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NUMERICAL MODEL OF CIRCULATION IN CHESAPEAKE BAY AND THE CONTINENTAL SHELF

Kurt W. Hess

Washington, DC November 1986

U.S. DEPARTMENT OF COMMERCE National Oceanic and Atmospheric Administration National Environmental Satellite, Data, and Information Service

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NUMERICAL MODEL OF CIRCULATION IN CHESAPEAKE BAY AND THE CONTINENTAL SHELF

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ABSTRACT. A three-dimensional numerical model of coastal and estuarine circulation has been updated to include water temperature as a prognostic variable and to incorporate variable-width channels. The model has been applied to the Chesapeake Bay and local continental shelf, and calibrated with the mean tides and currents. Verification included comparison with predicted tides at Hampton Roads, VA, and currents at the mouth of the Bay. In applications, the natural period of the Bay was found to be 1.70 days, which is close to estimates based on data, and flow at the mouth was found to adjust to increased flow from the Susquehanna River within one to three tidal cycles. Future plans are also discussed.

1. INTRODUCTION

The Marine Environmental Assessment Division (MEAD) has continued development and testing of the general threedimensional free-surface numerical circulation model MECCA (Model for Estuarine and Coastal Circulation Assessment) originally described by Hess (1985a). Models such as this have become important tools for studying the physical dynamics of estuaries and exploring the consequences of various human uses of the resource. MECCA uses finite difference approximations to the momentum, mass, continuity, and concentration equations to simulate three-dimensional water currents and salinities at 10 levels in a shallow water domain at time scales in the range of a few minutes to several months, and space scales in the range of a few kilometers to a few hundred kilometers. The first version of the model was applied to Chesapeake Bay to simulate tidal and density-driven currents, and to study the reduction in salinity in the Bay during the high river discharge conditions following the passage of hurricane Agnes over the watershed in 1972. Α user's guide for that version is now available (Hess, 1985b).

MECCA was subsequently applied to a larger domain which includes Chesapeake Bay and a portion of the local continental shelf. Circulation in the Bay-shelf region is important for assessing the biological productivity of the estuary. Results of a detailed study which used wind forecasts and satellite-derived temperatures to provide input to the circulation model, which then generated currents to simulate trajectories of hypothetical biological drifters was presented in Johnson et al. (1986a). The experience in applying and testing MECCA in that study was directly responsible for several refinements in the modeling procedure. The improved version of MECCA is the subject of this memorandum.

The new version of the model incorporates two substantial advancements: the mass and momentum equations now allow for variable-width channels, and the set of prognostic variables now includes temperature. Channels of variable width allow rivers, which generally have widths much narrower than a typical grid cell width, to be explicitly included in the numerical grid This improvement is significant in the Chesapeake Bay scheme. region since the estuary has numerous tributaries. Temperature is an important variable in the estuarine environment, both for the computation of water density, and for proposed future water quality and biological applications. Also, among many other changes, the derivation of the vertically-averaged velocity components was refined. Previously (Hess, 1985a), the horizontal velocity equation was first vertically averaged, then converted to the dimensionless coordinate system; here, the velocity equation is converted to the dimensionless coordinate system first, then vertically integrated. The new derivation is preferable because several of the terms resulting from the integration are greatly simplified.

2. THE MODEL EQUATIONS

The heart of any modeling system is the set of equations, along with an explanation of the types of phenomena the equations are designed to simulate. The model equations describing coastal circulation employed here are nearly the same as those listed in Hess (1985a), with the main exception that the thermodynamic energy equation has been included to predict temperature changes. Our ultimate goal is to simulate short-term (sub-tidal period) changes in the state of an estuary, including daily heating cycles and mixing due to sudden (storm related) wind events, and to compare modeled results with observational data for the same time period. Secondarily, we wish to run a simulation model for intervals as long as a season to study changes which become evident only at that time scale. While the philosophy of keeping a modeling system simple is commendable, the behavior of a coastal estuary is, in reality, quite complex, and so the modeling system must also be complex if it is to adequately represent that estuary. We also note that it is much simpler to reduce a three-dimensional model to a two-dimensional one than to accomplish the reverse.

The major feature of this modeling approach is the use of split-mode velocity equations (Madela and Piacsek, 1977; Sheng, 1983; Blumberg and Mellor, 1983). The external, or barotropic, velocity mode (which is simply the vertically-averaged velocity) is subtracted from the total velocity to get the internal, or baroclinic, velocity mode. Because the internal mode stability requirement in the numerical solution scheme is less stringent than that for the external mode, the internal mode can be updated less often, resulting in significant savings in computer time. In practice, the internal mode timestep we used was about four times the external mode timestep.

Another feature of the model equations is the use of a dimensionless vertical coordinate, also known as a sigma coordinate (Phillips, 1957). This terrain-following coordinate gives a better representation of the bathymetry and avoids the problem in a fixed-grid system of having the surface elevation fall below the next-to-surface level. A disadvantage is that all terms in the model equations involving derivatives become somewhat more complex.

The set of model equations is derived in the following manner (a summary of the equations at each step of the development appears in Appendix A). The equations for flow within a continuum (set A) are first integrated widthwise to get the variable width form (set B). Then the equations are transformed with a new vertical coordinate (set C). Next the velocity and continuity equations are integrated over the vertical to get a set of external mode equations (set D). Lastly, the verticallyintegrated equations are subtracted from set C velocity equations to get the internal mode velocity equations (set D).

2.1 The External Mode Momentum and Mass Equations

The momentum equations apply to flow in the northern hemisphere in a right-handed Cartesian coordinate system with the z axis directed upward and the x axis in the horizontal tangent plane with arbitrary orientation. Metric units are used throughout unless specifically noted. In the following list of equations, mathematical notation is used for partial differentiation of the dependent variable with respect to the independent variable, so that, for example,

$$f_{,x} = \partial f/\partial x$$
 and $f_{a,zz} = \partial^2 f_a/\partial x^2$ (2.1)

For the external mode, the horizontal momentum equations used in the model (Appendix A) are

$$(H\underline{u}), t + B_{x}^{-1}(HB_{x}\Theta_{uu}\underline{uu}), x + (H\Theta_{uv}\underline{uv}), y = -gHh, x - \alpha_{0}Hp_{a}, x$$

$$- G^{*}_{x} + f\underline{v} + B_{x}^{-1}(2A_{h}HB_{x}\underline{u}, x), x + (A_{h}H[\underline{v}, x + \underline{u}, y]), y$$

$$+ T_{sx} - T_{bx} - B_{x}^{-1}C_{ws}H\Theta_{su}\underline{u}|\underline{u}|$$

$$(2.2)$$

and

$$(\underline{H}\underline{v})_{,t} + (\underline{H}\Theta_{uv}\underline{uv})_{,x} + \underline{B}_{y}^{-1}(\underline{H}B_{y}\Theta_{vv}\underline{vv})_{,y} = -gHh_{,y} - \alpha_{o}Hp_{a,y}$$

$$- G^{*}_{y} - f\underline{u} + \underline{B}_{y}^{-1}(\underline{2}A_{h}HB_{y}\underline{v}_{,y})_{,y} + (A_{h}H[\underline{v}_{,x} + \underline{u}_{,y}])_{,x}$$

$$+ T_{sy} - T_{by} - \underline{B}_{y}^{-1}C_{ws}H\Theta_{sv}\underline{v}|\underline{v}| \qquad (2.3)$$

and the continuity equation is

$$h_{t} + B_{x}^{-1}(B_{x}H\underline{u})_{x} + B_{y}^{-1}(B_{y}H\underline{v})_{y} = 0$$
 (2.4)

where \underline{u} and \underline{v} are the vertically-averaged (external mode) velocities in the x and y directions, respectively, B_{X} and B_{y} are the widths of flow in the x and y directions, respectively, and H is the total water depth and consists of the sum of the water level above mean sea level, h, and the depth below mean sea level, d. The gravitational acceleration is g, α_{O} is a reference specific volume, p_{a} is the atmospheric pressure, f the Coriolis acceleration, A_{h} the lateral eddy viscosity coefficient, A_{v} the vertical eddy viscosity coefficient, T_{S} the wind stress per unit density at the air-water interface, T_{b} the stress per unit density at the water-bottom interface, and C_{WS} the water-side interfacial friction coefficient. The variables G and G^{*} incorporate horizontal density gradients, and the θ terms include effects of the deviation of the horizontal velocity from the vertical mean; these terms are defined in the next section in eqs. 2.10-2.13.

In the presence of a channel, the diffusion terms involving the cross derivatives (the sixth terms on the right sides of 2.2 and 2.3) are dropped; with no channel, $B_X = B_y = 1$ and $C_{WS} = 0$. Note that these equations are similar to those in Oey et al. (1985), but ours contain the effects of both the horizontal density gradient and the internal mode velocities.

