FACTORS INFLUENCING EQUILIBRIUM
OF A MODEL SAND BEACH

by

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ABSTRACT

Characteristics of a two-dimensional model beach subjected to wave action and with initial slope 1:60 were investigated. The hypothesis that the beach profile reaches a repeatable equilibrium form was tested. It was found that the entire profile did not have a repeatable equilibrium shape, possibly due to minor uncontrollable water level variations. Certain features of the profile were found to exhibit better qualities than others. The maximum height of the beach crest above stillwater was determined to have a repeatable and stable shape. For other test data with initial slopes 1:20 and 1:30, the maximum height of the beach crest was found to correlate well with the deepwater wavelength. The initial slope was determined to have a significant effect on the determined relations.

PREFACE

Research described in this report was conducted as part of the research program in the Coastal, Hydraulic and Ocean Engineering Group at Texas A&M University.

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INTRODUCTION

An understanding of the relationships which govern beach deformation is essential for coastal engineers and others concerned with coastal protection. Beach processes occurring in nature are, however, extremely complex. The large number of uncontrolled variables and their interactions are difficult to resolve. In addition, lack of adequate measuring techniques and of instrumentation make the determination of fundamental relationships in the course of field studies difficult. Consequently, such relationships are most frequently studied by means of a small-scale hydraulic model. Ideally, the mechanism of sediment transport is reproduced to scale in the model. However, model results must be interpreted with caution because knowledge of how the sediment transport mechanism is scaled is not complete. These problems in interpretation of small-scale model results will be discussed.

Coastal deformations can be classified into two general groups: long-term and short-term. Long-term changes are those changes in coastline which occur over hundreds of years and result in a general prograding or recession of the shoreline. Of more interest to the coastal engineer is the short-term change which is associated with the variable wave climate and resulting sediment motion. Wave-induced sediment motion can be divided into two components: motion along shore and motion normal to the coast. It is with the latter, or onshore-offshore motion, with which this report is concerned.

As waves progress onto a beach, sediment motion occurs as a function of the wave characteristics. The resulting changes in the beach have a feedback control on the incident waves. For example, changes in depths caused by breaking waves result in changes in locations where the waves break. Intuitively, it would seem that when a given beach is subject to waves of constant characteristics for a sufficient length of time, an equilibrium state will develop. In nature, variable meteorological conditions and resulting variable wave conditions probably seldom allow an equilibrium to be attained. In the hydraulic model, where one has control over wave parameters, such as wave height and wave period, and beach parameters, such as grain size and size distribution, the concept of the equilibrium profile can be studied more readily. Thus, in nature the equilibrium profile needs to be defined in terms of statistical averages. However, in the laboratory the equilibrium profile is defined as a stable configuration on which the oscillatory motion of a given sediment particle is about a mean position, the sorting action of waves presumably having reached an equilibrium. The net transport across any section parallel to the beach is zero.

REVIEW OF THE LITERATURE

Early studies of wave action on model sand beaches include work by Meyer (1936) and Waters (1939). These studies did not consider the effects of tides, wind, or currents. In these two-dimensional models it was recognized that two general types of equilibrium profiles developed, designated "ordinary" and "storm". These designations refer to the fact that in nature, steep, short-period waves are associated with storms and less steep, swell waves are characteristic of mild summer conditions. The "ordinary" or "summer" profile is typically a smooth profile with accretion of material at the shoreline. The "storm" or "winter" profile has a characteristic offshore bar and is the result of erosion at the shoreline. The type of profile generated was found to be most dependent on the deepwater wave steepness, H_o/L_o. When steepness values were less than 0.025 the "ordinary" profile was found to occur, whereas for steepness values greater than this, the "storm" profile was observed. The "ordinary" profile is also referred to in the literature as the "normal" profile. Figure 1 shows a schematic diagram of these two types of profiles indicating the differences between them. Neither of the two early studies considered the effect of grain size or absolute wave height. Johnson (1949) summarized the results and pointed out that no simple relation exists between H_o/L_o and a beach slope to adequately define an equilibrium profile.

Rector (1954) studied equilibrium profiles of model beaches subjected to constant wave action. He examined sands of four different median diameters, and the relationship between time factor, wave and sand characteristics, and equilibrium profiles. It

