WAVE LENGTH AND WAVE CELERITY DURING SHORE-BREAKING

by

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FOREWORD

This work presents numerical solutions for the predicton of wave length and speed behavior during littoral shore-breaking. It is basic support methodology required in the develpment of a multiple shore-breaking wave transformation model described in subsequent work.

The work described herein constitutes partial fulfillment of contractual obligations with the Federal Coastal Zone Management Program (Coastal Zone Management Act of 1972, as amended) through the Florida Office of Coastal Management subject to provisions of contract CM-37 entitled "Engineering Support Enhancement Program". Under provisions of DNR contract C0037, this work was reviewed by the Beaches and Shores Resource Center, Institute of Science and Public Affairs, Florida State University. The document has been adopted as a Beaches and Shores Technical and Design Memorandum in accordance with provisions of Chapter 16B-33, Florida Administrative Code.

At the time of submission for contractural compliance, James H. Balsillie was the contract manager and Administrator of the Analysis/Research Section, Hal N. Bean was Chief of the Bureau of Coastal Data Acquisition, Deborah E. Flack Director of the Division of Beaches and Shores, and Dr. Elton J. Gissendanner the Executive Director of the Florida Department of Natural Resources.

Deborah E. Flack

Deborah E. Flack, Director
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ABSTRACT

Prediction of wave phase speed and, hence, wave length at shore-breaking has remained a controversial issue. Based on available field data (n = 47) and laboratory data (n = 40 to 71), a family of relationships are derived for predicting the wave length at shore-breaking. Assuming approximate linear wave speed attenuation, a method is derived for prediction of wave speed during the shore-breaking process.

INTRODUCTION

Wave height, H, wave length, L, wave period, T, and water depth, d, constitute basic hydraulic variables which form the basis for derivation of composite parameters (e.g., wave steepness, H/L, or wave steepness parameter $H/(g\ T^2)$) required in most coastal engineering design applications. It becomes not only desirable to be able to provide for determination of such parameters over a wide variety of conditions in order to accurately describe a natural process, but to be able to provide the simplest and most straightforward precedures possible.

As the number of basic variables becomes large, the solution of any problem invariably becomes proportionately more complex. It becomes desirable, therefore, to provide methods for predicting as many of the variables as is feasible. One such variable is the wave length.

As will become evident, determination of the local values of H and d as waves shore-propagate is complex, since ever following initial specification of their values, which may exhibit a wide range, H relative to d experiences additional and significant progressive transformations as shoaling continues. The wave period, though, once initially specified is considered to be conserved (i.e., remains invariant) across the shoaling bathymetry until shore-breaking occurs, and a simplifying condition emerges. The wave length, however, behaves in the same fashion as H, thereby introducing additional compexity. The wave length not only appears in many shoaling design wave equations (and usually just when one has little insight as to its local value short of tedious calculations for obtaining an approximation), but most importantly is related to the wave speed and wave energy.

It becomes important, therefore, to provide a method(s) for prediction of the wave length and wave speed. In this paper such prediction is investigated during the shore-breaking process.

DISCUSSION AND RESULTS

As shore-propagating waves approach the shoreline across shoaling bathymetry, the wave height tends to initially decrease due to a number of factors such as bottom friction, etc., and then begins to increase rapidly in height just before shore-breaking occurs. The transformation is illustrated in Figure 1 (notation is defined at the end of the paper). It is the increase in wave height which defines the shore-breaking[1] process. Shore-breaking wave mechanics are described by the alpha wave peaking concept (Balsillie, 1980, In Manuscript) and denoted by α_b in Figure 1. Alpha wave peaking, then, describes the "zone" of interest for investigation of the wave celerity and wave length.

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[1]Waves may break in deep or relatively "deeper" water due only to critically high wind stresses which cause waves to become critically steep (i.e., forced waves); shore-breaking waves occur primarily because water depths become critically shallow.

