AGU PUBLICATIONS

Geophysical Research Letters

RESEARCH LETTER

10.1002/2016GL068374

Key Points:

- GRAV-D airborne gravity improves the geoid to decimeters
- Twenty years average of satellite altimetric data over lakes should be accurate at 1 to 2 cm

Supporting Information:

- Supporting Information S1

Correspondence to:
Y.-M. Wang,
yan.wang@noaa.gov

Citation:
Li, X., J. W. Crowley, S. A. Holmes, and Y.-M. Wang (2016), The contribution of the GRAV-D airborne gravity to geoid determination in the Great Lakes region, *Geophys. Res. Lett.*, *43*, 4358–4365, doi:10.1002/2016GL068374.

Received 26 FEB 2016
Accepted 21 APR 2016
Accepted article online 26 APR 2016
Published online 14 MAY 2016

The contribution of the GRAV-D airborne gravity to geoid determination in the Great Lakes region

Xiaopeng Li[1], John W. Crowley[2], Simon A. Holmes[3], and Yan-Ming Wang[4]

[1]DST, Inc., Silver Spring, Maryland, USA, [2]Natural Resources Canada, Ottawa, Ontario, Canada, [3]SGT, Inc., Greenbelt, Maryland, USA, [4]NOAA, Silver Spring, Maryland, USA

Abstract The current official North American Vertical Datum of 1988 (NAVD 88) and the International Great Lakes Datum of 1985 (IGLD 85) will be replaced by a new geoid-based vertical datum in 2022. The Gravity for the Redefinition of the American Vertical Datum (GRAV-D) project collects high-quality airborne gravity data to improve the quality of the gravitational model that underpins the geoid model. This paper validates the contribution of GRAV-D data in the Great Lakes region. Using the lake surface height measured by satellite altimetry as an independent data set, Global Gravity Models (GGMs) with/without the GRAV-D data are compared. The comparisons show that the improvement reaches decimeters over Lake Michigan where the historic gravity data have significant errors. Over all lakes, except Lake Erie, the GRAV-D data improve the accuracy of the gravitational model to 1–3 cm.

1. Introduction

The U.S. National Geodetic Survey (NGS) is developing a highly accurate national gravimetric geoid model with a scheduled 2022 release. This model will provide the transformation from geodetic ("ellipsoidal") heights obtained from the Global Navigation Satellite System to orthometric heights and will serve as the zero-height surface for the next vertical datum of the USA. Toward this end, the Gravity for the Redefinition of the American Vertical Datum (GRAV-D) project is currently aiming to provide airborne gravity data over the United States and its territories. As of September 2015, NGS has flown 45% of the required surveys, which started in 2007. This airborne gravimetry will augment and correct existing terrestrial gravity data, to be ultimately combined with satellite gravity models [e.g., *Tapley et al*., 2004; *Rummel et al*., 2011] and other data (e.g., digital elevation models, satellite altimetry) to yield the targeted geoid accuracy: ±1–2 cm in flat areas and ±2–3 cm in higher elevations. *Smith et al*. [2013] and *Wang et al*. [2015] performed case studies in Texas and Iowa and demonstrated that such accuracies could be achieved with the GRAV-D data.

To improve the accuracy of its current geoid models over the USA using GRAV-D data, NGS computes new "Experimental Geoid" models (or "xGeoid" models) annually. One of the latest models, xGeoid15B (see NGS website), incorporates all GRAV-D data collected and processed prior to 31 March 2015. Its companion model, xGeoid15A, is intended to be identical to xGeoid15B, except that it excludes any contribution from GRAV-D data. The difference between these two models illustrates the contribution of the GRAV-D data. Over the Contiguous United States (CONUS), this contribution is most evident over Lake Michigan. Figure 1 shows the difference between xGeoid15A and xGeoid15B over the Great Lakes. Here the contribution from airborne gravimetry over Lake Michigan reaches the four decimeter level (Figure 1).

Satellite altimetry data can be used to compute the geodetic heights of the mean lake surface (MLS) along the ground tracks where they cross the lakes. These profiles of Mean Lake Surface Heights (MLSH), which define the mean height of the Lake with respect to the WGS84 ellipsoid, can then be used directly to assess the various global gravitational models (GGMs) that underpin the gravimetric geoid models. Accordingly, the structure of the paper is as follows. Section 2 introduces the principles used to assess gravity potential from GGMs using altimetry data. Section 3 describes the source and processing of the altimetric data. Section 4 outlines the various GGMs to be assessed. Section 5 presents the results from the altimetry analysis, focusing on Lake Michigan, since it is here that GRAV-D corrections to historic data show the most significant improvement. Section 5 then extends this analysis to the rest of the Great Lakes. Finally, discussions and conclusions are given in section 6.