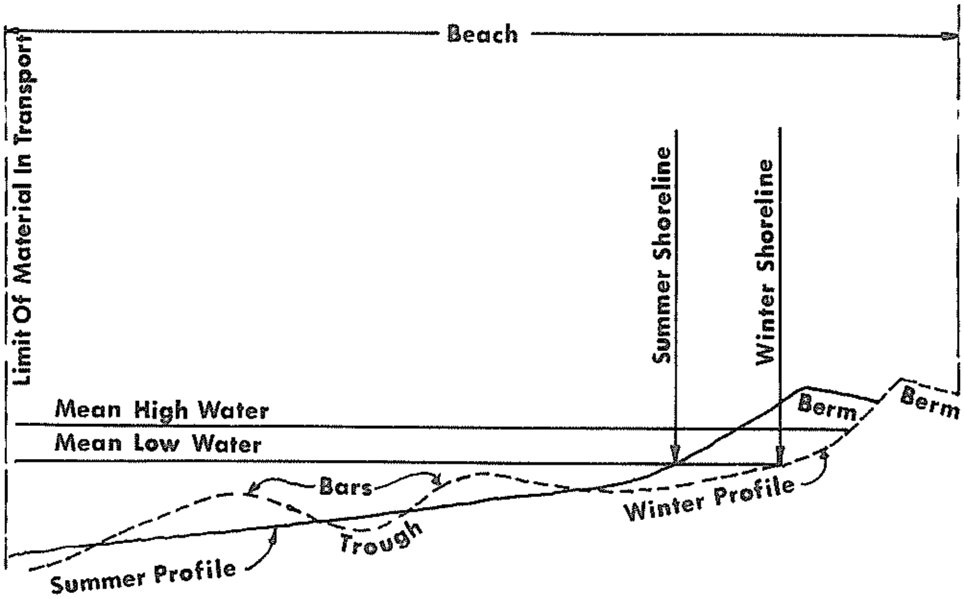


Figure 1. Beach Profile Showing Seasonal Distribution of Sand. After Bascom (1953).

was determined that the initial slope of the beach had little effect on the final profile shape. He also noted a sorting effect with the larger material showing a tendency to move shoreward. Empirical relationships for the equilibrium profile in the laboratory were given in terms of wave and sand characteristics.

Watts (1954) studied the effects on model beaches of varying water depth and wave period. He found that the introduction of tidal action into the model changed the foreshore and offshore slopes very little. It did inhibit the formation of troughs and bars and resulted in higher berms than constant depth tests. The variable period tests were found to greatly reduce offshore bar and trough formations.

Wiegel, Patrick, and Kimberley (1954) found in observing natural beaches that the critical value of wave steepness which separated storm and normal profiles in the wave tank was not applicable to prototype beaches. They found only storm profiles on a California beach during an observation period, even though the wave steepness value never exceeded 0.008. Saville (1957), using large-scale tests, confirmed these conclusions. He observed foreshore erosion in large-scale tests for H_0/L_0 values as low as 0.0064. The importance of absolute wave height was shown by these tests. In comparing small-scale tests with his large-scale tests, Saville found that the calibration of the small-scale model to the prototype by distortion of scales is a trial-and-error process.

Iwagaki and Sawaragi (1959) compared natural profiles with model profiles and found reasonable agreement between the two, landward of the critical point. They defined the critical point as the point on the profile seaward of which the profile is unaffected by the wave action. This is not the same as the depth of incipient motion which is seaward of their critical point.

Earattupuzha (1974) studied stable points on model beaches. Stable points are points where the developing profile remains unchanged. He related these points to the wave and beach parameters. For a storm profile there are two stable points. The first stable point is related to the plunge point of the breaking wave and the second stable point is the offshore limit of erosion on the original profile. On a normal profile no stable point exists in the strict sense. The point where the modified and original profiles intersect is moved continuously seaward as the profile accretes at the shoreline. There is a point, however, about which the modified profile appears to swing. Earattupuzha also classified beaches using the relationship between depth of lower threshold and the depth at which a wave can be approximated by a solitary wave. The depth of lower threshold is the depth at which fluid velocities are no longer strong enough to maintain sediment motion.

Iwagaki and Noda (1962) showed in a study of scale effects in two-dimensional model beaches that the profile type is dependent on wave height even when H_0/L_0 is held fixed. In addition, grain size is important. Figure 2 shows the results of their experiment work using different sand sizes and wave parameters. D_{50} is the median grain size.

Nayak (1970), in a study of equilibrium profiles in a wave tank, also considered scale effects. He modified the work of Iwagaki and Noda to include the effect of specific gravity of the bed material. His results are presented in Figure 3 as a dimensionless graph of