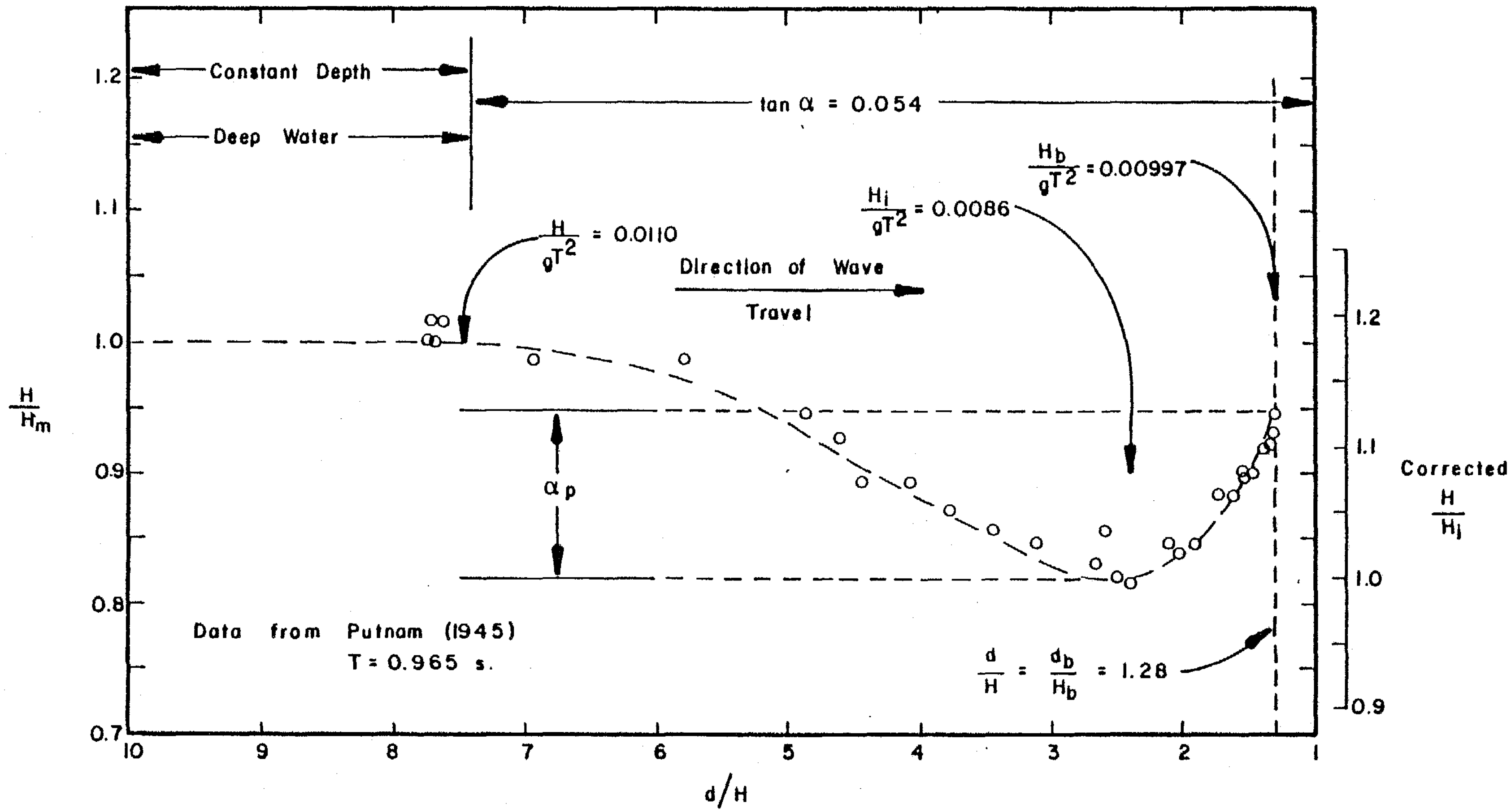

Figure 1. Illustration of wave transformation from deep water to shore-breaking, where the alpha wave peaking process (i.e., shore-breaking) is given the notation α_p.

The speed with which a group of waves, comprising a wave train, travels is not always equivalent to the speed of individual waves within the group. The individual wave speed, termed the phase speed, is given by:

$$c = L/T \qquad (1)$$

and the group wave speed, c_g , by:

$$c_g = n\,(L/T) \qquad (2)$$

In deep water (i.e., $d/L > 0.5$) $n = 0.5$, in intermediate wave depths (i.e., $0.04 < d/L < 0.5$) n increases in value to become, finally, $n = 1$ in shallow water (i.e., $d/L < 0.04$) where $c = c_g$.