2.2 The Internal Mode Momentum and Mass Equations

The internal mode velocities are defined as the departures from the vertical mean velocity, i.e.

$$\mathbf{u'} = \mathbf{u} - \underline{\mathbf{u}}$$
 and $\mathbf{v'} = \mathbf{v} - \underline{\mathbf{v}}$ (2.5)

For the internal mode, the momentum equations are

$$(Hu')_{t} + B_{x}^{-1}(B_{x}H[uu - \Theta_{uu}\underline{uu}])_{x} + (H[uv - \Theta_{uv}\underline{uv}])_{y}$$

+ $(\tilde{w}u')_{q} = G^{*}_{x} - G_{x} + fv' + B_{x}^{-1}(2A_{h}HB_{x}u',x)_{x}$
+ $(A_{h}H[v',x + u',y])_{y} + H^{-1}(A_{v}u',q)_{q} - T_{sx} + T_{bx}$
- $C_{ws}HB_{x}^{-1}(u|u| - \Theta_{su}\underline{u}|\underline{u}|)$ (2.6)

and

$$(Hv'), t + (H[uv - \Theta_{uv}\underline{uv}]), x + B_{y}^{-1}(B_{y}H[vv - \Theta_{vv}\underline{vv}]), y$$

+ $(\tilde{w}v'), q = G^{*}_{y} - G_{y} - fu' + (A_{h}H[v', x + u', y]), x$
+ $B_{y}^{-1}(2A_{h}HB_{y}v', y), y + H^{-1}(A_{v}v', q), q - T_{sy} + T_{by}$
- $C_{ws}HB_{y}^{-1}(v|v| - \Theta_{sv}\underline{v}|\underline{v}|)$ (2.7)

The internal-mode continuity equation is

$$B_{x}^{-1}(B_{x}u), x + B_{y}^{-1}(B_{y}v), y + (\tilde{w}), q = 0$$
(2.8)

The dimensionless vertical coordinate, q, is defined in terms of z as

$$q = (h - z)/H$$
 (2.9)

so that

$$\Theta_{u_{j}u_{j}} = \int_{-1}^{0} (u_{j}u_{j}/\underline{u}_{j}) dq \qquad (2.10)$$

$$\Theta_{su_{\underline{i}}} = \int_{-1}^{0} (u_{\underline{i}}/\underline{u}_{\underline{i}}) \left| [u_{\underline{i}}/\underline{u}_{\underline{i}}] \right| dq \qquad (2.11)$$

$$G_{x_{i}} = g[H_{j}^{o}(\rho - \rho_{o})dq]_{,x_{i}} + g(h_{,x_{i}} + qH_{,x_{i}})(\rho - \rho_{o})$$
(2.12)

$$G_{x_{i}}^{*} = \int_{-1}^{0} G_{x_{i}}^{*} dq$$
 (2.13)

where ho is the water density and $ho_{
m O}$ is a reference density. Also

$$\tilde{w} = Hdq/dt$$
 (2.14)

is a representative vertical velocity, approximately equal to the

actual vertical velocity (see eq. A.20 for the exact relationship).

2.3 The Hydrostatic, State, Salinity, and Temperature Equations

The hydrostatic approximation to the vertical momentum equation is

$$\mathbf{p}_{,\mathbf{q}} = -\rho \mathbf{g} \mathbf{H} \tag{2.15}$$

where p is the water pressure. A simplified equation of state (Mamaev, 1964) is

$$\rho = \rho_0 (1 + C_s S + C_T T)$$
(2.16)

where S is the salinity (parts per thousand), and T is the water temperature ($^{\circ}C$), and C_{s} and C_{T} are density coefficients. The equation of conservation of salinity is

$$(HS)_{t} + B_{x}^{-1}(B_{x}H[uS - D_{h}S_{,x}])_{,x} + B_{y}^{-1}(B_{y}H[vS - D_{h}S_{,y}])_{,y} + (\tilde{w}S - H^{-1}D_{v}S_{,q})_{,q} = 0$$
(2.17)

where D_h is the horizontal turbulent diffusivity and D_v is the vertical turbulent diffusivity. The thermodynamic conservation of heat equation is

$$(HT)_{t} + B_{x}^{-1}(B_{x}H[uT - D_{h}T, x])_{x} + B_{y}^{-1}(B_{y}H[vT - D_{h}T, y])_{y}$$

+ ($\tilde{w}T - H^{-1}D_{v}T, q$), q = HR (2.18)

where R is the internal heating term due to solar shortwave radiation (see Appendix B). We now have the set of equations for the dependent variables.

2.4 Parameterization

In order to scale the equations in a meaningful way (i.e. to a specific location in a real estuary), we must have realistic values for depth, geometry, and boundary inputs. Furthermore, stresses and turbulent parameters must also be evaluated. The wind stress formulation and the beta plane approximation are the same as in Hess (1985a). The bottom stress per unit density in the x direction is

$$T_{bx} = C_{wb}(u' + \underline{u})u_b$$
(2.19)

where C_{wb} is the water-bottom interfacial drag coefficient and u_b is the total water speed just above the bottom.

The vertical turbulent viscosity is approximated here using mixing length arguments. The use of higher-order approximations based on local turbulent kinetic energy balances does not appear to be justified at this time, given the lack of easilyinterpretable data on its variability and the extra computer resources required to implement such approximations (further developments in turbulent theory may alter this judgment, however). The instantaneous viscosity is formulated as the product of a mixing length, the local vertical velocity shear, and a reduction due to water column stability, and is

$$A_{\rm V} = [0.40z(1 - z/H)]^2 (u_{,z}^2 + v_{,z}^2)^{1/2} [C_0(1 + C_1R_1)^{-C_2}] + A_{\rm m}$$
(2.20)

where R_i is the Richardson number

$$R_{i} = -g\rho_{,z} / [\rho_{0}(u_{,z}^{2} + v_{,z}^{2})]$$
(2.21)

and A_m is the molecular kinematic viscosity (10⁻⁶ m²/s). Nominal values for C₀, C₁, and C₂ are 1.0, 5.0, and 1.0, respectively. The vertical turbulent mass diffusivity is defined as

$$D_{v} = [0.40z(1 - z/H)]^{2}(u_{z}^{2} + v_{z}^{2})^{1/2}[C_{3}(1 + C_{4}R_{1})^{-C_{5}}] + A_{m}$$
(2.22)

with nominal values for C_3 , C_4 , and C_5 are 0.005, 5.0, and 1.0, respectively.

Horizontal turbulent exchange coefficients are based on the local horizontal velocity shear and a length scale equal to the grid cell size (Tag et al., 1979), so that the momentum exchange coefficient is

$$A_{h} = C_{AH} \Delta L^{2} [2(\underline{u}, x^{2} + \underline{v}, y^{2}) + (\underline{u}, y + \underline{v}, x)^{2}]^{1/2} + A_{0} \qquad (2.23)$$

where C_{AH} is a coefficient with a nominal value of 0.01, ΔL is the grid cell size, and A_0 is a background value (1.0 m²/s). Also, we assume the equality of the horizontal mass (D_h) and momentum (A_h) transfer coefficients

$$D_{h} = A_{h} \tag{2.24}$$

At any external mode timestep, the exchange coefficients are computed by the above equations and averaged with the value from the previous timestep. This introduces a crude time dependency to the coefficients, but more importantly avoids wild numerical oscillations which may arise from the interaction of the velocity and the viscosity.

2.5 Thermal Boundary Conditions

The boundary condition on the temperatures at the air-water interface is

$$D_{\mathbf{V}}\mathbf{T}_{,\mathbf{Z}} = \mathbf{Q} \tag{2.25}$$

where Q is the sum of several terms which represent heating at the sea surface. These terms (Parkinson and Washington, 1979) are discussed in Appendix B. Also, because the surface slope is so small, the normal to the surface is taken to be parallel with the z direction. At the bottom, we have

$$D_v T_{,n} = C_{bed} u_b (T_b - T_{bed})$$
(2.26)

where n is the direction normal to the bottom, T_b is the water temperature just above the bottom, T_{bed} is the temperature of the sea bed just below the water-bottom interface, and C_{bed} is a dimensionless bulk heat transfer coefficient with a nominal value of 0.003. Here again the assumption of small bottom slopes allows us to use the z derivative in place of the n derivative in the above equation.

The internal heating term, R, accounts for the absorption of solar shortwave (i.e. visible range) energy in the water column. It is assumed that the energy reaching any depth, Q_Z , is related exponentially to the amount reaching the surface, Q_O , by

(2.27)

 $Q_z = Q_0 \exp(2.302z/d_{10})$

where d_{10} is the depth to which only 10 percent of the surface energy penetrates.

3. NUMERICAL SOLUTION OF THE MODEL EQUATIONS

The method chosen for solution of the equations is the application of finite difference techniques. The long history of this method has provided a wealth of theoretical and numerical information which has made it the method of choice for most three-dimensional coastal and ocean models (Parker, 1986; Sundermann and Lenz, 1983). Finite element techniques, although used occasionally for two-dimensional models (Wang et al., 1984), have not enjoyed widespread application in either marine or atmospheric modeling; mixed finite difference-finite element techniques (Davies, 1986) may be a possible compromise. Moreover, finite difference methods lend themselves easily to vectorization, and so are ideal for today's supercomputers.