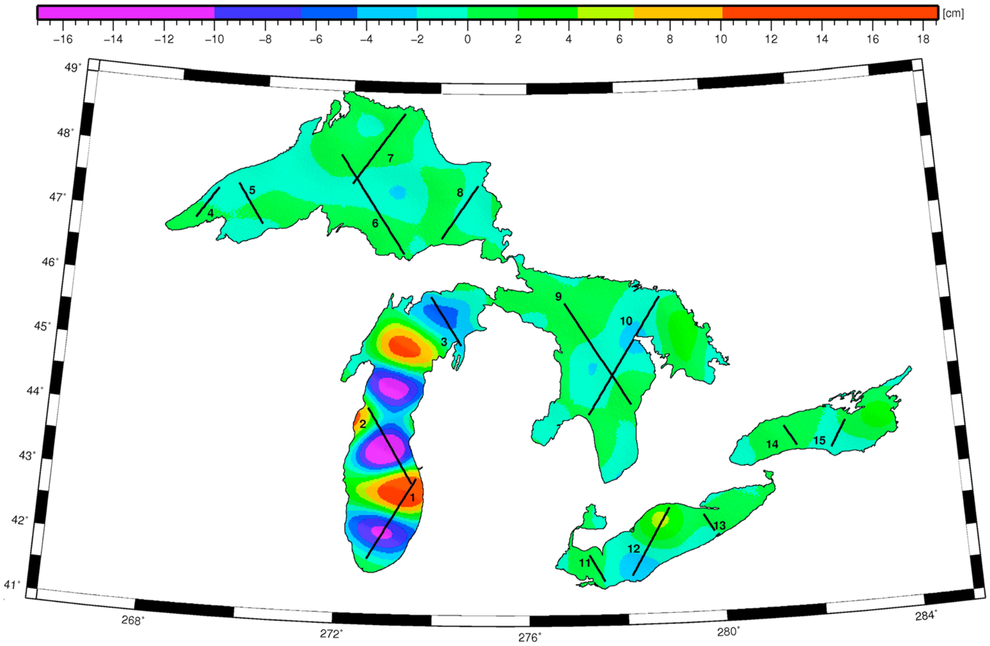

Figure 1. The difference between geoid models xGeoid15B and xGeoid15A over the Great Lakes. The contribution of airborne gravity is mostly seen over Lake Michigan. Also shown are the locations of the TPJO ground tracks.

2. The Geodetic and Dynamic Heights of Lake Surfaces

The rationale for using the altimetry data to assess GGMs stems from the simple hydrostatic assumption that the MLS approximates a "level" or "equipotential" surface of constant gravity potential W. Thus, the altimetry data, in measuring the MLSH for a given ground track, provides an independent estimate of the undulation of a single-level surface of the Earth's gravity field along this profile. Each MLS is assumed to align with a different level surface.

For each gravity model, the basic principle for the analysis applied herein is to determine the extent to which the potential field derived from the model aligns with the level surface as given by the altimetry. This is done here in two ways that are closely related. The first involves using a given GGM to compute values of potential (W_{MLS}) at the positions of the MLS as defined by altimetry. The extent to which these values deviate from a constant value highlights the discrepancies between a GGM and the independent altimetry. Expressing the geopotential differences in terms of physical heights makes the analysis more meaningful. For this, W_{MLS} is converted to a geopotential number, C_{MLS}:

$$C_{\mathrm{MLS}} = W_0 - W_{\mathrm{MLS}} \tag{1}$$

such that C_{MLS} is the potential difference between the mean lake surface and the W_0 potential value adopted for the geoid. For this NGS and the Canadian Geodetic Survey (CGS) have adopted $W_0 = 62{,}636{,}856.0\ \mathrm{m^2\,s^{-2}}$. From C_{MLS}, one can compute

$$H_{\mathrm{MLS}} = C_{\mathrm{MLS}} / \bar{g} \tag{2a}$$

$$H_{\mathrm{dMLS}} = C_{\mathrm{MLS}} / \gamma_{45} \tag{2b}$$

where $\bar{g}$ is the mean value of gravity down the plumbline between the MLS and the geoid. Since gravity is the gradient vector of potential W, dividing C_{MLS} by $\bar{g}$ yields the "orthometric height" H_{MLS} of the MLS, which is the distance from the geoid to the MLS along the plumbline. Along a truly level MLS, C_{MLS} is constant and $\bar{g}$ varies very slowly, and so orthometric heights are expected to be approximately constant here. However, to avoid estimating $\bar{g}$ inside the physical mass of the Earth and to avoid ambiguity with variable $\bar{g}$ along the altimetry profile, it is easier to replace $\bar{g}$ with a constant value of γ_{45}, herein set to $9.797645\ \mathrm{m\,s^{-2}}$. This yields the dynamic heights, H_{dMLS}, which are simply geopotential numbers that have been scaled to the units of height and so are constant along a true level surface. As such, H_{dMLS} is adopted for this analysis. For each

