Conditional Probabilities for Sequences of Wet Days at Phoenix, Arizona

PAUL C. KANGIESER

NOAA TECHNICAL MEMORANDA
National Weather Service, Western Region Subseries

The National Weather Service (NWS) Western Region (WR) Subseries provides an informal medium for the documentation and quick dissemination of results not appropriate, or not yet ready, for formal publication. The series is used to report on work in progress, to describe technical procedures and practices, or to relate progress to a limited audience. These Technical Memoranda will report on investigations devoted primarily to regional and local problems of interest mainly to personnel, and hence will not be widely distributed.

Papers 1 to 23 are in the former series, ESSA Technical Memoranda, Western Region Technical Memoranda (WRTM); papers 24 to 59 are in the former series, ESSA Technical Memoranda, Weather Bureau Technical Memoranda (WBTM). Beginning with 60, the papers are part of the series, NOAA Technical Memoranda NWS.

Papers 1 to 23, except for 5 (revised edition) and 10, are available from the National Weather Service Western Region, Scientific Services Division, P. O. Box 11188, Federal Building, 125 South State Street, Salt Lake City, Utah 84111. Papers 5 (revised edition), 10, and all others beginning with 24 are available from the National Technical Information Service, U.S. Department of Commerce, Sills Bldg., 5285 Port Royal Road, Springfield, Va. 22151. Price: $3.00 paper copy; $0.95 microfiche. Order by accession number shown in parentheses at end of each entry.

ESSA Technical Memoranda

WRTM 1   Some Notes on Probability Forecasting. Edward D. Diemer, September 1965. (Out of print.)
WRTM 2   Climatological Precipitation Probabilities. Compiled by Lucianne Miller, December 1965.
WRTM 3   Western Region Pre- and Post-FP-3 Program, December 1, 1965 to February 20, 1966. Edward D. Diemer, March 1966.
WRTM 4   Use of Meteorological Satellite Data. March 1966.
WRTM 5   Station Descriptions of Local Effects on Synoptic Weather Patterns. Philip Williams, Jr., April 1966 (revised November 1967, October 1969). (PB-178000)
WRTM 6   Improvement of Forecast Wording and Format. C. L. Glenn, May 1966.
WRTM 7   Final Report on Precipitation Probability Test Programs. Edward D. Diemer, May 1966.
WRTM 8   Interpreting the RAREP. Herbert P. Benner, May 1966 (revised January 1967). (Out of print.)
WRTM 9   A Collection of Papers Related to the 1966 NMC Primitive-Equation Model. June 1966.
WRTM 10   Sonic Boom. Loren Crow (6th Weather Wing, USAF, Pamphlet), June 1966. (Out of print.) (AD-479366)
WRTM 11   Some Electrical Processes in the Atmosphere. J. Latham, June 1966.
WRTM 12   A Comparison of Fog Incidence at Missoula, Montana, with Surrounding Locations. Richard A. Dightman, August 1966. (Out of print.)
WRTM 13   A Collection of Technical Attachments on the 1966 NMC Primitive-Equation Model. Leonard W. Snellman, August 1966. (Out of print.)
WRTM 14   Application of Net Radiometer Measurements to Short-Range Fog and Stratus Forecasting at Los Angeles. Frederick Thomas, September 1966.
WRTM 15   The Use of the Mean as an Estimate of "Normal" Precipitation in an Arid Region. Paul C. Kangieser, November 1966.
WRTM 16   Some Notes on Acclimatization in Man. Edited by Leonard W. Snellman, November 1966.
WRTM 17   A Digitalized Summary of Radar Echoes Within 100 Miles of Sacramento, California. J. A. Youngberg and L. B. Overaas, December 1966.
WRTM 18   Limitations of Selected Meteorological Data. December 1966.
WRTM 19   A Grid Method for Estimating Precipitation Amounts by Using the WSR-57 Radar. R. Granger, December 1966. (Out of print.)
WRTM 20   Transmitting Radar Echo Locations to Local Fire Control Agencies for Lightning Fire Detection. Robert R. Peterson, March 1967. (Out of print.)
WRTM 21   An Objective Aid for Forecasting the End of East Winds in the Columbia Gorge, July through October. D. John Coparanis, April 1967.
WRTM 22   Derivation of Radar Horizons in Mountainous Terrain. Roger G. Pappas, April 1967.
WRTM 23   "K" Chart Applications to Thunderstorm Forecasts Over the Western United States. Richard E. Hambidge, May 1967.