The finite difference approximation to a particular differential equation can be derived in any of several ways

(Roache, 1972). Our approach is to write the Taylor series expansion for each derivative at a grid point and select the difference which is accurate to the second order. The staggered placement of variables is used in horizontal space to allow small spatial waves to propagate away from their point of origin. Staggered placement in the vertical is used so that the horizontal velocity is defined at the surface and bottom. The advantage at the top is having a surface velocity for the advection of surface-borne pollutants; at the bottom, the velocity is required for the interfacial stress terms. The equations obtained from the Taylor series expansion are virtually identical to those derived from the control volume approach, in which the variables represent averages over the grid volume.

One of the most difficult and fundamental problems in applying a finite difference scheme is deciding how to partition what are continuous, interactive, simultaneous physical processes in nature into several discrete, isolated, sequential numerical procedures. We chose the following series of steps. First, the water levels and the external-mode horizontal velocities are computed by an alternating direction technique. These updates are performed at every external-mode timestep, which in the present application to Chesapeake Bay and the adjacent shelf is 360 seconds. At every third external-mode timestep the horizontal turbulent viscosity coefficient is updated. At every fourth external-mode timestep (or at each internal-mode timestep) the internal-mode horizontal and vertical velocities are updated, one horizontal gridmesh cell at a time. Finally, at each internal-mode timestep, the salinity and temperature are updated. At every third internal-mode timestep the vertical turbulent viscosity and diffusivity are updated.

The foundation of the numerical solution is the calculation of the external-mode velocities and the water levels; the computation technique is similar to those in classical vertically-integrated models (Leendertse, 1967). MECCA employs an alternating-direction implicit Abbott method (Sobey, 1970) to compute <u>u</u> and an approximation to h on the first sweep, then <u>v</u> and h on the second sweep. The application is relatively straightforward, but with non-linear advective terms approximated with upstream differencing and the bottom stress partitioned into internal- and external-mode components (see eq. 2.19).

The internal-mode velocities, salinities, and temperatures are updated less often than the external-mode velocities, thus saving computer time. The non-linear advective terms in the internalmode velocity equation have only a small contribution, so they are neglected. The numerical solution of each of these variables is based on an implicit calculation over the vertical (to augment numerical stability), with top and bottom fluxes providing the boundary conditions. In the present configuration there are 10 levels, equally spaced over the vertical, with one each at the air-water interface and the water-bottom interface. The vertical spacing is arbitrary but fixed, and is a result of using the dimensionless vertical coordinate. By contrast, in a layered system the spacing changes in response to local continuity imbalances; severe problems can arise if the layer thickness drops to zero.

The vertical density structure at each grid is checked for static stability. Salinity and temperature at all levels in one horizontal grid are first computed before moving to the next grid. After this update at all levels is completed, the stability between each level is checked. If the density at one level is lower than the density at the level just above, the salinities and temperatures at the two levels are made uniform in a way that preserves total mass over the column.

Another density check is made at the surface layer where there is atmospheric heating and cooling. Unrealistic behavior of the temperature was found during some simulations in which there was wind-induced cooling. Reductions of several degrees Celsius occurred during the cooling, followed by rapid increases caused by vertical mixing when the density structure became unstable. This aberration in the temperature history was eliminated by assuming instantaneous vertical mixing between the top two levels at all times, a situation which would normally occur anyway in the presence of wind waves.

Besides the usual square land and water grids in the gridmesh, we use triangular and riverine grids. Triangular grids (Fig. 1a), in which a diagonal represents the land-water boundary for better approximation of the coastline, have water flow through two sides and a water level along the diagonal. The grid has,



Figure 1. (a) Placement of variables on a triangular grid cell. (b) Placement of variables and definition of width in a variable-width riverine cell. therefore, only half the area of a square grid. It is a minor drawback that the numerical approximation of horizontal derivatives in triangular cells is only accurate to first order. Riverine grids (Fig. 1b) have flow in only one direction, a variable width, and a surface area smaller than that of a square grid.

4. APPLICATION TO CHESAPEAKE BAY AND THE LOCAL SHELF

A circulation model is a unique tool for studying many aspects of estuarine behavior, and was applied in this study to Chesapeake Bay and the local continental shelf as one way of assessing the biological productivity of the region. Specifically, we have simulated the motion of hypothetical blue crab larvae by assuming passive drift with the currents as provided by MECCA. The net motion of the larvae over 3 months under the influence of the wind and tidal currents was simulated, with the drifters hatching at the mouth of the Bay, rising to the surface, and subsequently being advected onto the continental shelf (Johnson et al., 1986b).

4.1. The Gridmesh

The first step in simulating the currents is the generation of a gridmesh to represent the distribution of the depths, currents, and other variables over the area of interest. To this end, the gridmesh shown in Fig. 2 was created. The area covered includes the Chesapeake Bay and the lower portions of four of its major tributaries and the local continental shelf out to approximately the 80-meter isobath and extending up and down the coast about 75 kilometers on each side of the mouth. Out on the shelf, where the depths are relatively great, large grid cells are desirable because less resolution is needed there and because the gravity wave speed there is relatively large (in an explicit numerical solution scheme for the external-mode velocity, the timestep is limited by the time it takes a gravity wave to cross the cell; although we are using an implicit numerical scheme, we use a timestep of the same order). In the Bay, small grid cells are desirable for increased resolution of the flow field. The grid cell size, 11.2 kilometers, was chosen as a compromise between these competing requirements. Furthermore, this size allows two cells to represent the Bay mouth. In the future, a mesh with variable grid sizes may provide a more suitable approach.

The number of vertical levels used in this Bay-shelf application is 10. The vertical variation in the horizontal velocity, the salinity, and the temperature at both the top and bottom boundaries is crucial in determining the fluxes across these surfaces. In addition, the vertical turbulent exchanges



Figure 2. Gridmesh covering Chesapeake Bay and the local continental shelf used in this study. Cell size is 11.2 km by 11.2 km. Cells marked with an "X" along the deep-water boundary require input water level conditions. Cells marked with a "<" along the northern and southern boundaries have radiation condition outflows specified; cells with the same mark at the heads of rivers require flowrate boundary conditions. are a strong function of the vertical gradients in the internalflow, so that the resolution provided by so many levels is fully justified in view of the complexity of the system.

4.2. The Boundary Conditions

Large amounts of data are required to run the model for a simulation of daily events over several months of real time. Examples of these are tide histories, wind speeds and directions, atmospheric temperatures, river flowrates, and riverine and oceanic velocities, salinities and temperatures. Additionally, there are several types of physical boundaries to consider. These are the oceanic, riverine, air-water, and the water-bottom boundaries. Each one of the general types of boundaries is discussed in turn.

Oceanic Boundaries

In the model grid domain we have selected, as in most coastal models, there are two types of oceanic boundaries: the deep water boundary and the lateral boundary. For both types it is assumed that the actual boundary runs through the center of the cell, so that water levels, salinities, and temperatures are required as input. Since the water level conditions at each are somewhat different, we will discuss the types of boundaries separately.

At the deep-water boundary, which runs approximately parallel to shore, the most important input value for driving the system is water level. Tidal water levels for any time are generated in MECCA by interpolation from a series of tide height-and-time values by

$$h_b(t) = 0.5((h_{i-1} - h_i)\cos(0.5\pi[t - t_{i-1}]/[t_i - t_{i-1}]))$$

+
$$(h_{j-1} + h_j)$$
 (4.1)

where h_b is the height at the boundary, h_i is the ith value of the tide (above m.s.l.), t is the time, and t_i is the ith time of occurance (after some reference time) of the tide extremum (high or low). The time t_i is the time of the extremum closest to, but later than, t. The above expression for the water level is also well suited for including water level perturbations due to inverse barometer effects and subtidal-frequency waves as this information becomes available.

Also at the deep-water boundary, the external- and internalmode velocities are computed as interior values from water levels at the boundary because of the staggered grid, and do not need to be specified separately.

The deep-water boundary temperatures and salinities over depth at grid cells are generated from a set of input values composed of the depth of the upper mixed layer, the depth of the bottom of the pycnocline, and values for salinity and temperature at four depths each: for the surface, the bottom of the mixed layer, the bottom of the pycnocline, and the bottom of the water column (Fig. 3). At any one cell, the index values of the depths, salinities and temperatures are found by linear interpolation between values at the end cells of the row (Points 2 and 3, Fig. 2). Finally, the nominal values of salinity and temperature at each level are found by interpolation between the closest index During outflow (flow directed outward from the values. computational region), the boundary values of salinity and temperature are generated from the interior field by either extrapolation or simple advection. During inflow, boundary values at each level are generated by linear interpolation between the value that occurred during the last episode of outflow, and the nominal value (assumed to occur 6.10 hours later).