GGM, potential values (W_{MLS}) computed at the positions of the MLS are converted to dynamic heights. Undulations of these computed heights about a constant value show the deviation of the potential W_{MLS} from its expected constant value.

The second, very similar, assessment procedure is to use each GGM to generate the geodetic heights of a level surface of constant potential W for all locations along the altimetry ground track. This also allows each GGM to be compared directly against the altimetry, since both are providing separate estimates of the level surface in terms of its geodetic height along the ground track. The level surfaces modeled by a GGM do not change shape significantly over a vertical displacement of <1 m, and so the choice of which level surface to model with each GGM does not affect the final results, so long as it is proximal to the MLS. Thus, for each combination of ground track and GGM, we choose that level surface for which the target potential value is equal to the mean value of W_{MLS} along the track. Numerically, for each point along the ground track, the geodetic heights of the target level surface (h_{W}) are computed for each model using

$$\mathrm{d}h \approx \left(W_{\mathrm{target}} - W_{\mathrm{MLS}}\right)/(\partial W/\partial h) \tag{3a}$$

$$h_W = \mathrm{MLSH} - \mathrm{d}h \tag{3b}$$

where W_{target} is the potential of the particular level surface selected for modeling by a GGM; W_{MLS} is the GGM value for gravity potential at the surface of the lake; $\partial W/\partial h$ is the GGM value for the derivative of gravity potential, along the geodetic normal and evaluated at the surface of the lake; dh is the vertical separation between the MLS from altimetry and the position of the GGM level surface; h_W is the geodetic height of the GGM level surface. As such, the less variable/undulating is dh, the better the GGM agrees with the altimetry. Moreover, the numerical approximation in (3a) can be verified for each station by using the GGM to generate a value for W at the geodetic height (h_W) determined in (3b) for that station, since these "check" values should very closely approximate W_{target}. This they do to the extent that the numerical error in the associated computed dynamic height from equation (2b) is much less than 1 mm from its expected constant value.

3. Satellite Altimetry Data

The lake surface height has been measured by the TOPEX/Poseidon 10 day exact repeat mission [*Fu and Cazenave*, 2001]. Jason 1 and 2 continue the same configuration and form a decade-long water level monitoring system. Merged TOPEX/Poseidon/Jason1/OSTM (Jason2) (TPJO) altimetric data [*Beckley et al.*, 2013], from NASA's MEaSUREs project (https://podaac.jpl.nasa.gov), at 1 Hz rate (~5.8 km), were used to compute the MLSH along the ground tracks. These are plotted in Figure 1 and have lengths ranging from 29 to 230 km. The TPJO time series has 822 ten day repeat cycles from 1992 until the present time. The first 10 cycles are known to have had an off-nadir attitude problem (B. Beckley, personal communication, 2015). Only the remaining cycles from 11 to 822 were used to form the MLSH along the tracks.

Water levels are constantly changing within the Great Lakes and can change by a few centimeters over a 10 day period and by as much as 1 m over the 22 years of data. This makes outlier detection difficult. Therefore, a spline regression analysis was applied to each track based on the assumption that the lake shape does not change. The residuals of each regression were used to detect possible outliers, i.e., if the residual of the misfit is outside the 3σ confidence bounds of the regression lines, the data point is removed. The arithmetic mean at each point from all cleaned cycles was then used to form the mean tracks crossing the lake. Only about 400 to 600 lake surface heights of the 812 repeats at each point along the ground track were used due to winter ice and data dropout. The unmodeled errors in environmental corrections, satellite orbit, and instruments as well as time-varying lake surface height due to meteorological effects should be reduced greatly due to their random nature and 22 years of repeated observations.

The standard deviations (STDs) of the MLSHs along tracks 1 and 2 are in the range of ±30–40 cm without removing the lake water level changes. Once these cycle-by-cycle biases are removed, the STDs are reduced to ±2–4 cm at every location. This demonstrates that the along-track profile of the lake surface does not significantly alter its shape from cycle to cycle and so is a good estimate of repeatability of altimeter observations over the lakes.