ESSA Technical Memoranda, Weather Bureau Technical Memoranda (WBTM)

WBTM 24   Historical and Climatological Study of Grinnell Glacier, Montana. Richard A. Dightman, July 1967. (PB-178071)
WBTM 25   Verification of Operational Probability of Precipitation Forecasts, April 1966-March 1967. W. W. Dickey, October 1967. (PB-176240)
WBTM 26   A Study of Winds in the Lake Mead Recreation Area. R. P. Augulis, January 1968. (PB-177830)
WBTM 27   Objective Minimum Temperature Forecasting for Helena, Montana. D. E. Olsen, February 1968. (PB-177827)
WBTM 28   Weather Extremes. R. J. Schmidli, April 1968 (revised July 1968). (PB-178928)
WBTM 29   Small-Scale Analysis and Prediction. Philip Williams, Jr., May 1968. (PB-178425)
WBTM 30   Numerical Weather Prediction and Synoptic Meteorology. Capt. Thomas D. Murphy, U.S.A.F., May 1968. (AD-673365)
WBTM 31   Precipitation Detection Probabilities by Salt Lake ARTC Radars. Robert K. Belesky, July 1968. (PB-179084)
WBTM 32   Probability Forecasting--A Problem Analysis with Reference to the Portland Fire Weather District. Harold S. Ayer, July 1968. (PB-179289)
WBTM 33   Objective Forecasting. Philip Williams, Jr., August 1968. (AD-680425)
WBTM 34   The WSR-57 Radar Program at Missoula, Montana. R. Granger, October 1968. (PB-180292)
WBTM 35   Joint ESSA/FAA ARTC Radar Weather Surveillance Program. Herbert P. Benner and DeVon B. Smith, December 1968 (revised June 1970). (AD-681857)
WBTM 36   Temperature Trends in Sacramento--Another Heat Island. Anthony D. Lentini, February 1969. (Out of print.) (PB-183055)
WBTM 37   Disposal of Logging Residues Without Damage to Air Quality. Owen P. Cramer, March 1969. (PB-183057)
WBTM 38   Climate of Phoenix, Arizona. R. J. Schmidli, P. C. Kangieser, and R. S. Ingram. April 1969. (Out of print.) (PB-184295)
WBTM 39   Upper-Air Lows Over Northwestern United States. A. L. Jacobson, April 1969. (PB-184296)
WBTM 40   The Man-Machine Mix in Applied Weather Forecasting in the 1970s. L. W. Snellman, August 1969. (PB-185068)
WBTM 41   High Resolution Radiosonde Observations. W. S. Johnson, August 1969. (PB-185673)
WBTM 42   Analysis of the Southern California Santa Ana of January 15-17, 1966. Barry B. Aronovitch, August 1969. (PB-185670)
WBTM 43   Forecasting Maximum Temperatures at Helena, Montana. David E. Olsen, October 1969. (PB-185762)
WBTM 44   Estimated Return Periods for Short-Duration Precipitation in Arizona. Paul C. Kangieser, October 1969. (PB-187763)
WBTM 45/1 Precipitation Probabilities in the Western Region Associated with Winter 500-mb Map Types. Richard A. Augulis, December 1969. (PB-188248)

NOAA Technical Memorandum NWSTM WR-86

CONDITIONAL PROBABILITIES FOR SEQUENCES OF WET DAYS AT PHOENIX, ARIZONA

Paul C. Kangieser [1]
Climatologist for Arizona
Weather Service Forecast Office
Phoenix, Arizona

[1] Mr. Kangieser retired from the National Weather Service on May 12, 1973. Of his 30-year total Federal service, he had been the Climatologist for Arizona for the past 17 years.