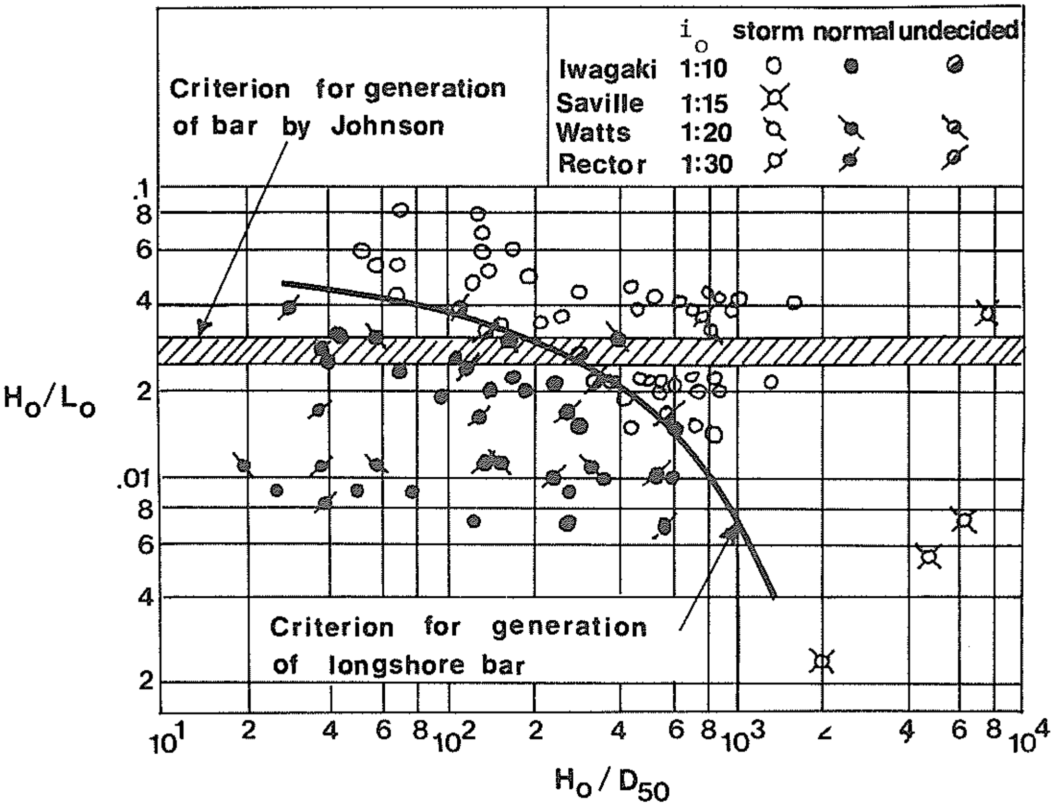


Figure 2. Criterion for Bar Generation.
After Iwagaki and Noda (1962).

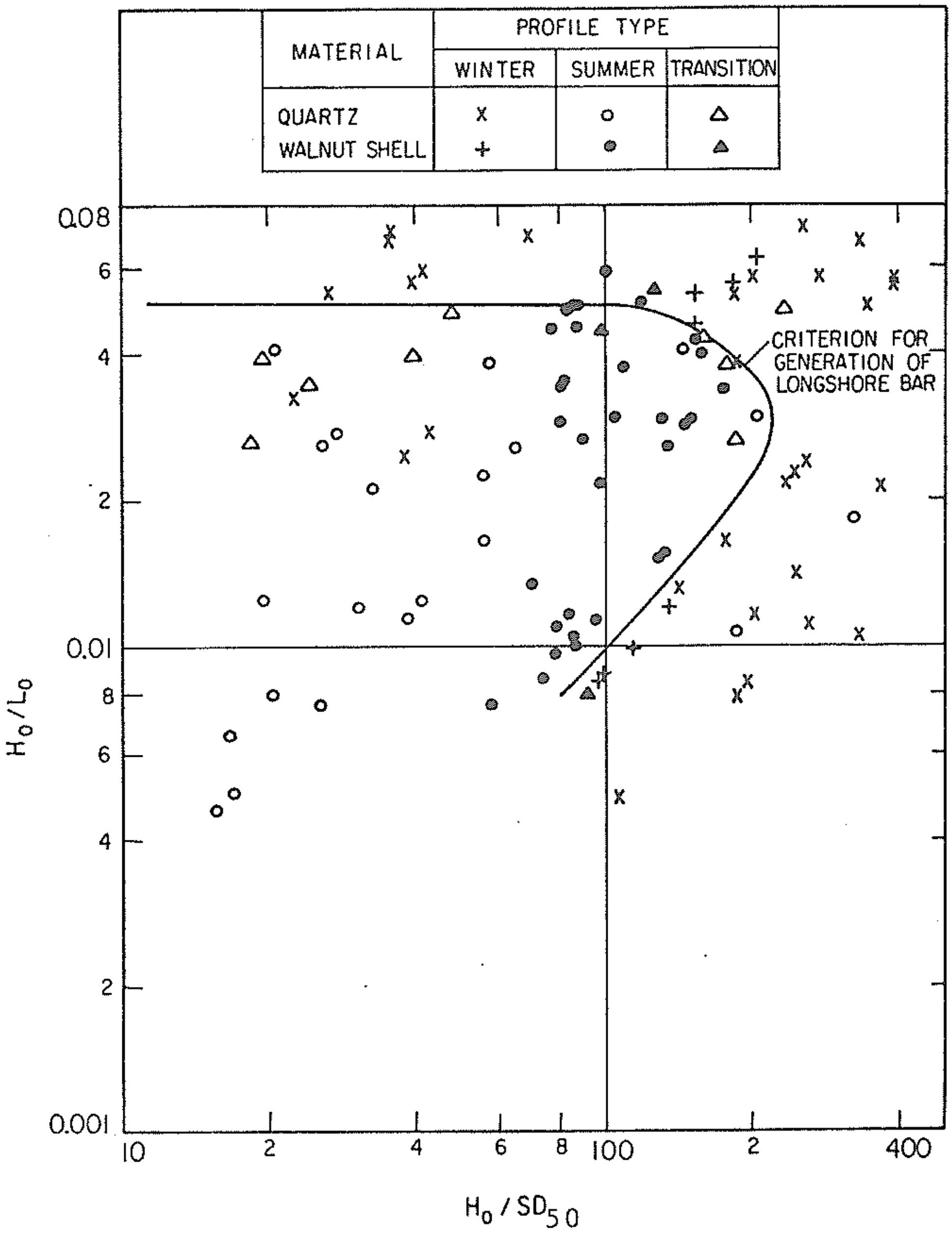


Figure 3. Criterion for Generation of Longshore Bar. After Nayak (1970).