In contrast to the deep water boundary, the lateral boundary, which runs approximately normal from shore, requires a different set of values. Water levels at the lateral boundaries are specified using the condition of outward radiation of mechanical energy (Sommerfeld, 1949). Since observations (Moody et al., 1984) suggest that the tide waves propagates landward in a direction nearly normal to the shoreline, it is assumed that the



Figure 3. Schematic representation of the vertical variation of temperature and salinity as specified in the oceanic boundary conditions.

tidal level at the lateral boundary, h_L , will rapidly adjust itself toward the value of the tidal level at the nearest deepwater boundary cell, h_b . Specifically, this condition (Davies, 1983) is

 $h_{\rm L} = h_{\rm b} + U/(gH)^{1/2}$ (4.2)

where U is the outward normal flowrate per unit width.

Salinities and temperatures at the lateral boundaries are specified in exactly the same way as at the deep-water boundary.

Riverine Boundaries

The upper ends of rivers are represented by grid cell conditions for either water level or velocity, and for salinity and temperature. There are two types of boundaries possible: a river channel cross section and a waterfall.

The river channel section boundary is assumed to be located at the grid cell side, where either flowrates or velocities are specified. Since river flowrate, Q_R , is commonly given by the U.S. Geological Survey, the external-mode velocity is expressed as

 $\underline{\mathbf{u}} = \mathbf{Q}_{\mathbf{R}} / (\mathbf{H} \Delta \mathbf{L}) \tag{4.3}$

where H is the local total depth. The internal-mode velocity is specified by the analytic function

$$\mathbf{u}^{*} = \mathbf{u}_{O} \cos\left(\pi \mathbf{q}\right) \tag{4.4}$$

where u_o is the value at the surface. Salinities or temperatures, C, over depth are specified by the analytic expression

$$C(q) = C_{top} - (C_{top} - C_{bottom})(1 - \cos[\pi q])/2$$
 (4.5)

The other possible riverine condition assumes that the boundary coincides with the center of the grid cell, and simulates the presence of a waterfall. The water level is specified as

$$h_{t} = Q_{R} / (\Delta L^{2}) \tag{4.6}$$

and concentration is found from

$$(HC)_{\pm} = C_f Q_R / (\Delta L^2)$$
(4.7)

where C_f is the salinity or temperature of the falls. This condition is experimental and was not used in the model runs.

<u>Air-water Interface</u>

Wind speed and direction data is entered as a series of values, taken from the National Weather Service's Limited-area Fine-mesh Model (LFM). Twice daily wind data from the LFM's lowest layer (the boundary layer) from several atmospheric model grid points for the region are used as input to a regression equation (Johnson et al., 1986a) to generate 10-meter winds at Norfolk VA Naval Air Station. These time-varying Norfolk winds are then used at all grids in the circulation model.

Twice-daily boundary layer atmospheric temperatures are also used in the surface heating equations (Appendix B). Atmospheric pressure, relative humidity, and fraction of sky cover are taken as constants ($p_a = 101400$ Pa, Rh = 0.7, n = 0.10).

Water-bottom Interface

At the bottom of the water column, the sea bed temperature, T_{bed} , represents the temperature of the first few meters of material below the water-bottom interface. Not much is known about the time variability of this value, so it is assumed temporally constant (although it does vary in the horizontal).

4.3 Model Initialization

Initial values for the model's variables are set in one of two ways, depending on whether an antecedent run exists. In the modeling environment, we consider a "run" to consist of a single computer job submission simulating a finite interval of time. Several runs strung together in series constitute a "sequence".

At the beginning of the first run in the sequence, water levels and all velocities are set to zero, and salinities and temperatures are set by interpolation from the boundary values. The vertical turbulent diffusion coefficient, A_V , is set to a relatively large value (0.01 m²/s) at all cells to suppress oscillations which can develop when a fluid system at rest is subjected to sudden forcing. For the same reason, the horizontal diffusion coefficients are also set to relatively large values (500 m²/s in the Bay and 50 m²/s on the shelf).

For subsequent runs in the sequence, all the above variables are set to their respective values at the end of the previous run.

Some special initialization procedures are used in the first run in the sequence for the tides, winds, and concentrations. The tidal water levels are not applied at full force during the first 24 hours, but are reduced by multiplication by a simple ramp function. The ramp function is zero at t = 0, increases linearly to unity at 24 hours, and is unity thereafter. The winds are treated in the same way. The concentrations are not updated during the first 24 hours, but the exchange coefficients are. This allows the vertical and horizontal exchange coefficients to incorporate the velocity shears more accurately.

Since the concentrations, and hence densities, are not updated during the first 24 hours of the first run, a serious problem could develop in the momentum equations. The horizontal pressure gradients in the momentum equations (the G and G* terms) incorporate the initial mass distribution, which is after all just a guess. These terms could be orders of magnitude too large, causing unrealistic horizontal currents; moreover, there is no self-adjustment process since the concentrations are not being updated. To prevent these problems, the pressure gradient terms are multiplied by another ramp function. This function is zero for times less than 24 hours, increases linearly to unity at 36 hours, and is unity thereafter.

5. SIMULATION OF TIDES AND TIDAL CURRENTS

The astronomical tides are a major source of energy to Chesapeake Bay, and so are the phenomenon we have chosen to study first. For the first part of the investigation, we have simulated the mean tides in the Bay. Mean tides are determined by the National Ocean Service (NOS) from a long series of observations. In the model, the mean tide is generated by a forcing tide with a constant range and a constant period of 12.40 hours; this period is close to that of the M_2 tidal constituent (12.42060 hours), and in 31 days there are exactly 60 cycles.

5.1 Preliminary Tests

The first test of the model of the Bay-shelf system determined the time necessary for the tides and currents at selected locations in the gridmesh to reach a repeatability condition. Repeatability is defined here as occuring when successive high (or low) waters are within 0.001 meter, or when successive flood (or ebb) currents are within 0.001 m/s. With a timestep of 360 seconds, repeatability was reached after 5 to 6 tidal cycles (Fig. 4).

The next test was designed to shown how large a timestep is allowable to maintain realistic results. As a first approximation, the timestep should be about the size of the limiting explicit timestep

$$T = \Delta L / (2gH_{max})^{1/2}$$

(5.1)

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Figure 4. Plots of the amplitudes of successive high and low waters at Hampton Roads, VA, and Baltimore, MD, showing that a repeatability condition is reached after about five tidal cycles.

which for our basin is 289.9 seconds. An implicit solution scheme like the one we are using has been shown to be stable regardless of the timestep size (Sobey, 1970; Leendertse, 1967) for the restrictive case of uniform depth and without advective, Coriolis, bottom friction, pressure, and surface forcing terms. In a real basin, however, these conditions are not met, so there is no guarantee that the numerical scheme will be stable; we thus chose a timestep that was close to the limit for an explicit scheme (eq. 5.1). Moreover, an implicit numerical solution will generally have small wave deformation when the timestep is not greater than the limiting explicit timestep. Several test runs were completed, and the results show that a timestep of 360 seconds gives acceptable results; that is, water levels were within 0.01 meters and speeds were within 0.01 m/s after five tidal cycles for timesteps of 180, 360, and 720 seconds.

In another preliminary test performed we compared the numerical and analytic solutions for one-dimensional, frictionless, linearized flow in a variable-width channel. The tide range along the length of the channel with a uniformly-varying width, closed at one end and tidally forced at the other is given in terms of Bessel functions (see Lamb, 1932, Paragraph 186 for a similar example) by

 $h = A\cos(\sigma t) [\{ (1 - cY_{O}(kL_{2}))/J_{O}(kL_{2}) \} J_{O}(kx) + cY_{O}(kx)] (5.2)$

where σ is the tide's angular frequency, A is the tide amplitude at the mouth, L₂ is the distance from the origin to the open end of the channel, $k = \sigma/(gH)^{1/2}$, H is the channel depth, and

$$c = J_1(kL_1) / [Y_0(kL_2)J_1(kL_1) - Y_1(kL_1)J_0(kL_2)]$$
(5.3)

where L_1 is the distance from the origin to the closed end of the channel (Fig. 5a). Note that the analytic solution is independent of the actual channel width. The difference between the modeled and the analytic solutions was less than 2 percent at all grid points (Fig. 5b).

5.2 Model Calibration for Mean Tides

The first three steps in model development are generation of the equations, comparison of modeled to analytic solutions, and selection of the timestep. The next two steps are calibration and verification. Calibration involves selecting a set of test data and then matching model output to that data by judiciously adjusting model input parameters. This adjustment is permissible because (a) the finite-difference equations and grid are only an approximation to the actual system, and (b) many of the processes, especially the turbulent exchanges, are poorly understood and are thus represented by simple relationships with adjustable coefficients. Verification involves comparing model output with data for a different time period or another case, but using the model parameters obtained during the calibration step.

The calibration data selected for the Bay-shelf region were the mean tidal water levels and currents as tabulated by the National Ocean Service (NOS, 1985a,b). These mean values are averages over a long series of observations and are available for selected points around the Bay. For verification data, we selected NOS predicted water levels and currents for a specific period of time in June-July, 1980.