According to small amplitude (Airy) wave theory, the phase speed and wave length in any depth of water may be given by:

$$c = \frac{L}{T} = \frac{g\,T}{2\,\pi} \tanh \frac{2\,\pi\,d}{L} \qquad (3)$$

which shall later be evaluated.

In this work the alpha wave peaking process is assumed to occur in shallow water where $n = 1$. In order to determine the transformation of c and L during shore-breaking one, first, needs to have knowledge of the governing boundary conditions.

Terminal Boundary Conditions

Equation (3) is often applied to predict conditions at shore-breaking which defines the terminus of the process. A more common application from small amplitude wave theory, is given by:

$$c_b = \frac{L_b}{T} = \sqrt{g\,d_b} \qquad (4)$$

or where solitary wave theory is applied, by:

$$c_b = \frac{L_b}{T} = \sqrt{g\,(d_b + H_b')} \qquad (5)$$

where H_b' is that portion of the wave height at shore-breaking above the design water level. About equation (5), Smith (1976) states: "Although this equation is widely used in the literature on wave theories and is generally accepted, few discussions have been presented which establishes its validity." The same appears to be true of equation (4), while a general misunderstanding about equation (3) seems to have been proliferated in the literature.

Van Dorn (1978) found that at shore-breaking, the wave speed was always greater than the small amplitude speed of equation (4), and smaller than the solitary wave celerity of equation (5). He reports that:

$$c_b = \sqrt{2\,g\,H_b'} \qquad (6)$$

which was found to "..... agree roughly with that predicted for limiting Stokes waves in deep water .....".

Available field and laboratory data (see the table) are used to evaluate the above equations. The wave celerity is analyzed in terms of the wave length rather than the wave speed since the length yields a much wider range of values. The data are plotted in Figure 2. The figure illustrates that equation (3) does not appear to predict L_b and, hence, c_b , with the precision of the other fitted relationships. It is to be recalled, however, that equation (3) was developed from theoretical considerations to represent an upper limit envelope curve (see Figure 8 of Bretschnieder, 1960). In addition, because equation (3) is an algorithm it is awkward to apply and not generally recommeded for use in design work. Equations that more successfully predict expected values are given by:

Table of statistics relating measured and predicted wave lengths at the shore-breaking position.

	n	$L_b = m T \sqrt{g d_b}$ m	r	$L_b = m T \sqrt{g(d_b + H_b)}$ m	r	$L_b = m T \sqrt{g H_b}$ m	r
FIELD DATA							
Gaillard (1904)	26	0.9462	0.9608	0.7514	0.9587	1.241	0.9404
Balsillie and Carter (1980)	21	1.226	0.9692	0.9101	0.9695	1.359	0.9689
Field Results	47	0.9832	0.9737	0.7737	0.9757	1.259	0.9720
LABORATORY DATA							
Galvin and Eagleson (1965)	24	1.342*	0.3603*	0.8843*	0.3842*	1.178	0.3658
Eagleson (1965)	7	1.084*	0.8399*	0.8131*	0.8850*	1.233	0.7985
Van Dorn (1976, 1978)	12	1.264	0.9636	0.8895	0.9889	1.254	0.9933
Buhr Hansen and Svendsen (1979)	28	1.113	0.9947	0.8036	0.9962	1.162	0.9969
Laboratory Results	40-71	1.205	0.9858	0.8561	0.9947	1.211	0.9933
Total Results	87-118	0.9972	0.9801	0.7794	0.9210	1.254	0.9836
Weighted m	87-118	1.111	------	0.829	------	1.251	------
Adjusted m**	87-118	1.1176	------	0.8374	------	1.2644	------

NOTES: Unless otherwise indicated all m and r are from regression analyses.
* These results represent d_b referenced to MWL and are not used in determination of m, all others used in the analysis are referenced to SWL.
** Adjusted values were determined such that equations (7) through (11) all yield consistent results.

