated into the axially symmetric vortex, resulting in mean-flow intensification as the asymmetric PV filamented. Subsequent analyses (Hendricks et al. 2004; Montgomery et al. 2006) of full-physics numerical simulations extended the latter paradigm to include convectively generated PV anomalies.

AB is not strictly applicable for wavenumbers greater than 1. Wavenumber-2 instabilities do indeed exist (Terwey and Montgomery 2002); however, their impact on the vortex is difficult to understand in the AB context. In an alternative to WKB or piecewise continuous analyses, Willoughby (1978) analyzed continuous trains of forced IB waves that conserved the tangential wavenumber and rotation frequency with respect to the ground as they propagated radially. Here we apply this approach through Fourier synthesis of a spectrum of intermittently forced VRWs.

The hypothesis that spiral bands are VRWs has appealing aspects. They are advected downwind as a train of cyclonic and anticyclonic vortices moving around the vortex with less than the mean tangential wind speed in a passband between the one-dimensional VRW frequency and zero frequency. They propagate wave energy outward, even though their radial phase velocity is directed inward. If they are excited near the RMW, they should transport angular momentum inward toward the locus of forcing and carry wave energy outward toward the critical radius. Thus, one would expect VRWs excited in the eyewall to accelerate the flow there and to decelerate the flow at the critical radius. Somewhat paradoxically, deceleration at the critical radius increases the vorticity by reducing, or even reversing, the anticyclonic shear outside the radius of maximum wind. The waves' energy is absorbed as their Doppler-shifted frequencies and group velocities approach zero. There, cyclonic and anticyclonic vorticity filaments become so narrow and closely packed that their influences mask each other in Poisson inversions to obtain the streamfunction or geopotential.

Inner and outer spiral rainbands are distinctive features in TC imagery. Numerical simulations seem to link their properties to VRWs. Extending the synthesis of Willoughby et al. (1984) and Willoughby (1988), Houze (2010) categorized observed rainbands as "distant," "primary," and "secondary." Distant rainbands form outside the vortex core, far from the storm center. Although it may be argued that they are not VRWs since the mean-flow radial vorticity gradient there is so weak, Li and Wang (2012) simulated outward-propagating wavenumber-1 spirals that seemed consistent with VRW dynamics. The primary rainband is a (predominantly) azimuthal wavenumber-1 feature, essentially stationary within the vortex core; however, it is not generally interpreted to be a VRW. Secondary rainbands are tightly wound spiral bands just outside the eyewall. They are much smaller than the primary rainband, with VRW-like radial and azimuthal propagation.

Radar observations show that secondary rainbands often have properties consistent with VRWs. For example, spiral bands of vorticity in Hurricane Olivia (1994) were located near the 20-km radius. Deep convection in the eyewall may have forced VRWs, leading to outward energy fluxes and inward momentum fluxes consistent with the Eliassen-Palm theorem (Reasor et al. 2000; Black et al. 2002). However, other factors, such as local vertical shear, could have contributed to the storm's intensity changes, and it was not apparent that the bands alone caused the vortex to spin up. In another example, reflectivity radar observations during Hurricane Elena's (1985) rapid intensification and weakening showed wavenumber-2 spiral bands that propagated outward while rotating cyclonically more slowly than the mean flow, consistent with the VRW theory (Corbosiero et al. 2005, 2006).

Research aircraft data collected in Hurricanes Rita and Katrina during the Hurricane Rainband and Intensity Change Experiment of 2005 (RAINEX; Judt and Chen 2010) appeared to show VRWs as the intense TCs experienced secondary eyewall replacements. An outer PV maximum generated by convective forcing became pronounced as the eyewall contracted and the secondary wind maximum developed.

Romine and Wilhelmson (2006) report small-scale spiral bands that may have stemmed from shearing instability in a numerical simulation of Hurricane Opal (1995). These waves may have influenced hurricane intensity changes through the transport of angular momentum into the core, but axially symmetric convection seems to have been at least equally important. A 24-h MM5 simulation reproduced formation of spiral bands observed during the first stage of Hurricane Andrew's rapid deepening (Chen and Yau 2001). Analysis of the latent heat release and PV anomalies suggested that convectively forced VRWs caused acceleration of the mean wind in the lower and middle troposphere, both inside and outside the eyewall, and deceleration in the upper troposphere above the eyewall (Chen et al. 2003).

Qiu et al. (2010) simulated inward-propagating spiral rainbands and outward-propagating features interpreted to be VRWs. Convection in the spiral bands excited perturbations that moved PV toward the inner core of the vortex, while VRWs became elongated tangentially and compressed radially as they moved outward. Simultaneously, the primary eyewall shrank gradually, outer spiral bands shifted inward, and the vortex formed an intensifying secondary eyewall. The secondary eyewall became the new primary eyewall, which intensified rapidly, ostensibly as a result of the VRW filamentation. In the interpretation of Qiu et al. (2010), VRWs accelerated the mean flow in two ways: by the axisymmetrization process of Montgomery and Kallenbach (1997) and by enhancement of convective PV generation near the stagnation radius.

VRW's role in intensity change has been explored through initial-value simulations. Martinez et al. (2010a) initialized a BND vortex with a wavenumber-4 vorticity asymmetry. Wave angular momentum transports caused the mean vortex to accelerate inside its radius of maximum winds and decelerate outside. A moat of lowperturbation vorticity amplitude formed between the vortex core and VRW spirals that had propagated outward to the edge of the mean-flow vorticity skirt, suggesting a possible mechanism for initiation of outer eyewalls. Follow-on experiments (Martinez et al. 2010b) with either an elliptical (wavenumber 2) initial vorticity distribution on a weak stable vortex or initial random perturbations on a stronger, barotropically unstable vortex also predicted strengthening and contraction of the eyewall wind maximum through wave-mean flow interactions. Quasimodes (Schecter et al. 2000; Schecter and Montgomery 2003, 2004) are continuous spectrum asymmetries that behave much like true eigenmodes. They were important to intensification of the weaker vortex. A follow-on study with stable and unstable BND mean vortices confirmed these results and related formation of long-lasting elliptical eyes to a slowly damped quasimode (Menelaou et al. 2013). Initialization of a weak vortex with thermal anomalies in a dry version of the WRF Model showed that they may have a role in tropical cyclogenesis (Menelaou and Yau 2014).

Nguyen et al. (2011) argued that TC intensification proceeds in alternating symmetric and asymmetric episodes, consistent with Kossin and Eastin (2001). During the symmetric stage, the eyewall develops a ringlike PV structure in response to circularly symmetric heating. Then, barotropic instability develops, leading to redistribution of PV. During this asymmetric stage, the central pressure falls coincident with some weakening of the maximum wind. Wave momentum transports remove the eyewall PV maximum, setting the stage for renewed symmetric intensification.