4. GGMs Assessed Over the Great Lakes

The current NGS recipe for computing xGeoid15A/B is first to generate an appropriate reference GGM, and then augment the high-frequency information with (a) NGS terrestrial data and (b) forward modeling of a digital elevation model. Thus, the reference GGMs contain all the geopotential information for the final geoid, except these two final inputs. The only difference between the xGeoid15A and xGeoid15B models is the respective reference GGM upon which xGeoid15A/B are based. As such, a plot showing the geoidal differences between the two reference GGMs would be virtually indistinguishable from Figure 1, which shows the differences between the final geoid models xGeoid15A and xGeoid15B. Additionally, the absence of quality terrain and NGS terrestrial gravity data over the Great Lakes means that the xGeoid15A and xGeoid15B geoid models correspond very closely with their corresponding reference GGMs in these areas.

For xGeoid15B, its reference GGM, xGeoid15B$^{\mathrm{ref}}$, is composed of GOCO05S [*Mayer-Gürr and the GOCO consortium*, 2015] satellite gravity information, EGM2008 [*Pavlis et al.*, 2012], and the GRAV-D data. For xGeoid15A, its reference GGM, xGeoid15A$^{\mathrm{ref}}$, is identical to xGeoid15B$^{\mathrm{ref}}$, except that it excludes the GRAV-D data. Thus, xGeoid15B$^{\mathrm{ref}}$ contains all of the GRAV-D data that are propagated into xGeoid15B and is the sole vehicle by which this propagation occurs. See the NGS website for more detail. Both xGeoid15A$^{\mathrm{ref}}$ and xGeoid15B$^{\mathrm{ref}}$ have the same structure and resolution as EGM2008. All are essentially ellipsoidal harmonic models complete to degree $N_{\mathrm{max}} = 2159$, transformed to spherical harmonics complete to degree $N_{\mathrm{max}} = 2190$ and order $m = 2159$.

Two additional models are assessed herein: EIGEN6C4 [*Förste et al.*, 2014] and CGG2013i08 [*Huang and Véronneau*, 2013; *Véronneau and Huang*, 2016]. EIGEN6C4 is a spherical harmonic GGM like EGM2008. However, it is complete to degree and order $N_{\mathrm{max}} = 2190$ and so contains additional coefficients beyond those in the other three GGMs considered here. EIGEN6C4 incorporates GRACE and GOCE information, as well as EGM2008 over land to support the solution beyond the spectral resolution of the satellite data. As such, over land and including the Great Lakes, the data input for EIGEN6C4 is comparable to xGeoid15A$^{\mathrm{ref}}$.

CGG2013i08 is the latest Canadian Gravimetric Geoid model from CGS. It is not a GGM and so cannot be used directly to generate values for potential W_{MLS} directly at the MLS. However, with some additional information, the model can be used to infer H_{dMLS} for the MLS. CGG2013i08 infers orthometric heights (H_{MLS}) of any point along the mean lake surface using

$$H_{\mathrm{MLS}} = \mathrm{MLSH} - N_{\mathrm{CGG}} \tag{4}$$

where N_{CGG} is the CGG2013i08 geoid value for the ground track position. The intent here is to substitute these H_{MLS} values into equation (2a) to compute the geopotential number C_{MLS}. Equation (2b) then provides the dynamic heights of the MLS. For this it is necessary to estimate $\bar{g}$ in equation (2a). Values for $\bar{g}$ are estimated by first using EGM2008 to generate a gravity value at the MLS and then by using a Helmert-type approximation [*Hofmann-Wellenhof and Moritz*, 2005] to calculate $\bar{g}$ (note that any uncertainty regarding density in the Helmert approximation leads to negligible differences in $\bar{g}$ due to the short orthometric heights involved).

5. Results

Section 4 provided four GGMs and one geoid model to be assessed:(1) EGM2008, (2) EIGEN6C4, (3) xGeoid15A$^{\mathrm{ref}}$, (4) xGeoid15B$^{\mathrm{ref}}$, and (5) CGG2013i08. Since track 3 (Figure 1) is relatively short and crosses islands, tracks 1 and 2 are selected to assess the various GGMs and geoid model over Lake Michigan. First, for the four GGMs only, each model is used to generate a level surface that is proximal to the MLS, as detailed in section 2. The geodetic heights of these level surfaces along the ground tracks (h_W), computed according to equations (3a) and (3b), are compared against the corresponding altimetry MLSH values. The results are plotted in Figure 2 (top).