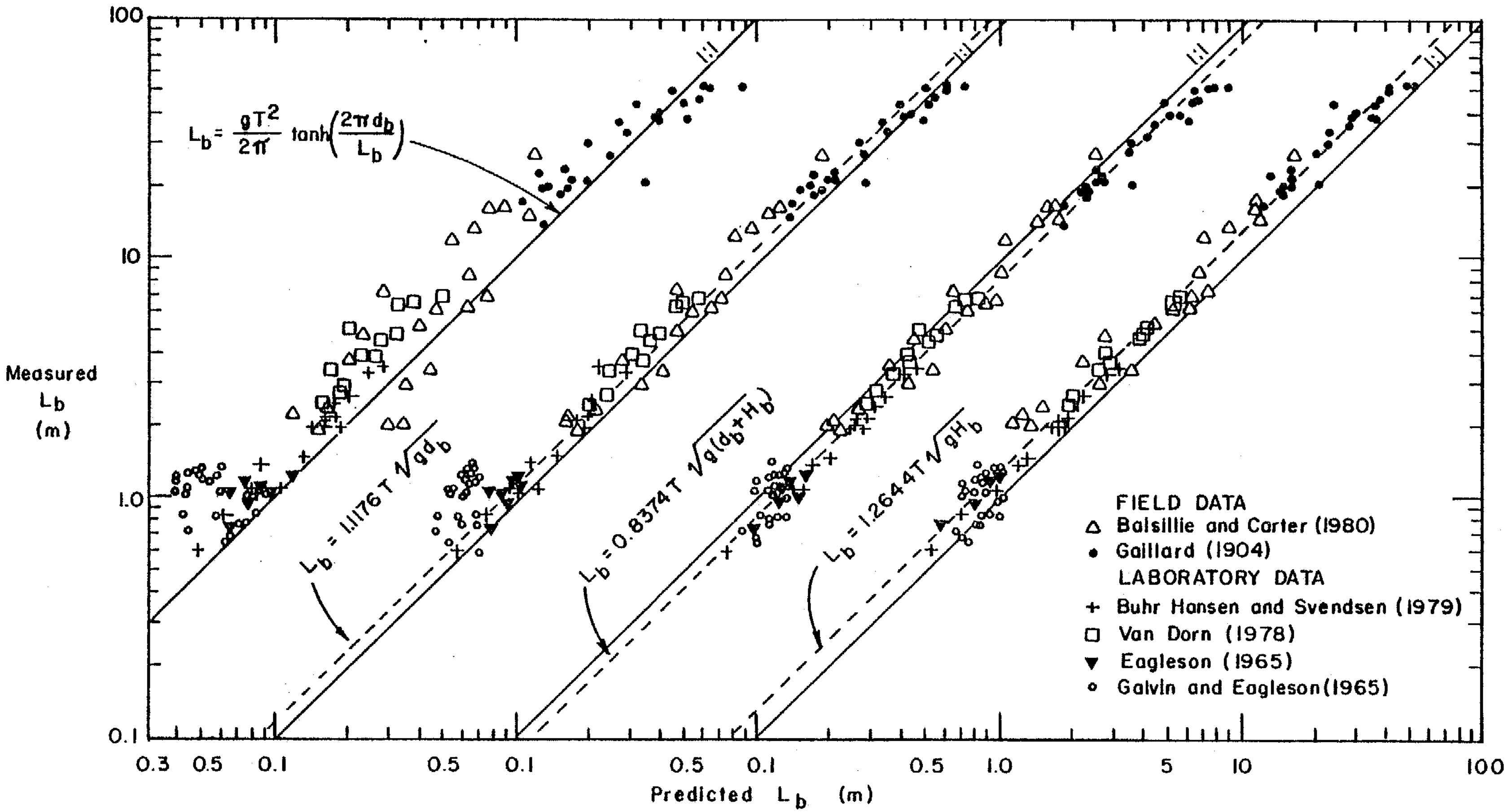

Figure 2. Evaluation of relationships for prediction of the wave length at the shore-breaking position.

$$L_b = T\sqrt{1.249\ g\ d_b} \qquad (7)$$

$$L_b = T\sqrt{1.60\ g\ H_b} \qquad (8)$$

and

$$L_b = T\sqrt{0.701\ g\ (d_b + H_b)} \qquad (9)$$

Where $d_b = 1.28\ H_b$ (McCowan, 1894; Munk, 1949; Balsillie, In Manuscript) and $H_b' = 0.84\ H_b$ (Balsillie, In Manuscript), the previous three relationships can be modified to yield two additional equations:

$$L_b = T\sqrt{1.904\ g\ H_b'} \qquad (10)$$

and

$$L_b = T\sqrt{0.755\ g\ (d_b + H_b')} \qquad (11)$$

and we now have a family of design relationships for prediction of L_b and c_b.