H_0/L_0 vs. H_0/SD_{50} where S is the specific gravity of the material and D_{50} is the median grain size. From the figure it can be seen that storm profiles were developed by low as well as high values of wave steepness for certain combinations of grain size and specific gravity. An intermediate range of H_0/L_0 produced a normal profile. Nayak also studied the wave reflection characteristics of model beaches.

Noda (1972) studied the scale effects involved in two-dimensional model beaches using a number of different materials. His results relate model distortion to the material used in the model. The results are given in Figure 4 and in Equations (1) and (2). As can be seen, the experimenter has the freedom to choose two out of four basic parameters: n_D, $n_{\gamma'}$, λ, μ,

where $n_{\text{parameter}} = \dfrac{\text{model parameter}}{\text{prototype parameter}}$

D = median diameter of material

γ' = relative specific weight of bed material $(\gamma_s - \gamma_f)/\gamma_f$

λ = horizontal scale = x_m/x_p

μ = vertical scale = y_m/y_p

$$n_D n_{\gamma'}^{(1.85)} = \mu^{(0.55)} \qquad (1)$$

$$\lambda = \mu^{(1.32)} n_{\gamma'}^{(-0.386)} \qquad (2)$$

Subscripts m and p refer to model and prototype, and subscripts s and f refer to sediment and fluid properties.

As can be seen from the graph, if the material used in the model is sand ($n_{\gamma'} = 1$), the horizontal scale must always be different from

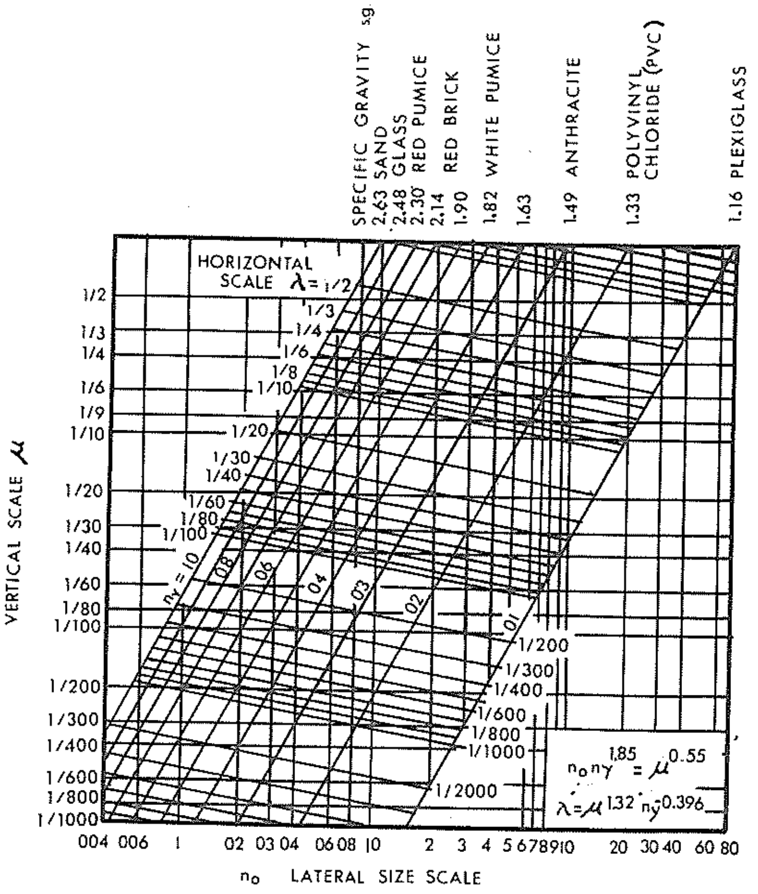


Figure 4. Graphical Representation of Model Law. After Noda (1972).

the vertical scale. Also it can be seen that if the same material is used in the model and the prototype, then $\lambda = \mu = 1$ or the model dimensions must equal the prototype dimensions. This indicates that one must use the smallest size sand available in order to model waves of any reasonable magnitude. Cohesive materials are to be avoided, due to non-similarity of sediment characteristics.

The most recent classification scheme is that proposed by Sunamura and Horikawa (1974). This classification is based on displacement of initial shoreline as well as bar formation:

Type I: shoreline erodes and offshore bar is present
Type II: shoreline accretes and offshore bar is present
Type III: shoreline accretes but no offshore bar is present.

Figure 5 indicates their classification schematically. They found the dominant parameters in beach profile formation to be H_o/L_o, $\tan \beta$, and d/L_o. Where H_o = deepwater wave height, L_o = deepwater wave length, $\tan \beta$ = initial slope and d = mean grain size. Using a number of previous tests on equilibrium profiles found in the literature they found Type III to occur for:

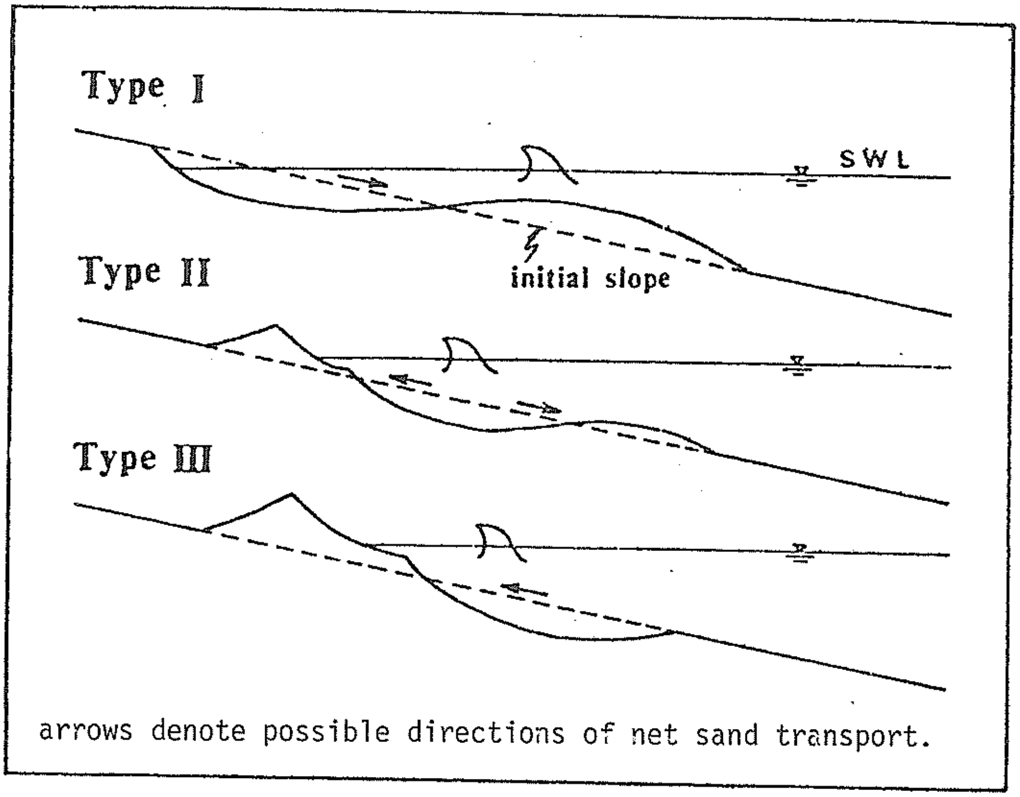
$$H_o/L_o \leq 4(\tan \beta)^{-0.27}(d/L_o)^{0.67}$$

Type I was found to occur for:

$$H_o/L_o \geq 4(\tan \beta)^{-0.27}(d/L_o)^{0.67}$$

The Type II profile was found in a complex intermediate region given by:

$$4(\tan \beta)^{-0.27}(d/L_o)^{0.67} \leq H_o/L_o \leq 8(\tan \beta)^{-0.27}(d/L_o)^{0.67}$$



arrows denote possible directions of net sand transport.

Figure 5. New Beach Profile Classification After Sunamura and Horikawa (1974).

Figure 6 indicates the agreement of previous model tests with this classification. As both Type II and III profiles have accreted shorelines, the division between erosion and accretion is the Type I - Type II borderline or:

$$H_o/L_o = 8(\tan \beta)^{-0.27}(d/L_o)^{0.67}$$

Applying this criterion to field data they found the border between erosion and accretion to shift to a new position:

$$H_o/L_o = 18(\tan \beta)^{-0.27}(d/L_o)^{0.67},$$

possibly due to scale effect. As will be pointed out later, the principal type of profile observed in this study is the Type II profile. For this reason, this classification scheme will be adhered to in this report, as other classifications have no provision for the type of profile observed.

More recent studies of scale effects indicate that the problem has not been solved. Paul, Kamphuis and Brebner (1972) indicated that it is impossible to attain exact similarity between a prototype sand beach and its corresponding lightweight sediment model. In testing "bakelite" beaches they found no winter profile to occur even for high H_o/L_o values. They concluded that a generalized criterion for bar formation for all materials would be impossible and that a separate criterion must exist for each material. This is due to the differences in porosity, angularity, and specific gravity of various materials.

Collins and Chestnutt (1975) attempted to verify the model law proposed by Noda. Using "rocklite", a commercial ceramic sand, and pumice, he attempted to model profiles which were obtained in a larger