TABLE OF CONTENTS

CONDITIONAL PROBABILITIES FOR SEQUENCES OF WET DAYS
AT PHOENIX, ARIZONA

I. INTRODUCTION

Erickson [1] presented data for Washington showing the probability of
a thunderstorm day, given the knowledge that a given number of conse-
cutive thunderstorm days have already occurred. Mr. Robert S. Ingram,
Meteorologist in Charge of the Weather Service Forecast Office in
Phoenix, Arizona, suggested that similar data be prepared for Phoenix
using precipitation days.

A good model for such a study seemed to be a study prepared at the
National Weather Records Center (now National Climatic Center) for
NASA's George C. Marshall Space Flight Center, Alabama [2]. [1]
yields only the probability of one more day with a thunderstorm,
given that "i" consecutive days with thunderstorms have just occurred,
whereas [2] yields the probability of "k" additional days with thunder-
storms given that "i" consecutive days with thunderstorms have just
occurred.

The method of actually counting the number of consecutive days with
thunderstorms was used in [2], which is the most desirable procedure,
but is quite laborious without a computer. In processing the Phoenix
data, the method of counting runs used by Jorgensen [3] was used.
With the thought that similar data may be of value to forecasters
in other states, the method is described in some detail below.

Jorgensen's method consists of counting the number of runs of 1-day,
2-days, 3-days, etc., duration and assigning their occurrence to the
month in which most of the run's days occurred. The method used in
[2] was to actually count the number of times that 1-day, 2-conse-
cutive days, 3-consecutive days, etc., with a given type of weather
occurred. The difference in the two methods may be better visualized
by realizing that whenever Jorgensen tallied one run of 3 days length,
the authors of [2] tallied one period of 3 consecutive days, two
periods of 2 consecutive days, and 3 periods of 1 day with a given
type of weather. It is a simple matter to convert run data into the
type of data used in [2], and if all runs ended on the last day of the
month (if that is the unit of stratification used), then the two
methods would yield identical results.

II. DATA

Data used for the Phoenix study were the combined record for the 77-year
period 1896-1972. These data were divided into two seasonal periods:

(1) July and August, yielding a sample of 4774 days over the 77-year period, and (2) December, January, February and March, giving a total number of 9317 days. The number of runs of various length of days with precipitation were counted, assigning the run to the month in which most of the run's days occurred. A day with "precipitation" was defined as a day on which at least a trace of precipitation occurred in the 24-hour period between midnight and midnight.

III. PROCEDURES AND RESULTS

The method of converting the run data to the number of times precipitation occurred on 1 day, 2 consecutive days, 3 consecutive days, etc., may be illustrated by the following examples using the July and August data in Table 1a. Let F_1 be the number of times precipitation occurred on 1 day and F_2 be the number of times precipitation occurred on two consecutive days. Then

$$F_1 = 343(1) + 224(2) + 111(3) + 52(4) + 28(5) + 11(6) +$$
$$4(7) + 3(8) + 1(9) + 1(10) = 1609$$

$$F_2 = 224(1) + 111(2) + 52(3) + 28(4) + 11(5) + 4(6) +$$
$$3(7) + 1(8) + 1(9) = 831$$

Continuing in the same manner for the rest of the data in Tables 1a and 1b, yields the data in the F_k columns in Tables 2a and 2b.

To get the data in Tables 3a and 3b, proceed as follows. First, let us define P(k/i) as the probability of "k" additional days with precipitation (trace or more) given that "i" consecutive days with precipitation have just occurred. The solidus, "/", is shorthand notation for "given that".

P(k/o) is the probability of "k" consecutive days of rain, when it is not known whether the previous day was wet or dry. In other words, P(k/o) is the probability of "k" consecutive days of precipitation and may be read simply as P(k) = probability of "k" consecutive days of precipitation occurring anywhere in the whole sample of T days.

$$P(k) = \frac{F_k}{T} \qquad (1)$$

For Table 2a, T = 4774 days, while for Table 2b, T = 9317 days. The values of P(k) from Tables 2a and 2b are transferred to row 1 (i = o) of Tables 3a and 3b.