Initial Boundary Condition

With the exception of the results of Buhr Hansen and Svendsen (1979), there is little, if any, data available which will allow for determination of the wave speed at the point of initiation of the alpha wave peaking process (i.e., at c_i). Based on other alpha wave peaking investigations (Balsillie, 1980, In Manuscripts), it may be reasonable to assume that c_i can be related to c_b. However, the problem is encountered that the difference between c_i and c_b is slight, at least compared to natural variability in the data and possible measurement errors.

Another approach using theoretical reasoning may provide useful results. Solitary wave theory would appear to be applicable. Use of Solitary wave theory poses problems, however, since it is assumed that the entire wave lies above the still water level (SWL), a condition that does not apply during alpha wave peaking for periodic waves. Correction of this artifact is necessary.

The total energy of a wave is the sum total of its kinetic and potential energies. The kinetic energy is that portion of the total energy due to water particle velocities associated with wave motion. Potential energy is that portion of the total energy resulting from the wave fluid mass lying above the SWL. "Total energy in a solitary wave is about evenly divided between kinetic and potential energy" (U. S. Army, 1975), and is given by:

$$E_T = \frac{8}{3\sqrt{3}} \rho_f \, g \, H^{3/2} \, d^{3/2} \qquad (12)$$

However, since the wave crest does not lie totally above the SWL, then at the initiation of alpha wave peaking:

$$E_{Ti} = E_{Ki} + E_{Pi} \qquad (13a)$$

$$E_{Ti} = \frac{8}{6\sqrt{3}} \rho_f \, g \, H_i^{3/2} \, d_i^{3/2} + \left(\frac{H'}{H}\right)_{i\alpha} \frac{8}{6\sqrt{3}} \rho_f \, g \, H_i^{3/2} \, d_i^{3/2} \qquad (13b)$$

and

$$E_{Ti} = \frac{8}{6\sqrt{3}} \rho_f \, g \, H_i^{3/2} \, d_i^{3/2} \left[1 + \left(\frac{H'}{H}\right)_{i\alpha}\right] \qquad (13c)$$

where at initiation of the alpha wave peaking process, E_{Ti} is the total wave crest energy, E_{Ki} is the kinetic energy, E_{Pi} is the potential energy, and $(H'/H)_{i\alpha}$ is the percent of the wave crest height lying above the SWL.

Similarly, at the shore-breaking position, where $H_b'/H_b = 0.84$ (Balsillie, In Manuscript):

$$E_{Tb} = E_{Kb} + E_{Pb} \tag{14a}$$

$$E_{Tb} = \frac{8}{6\sqrt{3}} \rho_f \, g \, H_b^{3/2} \, d_b^{3/2} + \frac{H_b'}{H_b} \frac{8}{6\sqrt{3}} \rho_f \, g \, H_b^{3/2} \, d_b^{3/2} \tag{14b}$$

and

$$E_{Tb} = 1.84 \frac{8}{6\sqrt{3}} \rho_f \, g \, H_b^{3/2} \, d_b^{3/2} \tag{14c}$$

Now, by applying the Rayleigh assumption (Eagleson and Dean, 1966) given by:

$$c_i \, E_{Ti} = c_b \, E_{Tb} \tag{15}$$

in combination with equations (13) and (14), then:

$$\frac{c_i}{c_b} = \frac{L_i/T}{L_b/T} = \frac{L_i}{L_b} = 1.84 \left[1 + \left(\frac{H'}{H}\right)_{i\alpha}\right]^{-1} \tag{16}$$

where $(H'/H)_{i\alpha}$ may be predicted according to Balsillie (In Manuscript) by:

$$\left(\frac{H'}{H}\right)_{i\alpha} = 0.54 + 10.34 \left(\frac{H}{g\,T^2}\right)_i^{1.014} \tag{17}$$

or by:

$$\left(\frac{H'}{H}\right)_{i\alpha} = 0.84 - 0.307 \left\{\tanh\left(0.3\left[\left(\frac{d}{H}\right)_i' - 1.28\right]\right)\right\} \qquad (18)$$

which provides the percentage of the wave height at the initiation of alpha wave peaking lying above the SWL.