First and most importantly, Figure 2 shows that xGEOID15B$^{\mathrm{ref}}$, the only model that incorporates the GRAV-D data, tracks the altimetry almost to the 1 cm level. As expected, EIGEN6C4 and xGeoid15A$^{\mathrm{ref}}$ track each other very closely. This is not surprising, since over Lake Michigan both are essentially combinations of EGM2008 and the GOCE/GRACE data, although the computational recipes differ. Thus, all three models suffer from the errors in the historic gravity data that supported EGM2008 here. To illustrate this consideration,

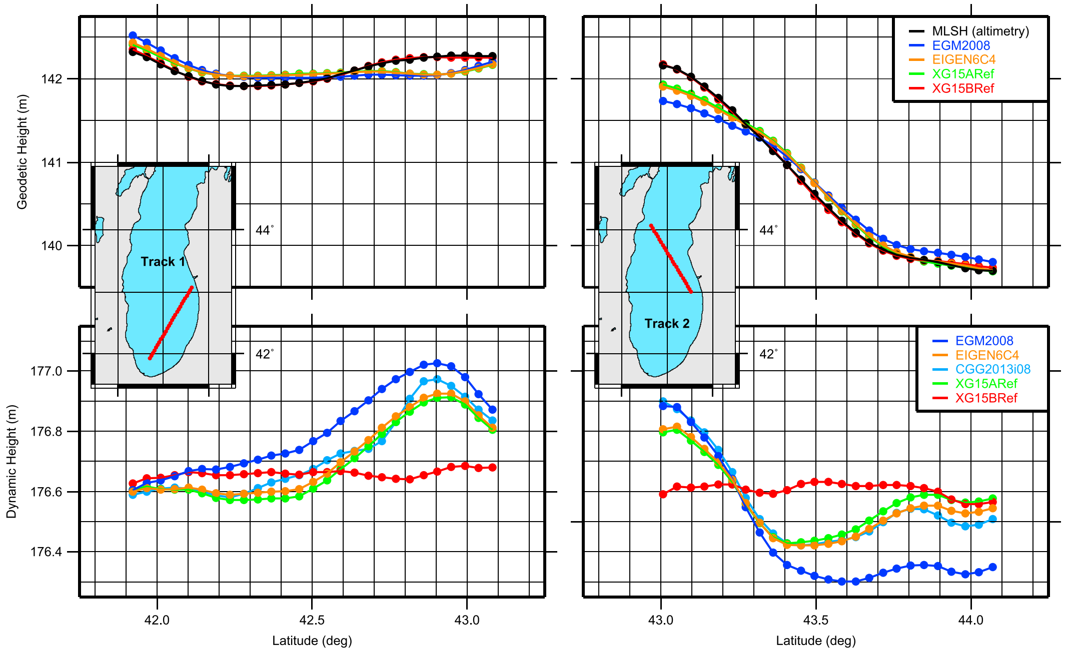

Figure 2. (top) Geodetic and (bottom) dynamic heights for tracks (left) 1 and (right) 2 of Lake Michigan. xGeoid15B$^{\mathrm{ref}}$ provides the flattest dynamic height and is therefore the best representation of a level surface.

Figure 3 shows the discrepancy between: the NGS GRAV-D gravimetry, after it has been harmonically modeled and downward continued to each lake or topographic surface, and (minus) the EGM2008 gravity values. Note the prominent macrofeatures in the center of Lake Michigan, where the GRAV-D airborne data deviates substantially from EGM2008. Figure 4 shows the same downward continued NGS GRAV-D gravimetry but this time in terms of its discrepancies with the actual NGA terrestrial survey data that was used across Lake Michigan for EGM2008. Figure 4 clearly reveals the offending surveys that are responsible for the alternating positive/negative/positive (north to south) macrofeatures that are observed in Figure 3.