During the calibration process, we tested the influence of (primarily) the following on water levels and current speeds: grid cell depths and widths, oceanic boundary tidal range, bottom friction (C_{wb}), and horizontal friction (C_{AH}). For this calibration, the water was assumed to have zero salinity and a temperature of 0°C, so that both the horizontal and the vertical density gradients were zero. The side interfacial friction coefficient, C_{ws} , was set to zero. The vertical turbulent viscosity parameter, C_0 (see eq. 2.20), also contributes to the bottom friction through the bottom boundary condition on the velocity; this parameter's value was kept at 1.0, with the recognition that for model simulations with varying density (hence non-zero Richardson number) the value of C_0 may need refining. Values resulting from the calibration include $C_{wb} = 0.003$ and $C_{AH} = 0.01$.







(b)

Figure 5. (a) Schematic of the variable-width channel, and (b) a comparison of the numerical and analytic solution for the tide range along the channel. The amplitude is the ratio of the tide range at the mouth to the range at any other point. Here $\sigma = 2.9088 \times 10^{-4}$, g = 10 m/s², H = 100 m, A = 0.10 m, L₁ = 150 km, and L₂ = 250 km.

Based on these tests and on the known behavior of analytic solutions of tides in simple basins, we can generalize our observations made during the model calibration step as follows. Increasing the oceanic tide range increases the interior tides and currents proportionately. Increasing water depths between locations causes the tide wave to propagate more rapidly, and so decreases the time lag between phases at these locations. Increasing the bottom friction causes greater loss of energy, and so reduces the currents and tides, with greater reduction in the Bay further from the mouth; bottom friction has a small effect on the phase. On the open shelf, the horizontal friction was much more efficient in damping tides than the bottom friction. If a river's length is extended, the tide range at the river mouth is reduced. If a river's width is decreased at a point, the tide range is lowered upstream of that point and raised downstream. Parenthetically, we should add that the location of the major tide gauging station in the lower Bay, at Hampton Roads, VA, is inside the mouth of a river with a restricted entrance, and is therefore a poor location for representing the tides in the adjacent estuary and shelf.

The tide ranges and phases at several Bay locations resulting from the calibration procedure are shown in Fig. 6a,b. Modeled mean tide ranges are within 0.10 m for most stations, and modeled mean high water occurs within 0.5 hours for most stations. Using NOS tide tables for several days of data, we determined that for comparison purposes the tide phase at Sandy Hook, NJ, was approximately 1.44 hours ahead of that at Hampton Roads, VA, and the phase at Baltimore Harbor, MD, was 10.10 hours behind.

The calibrated results for flood and ebb currents are shown in Fig. 6c. Most current speeds were within 0.05 m/s of the observed mean values. Without calculating the flood and ebb directions, we know from previous work in similar models that they are within 10° or 20° of the observed. For comparison purposes we estimated that the flood at Baltimore Harbor (off Sandy Point, MD) was 8.84 hours after flood at the mouth of the Chesapeake, and the ebb was 8.64 hours later.

Some data for deep-water currents on the shelf are given in Moody et al. (1984). Current measurements at three depths at a location known as station MAB, 80 kilometers due east of the Bay mouth in 28 meters of water show that the major axis of the current ellipse is oriented approximately toward the Bay entrance. This orientation is evidence that the tide wave propagates inward normal to the coastline; a wave propagating along the coast (a Kelvin wave) will have its current ellipse oriented parallel to the coast. A comparison of the currents (Fig. 7) shows that at all depths the MECCA currents have a strength of 0.20 m/s along the major axis, and 0.10 m/s along the minor axis; the orientation is constant. By contrast, the observed currents change in orientation and strength from one



Figure 6. Calibrated values for (a) mean tide range, (b) mean time of high water after high water at Hampton Roads, VA, and (c) calibrated values for mean flood and ebb speeds.





Figure 7. Modeled and observed currents at three depths on the continental shelf. Data are from Moody et al. (1984).

depth to another, and are strongest (0.20 m/s) at mid-depth. The near-surface and near-bottom currents are weaker (0.15 m/s), and the major axis is rotated clockwise with depth. This change of orientation is due to friction and the earth's rotation; possibly MECCA's turbulent parameterization is not sufficient for the open shelf area. Tidal currents on the shelf are relatively weak, however, so the differences are unimportant for our purposes.

5.3 Relationship Between Oceanic and Interior Tides

The last detail to be addressed for the calibration is the relationship of the tide at the oceanic boundary and the tide inside the Bay. A table of values was generated by running MECCA with several values of tide range, and examining the tide at Hampton Roads, Va. A graph of high and low waters is shown in Fig. 8. When the data for high and low waters is combined, an approximate relationship for the ranges is found to be

$$R_{\rm HR} = 0.9 R_0^{2/3} \tag{5.4}$$

where R_{HR} is the range at Hampton Roads (meters) and R_0 is the range at the oceanic boundary (meters). The cases tested were limited oceanic ranges not greater than 1.6 meters. The time of high water at Hampton Roads was roughly 2.2 hours after high water at the oceanic boundary.



Figure 8. Modeled tide ranges at the oceanic open boundary vs. modeled tide ranges at Hampton Roads.

5.4 Model Verification for Tides and Tidal Currents

The calibrated model was verified by comparing the water levels and currents from the circulation model with those in the NOS tables for the period 6 - 10 July, 1980. Winds were set to zero, river flowrates were set to their historical means (Table 1), and a 10-day spinup (June 26 - July 5) was used. Another possible comparison would be between modeled tides and water levels obtained by direct observation; this comparison was not made because observed tides contain not only the astonomical tide but also wind and atmospheric pressure-induced tides and other shelf influences. The chosen method of comparison isolates the astronomical tide as the single factor in water level changes.

The boundary conditions for the period of interest were generated in several steps. First, the tide times and heights for the period 26 June - 10 July, 1980, were taken from the table for Hampton Roads, and the mean water level over that time was computed by simple averaging. The mean level was then subtracted from the individual heights to get a series of normalized highs and lows. A third series of highs and lows for the oceanic boundary was then created by calculating the deep-water high or low value corresponding to the normalized value by interpolation with the data plotted in Fig. 8; the time of the oceanic high or low was set as 2.2 hours earlier than the tabulated value.

Figure 9 shows a comparison of the modeled and normalized water levels and currents for Hampton Roads, VA. Water levels (Fig. 9a) for any time between the times of the normalized values were generated by an interpolation function analogous to eq. 4.1. The most obvious conclusion we can make by looking at the curves is that the phases of the tides are very close, with MECCA water levels being slightly ahead (by about 0.5 meters) for the higher of the two daily highs, but quite accurate for the lower high.

Table 1. Historical mean flowrates used in the riverflow experiment. The York (Mattaponi and Pamunkey) River is not modeled in the gridmesh. Data are from U.S. Geological Survey and include the 1985 water year.

River	Long-term Mean (m ³ /s)	Mean Used in MECCA (m ³ /s)
Susquehanna	1212	1210
Potomac	327	320
Rappahannock	47	50
York	45	-
James	214	210

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Figure 9. Comparison of model output (dashed lines) and National Ocean Service predictions (solid lines), interpolated from tabulated values, for 6-10 July, 1980, for (a) tides at Hampton Roads, VA, and (b) currents at the entrance to Chesapeake Bay.

Modeled and NOS currents for the mouth of Chesapeake Bay for 6 - 10 July, 1980, are shown in Fig. 9b. NOS values are generated by an interpolation function analogous to eq. 4.1, but using current speeds for the NOS station at the Chesapeake Bay Mouth. Since the NOS currents are based on measurements from a near-surface current meter, the MECCA current used in the comparison is the one at the surface. The phase of the modeled currents is close to, but slightly behind, that of the table. Throughout most of the 5-day simulation, the MECCA ebb (positive) currents are smaller than the NOS ebbs by about 0.15 m/s, while the flood (negative) currents are about 0.20 m/s larger.

Modeled and NOS tides at Baltimore are shown in Fig. 10a. Here the difference between the two is more pronounced, with the modeled low waters generally 0.10 to 0.20 meters above the NOS low waters, and the modeled high waters sometimes higher, sometimes lower than the NOS levels. Since the diurnal inequality (the fact that one high water is significantly higher than the other) is more pronounced at Baltimore than at Hampton Roads, it seems that some tidal components are amplified as they travel up the Bay more than other components. The fact that the model does not reproduce this phenomenon may be due to the use of the series of high and low waters and the interpolation function to supply boundary tides, rather than the use of the sum of a series of tidal constituents (see Section 7.2).

Modeled and NOS currents at Sandy Point, MD, are shown in Fig. 10b. As with the water level variation at Baltimore, the modeled currents here do not accurately capture the diurnal inequality displayed in the NOS prediction. The flood and ebb during the lesser of the two tidal cycles is adequately reproduced (to within 0.10 m/s), but the magnitudes of the modeled currents are smaller than the corresponding predicted currents during the stronger of the two cycles by 0.10 to 0.30 m/s.

These results show that MECCA reproduces the NOS predicted tides and currents in the lower Bay with an acceptable level of accuracy, but that further work is necessary to have sufficiently accurate tides and currents in the upper Bay.