In contrast with the vorticity dynamics considered previously, Nolan and Montgomery (2002) and Nolan and Grasso (2003) initialized perturbations on symmetric vortices as asymmetric and symmetric thermal anomalies. The adjustment process occurred in two stages: adjustment to balance and axisymmetrization of the resulting vorticity perturbation. Asymmetric thermal perturbations weakened the mean vortex in most cases, whereas symmetric thermal perturbations strengthened it—but only about as much as would be expected from a balanced response to the heat added. In follow-on simulations (Nolan et al. 2007), convectively induced wave momentum fluxes appear to have weakened the vortex.

2. Vortex Rossby wave dynamics

While symmetric heating appears to be the dominant factor of TC intensity change, asymmetric heating controlled by shearing environmental flows or internal dynamics, including VRWs, may also be important (Willoughby et al. 2007). As discussed previously, asymmetric convection can excite Rossby waves that affect vortex development through eddy fluxes of angular momentum, as can waves that arise from reversals of the vorticity gradient inside the RMW that satisfy the necessary condition for barotropic instability (e.g., Kossin et al. 2000). The BND formulation explicitly excludes the RIB resonance between VRWs and IB waves that can cause both wave trains to grow (Schecter and Montgomery 2003, 2004).

Here, we examine nondivergent VRWs forced by imposed vorticity sources and sinks that crudely represent eyewall convection (Cotto 2012). VRWs' radial phase and group velocities can be directed either outward or inward, but, as shown above, their tangential phase velocity is always directed upstream in a vortex with outwardly decreasing axially symmetric vorticity. The phase and group velocities are relatively slow ($<10 \text{ m s}^{-1}$) in comparison with the ($\sim 20-50 \text{ m s}^{-1}$) meanflow wind in the vortex core. Thus, they are advected around the vortex as a wave train of cyclonic and anticyclonic vortices that move downstream with somewhat less than the mean-flow speed. The BND model is the simplest physical system that embodies VRWs' rotational dynamics.

The analysis begins with the linearized momentum and continuity equations in cylindrical coordinates (Fig. 1). Here, $V_0(r)$ is the mean tangential wind; r is radius; λ is azimuth angle (reckoned cyclonically from north); u is radial perturbation wind, positive outward; v is the tangential perturbation wind, positive cyclonically; ϕ is perturbation geopotential; and f_0 is the Coriolis parameter:

$$\frac{\partial u}{\partial t} + \frac{V_0}{r} \frac{\partial u}{\partial \lambda} - \left(\frac{2V_0}{r} + f_0\right)v + \frac{\partial \phi}{\partial r} = F_r, \quad (1a)$$

$$\frac{\partial v}{\partial t} + \frac{V_0}{r} \frac{\partial v}{\partial \lambda} + \left(\frac{\partial V_0}{\partial r} + \frac{V_0}{r} + f_0\right)u + \frac{1}{r}\frac{\partial \phi}{\partial \lambda} = F_{\lambda}, \text{ and }$$

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{1}{r} \frac{\partial v}{\partial r} = 0, \quad (1c)$$



FIG. 1. Vortex-centered cylindrical coordinates with azimuth reckoned cyclonically from north.

where $\partial/\partial t + (V_0/r)\partial/\partial\lambda$ is the linearized Lagrangian derivative, $\xi_0 = 2V_0/r + f_0$ is the inertia parameter, $\zeta_0 = \partial V_0/\partial r + V_0/r + f_0$ is the mean-flow absolute vorticity, and F_r and F_λ are the imposed forcing derived from a vector forcing potential **A** such that $F_r = -r^{-1}(\partial \mathbf{A}/\partial\lambda)$ and $F_\lambda = \partial \mathbf{A}/\partial r$, so that the forcing is purely rotational.

The vorticity equation is formed by cross differentiating the momentum equations and substituting from the continuity equation:

$$\begin{pmatrix} \frac{\partial}{\partial t} + \frac{V_0}{r} \frac{\partial}{\partial \lambda} \end{pmatrix} \begin{pmatrix} \frac{\partial v}{\partial r} + \frac{v}{r} - \frac{1}{r} \frac{\partial u}{\partial \lambda} \end{pmatrix} + u \frac{\partial \zeta_0}{\partial r}$$

$$= \frac{\partial^2 \mathbf{A}}{\partial r^2} + \frac{1}{r} \frac{\partial \mathbf{A}}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \mathbf{A}}{\partial \lambda^2} \equiv Q.$$
(2)

The strictly nondivergent flow can be represented with a streamfunction ψ such that $u = -r^{-1}(\partial \psi / \partial \lambda)$ and $v = \partial \psi / \partial r$:

$$\left(\frac{\partial}{\partial t} + \frac{V_0}{r}\frac{\partial}{\partial\lambda}\right)\left(\frac{\partial^2\psi}{\partial r^2} + \frac{1}{r}\frac{\partial\psi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\psi}{\partial\lambda^2}\right) - \frac{1}{r}\frac{\partial\psi}{\partial\lambda}\frac{\partial\zeta_0}{\partial r} = Q.$$
(3)

Sinusoidal Fourier components with tangential wavenumber *n* and frequency ω are represented in terms of complex exponentials and radial structure functions $\psi(r, t, \lambda) = \text{Re}[\Psi(r)e^{i(\omega t - n\lambda)}]$, where Ψ is a function of *r* alone. The unforced left-hand side of (3) simplifies to obtain the dispersion relation for free waves:

$$\left(\omega - \frac{nV_0}{r}\right) \left(\frac{d^2\Psi}{dr^2} + \frac{1}{r}\frac{d\Psi}{dr} - \frac{n^2}{r^2}\Psi\right) + \left(\frac{n}{r}\frac{\partial\zeta_0}{\partial r}\right)\Psi = 0,$$
(4)

which may be solved for the local Doppler-shifted frequency Ω :

$$\Omega = \left(\omega - \frac{nV_0}{r}\right) = -\frac{\left(\frac{n}{r}\frac{\partial\zeta_0}{\partial r}\right)\Psi}{\left(\frac{d^2\Psi}{dr^2} + \frac{1}{r}\frac{d\Psi}{dr} - \frac{n^2}{r^2}\Psi\right)}.$$
 (5)

By assuming a convenient functional form for $\Psi(r)$, we can write the expression for Ω in terms of tangential and local radial wavenumbers. For example, expressing $\Psi(r)$ as a zero-order Hankel function $\Psi = H_0(k_r r)$, where k_r represents the radial wavenumber, yields a locally valid (in the WKB sense) dispersion relation. Since $\partial \zeta_0 / \partial r$ and V_0 are functions of r, k_r must be a slowly varying function of radius,

$$\Omega = \left(\omega - \frac{nV_0}{r}\right) = \frac{\left(\frac{n}{r}\frac{\partial\zeta_0}{\partial r}\right)}{\left(k_r^2 + \frac{n^2}{r^2}\right)}.$$
(6)

Since the Doppler-shifted frequency is always negative, when $k_r > 0$, the phase velocity C_r is inward and the group velocity C_{gr} is outward; when $k_r < 0$, C_r is outward and C_{gr} is inward, as shown in Fig. 2.