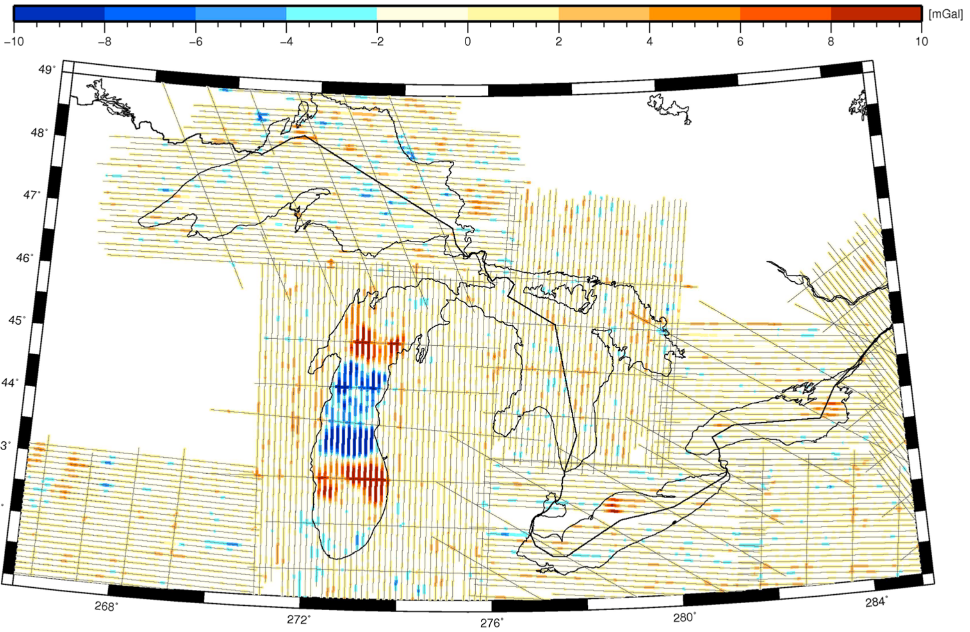

Figure 3. Discrepancy between downward continued GRAV-D track line data and EGM2008 gravity values. Nominal track spacing is 10 km. Adjusted GRAV-D crossovers match to <2 mGal RMS.

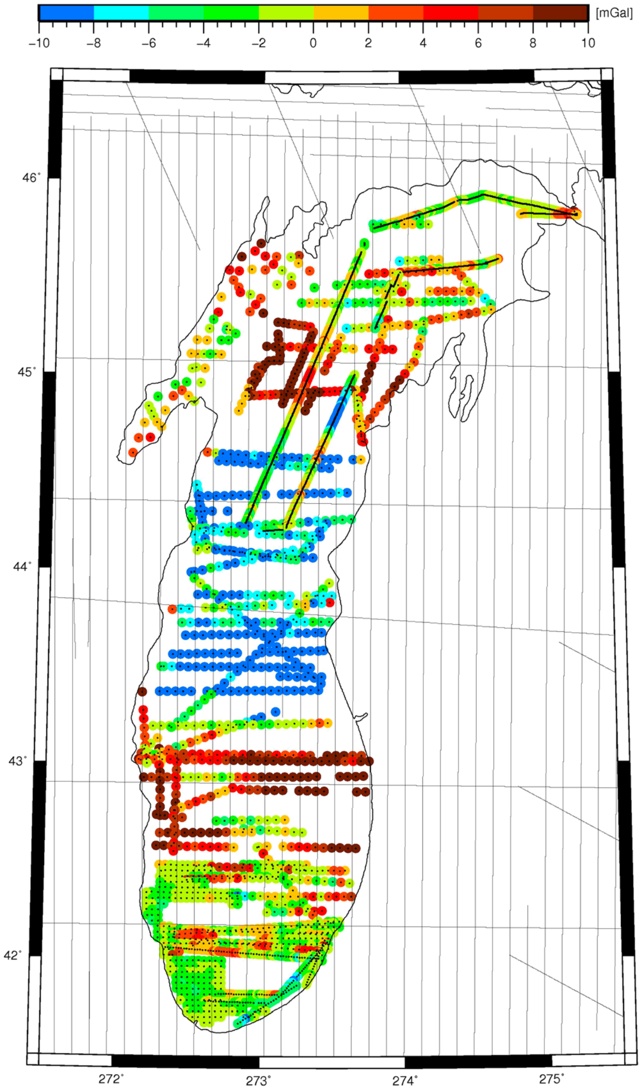

Figure 4. Discrepancy between downward continued GRAV-D data and historic terrestrial gravity surveys supporting EGM2008 over Lake Michigan. GRAV-D airborne tracks are superimposed.

The second set of results involves the computation of the dynamic height (H_{dMLS}) of the MLS from each of the four GGMs and CGG2013i08. Values for H_{dMLS} are computed according to equation (2b), as described previously and shown in Figure 2 (bottom). Recall that values for H_{dMLS} are expected to be constant for a level surface. Any deviation from constant flatness reflects disagreements between the level surface defined by the altimetry (MLS) and those inferred by the GGMs (or CGG2013i08). Here we observe that both EIGEN6C4 and xGeoid15A$^{\mathrm{ref}}$ show a 10 cm improvement over EGM2008 along track 1, and a 20 cm improvement (from 43.5° to 44° latitude) along track 2. The standard deviation (STD) along each track provides a metric for assessing the quality of the model, with low values indicating the desired level lake surface and large values indicating significant departures from a level surface.