-2-

To complete column 1 in Table 3a using data in Table 2a, note that

$$P(1/1) = \frac{831}{1609} = 0.516$$

$$P(1/2) = \frac{396}{831} = 0.477, \text{ etc.}$$

To complete column 2, we use

$$P(2/1) = \frac{396}{1609} = 0.246$$

$$P(2/2) = \frac{185}{831} = 0.223, \text{ etc.}$$

The remainder of Tables 3a and 3b may be filled out in this manner using values of F_k from Tables 2a and 2b.

After the values of $P(k/i)$ are entered in Tables 3a and 3b, the data may be checked using the following conditional probability formula.

$$P(k/i) = P(1/i) \times P(1/i+1) \times P(1/i+2) \times \ldots \times P(1/i+k-1) \qquad (2)$$

For example, from Table 3a,

$$P(5/3) = P(1/3) \times P(1/4) \times P(1/5) \times P(1/6) \times P(1/7)$$

$$= (.467)(.459)(.435)(.459)(.471) = 0.020.$$

Thus, every value in Tables 3a and 3b for $k \geq 2$, can be checked using the probabilities in the $K = 1$ column in each table.

IV. DISCUSSION

The advantage of the method described here over the assumption that the data may be adequately represented by a low-order Markov function (see [4]) is the following: If a tendency exists for the probability of additional precipitation days to increase after several precipitation days have occurred, the method just described will reveal such a tendency.   For example, Jorgensen found that a 72% probability existed for at least one more "rain" day in San Francisco if 4 had just occurred, whereas the corresponding probability after only 1 "rain" day was only 58%. Such a probability distribution contains information of great value to the forecaster and would not be revealed by a study such as [4].

-3-

The Phoenix data seem to show a tendency for an 8-day sequence of precipitation in both Tables 3a and 3b. Note for example, the relatively larger values in Table 3b of P(1/7), P(2/6), P(3/5), and P(4/4) compared to the values immediately above or below them in the same columns. This pattern is also present to a lesser degree in Table 3a, and may indicate a tendency for precipitation to persist for 8-day periods in Phoenix, a tendency that may warrant a more extensive investigation by spectrum analysis.

V. TABLES

TABLES 1a AND 1b

Table 1a is based on data for July and August 1896-1972.

Table 1b is based on data for December, January, February and March 1896-1972.

X = Length of run of days with precipitation

Y = Number of runs of length X during 77-year period

Table 1a			Table 1b	
X	Y		X	Y
1	343		1	473
2	224		2	253
3	111		3	114
4	52		4	49
5	28		5	16
6	11		6	11
7	4		7	3
8	3		8	1
9	1		9	2
10	1		10	1

TABLES 2a AND 2b

Table 2a is based on data for July and August 1896-1972.

Table 2b is based on data for December, January, February and March 1896-1972.

k = Number of consecutive days with precipitation

F_k = Number of times k consecutive days with precipitation occurred during the 77-year period

In Table 2a $P(k) = F_k / 4774$. In Table 2b $P(k) = F_k / 9317$.

	Table 2a				Table 2b	
k	F_k	P(k)		k	F_k	P(k)
1	1609	.337		1	1720	.185
2	831	.174		2	797	.086
3	396	.083		3	347	.037
4	185	.039		4	150	.016
5	85	.018		5	67	.007
6	37	.008		6	33	.004
7	17	.004		7	15	.002
8	8	.002		8	8	.001
9	3	.001		9	3	.0003
10	1	.0002		10	1	.0001

Probability, $P(k/i)$, of "k" additional days with precipitation (trace or more) given that "i" consecutive days with precipitation have just occurred. Phoenix WSFO, Arizona.