The method of Lindzen and Kuo (1969) is well suited for solving second-order partial differential equations, like (4), with boundary conditions imposed at both ends of the domain. Following Willoughby (1977), each Fourier component is characterized by ω , its spatially constant rotation frequency with respect to the ground. Thus, the wave pattern rotates with a constant angular velocity and the Doppler-shifted frequency varies with radius as the tangential advection changes. For a specified $V_0(r)$, the coefficients in (4) are known. Replacing the radial derivatives with second-order finite differences on a 1-km radial grid transforms (4) to a tridiagonal algebraic system. The Lindzen-Kuo algorithm works like a conventional tridiagonal solver. It reduces the system to lower-diagonal form on an outward pass and the back substitutes from the vortex periphery to obtain Ψ at each radial grid point. The algorithm requires boundary conditions at the vortex center and r =4000 km, the outer boundary of the domain. Since these boundary points lie far outside the VRW waveguide, $\Psi = 0$ is appropriate at both ends.



FIG. 2. Schematic VRW dispersion relation showing Dopplershifted frequency as a function of radial wavenumber for different tangential wavenumbers.

Once $\Psi(r)$ is known, a derivation analogous to that for vorticity, but with reversed order of differentiation, yields the divergence equation. It is actually a Poisson equation for the geopotential with forcing that is a function of $\Psi(r)$. It is readily solved using the Lindzen– Kuo algorithm.

The Eliassen–Palm relation is an excellent tool for gaining insight into wave energy and angular momentum fluxes as well as wave–mean flow interactions. The derivation begins with the linearized tangential momentum equation written in terms of the tangential phase speed and tangential wavenumber. Multiplication by minus the Doppler-shifted tangential rotation speed times the perturbation tangential wind plus the geopotential, simplifying and integrating around the vortex, so that the exact tangential derivatives vanish, yields

$$-\frac{\Omega r}{n}\langle uv\rangle + \langle u\phi\rangle = 0, \quad \text{or} \quad \Omega r\langle uv\rangle = n\langle u\phi\rangle.$$
(7)

In (7), angle brackets denote integration around a circle of constant radius to compute radial eddy fluxes. The product of the Doppler-shifted frequency with the eddy angular momentum flux equals the product of the tangential wavenumber with the eddy geopotential flux. Since for VRWs $\Omega < 0$, this relation shows that an outward eddy flux of wave energy, $\langle u\phi \rangle > 0$, requires an inward flux of angular momentum, $r\langle uv \rangle < 0$.

The propagation of wave energy is naturally away from the source. The dispersion relation (6) describes propagating waves in a passband between $\Omega = 0$ and the frequency of a tangentially propagating onedimensional VRW, $\Omega_{1D} = (\partial \zeta_0 / \partial r)(n/r)^{-1}$. The radius at which $\Omega(r_{1D}) = \Omega_{1D}$ is a turning point of (4), such that $k_r(r_{1D}) = 0$, and k_r becomes imaginary on the highfrequency side where $|\Omega| > |\Omega_{1D}|$ —that is, where Ω is more negative than Ω_{1D} . Beyond the turning point, the waves' radial structure is evanescent. For forcing in

the eyewall, energy propagates both inward toward the turning radius and outward toward the critical radius. Near the critical radius, at the outer edge of the waveguide, where the group velocity is almost zero, the waveenergy packets stagnate and are eventually absorbed. At the inner edge of the waveguide, the frequency is Doppler shifted to Ω_{1D} . As Fig. 2 shows inward- and outward-propagating branches of the dispersion relation meet at the turning point. Thus, the wave energy can jump from propagating inward, toward the turning point, to propagating outward, away from it. Generally, the waves are at least partially reflected, but the details can depend upon proximity of domain boundaries (e.g., the origin) or the presence of other turning points on the high-frequency side of r_{1D} . The reflected energy propagates outward, back across the RMW to the critical radius where it, too, is absorbed.

The initially inward-propagating energy packets support an outward angular momentum flux that is balanced by angular momentum carried by the waves reflected from the turning radius. The initially outwardpropagating packets carry energy toward the critical radius and angular momentum toward the locus of forcing. Thus, there is a divergence of wave energy from the source, a convergence of wave energy around the critical radius, a divergence of angular momentum from the neighborhood of the critical radius, and a convergence of angular momentum near the RMW. This convergence intensifies the strongest winds in the eyewall. The present barotropic, nondivergent model is the simplest one that captures these rotational dynamics.

The mean tangential wind (Fig. 3a) is based upon that of Wood et al. (2013). It is a continuous, differentiable function. As used here, it is formulated somewhat differently from Wood et al. (2013). The parameters are $n_{\rm in}$, which defines the power-law variation of wind inside the RMW; n_{out} , which defines the inverse power law outside the RMW; and L (the same as λ of Wood et al.), which defines the width of the transition across the RMW. Values used are $n_{in} = 1$, $n_{out} = 0.5$, and L = 0.25, which correspond to roughly to Riehl's (1963) model of the steady-state hurricane and represent the inner structure of observed hurricanes reasonably well. Defined in this way, the cyclostrophic relative vorticity (not shown) would be cyclonic everywhere, so that in the limit of large radius, a circular path around the vortex would enclose infinite cyclostrophic relative vorticity. The absolute vorticity of the gradient wind (Fig. 3b) is large and spatially uniform inside the radius of maximum wind. It decreases sharply to less than 1/10 of the maximum 50 km from the center and is comparable with the Coriolis parameter by 200 km. At larger radii the relative vorticity is negative so that the circulation based



FIG. 3. Wood et al. (2013) vortex as implemented here with 50 m s^{-1} maximum wind at 25-km radius. (a) The cyclostrophic wind (dashed) increases linearly with radius inside the eye and decreases inversely with the square root of radius outside the eye. The corresponding gradient wind (solid) decreases somewhat more rapidly so that it has zero circulation at large radius. (b) Absolute vorticity computed from the gradient wind and Coriolis parameter at 20°N.

upon the gradient wind approaches zero in the limit of large radius.

Within a few hundred kilometers of the vortex center, the modeled cyclostrophic wind is close to the gradient wind. The wind used in the VRW model is the gradient wind as calculated in Willoughby (2011). Although the cyclostrophic wind has unbounded circulation at arbitrarily large radius, it supports computation of a welldefined pressure wind relation because it specifies the radial geopotential gradient, with which the gradient wind is consistent. The circulation theorem implies that an arbitrarily large path encircling the vortex must contain zero net gradient-wind vorticity. Thus, beyond some radius, both the relative vorticity and its radial gradient must reverse. The latter condition implies the



FIG. 4. Real (solid) and imaginary (dashed) parts of the forcing time series, for a complete forcing cycle, composed of two rotation periods when forcing is active and six subsequent rotation periods when it is quiescent.

existence of a second VRW waveguide in the outer part of the gradient-balance vortex. In this outer waveguide, VRWs should propagate downstream relative to the mean flow with very low frequencies.