The GOCE data helped to reduce the STD of the dynamic height from ±14.1 cm for EGM2008 to ±12.8 cm for xGeoid15A$^{\mathrm{ref}}$ along track 1, and from ±20.2 cm for EGM2008 to ±11.4 cm for xGeoid15A$^{\mathrm{ref}}$ along track 2. However, the most significant improvement comes from incorporating the GRAV-D data into xGEOID15B$^{\mathrm{ref}}$. For xGEOID15B$^{\mathrm{ref}}$, the STD of the dynamic heights across the lake is reduced to merely ±1.3 cm and ±2.2 cm for tracks 1 and 2, respectively. This is an order of magnitude improvement over all other gravity models. The standard errors of altimetric data are estimated at 1.5 cm. The discrepancies of 1.3 and 2.2 cm between the altimetric data and xGeoid15B$^{\mathrm{ref}}$ for tracks 1 and 2 imply that the accuracy of xGeoid15B$^{\mathrm{ref}}$ should be at the level of 1–2 cm.

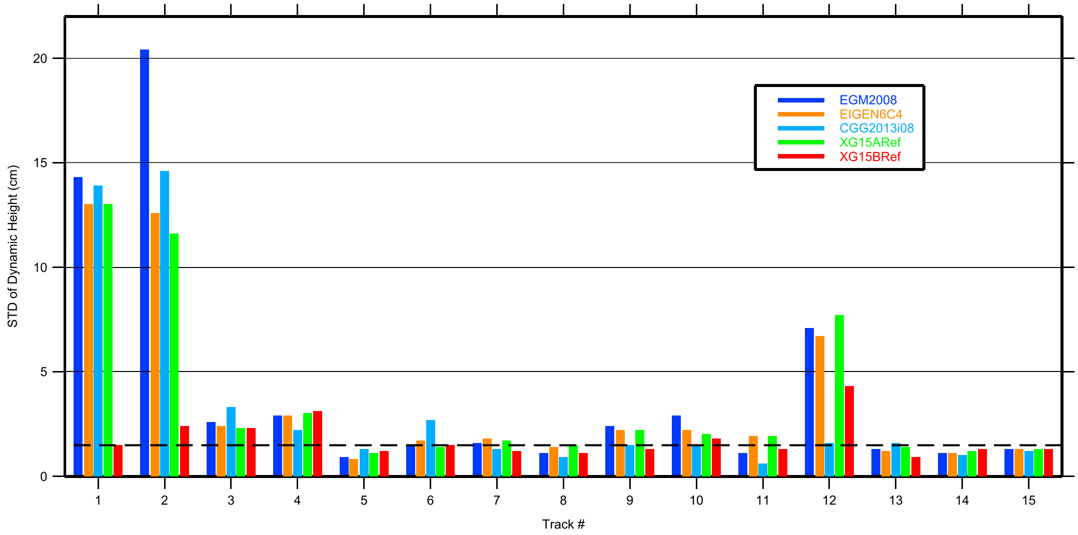

Figure 5. The STD of dynamic height for each track and model. The dashed black line indicates the standard errors of altimetric data (~1.5 cm).

The comparisons of the GGMs and CGG2013i08 against the altimetry, using dynamic heights computed for the MLS, are extended to the remaining tracks of the Great Lakes. Here again, GRAV-D data are shown to improve the xGeoid15B^{ref} GGM over the lakes, even if these results are not as significant as over Lake Michigan. Results are summarized in Figure 5 and Table S1 (supporting information). These show clearly that the xGeoid15B^{ref} GGM agrees with altimetric data to 2–3 cm or better along all the tracks, except track 12 over Lake Erie. The large discrepancy over Lake Erie for xGeoid15B^{ref} is likely due to poor terrestrial data that supported EGM2008, but which lies beyond the resolution of GRAV-D data to correct. Even so, improvement shown by the xGeoid15B^{ref} GGM is clear: the GRAV-D data reduce the STD of the dynamic heights from 7.5 cm in xGeoid15A^{ref} to 4.1 cm.

6. Discussion and Conclusions

Incorporating the GRAV-D data into the gravity models over the Great Lakes yields significant improvements over Lake Michigan. These improvements were verified using independent data from satellite altimetry.