6. MODEL APPLICATIONS

Now that we have a model which has been verified for tidal water levels and currents (at least in the lower Bay), we can use it to test hypotheses or to answer questions about physical processes occurring in the Bay. A few of many possible applications are now discussed; conclusions should be regarded as tentative until verification is complete.



Figure 10. Comparison of model output (dashed lines) and National Ocean Service predictions (solid lines), interpolated from tabulated values, for 6-10 July, 1980, for (a) tides at Baltimore Harbor, MD, and (b) currents just offshore of Sandy Point, MD.

6.1 Natural Period of Chesapeake Bay

A numerical experiment was performed to determine the natural period of Chesapeake Bay. The natural period is important for intrepreting the Bay's response to environmental forcing at all frequencies. The experiment involved initializing the Bay at rest with a water elevation above mean sea level that was zero on the shelf and at the Bay mouth, and increased linearly to 1.0 meter below m.s.l. at the head of the Bay. The subsequent time history of the water levels and currents was then modeled. The water level histories at several stations in the upper Bay are shown in Fig. 11. The natural period at these stations was determined by the time interval between successive crossings of the zero-elevation axis. The mean period was found to be 1.70 days.

There is an analogue to this hypothetical situation in nature, and it occurs after the passage of a hurricane. Upon the approach of a cyclonic storm from the southeast, winds from the north drive water from the Bay, causing low waters in the upper portions of the Bay. After the storm passes and the winds have diminished, the water level will rebound toward its natural configuration. Records of water levels at the Baltimore tide gage during hurricanes (Harris, 1963: Neumann and Pierson, 1966) show a time history much like that produced by the model (Fig. 12). Wang (1979) has also estimated the natural period of the Bay from the time required for the tide to propagate up the Bay. The results are shown in Table 2, and are fairly consistent, ranging from 1.14 days for the 1936 storm to Wang's estimate of 2.07 days.



Figure 11. Water level history at several model grid cells near Baltimore, MD, from the free oscillation test.



Figure 12. Storm tracks and water level histories at several East Coast stations including Baltimore, MD, during the passage of the hurricanes of 1936 and 1944 (figure from Neumann and Pierson [1966] used by permission of copyright owner).

Source	Period (days)
Model	1.70
Hurricane of 1936	1.96
Hurricane of 1944	1.14
Wang (1979)	2.07

Table 2. Estimates of the natural period of Chesapeake Bay.

6.2 Effect of Suddenly Increased Riverflow

Another simple application of MECCA involves studying the rate of propagation of a flood wave down the Bay. In this test, the flowrates of the major rivers were set to their historical means (Table 1). After allowing the modeled variables to come to a repeatability condition with tidal and river input, we doubled the Susquehanna River flowrate over the course of one day. Figure 13 shows the Susquehanna flowrate and the net Susquehanna flowrate at the mouth during the next several days. The net flowrate at the mouth was defined as the mean over a tidal cycle, and the net Susquehanna flowrate is the mean minus the sum of the flowrates of the other rivers.

The results show that the increase in the hydrograph, the flood, was felt at the Bay mouth one to three tidal cycles later, showing that the increased flow propagates downbay at the gravity wave speed. Naturally, constituents of the water such as sediments move much more slowly, at a speed approximately equal to the flowrate divided by the cross-sectional area.

6.3 Larval Drift

An obvious use of a numerical circulation model is the simulation of transport of Lagrangian drifters under the action of the currents to assess the fate of water-borne species such as pollutants or fish larvae. Lagrangian particles are assumed to move at the same velocity as the MECCA currents, and their positions are updated by numerical time-integration of these currents. Drift experiments and studies of the motion of blue crab larvae during the fall of 1980 were described by Johnson et al., (1986b).



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Figure 13. Flowrate at the mouth of Chesapeake Bay and flowrate in the Susquehanna River showing the time an increase in flow in the upper Bay can be observed at the mouth.

Temperature Spinup Experiment 6.4

A number of comparisons of modeled and observed surface water temperatures were described in Johnson et al. (1986a). Specifically, temperatures for the period 9 - 23 April, 1982 were simulated and analyzed. Differences between the modeled output and the data were thought to be caused by the use of an unrepresentative value for d_{10} (6 meters was used, whereas a value from 0.3 to 3.0 meters is probably more realistic) , the use of 12-hourly LFM atmospheric temperatures rather than hourly observed temperatures, or the use of an insufficiently long spinup time. Experiments were then performed on the initialization and spinup of the temperatures.

The spinup experiment began with the Bay at an initialized temperature, and with mean tides and currents at a repeatability condition. A daily heating cycle, representative of April, was used each day, and the winds applied were of constant strength (7.0 m/s) but varied in direction uniformly over a 2-day period. Air temperatures varied from 20°C in the day to 10°C at night. Water temperatures at two locations in the region, at four depths (0, 33, 66, and 100 percent of the depth), for a period of 120 days are shown in Fig. 14.



Figure 14. Water temperatures at four depths for two location in the Bay/shelf region: (a) mid-Bay, and (b) mid-shelf. The mid -Bay station has a depth of 8 meters, and the mid-shelf station has a depth of 38 meters.

The plots show that the surface temperature comes to a condition close to repeatability relatively quickly, within 5 to 10 days. This is undoubtedly due to the wind mixing there, and the absorption of atmospheric heating by the full depth of the upper layer. The temperature changes after that time are small; in the Bay, the temperature range over the two-day (wind-related) cycle is about 1.5° C, and at the mid-shelf location it is 1.0° C. In addition, the bottom temperature changes very little over the 4-month period, because it rapidly comes to equilibrium with the sea bed temperature (cf. eq. 2.26).

The temperatures below the surface and above the bottom, within the core of the flow, show a rapid change over the first 50 days, and a slower rate of change thereafter. A notable exception is the near-surface temperatures at the mid-Bay station; at 90 days, there is a large ($2^{\circ}C$) drop over a few days. This change appears to be an overturning event, as the below-surface warms up to a point when it becomes less dense than the waters above and below it. Apparently, the conditions chosen for the experiment, if sustained long enough, would lead to temperatures nearly uniform in the upper half of the water column.

7. SUMMARY AND FUTURE PLANS

A three-dimensional numerical model has been developed and applied to an estuarine-shelf hydrodynamical system. The model was designed to simulate tidal, density-driven, river discharge, and wind-driven currents. The model's tidal currents have been calibrated with NOS observed mean tidal currents, and the process of verifying the currents is well along. Much work remains, however, before estimates of confidence can be placed on the computed salinities and temperatures.

Experience with model testing and application has pointed to several topics which ought to be developed further. These include the use of variable grid sizes, the use of tidal series for boundary condition water levels, vectorization of the computer code, and extension of the model to other basins. These topics are described in greater detail below.

7.1 Grid Stretching

Variable grid sizes can be obtained by stretching the grid in one or more directions with a piecewise coordinate transformation such as (Schmalz, 1985)

 $\mathbf{x} = \mathbf{a} + \mathbf{b}\mathbf{r}^{\mathbf{C}} \tag{7.1}$

where x is the coordinate in real space, and r is the coordinate

in the uniform (model) domain. Using the transformation (7.1), we get

$$f_{X} = [cbx^{c-1}]^{-1}f_{r}$$
(7.2)

The general strategy is to substitute the expression (7.2) for the x-derivative wherever it appears in the model equations (the y and q coordinates can be treated similarly). Grid stretching by coordinate transformation is preferable to the use of unequally spaced points because the latter will reduce some second-order finite-difference approximations to first-order (Roach, 1972).

Further refinements of this general technique include the use of sheared coordinates (Jelesnianski, 1976), or boundary-fitted coordinates (Spaulding, 1984). These coordinate systems incorporate coastline geometry more accurately, and so can give more accurate results.

7.2 Use of Tidal Series for Boundary Conditions

At any location, the tidal water level variation can be expressed as

$$\mathbf{h} = \mathbf{h}_{0} + \sum f_{n}(\mathbf{t}) \mathbf{A}_{n} \cos(\omega_{n} \mathbf{t} + [\mathbf{V}_{0} + \mathbf{u}]_{n} - \mathbf{k}_{n})$$
(7.3)

where h_0 is a reference level, f_n is the lunar node function for tidal constituent n, A the constituent amplitude, ω the frequency, $V_0 + u$ the equilibrium argument, and k the epoch. This approach was used successfully by Hess (1976) and Hess and White (1974) for Narragansett Bay, RI, and can be useful here to provide more accurate tidal boundary conditions.

7.3 Vectorization of the Code

Investigations are presently underway to determine what steps are necessary to run the model on NOAA's Cyber 205 at the National Meteorological Center. Extensive rewriting of the computer code is anticipated to conform to the machine's compiler, but the result should be a version which runs much faster than the VAX-based code presently used.