Comparison of results from equation (16) with the measured data of Buhr Hansen and Svendsen (1979) suggests that further calibration of equation (16) is necessary. The data of Figure 3 indicate that a correction factor, Φ , for equation (16) may be given by:

$$\Phi = \frac{c_i \text{ measured}}{c_b \text{ predicted}} = 0.7078 \left(\frac{d}{H}\right)_i^{'\,0.4353} \qquad (19)$$

where $(d/H)_i'$ is the relative water depth at the initiation of alpha wave peaking (Balsillie, In Manuscript) according to:

$$\left(\frac{d}{H}\right)_i' = 1.28 - 1.56 \ln\left\{\tanh\left[65\left(\frac{H}{g\,T^2}\right)_i\right]\right\} \qquad (20)$$

and equation (16) appears in the final form:

$$\frac{c_i}{c_b} = \frac{L_i/T}{L_b/T} = \frac{L_i}{L_b} = 1.84\,\Phi\left[1 + \left(\frac{H'}{H}\right)_{i\alpha}\right]^{-1} \qquad (21)$$

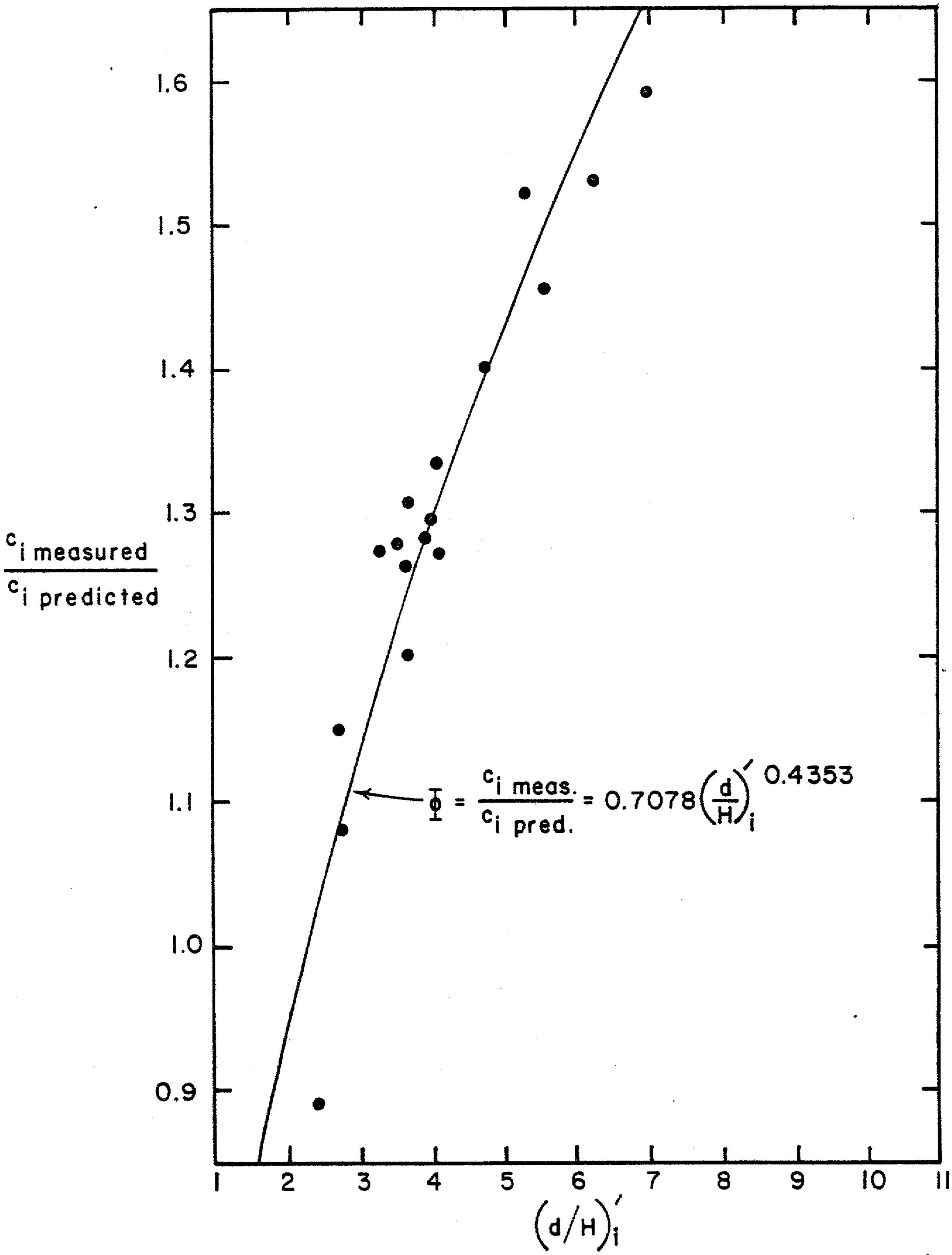

Figure 3. Comparison of predicted and measured wave speeds at the point of initiation of alpha wave peaking; predicted data from equation (16), measured data from Buhr Hansen and Svendsen (1979).

Wave Celerity and Wave Length Transformation during Alpha Wave Peaking

The data of Buhr Hansen and Svendsen (1979) for a slope of 0.0292 suggest that the transformation of c, the local wave celerity during shore-breaking, is non-linear but only very slightly so. For three bed slopes of 0.022, 0.040 and 0.083, Van Dorn (1978) illustrates that the transformation of c is only slightly non-linear and that it accelerated at a rate close to -0.5 g tan α_b.

In addition, Van Dorn's result requires that the transformation of c is dependent on the bottom slope, as would be expected. Unfortunately, Van Dorn did not publish his transformation data, and only the data of Buhr Hansen and Svendsen, for a single slope, are available. Fortunately, however, from the above it is possible to assume that the behavior of c is essentially linear and can be determined as a function of the relative water depth. Accordingly, where shore-breaking occurs when $d_b/H_b = 1.28$, then:

$$c = c_i - \left[\left(1 - \frac{(d/H) - 1.28}{(d/H)_i' - 1.28}\right)\left(c_i - c_b\right)\right] \qquad (22)$$

which is valid during the alpha wave peaking process where $(d/H)_i' > (d/H) > 1.28$.

CONCLUSIONS

Three issues concerning the prediction of the wave length and the wave speed during the shore-breaking wave process have been addressed.