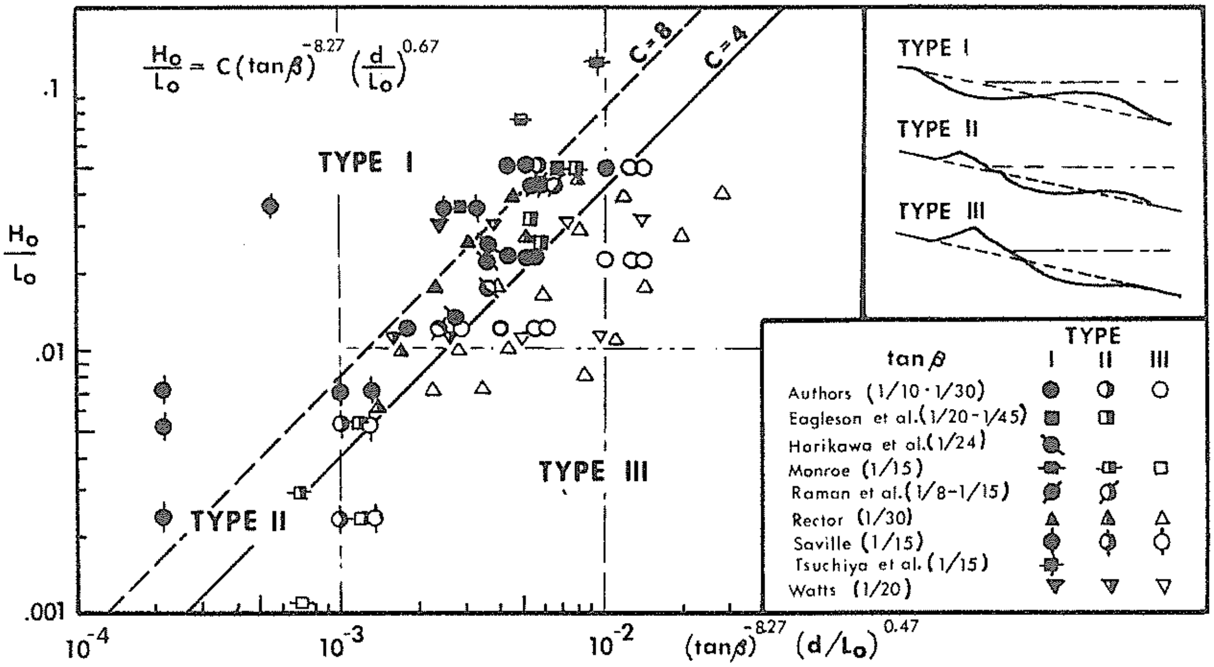


Figure 6. Classified Laboratory Beach Profiles.
After Sunamura and Horikawa (1974).

tank. Considering the large tank as the prototype, he found the slope of the foreshore to be reproduced correctly, but the offshore and surf zone profiles were not reproduced. In addition, shoreline changes were different in the model compared to the prototype. He concluded that Noda's model law may be valid only for predicting the foreshore slope.

The above studies were concerned only with equilibrium profile, and little mention has been made of the time development of the profile. Fairchild (1959) studied suspended load transport on model beaches and noted the importance of water temperature on the variability of suspended load. He observed a 50% to 75% variation in suspended load over a range of temperatures [53-80°F (11.7-26.7°C)]. Saville (1957) used this argument to explain differences in time development in his large-scale outdoor tests during different parts of the year. Chestnutt (1975) found an increase in shoreline recession in several outdoor model experiments to coincide with a drop in temperature. Thus, the hypothesis that colder, more viscous water has an increased capacity to transport sediment has not been quantified, but several investigations support the hypothesis. It is obvious that water temperature will have to be considered in studies involving time development of model beach profiles, and ultimately to determine the relations between model and prototype sediment time scales.

As can be seen, numerous laboratory studies have been made of the "equilibrium profile". However, relatively few investigations have been made with the objectives of the Phase I part of this research, which is to determine the effects of laboratory assumptions on data obtained in the actual study. Chestnutt (1975) studied laboratory effects in an outdoor tank. His conclusions prompted the Phase I part of this research. For a Type I profile, he concluded that equilibrium was not reached in tests for as long as 375 hours. In addition, he found that variables such as initial tank length, initial slope and water temperature may be important variables affecting profile shape. Table IV in Appendix A compares initial conditions for Chestnutt's test with initial conditions for the Phase I tests conducted in this report. In another report, Collins and Chestnutt (1975), found that the position of the offshore bar on stable profiles generated by identical wave conditions was not repeatable, thus seriously questioning the concept of the "equilibrium profile". In this same report it was concluded that grain shape and size distribution of the model material are additional important variables. Narrow grain size distributions and smooth spherical shapes produce unstable profiles having bars which periodically grow and disappear. In an earlier report, Chestnutt and Galvin (1974) studied the reflection characteristics of a developing beach profile. The actual test data are part of the data presented in Chestnutt (1975). They found reflection coefficient variations correlated with profile changes. Additionally, they studied two-dimensionality in tanks of two widths. Results indicated that beach behavior was two-dimensional but not in a 10-foot tank.

SCOPE OF THE PRESENT STUDY

The present study is concerned with the normal component of sediment motion and its role in the formation of the equilibrium profile. The actual process of beach profile formation is of a variable three-dimensional nature in which equilibrium conditions are seldom observed. No suitable wave theory is available to describe the water particle motions within the breaker zone. Neither the exact interactions between the complex water and sediment motions, nor the mechanism of sediment entrainment are fully understood. Thus, the actual problem is beyond the scope of present mathematical capabilities. As a result, in a laboratory study of beach profile formation a large number of assumptions are made. These assumptions can be categorized into three groups: those associated with laboratory problems, those associated with simplifications, and those associated with scale effects.

Laboratory Assumptions:

1. The profile comes to an equilibrium shape and can be represented two-dimensionally.
2. The process of profile formation is repeatable within laboratory controls.
3. The dominant process variables are controllable in the laboratory and are known exactly.

Phenomenological Assumptions:

4. The dominant process in beach profile formation is onshore-offshore transport.
5. The model wave characteristics represent the dominant pro-

totype waves.