Table 3a (July and August)

k \ i	1	2	3	4	5	6	7	8	9	10
0	.337	.174	.083	.039	.018	.008	.004	.002	.001	.0002
1	.516	.246	.115	.053	.023	.011	.005	.002	.001	
2	.477	.223	.102	.045	.020	.010	.004	.001		
3	.467	.215	.093	.043	.020	.008	.003			
4	.459	.200	.092	.043	.016	.005				
5	.435	.200	.094	.035	.012					
6	.459	.216	.081	.027						
7	.471	.176	.059							
8	.375	.125								
9	.333									

Table 3b (December, January, February and March)

k \ i	1	2	3	4	5	6	7	8	9	10
0	.185	.086	.037	.016	.007	.004	.002	.001	.0003	.0001
1	.463	.202	.087	.039	.019	.009	.005	.002	.001	
2	.435	.188	.084	.041	.019	.010	.004	.001		
3	.432	.193	.095	.043	.023	.009	.003			
4	.447	.220	.100	.053	.020	.007				
5	.493	.224	.119	.045	.015					
6	.455	.242	.091	.030						
7	.533	.200	.067							
8	.375	.125								
9	.333									

VI.   REFERENCES

1. Erickson, Kenneth T. *Climatological Probability of Thunderstorms at Four Washington Locations,* Western Regional Technical Attachment No. 72-32, August 22, 1972.

2. Lee, Russel F., James W. Ownbey, and Frank T. Quinlan. *Thunderstorm Persistence at Cape Kennedy, Florida,* NASA CR-61259, National Weather Records Center, January 1969.

3. Jorgensen, D. L. *Persistency of Rain and No-Rain Periods During the Winter at San Francisco,* Monthly Weather Review, Vol. 77, No. 11, pp. 303-307, November 1949.

4. Heermann, Dale F., Morris D. Finkner, Edward A. Hiler. *Probability of Sequences of Wet and Dry Days for Eleven Western States and Texas,* Excerpt from Technical Bulletin 117. Colorado State University Experiment Station, Agricultural Research Service, U.S.D.A., and National Weather Service.

Western Region Technical Memoranda: (Continued)

No. 45/2   Precipitation Probabilities in the Western Region Associated with Spring 500-mb Map
            Types. Richard P. Augulis. January 1970. (PB-189434)
No. 45/3   Precipitation Probabilities in the Western Region Associated with Summer 500-mb Map
            Types. Richard P. Augulis. January 1970. (PB-189414)
No. 45/4   Precipitation Probabilities in the Western Region Associated with Fall 500-mb Map Types.
            Richard P. Augulis. January 1970. (PB-189435)
No. 46   Applications of the Net Radiometer to Short-Range Fog and Stratus Forecasting at Eugene,
            Oregon. L. Yee and E. Bates. December 1969. (PB-190476)
No. 47   Statistical Analysis as a Flood Routing Tool. Robert J. C. Burnash . December 1969.
            (PB-188744)
No. 48   Tsunami. Richard A. Augulis. February 1970. (PB-190157)
No. 49   Predicting Precipitation Type. Robert J. C. Burnash and Floyd E. Hug. March 1970.
            (PB-190962)
No. 50   Statistical Report of Aeroallergens (Pollens and Molds) Fort Huachuca, Arizona 1969.
            Wayne S. Johnson. April 1970. (PB-191743)
No. 51   Western Region Sea State and Surf Forecaster's Manual. Gordon C. Shields and Gerald B.
            Burdwell. July 1970. (PB-193102)
No. 52   Sacramento Weather Radar Climatology. R. G. Pappas and C. M. Veliquette. July 1970.
            (PB-193347)
No. 53   Experimental Air Quality Forecasts in the Sacramento Valley. Norman S. Benes. August
            1970. (PB-194128)
No. 54   A Refinement of the Vorticity Field to Delineate Areas of Significant Precipitation.
            Barry B. Aronovitch. August 1970.
No. 55   Application of the SSARR Model to a Basin Without Discharge Record. Vail Schermerhorn
            and Donald W. Kuehl. August 1970. (PB-194394).
No. 56   Areal Coverage of Precipitation in Northwestern Utah. Philip Williams, Jr., and Werner
            J. Heck. September 1970. (PB-194389)
No. 57   Preliminary Report on Agricultural Field Burning vs. Atmospheric Visibility in the
            Willamette Valley of Oregon. Earl M. Bates and David O. Chilcote. September 1970.
            (PB-194710)
No. 58   Air Pollution by Jet Aircraft at Seattle-Tacoma Airport. Wallace R. Donaldson. October
            1970. (COM-71-00017)
No. 59   Application of P.E. Model Forecast Parameters to Local-Area Forecasting. Leonard W.
            Snellman. October 1970. (COM-71-00016)