Dynamic heights for each GGM and the CGG2013i08 Canadian Geoid were computed along the altimetric MLS profile and used to assess the quality of each model. Undulations in the dynamic heights reflect disagreements between the altimetry-derived level MLS and those inferred by the GGMs and CGG2013i08. The STD of the dynamic height across each track was used as a metric for comparing the different models. It was shown that the addition of GOCE data to EGM2008, yielding xGeoid15A^{ref} and EIGEN6C4, improved both models over Lake Michigan. A more significant improvement, however, came from augmenting xGeoid15A^{ref} with GRAV-D gravity data, yielding xGeoid15B^{ref}. Here the standard deviation in dynamic height was reduced by an order of magnitude from previous models and a ±1–2 cm relative accuracy was achieved.

While less significant, improvements were obtained for the rest of the lakes when using xGeoid15B^{ref} and dynamic height STDs of 2.9 cm or less were obtained for every track, except track 12 over Lake Erie, where short wavelength errors from terrestrial data supporting EGM2008 still persist. These results demonstrate that the GRAV-D data improves the accuracy of GGMs, and their associated geoid models, in the test areas. Although the GRAV-D data in other regions have been independently verified as improving the geoid [*Smith et al.*, 2013; *Wang et al.*, 2015], this is the first time that such a significant improvement by airborne gravity in such an extensive area in the U.S. has been demonstrated. This is encouraging as NGS prepares to transition to a geoid-based vertical datum in 2022.

Finally, this study demonstrates that satellite altimetry is not only useful to monitor ocean-circulation and global change as has been documented in the literature, but it is also a valuable tool for the validation of Earth's gravity field over lakes and large water bodies.

Acknowledgments
The authors would like to thank the entire GRAV-D team for their tremendous efforts in collecting and processing the high-quality airborne gravity data. In addition, the paper benefited from discussions with M. Véronneau and J. Huang of CGS, Brian Beckley of SGT Inc., and reviews from two anonymous reviewers and Jeroen Ritsema. Earth Science Sector (ESS) contribution number 20150462.

References

Beckley, B., R. Ray, S. Holmes, N. Zelensky, F. Lemoine, X. Yang, S. Brown, S. Desai, G. Mitchum, and J. Hausman (2013), Integrated multi-mission ocean altimeter data for climate research complete time series version 2. [Available at ftp://podaac.jpl.nasa.gov/allData/merged_alt/preview/L2/docs/multi_alt_handbook_v2.pdf.]

Förste, C., et al. (2014), EIGEN-6C4—The latest combined global gravity field model including GOCE data up to degree and order 1949 of GFZ Potsdam and GRGS Toulouse, 2014 European Geosciences Union Meeting, Vienna.

Fu, L.-L., and A. Cazenave (Eds.) (2001), *Satellite Altimetry and Earth Sciences: A Handbook of Techniques and Applications*, Academic Press, San Diego, Calif.

Hofmann-Wellenhof, B., and H. Moritz (2005), *Physical Geodesy*, Springer, Austria.

Huang, J., and M. Véronneau (2013), Canadian gravimetric geoid 2010, *J. Geod.*, *87*, 771–790, doi:10.1007/s00190-013-0645-0.

Mayer-Gürr, T., and the GOCO consortium (2015), The combined satellite gravity field model GOCO05s, 2015 European Geosciences Union Meeting, Vienna.

Pavlis, N. K., S. A. Holmes, S. C. Kenyon, and J. K. Factor (2012), The development and evaluation of the Earth Gravitational Model 2008 (EGM2008), *J. Geophys. Res.*, *117*, B04406, doi:10.1029/2011JB008916.

Rummel, R., W. Yi, and C. Stummer (2011), GOCE gravitational gradiometry, *J. Geod.*, *85*, 777–790.

Smith, D. A., S. A. Holmes, X. Li, S. Guillaume, Y. M. Wang, B. Bürki, D. R. Roman, and T. M. Damiani (2013), Confirming regional 1 cm differential geoid accuracy from airborne gravimetry: The geoid slope validation survey of 2011, *J. Geod.*, *87*(10–12), 885–907.

Tapley, B. D., S. Bettadpur, M. Watkins, and C. Reigber (2004), The gravity recovery and climate experiment: Mission overview and early results, *Geophys. Res. Lett.*, *31*, L09607, doi:10.1029/2004GL019920.

Véronneau, M., and J. Huang (2016), The Canadian Geodetic Vertical Datum of 2013 (CGVD2013), *Geomatica*.

Wang, Y. M., et al. (2015), Results of analysis of the geoid slope validation survey 2014 in Iowa, 26th General Assembly of the International Union of Geodesy and Geophysics (IUGG) Prague Czech Republic, June 22–July 2, 2015.