3. VRW forcing

The wavenumber-2 forcing used here is cyclic with alternating active and quiescent intervals within a 17648-s cycle (Fig. 4). While the forcing is active, it rotates with a period of 2206 s. It turns on and remains active for 4412 s, or two rotation periods equivalent to one-fourth of the total cycle. This is the active time in Fig. 4. During the remaining three-fourths of the cycle, equivalent to six rotation periods, it is quiescent. The forcing remains off until the beginning of the next cycle. During this time, the waves propagate within the waveguide and ultimately dissipate, returning to the initial startup configuration by the end of the cycle.

The forcing is represented as a superposition of sinusoidal Fourier components (e.g., Churchill 1963). The harmonics of the complex forcing interfere constructively during the active phase and destructively during the quiescent phase. Each harmonic has a constant amplitude, rotation frequency, and relative phase determined by its complex Fourier coefficient. The frequency of the *n*th harmonic is *n* times the frequency of the fundamental. A spectrum of 28 harmonics is adequate to represent the forcing with minimal Gibbs phenomenon.

The forcing spectrum is composed of harmonics -6 to 22. Peak spectral amplitude lies in the eighth harmonic, which has the specified rotation frequency ω_8 . In the



FIG. 5. Wavenumber-2 Doppler-shifted frequency as a function of radius. The solid curve is the most strongly forced eighth harmonic, and the dashed curves are the less strongly forced fourth, sixth, tenth, and twelfth harmonics. The remaining solid curves indicate the one-dimensional VRW Rossby wave frequency and zero frequency. They are the boundaries of the VRW passband as functions of radius. The circle and letter F denote the frequency and center position of the strongest vorticity forcing.

case shown, the eighth harmonic's frequency is 0.6 of the orbital frequency of air moving with the wind at the radius where the forcing is applied. Only harmonics 4–12 contain power levels that contribute significantly to the wave energy.

Vortex Rossby waves can propagate only when their Doppler-shifted frequency is within the passband $\Omega_{1D}(r) < \Omega(r) < 0$. Geometrically, the loci of these Doppler-shifted frequencies delimit a relatively narrow, annular VRW waveguide (Fig. 5): Ω_{1D} is the most negative frequency possible for sinusoidal vorticity waves; Ω is well defined outside the waveguide, but the waves are radially evanescent when $\Omega > 0$ or $\Omega < \Omega_{1D} < 0$.

It is possible to make the (for example) wavenumber-2 waveguide reasonably wide by tuning the fundamental frequency. Harmonics 4, 6, 8, 10, and 12 propagate in a set of waveguides that extends from a minimum of 14-km radius for the twelfth harmonic to a maximum of 40 km for the sixth harmonic (Fig. 5). Locations of the turning point and critical radii differ for each harmonic. The waveguide for the eighth harmonic extends from 17 to 33 km. Since the frequencies of the higher harmonics are less negative than those of the lower harmonics, their critical radii and cut-off frequencies are located closer to the center. By design, the waveguides are widest for harmonics 6, 8, and 10. The eighth harmonic can propagate from 17 to 33 km for a total waveguide width of about 16 km. The less-negative frequency of the twelfth harmonic moves the turning radius inward to 14 km, but



FIG. 6. Fourier-synthesized wavenumber-2 forcing at 2206 s, when it is strongest.

it also causes the critical radius to lie at 23 km. Thus, this harmonic can propagate in a waveguide only 9 km wide.

The fourth harmonic is unusual. Its relatively large negative frequency places the critical frequency farther away from the center, at about 53 km. Before it reaches the critical radius, however, the fourth harmonic encounters another turning point at 29 km and then reenters the waveguide at 43 km. Thus, this wave is trapped between two inner turning radii at r = 20 and 29 km and between the outer turning radius and the critical radius. The evanescent wave may be able to "tunnel" a small amount of energy into this outer waveguide. Since harmonics lower than four have negative frequencies that are everywhere below Ω_{1D} , they are radially evanescent.

4. Wavenumber-2 harmonics

The Fourier-synthesized forcing varies sinusoidally within an annular domain $10 \le r \le 30$ km (Fig. 6) and reaches its maximum at t = 2206 s. The streamfunction and vorticity form the basis for analysis of the wave properties. The most strongly forced eighth harmonic is representative. In Fig. 7, the top panels show the radial structure of the real and imaginary parts of the vorticity (Fig. 7a) and streamfunction (Fig. 7b). Figure 7c shows the wave fluxes of angular momentum and energy as functions of radius. The bottom panels show maps of vorticity (Fig. 7d) and streamfunction (Fig. 7e) for this Fourier component. It is important to keep in mind that the only time variation that any of individual Fourier



FIG. 7. Wavenumber-2, eighth harmonic: (a) real (solid) and imaginary (dashed) parts of the vorticity, (b) real and imaginary parts of the streamfunction, (c) Eliassen–Palm fluxes of angular momentum and geopotential, (d) vorticity field, and (e) streamfunction field. The black circles in (d) and (e) mark the VRW critical radius for this component.



FIG. 8. Fields of complete wavenumber-2 (a),(c),(e),(g) vorticity and (b),(d),(f),(h) streamfunction at (a),(b) 1800, (c),(d) 3600, (e),(f) 5400, and (g),(h) 7200 s.

components exhibit is rotation with its specified frequency.

Since the eighth harmonic is the most strongly forced, it has the largest amplitude. Its vorticity field is qualitatively consistent with previous descriptions (e.g., Montgomery and Kallenbach 1997). It is organized into teardrop-shaped gyres that line up with the strongest forcing. Triangular evanescent tails extend inward beyond the inner waveguide boundary to the center. In the neighborhood of the critical radius, at approximately 33 km, the vorticity filaments wrap tightly around the center. The streamfunction is significantly different. It is 180° out of phase with the vorticity because the vorticity is the Laplacian of the streamfunction. Since Poisson solutions represent powerful smoothing, ψ is much less noisy than the vorticity, exhibiting more or less elliptical gyres with a hint of trailing structure and induced irrotational extensions beyond the critical radius. As expected from the Eliassen–Palm relation for $\Omega < 0$, $\langle u\phi \rangle > 0$ and $r \langle uv \rangle < 0$. The wave energy and momentum propagate in opposite directions. The radial geopotential flux represents energy propagating outward with the radial group velocity. Although the angular momentum flux is essentially confined to the waveguide, some energy flux tunnels outside of it.

The results for all the harmonics are comparable. The maxima of forcing, vorticity, and streamfunction fall within the waveguides. By design, the eighth harmonic number has the widest radial interval. Inside r = 21-26 km, the wave momentum fluxes for all harmonics are convergent so that they act to accelerate the mean flow inward from the RMW. In the outer part of the waveguide, they are divergent. Thus, the wave–mean flow interaction concentrates angular momentum from the outer part of the waveguide into the inner part.