                            NOAA Technical Memoranda NWS

No. 60   An Aid for Forecasting the Minimum Temperature at Medford, Oregon. Arthur W. Fritz,
            October 1970. (COM-71-00120)
No. 61   Relationship of Wind Velocity and Stability to SO_2 Concentrations at Salt Lake City, Utah.
            Werner J. Heck, January 1971. (COM-71-00232)
No. 62   Forecasting the Catalina Eddy. Arthur L. Eichelberger, February 1971. (COM-71-00223)
No. 63   700-mb Warm Air Advection as a Forecasting Tool for Montana and Northern Idaho. Norris E.
            Woerner, February 1971. (COM-71-00349)
No. 64   Wind and Weather Regimes at Great Falls, Montana. Warren B. Price, March 1971.
No. 65   Climate of Sacramento, California. Wilbur E. Figgins, June 1971. (COM-71-00764)
No. 66   A Preliminary Report on Correlation of ARTCC Radar Echoes and Precipitation. Wilbur K.
            Hall, June 1971. (COM-71-00829)
No. 67   Precipitation Detection Probabilities by Los Angeles ARTC Radars. Dennis E. Ronne, July
            1971. (COM-71-00925)
No. 68   A Survey of Marine Weather Requirements. Herbert P. Benner, July 1971. (COM-71-00889)
No. 69   National Weather Service Support to Soaring Activities. Ellis Burton, August 1971.
            (COM-71-00956)
No. 70   Predicting Inversion Depths and Temperature Influences in the Helena Valley. David E.
            Olsen, October 1971. (COM-71-01037)
No. 71   Western Region Synoptic Analysis-Problems and Methods. Philip Williams, Jr., February
            1972. (COM-72-10433)
No. 72   A Paradox Principle in the Prediction of Precipitation Type. Thomas J. Weitz, February
            1972. (COM-72-10432)
No. 73   A Synoptic Climatology for Snowstorms in Northwestern Nevada. Bert L. Nelson, Paul M.
            Fransioli, and Clarence M. Sakamoto, February 1972. (COM-72-10338)
No. 74   Thunderstorms and Hail Days Probabilities in Nevada. Clarence M. Sakamoto, April 1972.
            (COM-72-10554)
No. 75   A Study of the Low Level Jet Stream of the San Joaquin Valley. Ronald A. Willis and
            Philip Williams, Jr., May 1972. (COM-72-10707)
No. 76   Monthly Climatological Charts of the Behavior of Fog and Low Stratus at Los Angeles
            International Airport. Donald M. Gales, July 1972. (COM-72-11140)
No. 77   A Study of Radar Echo Distribution in Arizona During July and August. John E. Hales,
            Jr., July 1972. (COM-72-11136)
No. 78   Forecasting Precipitation at Bakersfield, California, Using Pressure Gradient Vectors.
            Earl T. Riddiough, July 1972. (COM-72-11146)
No. 79   Climate of Stockton, California. Robert C. Nelson, July 1972. (COM-72-10920)
No. 80   Estimation of Number of Days Above or Below Selected Temperatures. Clarence M. Sakamoto,
            October 1972. (COM-72-10021)
No. 81   An Aid for Forecasting Summer Maximum Temperatures at Seattle, Washington. Edgar G.
            Johnson, November 1972. (COM-73-10150)
No. 82   Flash Flood Forecasting and Warning Program in the Western Region. Philip Williams, Jr.,
            Chester L. Glenn, and Roland L. Raetz, December 1972. (COM-73-10251)
No. 83   A Comparison of Manual and Semiautomatic Methods of Digitizing Analog Wind Records. Glenn
            E. Rasch, March 1973. (COM-73-10669)
No. 84   Southwestern United States Summer Monsoon Source--Gulf of Mexico or Pacific Ocean? John E.
            Hales, Jr., March 1973.
No. 85   Range of Radar Detection Associated with Precipitation Echoes of Given Heights by the WSR-57
            at Missoula, Montana. Raymond Granger, April 1973.