7.4 Running MECCA for Other Areas

Preliminary discussions about using the model for other estuaries or coastal areas are under way. Two geographic regions are being considered. The Pamlico/Albemarle Sound in North Carolina could be included as a possible extension of the Chesapeake Bay/shelf model gridmesh; this area is important as a blue crab fishery and may interact with Chesapeake Bay, although the nearness of the Gulf Stream could pose difficult problems. The San Francisco Bay and adjacent shelf is another area of interest; coastal currents may be modeled to study the motion and fate of marine species such as dungeness crab.

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APPENDIX A. Derivation of the Set of Model Equations

1. The Basic Equation in a Continuum (set A)

The basic equations in a Boussinesq fluid continuum (Phillips, 1980) are the horizontal momentum equations with Reynolds stresses on a rotating Cartesian right-handed frame of reference

$$u_{,t} + (uu)_{,x} + (uv)_{,y} + (uw)_{,z} = -\alpha_{0}p_{,x} + fv$$

+ $(2A_{h}u_{,x})_{,x} + (A_{h}[v_{,x} + u_{,y}])_{,y} + (A_{v}u_{,z})_{,z}$ (A.1)

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and

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$$v_{,t} + (vu)_{,x} + (vv)_{,y} + (vw)_{,z} = -\alpha_{0}p_{,y} - fu + (A_{h}[u_{,y} + v_{,x}])_{,x} + (2A_{h}v_{,y})_{,y} + (A_{v}v_{,z})_{,z}$$
(A.2)

where u, v, and w are the components on fluid velocity in the x, y, and z direction, respectively. The fluid pressure is p, α_0 is a reference specific volume, f the Coriolis acceleration, A_h the horizontal diffusivity, and A_v the vertical diffusivity. The hydrostatic approximation to the vertical momentum equation is

 $\mathbf{p}_{,\mathbf{Z}} = -\rho \mathbf{g} \tag{A.3}$

where ρ is the water density and g the gravitational acceleration. An approximation to the continuity, or mass conservation, equation which neglects sound waves is

$$u_{,x} + v_{,y} + w_{,z} = 0$$
 (A.4)

An equation of state for sea water is

$$\rho = \rho_0 (1 + C_S S + C_T T) \tag{A.5}$$

where ρ_0 is a reference density, S the salinity (parts per thousand), and T the temperature (^oC). Conservation of dissolved salt is

$$S_{t} + (uS)_{x} + (vS)_{y} + (wS)_{z} - (D_{h}S_{x})_{x} - (D_{h}S_{y})_{y}$$

- $(D_{v}S_{z})_{z} = 0$ (A.6)

where D_h and D_v are the horizontal and vertical diffusivities. The heat conservation equation is

$$T_{t} + (uT)_{x} + (vT)_{y} + (wT)_{z} - (D_{h}T_{x})_{x} - (D_{h}T_{y})_{y}$$

- $(D_{v}T_{z})_{z} = R$ (A.7)

where R is an internal heating term. Nominal values for the parameters are g = 9.81 m²/s, α_0 = 1.0 l/kg, C_S = 0.0008 ppt⁻¹, C_T = -0.00007 °C⁻¹, f = 2\Omegasin(latitude), and Ω = 0.0000729 s⁻¹. The eddy coefficients A_h, A_v, D_h, and D_v vary over space and in time.

2. Laterally-averaged Equations (set B)

The set of equations is now averaged laterally (over the direction normal to the flow in the horizontal plane). The limits of integration define the width of the flow, and specifically the width of an imbedded river. We define B_x and B_y as the width of flow in the x- and y-directions, respectively. Following Blumberg (1975, 1978) and Wang and Kravitz (1980), and assuming that B doesn't vary with z or t, we get

$$u_{,t} + B_{x}^{-1}(B_{x}uu)_{,x} + (uv)_{,y} + (uw)_{,z} = -\alpha_{o}p_{,x} + fv$$

+ $B_{x}^{-1}(2B_{x}A_{h}u_{,x})_{,x} + (A_{h}[v_{,x} + u_{,y}])_{,y}$
+ $(A_{v}u_{,z})_{,z} - C_{ws}B_{x}^{-1}u|u|$ (A.8)

and

$$v_{,t} + (vu)_{,x} + B_{y}^{-1}(B_{y}vv)_{,y} + (vw)_{,z} = -\alpha_{o}p_{,y} - fu + (A_{h}[u, y + v_{,x}])_{,x} + B_{y}^{-1}(2B_{y}A_{h}v_{,y})_{,y} + (A_{v}v_{,z})_{,z} - C_{ws}B_{y}^{-1}v|v|$$
(A.9)

Continuity is given by

$$B_{x}^{-1}(B_{x}u)_{,x} + B_{y}^{-1}(B_{y}v)_{,y} + w_{,z} = 0$$
 (A.10)

If a channel is present, the cross-flow derivatives in the diffusion terms (the third term on the right side of A.8, the fourth in A.9) are set to zero; if a channel is not present, $B_X = B_V = 1.0$ and $C_{WS} = 0$.

Conservation of salinity is expressed as

$$S_{,t} + B_{x}^{-1}(B_{x}uS - B_{x}D_{h}S_{,x})_{,x} + B_{y}^{-1}(B_{y}vS - B_{y}D_{h}S_{,y})_{,y} + (wS - D_{v}S_{,z})_{,z} = 0$$
(A.11)

The conservation of heat is given by

$$T_{,t} + B_{x}^{-1}(B_{x}uT - B_{x}D_{h}T_{,x}), x + B_{y}^{-1}(B_{y}vT - B_{y}D_{h}T_{,y}), y$$

+ (wT - D_vT_{,z}), z = R (A.12)

The hydrostatic approximation to the vertical momentum equation and the equation of state are unchanged.

3. The Equations in the Dimensionless Vertical Variable (set C)

The equations are now transformed into a new coordinate system, with the vertical coordinate

$$q = (h - z)/(h + d) = (h - z)/H$$
 (A.13)

where h is the water surface elevation above mean sea level, d the mean sea level depth, and H the total local depth. The transformation is accomplished by substituting an expression in the x,y,z-system, [], the expressions in the x,y,q-system, () as follows

$$[]_{,z} = H^{-1}()_{,q}$$
(A.14)

$$[]_{,x} = ()_{,x} - H^{-1}(h_{,x} + qH_{,x})()_{,q}$$
(A.15)

$$[], y = (), y - H^{-1}(h, y + qH, y)(), q$$
(A.16)

$$[],t = (),t - H^{-1}(1 + q)h,t(),q$$
(A.17)

Note that only the positions of the variables is transformed; the directions of u and v are unchanged. The momentum equations in the x, y, q-system are

$$(Hu), t + B_{x}^{-1}(HB_{x}uu), x + (Huv), y + H^{-1}(u\tilde{w}), q$$

= -gh, x - $\alpha_{0}p_{a,x} - G_{x} + fv + B_{x}^{-1}(2HB_{x}A_{h}u, x), x$
+ $(A_{h}H[v, x + u, y]), y + H^{-2}(A_{v}u, q), q - kHB_{x}^{-1}u|u|$ (A.18)

and

$$(Hv)_{t} + (Hvu)_{x} + B_{y}^{-1}(HB_{y}vv)_{y} + H^{-1}(v\tilde{w})_{q}$$

= -gh_{y} - \alpha_{o}p_{a,y} - G_{y} - fu + (A_{h}H[u_{y} + v_{x}])_{x}
+ B_{y}^{-1}(2HB_{y}A_{h}v_{y})_{y} + H^{-2}(A_{v}v_{q})_{q} - kHB_{y}^{-1}v|v| (A.19)

where

$$\tilde{w} = Hdq/dt = w - (1 + q)h_{,t} - u(h_{,x} + qH_{,x}) - v(h_{,y} + qH_{,y})$$
(A.20)

and

$$G_{\mathbf{X}} = g[H_{\int}^{O} (\rho - \rho_{O})dq]_{,\mathbf{X}} + g(h_{,\mathbf{X}} + qH_{,\mathbf{X}})(\rho - \rho_{O})$$
(A.21)

$$G_{y} = g[H_{q}^{(\rho)}(\rho - \rho_{0})dq]_{,y} + g(h_{,y} + qH_{,y})(\rho - \rho_{0})$$
(A.22)

The hydrostatic approximation to the vertical momentum equation is now

$$\mathbf{p},\mathbf{q} = -\rho \mathbf{g} \mathbf{H} \tag{A.23}$$

Continuity, or mass conservation, in the absence of sound waves is

$$h_{t} + B_{x}^{-1}(HB_{x}u)_{x} + B_{y}^{-1}(HB_{y}v)_{y} + H\tilde{w}_{q} = 0$$
 (A.24)

Conservation of salt is

$$(HS)_{t} + B_{x}^{-1}(B_{x}H[uS - D_{h}S_{,x}])_{,x} + B_{y}^{-1}(B_{y}H[vS - D_{h}S_{,y}])_{,y}$$

+ ($\tilde{w}S - H^{-1}D_{v}S_{,q}$), q = 0 (A.25)

The thermodynamic conservation equation is

$$(HT)_{t} + B_{x}^{-1}(B_{x}H[uT - D_{h}T,x])_{x} + B_{y}^{-1}(B_{y}H[vT - D_{h}T,y])_{y}$$