5. Complete wavenumber-2 solution

Fourier synthesis of the time evolution of wavenumber-2 vorticity (Fig. 8, left panels) and streamfunction (Fig. 8, right panels) represents snapshots of the complete cycle at an interval of 1800 s. The forcing turns on at t = 0. By 1800 s, the forcing is ramping up, and teardrop-shaped vorticity anomalies (Fig. 8a) are aligned with elliptical streamfunction gyres of opposite sign (Fig. 8b). Vorticity filamentation appears near the critical radius.

By 3600s, the vorticity and streamfunction are approaching their largest amplitude (Figs. 8c and 8d), even though the forcing has been decreasing for about 1200s. The vorticity maximum is located at approximately r = 22 km, just inside the critical radius. Filamentation

FIG. 9. Wavenumber-3 Doppler-shifted frequencies for the fourth, sixth, eighth, tenth, and twelfth harmonics, plotted as in Fig. 5.

becomes prominent in the neighborhood of the critical radius. The streamfunction gyres have begun to exhibit a trailing-spiral structure with substantial induced circulation outside the critical radii of all Fourier components. By 5400 s, long after the forcing has stopped, the vorticity becomes concentrated in filaments near the critical radius, and the streamfunction around the vortex center weakens as the wave energy propagates outward (Figs. 8e and 8f). The streamfunction is organized into trailing spirals centered on places where vorticity filaments of opposite sign do not overlap. In other places, where filaments of opposite sign do overlap, they mask each other in the Poisson inversion so that the streamfunction is much smaller.

Eventually, at 7200s, strongly filamented vorticity remains but the streamfunction amplitude has died away (Figs. 8g and 8h). Vorticity and streamfunction decrease in amplitude in the vortex core because most of the wave energy has propagated away. Mathematically, the Fourier components interfere destructively there. Near the critical radius, the vorticity anomalies are very elongated and tightly wound so that vorticity masking becomes dominant. The net vorticity in the neighborhood of any point near the critical radius is now virtually zero, so that net forcing of the Poisson solution for the streamfunction is weak, resulting in small amplitude even though significant vorticity is still present.

The wavenumber-2 results illustrate the relationships among the forcing, streamfunction, geopotential, and vorticity for the complete Fourier wave train. The results describe the evolution of streamfunction from elliptical gyres to trailing spirals and finally to damped filaments.





FIG. 10. Wavenumber-3, eighth harmonic: (a) vorticity radial structures, (b) streamfunction radial structures, (c) wave angular momentum and geopotential fluxes, (d) vorticity field, and (e) streamfunction field. Circles in (d) and (e) indicate the VRW critical radius.



FIG. 11. Wavenumber-4 Doppler-shifted frequencies for the fourth, sixth, eighth, tenth, and twelfth harmonics, plotted as in Fig. 5.

6. Wavenumber-3 and -4 solutions

Wavenumbers 3 and 4 behave somewhat differently from wavenumber 2 because the Doppler shift becomes stronger as tangential wavenumber increases. A higher ratio, 0.85, of frequency at peak amplitude to the orbital period of the wind yielded the widest possible waveguide for wavenumber 3. Nonetheless, only harmonics 6, 8, and 10 have significant radial extent within the Rossby wave passband (Fig. 9). Thus, the vorticity, streamfunction fields exhibit more gyres confined to a radially narrower waveguide (Fig. 10). As before, the solutions exhibit trailing streamfunction spirals. The vorticity is characterized by wedge-shaped sectors that become filamented in the neighborhood of the critical radius.

For wavenumber 4, waves rotating with a frequency equal to 0.85 of the wind's orbital frequency also have the widest waveguide, but its width was only a few kilometers. Only harmonics 6 and 8 fall within the VRW passband (Fig. 11). The eighth harmonic exhibits welldefined trailing spirals and vorticity filaments near the critical radius, and the streamfunction exhibits trailing spirals (Fig. 12). The sixth harmonic (not shown) has a similar vorticity pattern but is even more confined radially because much of the forcing lies inward from the edge of the waveguide so that the propagating VRWs connect with the forcing largely through radially evanescent perturbations. Its streamfunction and vorticity gyres are more elliptical.

More rapid filamentation speeds up the time evolution of higher wavenumbers (Fig. 13). At 1800s, while the forcing for wavenumber 4 is still active, the completesolution vorticity is a train of radially compressed, teardrop-shaped eddies that induce inclined elliptical gyres. As the forcing ramps down at 3600 s, the vorticity becomes filamented rapidly so that vorticity masking results in essentially no expression of the vorticity in the streamfunction.

The results for wavenumbers 3 and 4 differ slightly in structure. The frequencies needed to attain the widest possible waveguide become a larger fraction of the winds' orbital frequency and the widths of the waveguides decrease with increasing tangential wavenumber. Of course, the number of streamfunction gyres increases for higher wavenumbers even as the waveguide width decreases and filamentation acts much more quickly.

In general, wavenumber-2 solutions are most like observed spiral bands because the waveguide is wider. The streamfunction exhibits trailing-spiral geometry during the interval after most of the vorticity has propagated to the outer part of the waveguide but before it becomes strongly filamented. Wavenumbers 3 and 4 are so narrowly confined and filamented that they are poor candidates to represent observed rainbands. On the other hand, wavenumber 1 should have a much wider waveguide. Given its interaction with the motion of the mean vortex, it is an attractive subject for further study.

7. Conclusions

The analyses of vortex Rossby waves (VRWs) in previous studies were based on WKB models or idealized piecewise continuous mean flows. Here we use Fourier synthesis to simulate VRWs excited episodically by strictly asymmetric vorticity sources in the eyewall of a barotropic, nondivergent, but otherwise hurricane-like vortex. The analysis follows the motion and evolution of the resulting waves as they are advected downstream, propagate radially, and filament to form trains of cyclonic and anticyclonic trailing spirals.

The barotropic, nondivergent model captures much of the rotational dynamics of these bands and reveals the evolution of the waves as they propagate radially and their harmonics interfere constructively or destructively. Since there is no net symmetric forcing, it induces no net cyclonic vorticity directly, but the waves sustain eddy fluxes that could change the mean vortex nonlinearly.

Propagating waves can exist only in a frequency passband between the one-dimensional Rossby wave frequency and zero Doppler-shifted frequency. Geometrically, this passband defines an annular waveguide within which the forcing is localized. Waves that initially propagate inward are Doppler shifted to the onedimensional VRW frequency, reflected at the turning radius, and subsequently propagate outward. Both these



FIG. 12. Wavenumber-4, eighth harmonic: (a) vorticity radial structures, (b) streamfunction radial structures, (c) wave angular momentum and geopotential fluxes, (d) vorticity field, and (e) streamfunction field. Circles in (d) and (e) indicate the VRW critical radius.