6. The effect of tidal fluctuations on the prototype profile is negligible and a mean sea level can be used in the model.

Scale Effect Assumptions:

7. The mechanism of sediment transport is the same in the model and prototype.
8. The model profile represents a scaled version of the prototype beach profile.

The validity of these assumptions is of great concern to those studying the equilibrium profile in the relatively small-scale hydraulic model. Certain assumptions such as number 4 above can be easily fulfilled, if the longshore component of sediment transport is in an equilibrium state. This is valid for areas with relatively constant longshore sediment motion. Assumption number 6 above is valid for regions with limited tides. Figure 7 indicates that assumption 8 may be reasonable. The actual agreement between prototype and model profiles demands a knowledge of sediment time scales and a record of the dominant prototype waves responsible for the prototype profile. This involves assumption 5 and 7 as an entire topic of current research on equilibrium profiles. This research is primarily concerned with assumptions 1, 2 and 3. The validity of these assumptions is essential before proceeding to the actual modelling phase.

Paul, Kamphuis and Brebner (1974) point out that in order to achieve similarity between natural and model beach profiles, large distortions of scales is limited by the maximum repose angle of the model beach material. Referring to Figure 7, it can be seen that the

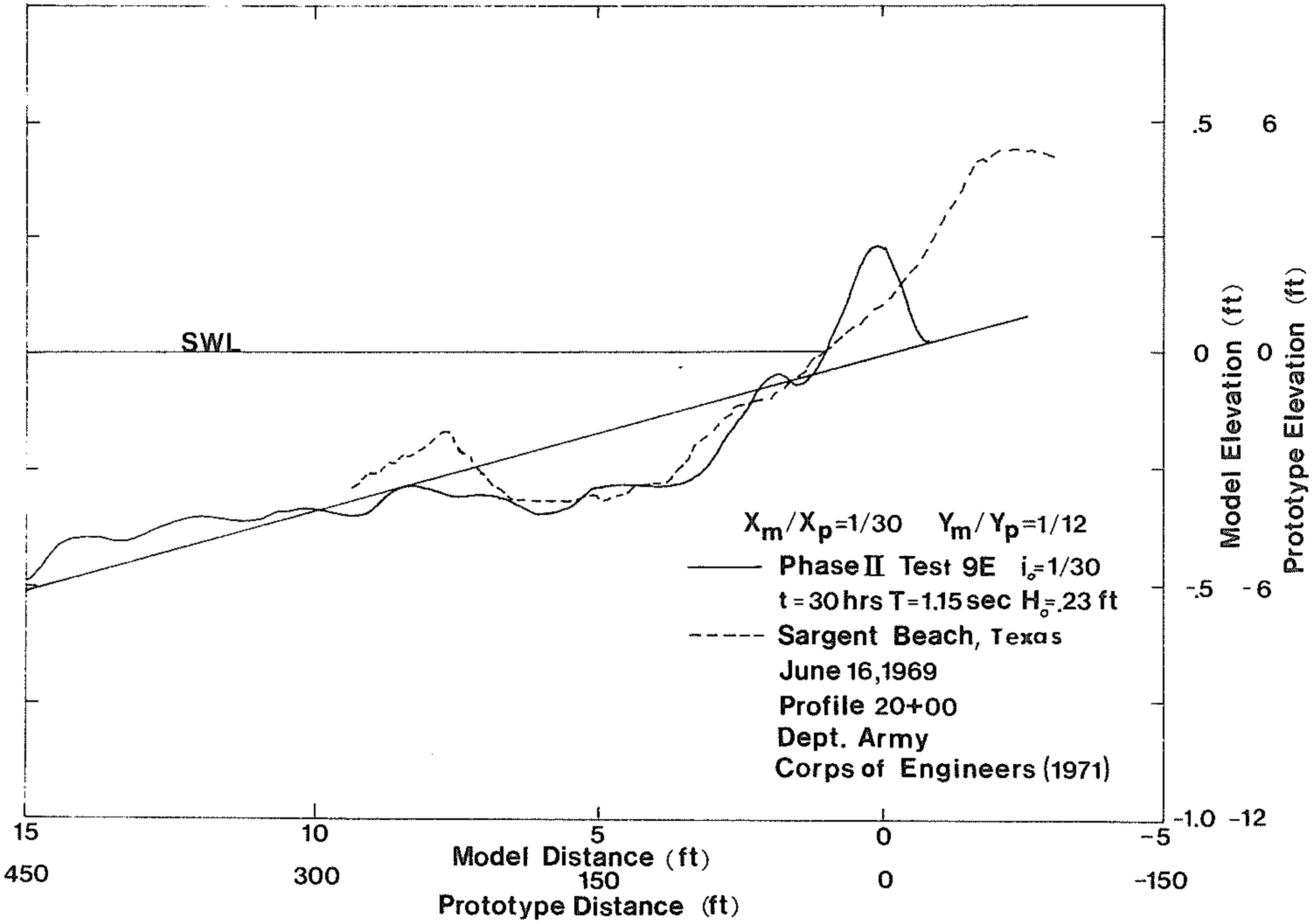


Figure 7. Comparison of Model and Prototype Beach Profiles by Trial-and-Error Fit.

distortion required for agreement in shape of an intial 1:30 model beach with a natural profile results in an unnatural repose angle at the foreshore. The maximum angle observed in the model at the foreshore. The maximum angle observed in the model at the foreshore is 14°. This angle distorted by a 2.5 distortion factor obtained from trial-and-error fit is 35°, an angle greater than the natural repose angle of quartz sand. For this reason, the flatter initial slope seems to be more desirable in the small-scale model. Hence a 1:60 initial slope, not generally found in the literature, was used in part of this research.