+ $(\tilde{w}T - H^{-1}D_{v}T,q)_{q} = HR$ (A.26)

4. The External Mode Velocity and Mass Equations

The external, or barotropic, mode of the horizontal velocity is identified here as the integrated value of the flow over the total depth. If we define the vertically-integrated velocities as

$$\underline{u} = \int_{-1}^{0} u dq \quad and \quad \underline{v} = \int_{-1}^{0} v dq \quad (A.27)$$

then the vertically-integrated (external mode) form of the velocity equations is

$$(H\underline{u})_{,t} + B_{x}^{-1}(HB_{x}H\Theta_{uu}\underline{u}\underline{u})_{,x} + (H\Theta_{uv}\underline{u}\underline{v})_{,y} = -gHh_{,x} - \alpha_{o}Hp_{a,x}$$
$$- G^{*}_{x} + f\underline{v} + B_{x}^{-1}(2A_{h}HB_{x}\underline{u}_{,x})_{,x} + (A_{h}H[\underline{v}_{,x} + \underline{u}_{,y}])_{,y}$$
$$+ T_{sx} - T_{bx} - B_{x}^{-1}C_{ws}H\Theta_{su}\underline{u}|\underline{u}| \qquad (A.28)$$

and

$$(\underline{Hv})_{,t} + (\underline{H\Theta}_{uv}\underline{uv})_{,x} + \underline{B_{y}}^{-1}(\underline{HB_{y}\Theta_{vv}\underline{vv}})_{,y} = -\underline{gHh}_{,y} - \alpha_{0}\underline{Hp}_{a,y}$$
$$- \underline{G^{*}}_{y} - \underline{fu}_{,y} + \underline{B_{y}}^{-1}(\underline{2A_{h}HB_{y}\underline{v}},\underline{y})_{,y} + (\underline{A_{h}H[\underline{v},\underline{x} + \underline{u},\underline{y}]})_{,x}$$
$$+ \underline{T}_{sy} - \underline{T}_{by} - \underline{B_{y}}^{-1}\underline{C_{ws}H\Theta_{sv}\underline{v}}|\underline{v}| \qquad (A.29)$$

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and the external-mode continuity equation is

$$h_{t} + B_{x}^{-1}(B_{x}H\underline{u})_{x} + B_{y}^{-1}(B_{y}H\underline{v})_{y} = 0$$
 (A.30)

where

$$G_{X}^{*} = \int_{-1}^{0} G_{X} dq$$
 and $G_{Y}^{*} = \int_{-1}^{0} G_{Y} dq$ (A.31)

and

$$\Theta_{uu} = \int_{-1}^{0} (uu/\underline{uu}) dq, \quad \Theta_{uv} = \int_{-1}^{0} (uv/\underline{uv}) dq, \quad \Theta_{vv} = \int_{-1}^{0} (vv/\underline{vv}) dq$$

$$(A.32)$$

$$\Theta_{uv} = \int_{-1}^{0} (u/\underline{uv}) dq, \quad \Theta_{vv} = \int_{-1}^{0} (vv/\underline{vv}) dq$$

$$(A.32)$$

$$\Theta_{\mathrm{SU}} = \int_{-1}^{1} (u/\underline{u}) |(u/\underline{u})| dq \qquad \Theta_{\mathrm{SV}} = \int_{-1}^{1} (v/\underline{v}) |(v/\underline{v})| dq \quad (A.33)$$

5. The Internal Mode Velocity Equations

We can define the internal mode velocities as

$$u' = u - \underline{u}$$
 and $v' = v - \underline{v}$ (A.34)

and then derive the internal mode velocity equations by subtracting the equations for the external mode velocity from the equations for the total velocity. This gives the internal-mode velocity equations

$$(Hu')_{t} + B_{x}^{-1}(B_{x}H[uu - \Theta_{uu}uu])_{x} + (H[uv - \Theta_{uv}uv])_{y}$$

+ $(\tilde{w}u')_{q} = G^{*}_{x} - G_{x} + fv' + B_{x}^{-1}(2A_{h}HB_{x}u',x)_{x}$
+ $(A_{h}H[v',x + u',y])_{y} + H^{-1}(A_{v}u',q)_{q} - T_{sx} + T_{bx}$
- $C_{ws}HB_{x}^{-1}(u|u| - \Theta_{su}u|u|)$ (A.35)

and

$$(Hv')_{t} + (H[uv - \Theta_{uv}uv])_{x} + B_{y}^{-1}(B_{y}H[vv - \Theta_{vv}vv])_{y} + (\tilde{w}v')_{q} = G^{*}_{y} - G_{y} - fu' + (A_{h}H[v', x + u', y])_{x} + B_{y}^{-1}(2A_{h}HB_{y}v', y)_{y} + H^{-1}(A_{v}v', q)_{q} - T_{sy} + T_{by} - C_{ws}HB_{y}^{-1}(v|v| - \Theta_{sv}v|v|)$$
(A.36)

The internal-mode continuity equation is

$$B_{x}^{-1}(B_{x}u)_{,x} + B_{y}^{-1}(B_{y}v)_{,y} + H^{-1}(\tilde{w})_{,q} = 0$$
 (A.37)

APPENDIX B. Water Surface Heat Flux Terms.

Following Parkinson and Washington (1979), we write the total heat downward flux at the air-water interface as

 $Q_{\rm T} = Q_1 + Q_2 \tag{B.1}$

where Q_1 is solar shortwave component which penetrates the water column below the surface, and Q_2 is the flux that is absorbed right at the surface in the "skin layer" (approximately the top millimeter). These components are treated separately.

1. The Solar Shortwave Component

The solar shortwave component is defined as

$$Q_1 = Q_{SS}(1 - A) f_C(n)$$
 (B.2)

where Q_{SS} is the flux at the sea surface under cloudless conditions, A is the albedo of the sea surface, f_C is a cloudiness function, and n is the fraction of sky covered by clouds. Here

$$Q_{SS} = S_C \cos^2(Z) / [0.10 + 1.085 \cos(Z) + 10^{-5} \{\cos(Z) + 2.7\} e_a]$$
(B.3)

where $S_{\rm C}$ is the solar constant (1353 w/m^2), $e_{\rm a}$ the atmospheric vapor pressure at the surface, and Z the solar zenith angle defined by

$$\cos(Z) = \sin(\emptyset)\sin(D) + \cos(\emptyset)\cos(D)\cos(H_a)$$
(B.4)

and

$$\emptyset$$
 = geographic latitude, (B.5)

 $D = declination = 23.44^{\circ} cos([172 - day in year]\pi/180)$ (B.6)

$$H_a = hour angle = (12 - solar hour)\pi/12$$
(B.7)

The solar hour is the hour of the day measured from midnight. The vapor pressure is found by

$$e_a = 611 \times 10^{(7.5[T - 273.16]/[T - 35.86])}$$
 (B.8)

where T is the atmospheric boundary layer temperature (K).

The cloudiness function used here is

$$f_{C}(n) = 1 - n$$
 (B.9)

2. The Flux at the Skin Layer

The surface flux skin layer is the sum of several terms

$$Q_2 = Q_L + Q_B + Q_e + Q_s$$
 (B.10)

where Q_L is the longwave radiation from the atmosphere to the sea, Q_B is the negative of the black body radiation from the water surface, Q_e the net downward evaporative, or latent, heat flux, and Q_s is the net downward sensible heat flux. Taking the terms one at a time, we have

$$Q_{\rm L} = C_{\rm sb} T_{\rm a}^{4} (1 - 0.26 \exp[-0.000777\{273 - T_{\rm a}\}^{2}])$$
(B.11)

where C_{sb} is the Stefan-Boltzman constant (5.670 x 10^{-8} w/m²/K⁴) and T_a is the representative atmospheric temperature (K).

The downward black body radiation from the sea is

$$Q_{\rm B} = -0.97 C_{\rm sb} T_{\rm w}^{4} \tag{B.12}$$

where 0.97 is the surface emissivity and T_W is the temperature at the sea surface.

The net downward evaporative heat flux is

$$Q_{e} = - \rho_{a} C_{e} V_{10} (q_{a} - q_{s}) L_{v}$$
(B.12)

where ρ_a is air density, C_e the transfer coefficient (0.00175), V₁₀ the wind speed at 10 meters, q_a the specific humidity of the air at 10 meters, q_s the specific humidity at the surface (where saturation is assumed), and L_v is the latent heat of vaporization (2.5 x 10⁶ J/kg). Specific humidity is defined as

$$q = 0.622e/[p - (1 - 0.622)e]$$
(B.13)

and relative humidity as

$$Rh = q/q_{\rm S} \tag{B.14}$$

where 0.622 is the ratio of the molecular weights of dry air and water vapor.

The sensible heat flux is approximated by the bulk formula

$$Q_{s} = \rho_{a}c_{p}c_{f}V_{10}(T_{a} - T_{w})$$
(B.15)

where c_p is the heat capacity of dry air (1004 J/kg/K), and C_f the transfer coefficient (0.00175).