FIG. 13. Fields of complete wavenumber-4 (a),(c) vorticity and (b),(d) streamfunction at (a),(b) 1800 and (c),(d) 3600 s.

waves and those that initially propagate outward become filamented and are ultimately absorbed when they reach the outer critical radius.

The waveguide is narrow because the waves' phase speeds are slow and the interval $0 > \Omega > \Omega_{1D}$ is narrow. Early in the forcing cycle, the streamfunction forms elliptical gyres centered on the vorticity extrema. The vorticity perturbations that accumulate near the critical radius stretch into narrow cyclonic and anticyclonic bands that become filamented as they wrap around the vortex. The structures of the corresponding streamfunction gyres, however, are predominantly relatively broad elliptical gyres or trailing spirals that show less filamentation than the vorticity. Later, when the vorticity has propagated outward and become increasingly filamented near the critical radius, they briefly become trailing spirals centered on places where filaments of opposite sign do not mask each other. Ultimately, progressive filamentation masks

nearly all of the vorticity so that the streamfunction amplitude becomes small.

As shown in the appendix, a TC-like vortex can support a quasimode-like feature (Schecter et al. 2000), composed of a transfinite set of VRWs with near-zero Doppler-shifted frequencies propagating in narrow, overlapping annular waveguides outside of the inner strong mean radial vorticity gradient. When the meanflow vorticity decreases outward, these waveguides are also bounded by inner turning radii and outer critical radii, such that the waves' frequencies lie between the local Ω_{1D} and zero. In initial-value simulations with timemarching models, the initial vorticity asymmetry projects onto this quasimode, is carried downwind, and filaments, but it cannot propagate far radially. Similarly, initial vorticity that does not project onto any VRW can filament and strengthen the mean vortex by inward angular momentum fluxes, as described by Möller and Montgomery (1999). The forced waves with wavenumbers 2 and

higher that are modeled here have somewhat wider waveguides, but they illustrate the challenges that slow tangential-phase propagation poses to interpretation of TC rainbands as VRWs. Wavenumber-1 VRWs, which are much less sensitive to the Doppler shift but are complicated by interaction with the mean-vortex vortex translation, may prove to be more promising candidates.

The foregoing characteristics make it difficult to identify vortex Rossby waves with observed hurricane rainbands, other than perhaps some inner rainbands. Nonetheless, they seem to be a viable mechanism for feeding vorticity and angular momentum into the bases of vertical hot towers (e.g., Hendricks et al. 2004; Montgomery et al. 2006). The wave momentum and geopotential fluxes are consistent with the Eliassen-Palm relation. Since the Doppler-shifted frequency is negative, they are oppositely directed. The computed waves transport net angular momentum inward and net wave energy outward. Angular momentum flux divergence near the critical radius decelerates the mean flow there and angular momentum convergence accelerates the mean flow at the locus of forcing. Both the quasimode and vorticity that do not project onto VRWs exhibit similar mean-flow interactions. Although vortex Rossby waves with wavenumbers 2 and higher can apparently influence intensity change, their slow phase velocity and narrow frequency range limit their effect to a narrow radial interval outward from the locus of forcing.

Acknowledgments. This research was supported by NSF Grants ATM-0454501 and AGS-1211172 to Florida International University.

APPENDIX

Doppler-Shifted Frequency of a Single Quasimode Component

When treated as an initial-value problem, vorticity perturbations on a mean vortex can project onto a countable set of radially propagating VRWs, as simulated here, onto a dense superposition of VRWs that we identify with the "quasimode" (e.g., Schecter et al. 2000) or onto nonpropagating vorticity. All of these phenomena are subject to filamentation that generally acts to strengthen the mean vortex, but only the first supports propagation over a significant radial distance.

Outside the eyewall, but close enough to the center for the cyclostrophic approximation to be valid, the axially symmetric Wood–White wind profile V_0 may be written the locally as $V_0(r) = V_{MAX}(R_{MAX}/r)^m$, where V_{MAX} is



FIG. A1. The radial extent of the waveguide for a quasimode component as defined by the variation of Doppler-shifted frequency between the one-dimensional VRW frequency and zero frequency. Two overlapping passbands for nearby rotation frequencies appear in gray.

the TC's maximum wind that occurs at radius R_{MAX} , and $m = \frac{1}{2}$ is an exponent (equivalent to n_{out} in section 2) that defines the profile shape. The relative vorticity of this profile is $\zeta = (1 - m)V_0/r$, and the radial vorticity gradient is $\partial \zeta/\partial r = (1 - m)\partial(V_0/r)/\partial r$. Thus, the Doppler-shifted frequency of a one-dimensional VRW with tangential wavenumber *n* becomes

$$\Omega_{1D} = [(1-m)\partial(V_0/r)/\partial r](n/r)^{-1}.$$
 (A1)

The two-dimensional Doppler-shifted frequency of a radially and tangentially propagating quasimode component is $\Omega(r) = (\omega - nV_0/r)$. $\Omega_{1D} = \Omega(r_{1D})$ defines the highest (i.e., most negative) Ω for a propagating VRW at r_{1D} , the boundary of the waveguide closest to the vortex center (Fig. A1). The lowest (least negative) frequency at the most distant waveguide boundary occurs when $\Omega(r_{1D} + \Delta r) = 0$, where Δr is the waveguide width. Since the waveguide is narrow, it is possible to use a Taylor series to approximate $\Omega(r)$. Given that ω is constant, the Doppler-shifted frequency at the waveguide's outer (VRW critical radius) boundary is approximately:

$$\Omega_{1\mathrm{D}} - n[\partial(V_0/r)/\partial r]\Delta r = 0, \qquad (A2)$$

which simplifies to

$$\Delta r = r_{1D}(1-m)/n^2$$
. (A3)

Thus, initial vorticity perturbations in these overlapping waveguides of the outer vortex where the mean vorticity gradient is weak and V_0/r is small project onto a transfinite set of VRWs that (in our interpretation) compose the quasimode (e.g., Schecter et al. 2000). The waveguides' small widths restrict radial propagation while promoting filamentation and absorption at their critical radii. The quasimode waveguides become narrower as the tangential wavenumber increases or as the wind profile becomes sharper (i.e., as *m* increases). For $m = \frac{1}{2}$, $\Delta r = r_{1D}/2$, $r_{1D}/8$, $r_{1D}/18$, and $r_{1D}/32$ for wavenumbers 1–4, respectively. In a Rankine vortex where m = 1, the wind varies inversely as radius so that the outer vortex is irrotational. No outer waveguides exist in this case because both the mean vorticity and its radial gradient are zero.