First, a family of relationships based on field and laboratory data have been defined for determination of L and c at the shore-breaking position. These relationships have been used to refine theoretical predictions from small amplitude (Airy) and solitary wave theories. The family of derived relationships provide for alternate data to more closely facilitate the needs of the coastal engineer whose completeness in data may differ from project to project. In addition, the commonly used algorithm given by equation (3) is assessed. Not only is the expression difficult to apply (i.e., that which it predicts requires itself to be predicted), but that in surf zone applications it has often been incorrectly applied since it is an upper-limit envelope curve fit. In view of the developments presented in this work, the continued use of equation (3) during shore-breaking is not recommended.

Second, based on consideration of solitary wave theory (with corrections for the potential energy since the wave crest does not lie totally above the SWL) and the Rayleigh assumption, the wave length and wave speed at the initiation of shore-breaking (i.e., beginning of alpha wave peaking where the wave crest begins to significantly increase in height and in profile view becomes asymmetrical and distorted) may be predicted according to equation (21).

Third, using the above two results as boundary conditions, the transformation of L and c during alpha wave peaking, assuming linearity in attenuation, may be predicted from equation (22).

NOTATION

Symbols

c	local individual wave (phase) speed.
c_g	group wave speed.
d	local water depth measured from the DWL.
DWL	design water level.
E_K	kinetic wave crest energy.
E_P	potential wave crest energy.
E_T	total wave crest energy.
g	acceleration of gravity.
H	local wave height.
H'	local wave height lying above the SWL.
L	local wave length.
m, r	coefficients.
n	number of data points comprising a sample.
SWL	that design water level represented by the still water level.
T	wave period.
$\tan\alpha$	bottom slope.
α_p	alpha peaking.
ρ_f	fluid mass density.
Φ	coefficient.

Subscripts

b	parameter value at the shore-breaking point (i.e., at the termination of alpha peaking).
i	parameter value at the beginning of shore-breaking (i.e., at the beginning of alpha peaking).
m	parameter value just before entering transitional water depth.

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