The insight of Möller and Montgomery (1999) describes what can happen in an irrotational vortex. The radial shear of angular velocity distorts the perturbations into trailing spirals. On the convex (downstream) side of a cyclonic trailing streamfunction spiral, for example, radial inflow and faster tangential flow correlate; whereas on the concave (upstream) side, radial outflow and slower tangential wind correlate. The result is an inward eddy flux of angular momentum even though no VRW propagation occurs. As a practical matter, it may be difficult to distinguish in observations or full-physics model results between filamentation of the densely packed VRWs that compose the quasimode and that of nonpropagating vorticity that does not project onto VRWs. In this context it is important to recall that vorticity filamentation leading to mean-flow acceleration is a property of vorticity, whether or not it projects onto propagating waves.

REFERENCES

- Andrews, D. G., and M. E. McIntyre, 1976a: Planetary waves in horizontal and vertical shear: The generalized Eliassen-Palm relation and the mean zonal acceleration. J. Atmos. Sci., 33, 2031–2048, doi:10.1175/1520-0469(1976)033<2031: PWIHAV>2.0.CO;2.
- —, and —, 1976b: Planetary waves in horizontal and vertical shear: Asymptotic theory for equatorial waves in weak shear. J. Atmos. Sci., 33, 2049–2053, doi:10.1175/ 1520-0469(1976)033<2049:PWIHAV>2.0.CO;2.
- Black, M. L., J. F. Gamache, F. D. Marks Jr., C. E. Samsury, and H. E. Willoughby, 2002: Eastern–Pacific Hurricanes Jimena of 1991 and Olivia of 1994: The effects of vertical shear on structure and intensity. *Mon. Wea. Rev.*, **130**, 2291–2312, doi:10.1175/1520-0493(2002)130<2291:EPHJOA>2.0.CO;2.
- Boyd, J. P., 1976: The noninteraction of waves with the zonally averaged flow on a spherical Earth and the interrelationships on eddy fluxes of energy, heat and momentum. J. Atmos. Sci., 33, 2285–2291, doi:10.1175/1520-0469(1976)033<2285: TNOWWT>2.0.CO;2.
- Chen, Y., and M. K. Yau, 2001: Spiral bands in a simulated hurricane. Part I: Vortex Rossby wave verification. J. Atmos. Sci., 58, 2128–2145, doi:10.1175/1520-0469(2001)058<2128: SBIASH>2.0.CO;2.
- —, G. Brunet, and M. K. Yau, 2003: Spiral bands in a simulated hurricane. Part II: Wave activity diagnostics. J. Atmos. Sci., 60, 1239–1256, doi:10.1175/1520-0469(2003)60<1239: SBIASH>2.0.CO;2.
- Churchill, R. V., 1963: Fourier Series and Boundary Value Problems. 2nd ed. McGraw-Hill Book Company, 248 pp.

- Corbosiero, K. L., J. Molinari, and M. T. Black, 2005: The structure and evolution of Hurricane Elena (1985). Part I: Symmetric intensification. *Mon. Wea. Rev.*, **133**, 2905–2921, doi:10.1175/ MWR3010.1.
- —, —, A. R. Aiyyer, and M. L. Black, 2006: The structure and evolution of Hurricane Elena (1985). Part II: Convective asymmetries and evidence for vortex Rossby waves. *Mon. Wea. Rev.*, **134**, 3073–3091, doi:10.1175/MWR3250.1.
- Cotto, A., 2012: Intermittently forced vortex Rossby waves. M.S. thesis, Dept. of Earth and Environment, Florida International University, 81 pp. [Available online at http:// digitalcommons.fiu.edu/etd/553.]
- Eliassen, A., and E. Palm, 1960: On the transfer of energy in stationary mountain waves. *Geofys. Publ. Geophys. Norv.*, 22 (3), 1–23.
- Gonzalez, I., III, A. Cotto, and H. E. Willoughby: 2015: Synthesis of vortex Rossby waves. Part II: Vortex motion and waves in the outer waveguide. J. Atmos. Sci., 72, 3958– 3974. doi:10.1175/JAS-D-15-0005.1.
- Guinn, T. A., and W. H. Schubert, 1993: Hurricane spiral bands. J. Atmos. Sci., 50, 3380–3403, doi:10.1175/1520-0469(1993)050<3380: HSB>2.0.CO;2.
- Hendricks, E. A., M. T. Montgomery, and C. A. Davis, 2004: The role of "vortical" hot towers in the formation of Tropical Cyclone Diana (1984). J. Atmos. Sci., 61, 1209–1232, doi:10.1175/ 1520-0469(2004)061<1209:TROVHT>2.0.CO;2.
- Houze, R. A., Jr., 2010: Clouds in tropical cyclones. *Mon. Wea. Rev.*, **138**, 293–344, doi:10.1175/2009MWR2989.1.
- Judt, F., and S. S. Chen, 2010: Convectively generated potential vorticity in rainbands and formation of the secondary eyewall in Hurricane Rita of 2005. J. Atmos. Sci., 67, 3581–3599, doi:10.1175/2010JAS3471.1.
- Kossin, J. P., and M. D. Eastin, 2001: Two distinct regimes in the kinematic and thermodynamic structure of the hurricane eye and eyewall. J. Atmos. Sci., 58, 1079–1090, doi:10.1175/ 1520-0469(2001)058<1079:TDRITK>2.0.CO;2.
- —, W. H. Shubert, and M. T. Montgomery, 2000: Unstable interactions between a hurricane's primary eyewall and a secondary ring of enhanced vorticity. J. Atmos. Sci., 57, 3893– 3917, doi:10.1175/1520-0469(2001)058<3893:UIBAHS>2.0.CO;2.
- Li, Q., and Y. Wang, 2012: Formation and quasi-periodic behavior of outer rainbands in a numerically simulated tropical cyclone. J. Atmos. Sci., 69, 997–1020, doi:10.1175/2011JAS3690.1.
- Lindzen, R. S., and H. L. Kuo, 1969: A reliable method for the numerical integration of a large class of ordinary and partial differential equations. *Mon. Wea. Rev.*, 97, 732–734, doi:10.1175/ 1520-0493(1969)097<0732:ARMFTN>2.3.CO;2.
- MacDonald, N. J., 1968: The evidence for the existence of Rossbylike waves in the hurricane vortex. *Tellus*, **20A**, 138–150, doi:10.3402/tellusa.v20i1.9993.
- Martinez, Y., G. Brunet, and M. K. Yau, 2010a: On the dynamics of two-dimensional hurricane-like convective rings vortex formation. J. Atmos. Sci., 67, 3253–3267, doi:10.1175/2010JAS3500.1.
- —, —, and —, 2010b: On the dynamics of two-dimensional hurricane-like vortex symmetrization. J. Atmos. Sci., 67, 3559– 3580, doi:10.1175/2010JAS3499.1.
- Menelaou, K., and M. K. Yau, 2014: On the role of asymmetric convective bursts to the problem of hurricane intensification: Radiation of vortex Rossby waves and wave-mean flow interactions. J. Atmos. Sci., 71, 2057–2077, doi:10.1175/JAS-D-13-0343.1.
 - —, —, and Y. Martinez, 2013: Impact of asymmetric dynamical processes on the structure and intensity change of two-dimensional hurricane-like annular vortices. J. Atmos. Sci., 70, 559–582, doi:10.1175/JAS-D-12-0192.1.

- Möller, J. D., and M. T. Montgomery, 1999: Vortex Rossby waves and hurricane intensification in a barotropic model. J. Atmos. Sci., 56, 1674–1687, doi:10.1175/1520-0469(1999)056<1674: VRWAHI>2.0.CO;2.
- —, and —, 2000: Tropical cyclone evolution via potential vorticity anomalies in a three-dimensional balance model. J. Atmos. Sci., 57, 3366–3387, doi:10.1175/1520-0469(2000)057<3366: TCEVPV>2.0.CO;2.
- Montgomery, M. T., and R. J. Kallenbach, 1997: A theory for vortex Rossby waves and its application to spiral bands and intensity changes in hurricanes. *Quart. J. Roy. Meteor. Soc.*, 123, 435–465, doi:10.1002/qj.49712353810.
- —, and J. Enagonio, 1998: Tropical cyclogenesis via convectively forced vortex Rossby waves in a three-dimensional quasigeostrophic model. J. Atmos. Sci., 55, 3176–3207, doi:10.1175/ 1520-0469(1998)055<3176:TCVCFV>2.0.CO;2.
- —, M. E. Nicholls, T. A. Cram, and A. B. Saunders, 2006: A vortical hot tower route to tropical cyclogenesis. *J. Atmos. Sci.*, 63, 355–386, doi:10.1175/JAS3604.1.
- Nguyen, C. M., M. J. Reeder, N. E. Davidson, R. K. Smith, and M. T. Montgomery, 2011: Inner-core vacillation cycles during the intensification of Hurricane Katrina. *Quart. J. Roy. Meteor. Soc.*, **137**, 829–844, doi:10.1002/qj.823.
- Nolan, D. S., and M. T. Montgomery, 2002: Nonhydrostatic, three-dimensional perturbations to balanced, hurricanelike vortices. Part I: Linearized formulation, stability, and evolution. J. Atmos. Sci., 59, 2989–3020, doi:10.1175/ 1520-0469(2002)059<2989:NTDPTB>2.0.CO;2.
- —, and L. D. Grasso, 2003: Nonhydrostatic, three-dimensional perturbations to balanced, hurricane-like vortices. Part II: Symmetric response and nonlinear simulations. J. Atmos. Sci., 60, 2717–2745, doi:10.1175/1520-0469(2003)060<2717:NTPTBH>2.0.CO;2.
- —, Y. Moon, and D. P. Stern, 2007: Tropical cyclone intensification from asymmetric convection: Energetics and efficiency. J. Atmos. Sci., 64, 3377–3405, doi:10.1175/JAS3988.1.
- Qiu, X., Z. Tan, and Q. Xiao, 2010: The roles of vortex Rossby waves in hurricane secondary eyewall formation. *Mon. Wea. Rev.*, 138, 2092–2109, doi:10.1175/2010MWR3161.1.
- Reasor, P. D., M. T. Montgomery, F. D. Marks Jr., and J. F. Gamache, 2000: Low-wavenumber structure and evolution of the hurricane inner core observed by airborne dual-Doppler radar. *Mon. Wea. Rev.*, **128**, 1653–1680, doi:10.1175/ 1520-0493(2000)128<1653:LWSAEO>2.0.CO;2.
- Riehl, H., 1963: Some relations between wind and thermal structure of steady state hurricanes. J. Atmos. Sci., 20, 276–287, doi:10.1175/1520-0469(1963)020<0276:SRBWAT>2.0.CO;2.

- Romine, G. S., and R. B. Wilhelmson, 2006: Finescale spiral band features within a numerical simulation of Hurricane Opal (1995). *Mon. Wea. Rev.*, **134**, 1121–1139, doi:10.1175/ MWR3108.1.
- Schecter, D. A., and M. T. Montgomery, 2003: On the symmetrization rate of an intense geophysical vortex. *Dyn. Atmos. Oceans*, 37, 55–88, doi:10.1016/S0377-0265(03)00015-0.
- —, and —, 2004: Damping and pumping of a vortex Rossby wave in a monotonic cyclone: Critical-layer stirring versus inertia-buoyancy wave emission. *Phys. Fluids*, **16**, 1334–1348, doi:10.1063/1.1651485.
- —, H. E. Dubin, A. C. Cass, C. F. Driscoll, I. M. Lansky, and T. M. O'Neil, 2000: Inviscid damping of asymmetries on a twodimensional vortex. *Phys. Fluids*, **12**, 2397–2412, doi:10.1063/ 1.1289505.
- Senn, H. V., and H. W. Hiser, 1959: On the origin of hurricane spiral bands. J. Meteor., 16, 419–426, doi:10.1175/ 1520-0469(1959)016<0419:OTOOHS>2.0.CO;2.
- Shapiro, L. J., and M. T. Montgomery, 1993: A threedimensional balance theory for rapidly rotating vortices. J. Atmos. Sci., 50, 3322–3335, doi:10.1175/1520-0469(1993) 050<3322:ATDBTF>2.0.CO;2.
- Terwey, W. D., and M. T. Montgomery, 2002: Wavenumber-2 and wavenumber-*m* vortex Rossby wave instabilities in a generalized three-region model. J. Atmos. Sci., 59, 2421–2427, doi:10.1175/ 1520-0469(2002)059<2421:WAWMVR>2.0.CO;2.
- Willoughby, H. E., 1977: Inertia-buoyancy waves in hurricanes. J. Atmos. Sci., 34, 1028–1039, doi:10.1175/1520-0469(1977)034<1028: IBWIH>2.0.CO;2.
- —, 1978: A possible mechanism for the formation of hurricane rainbands. J. Atmos. Sci., 35, 838–848, doi:10.1175/ 1520-0469(1978)035<0838:APMFTF>2.0.CO;2.
- —, 1988: The dynamics of the tropical cyclone core. Aust. Meteor. Mag., 36, 183–189.
- —, 2011: The golden radius in balanced atmospheric flows. Mon. Wea. Rev., 139, 1164–1168, doi:10.1175/2010MWR3579.1.
- —, F. D. Marks, and R. J. Feinberg, 1984: Stationary and moving convective bands in hurricanes. J. Atmos. Sci., 41, 3189–3211, doi:10.1175/1520-0469(1984)041<3189:SAMCBI>2.0.CO;2.
- —, E. N. Rappaport, and F. D. Marks, 2007: Hurricane forecasting: The state of the art. *Nat. Hazards Rev.*, 8, 45–49, doi:10.1061/(ASCE)1527-6988(2007)8:3(45).
- Wood, V. T., L. W. White, H. E. Willoughby, and D. P. Jorgensen, 2013: A new parametric tropical cyclone tangential wind model. *Mon. Wea. Rev.*, **141**, 1884–1909, doi:10.1175/ MWR-D-12-00115.1.