Thus, the scope of the present study was two-phase. First, a study of laboratory assumptions was undertaken, and secondly, based on the results of this study, an analysis of previously generated, model profile data was attempted. Reflection measurements were made to assess its suitability as an indicator of profile equilibrium.

TEST PROCEDURES

Facilities

All experiments were conducted in a glass-walled channel with variable beach slope. The dimensions, as indicated in Figure 8, were 2 feet wide x 3 feet deep x 120 feet long. Waves were generated by a wave generator of the oscillating pendulum type, with variable stroke and speed. A corrugated hardware cloth wave filter was situated a short distance from the wave generator to prevent reflected waves from the beach from being regenerated as secondary waves from the wave generator. Wave data were collected using a capacitance-type wave gauge and recorded on a Hewlett-Packard two-channel recorder. The gauge was calibrated statistically by moving it up and down known amounts with respect to still water. A one-inch grid system on the side of the tank, for the length of the beach profile, allowed visual and photographic determination of sand levels at given time and space intervals.

Procedure

Beach material used in all tests was uniform Ottawa sand with median diameter, D_{50} = 0.30mm and specific gravity γ 2.65. Sand size distribution is shown in Figure 9. For the first phase of this research, initial slope of the beach was 1:60. Sand was smoothed to an initial slope taped on the side of the tank. Profiles were measured at intervals of 0.5 feet marked on the side of the tank and recorded visually as deflections, positive and negative about the initial,

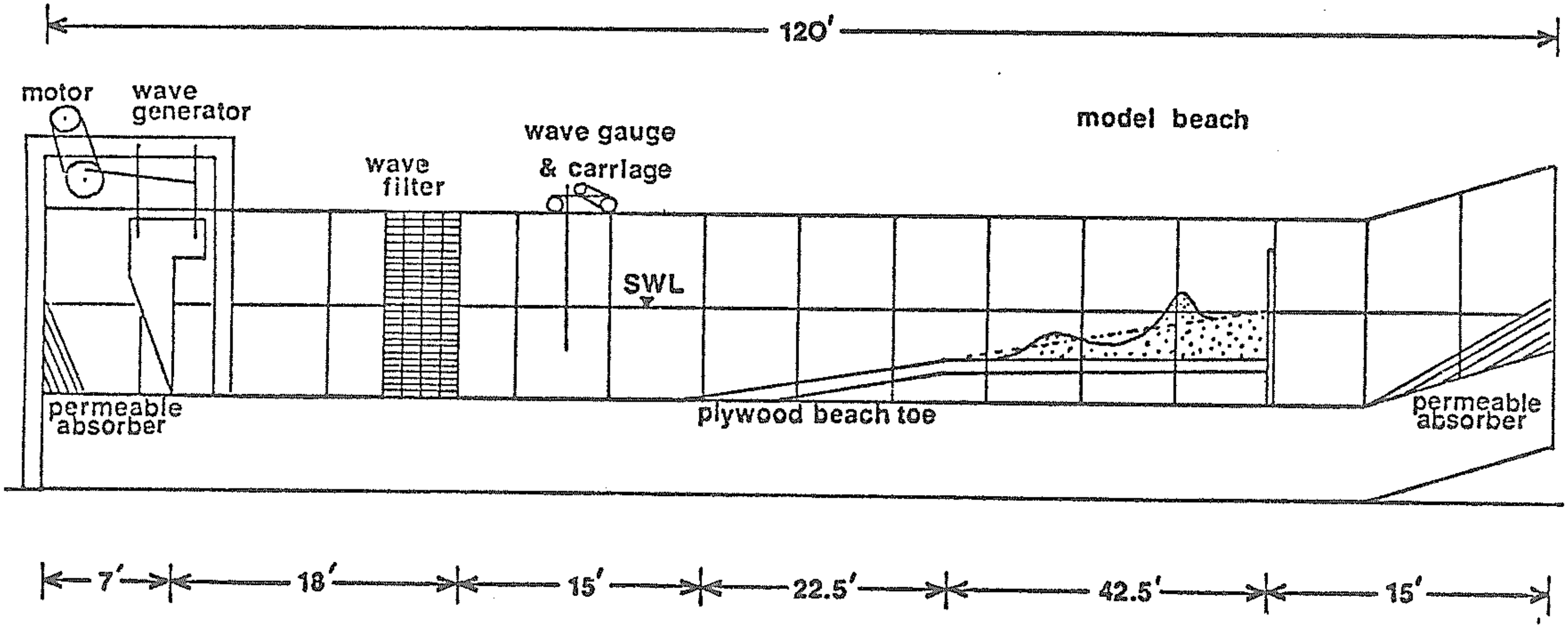


Figure 8. Wave Tank Used in the Study.

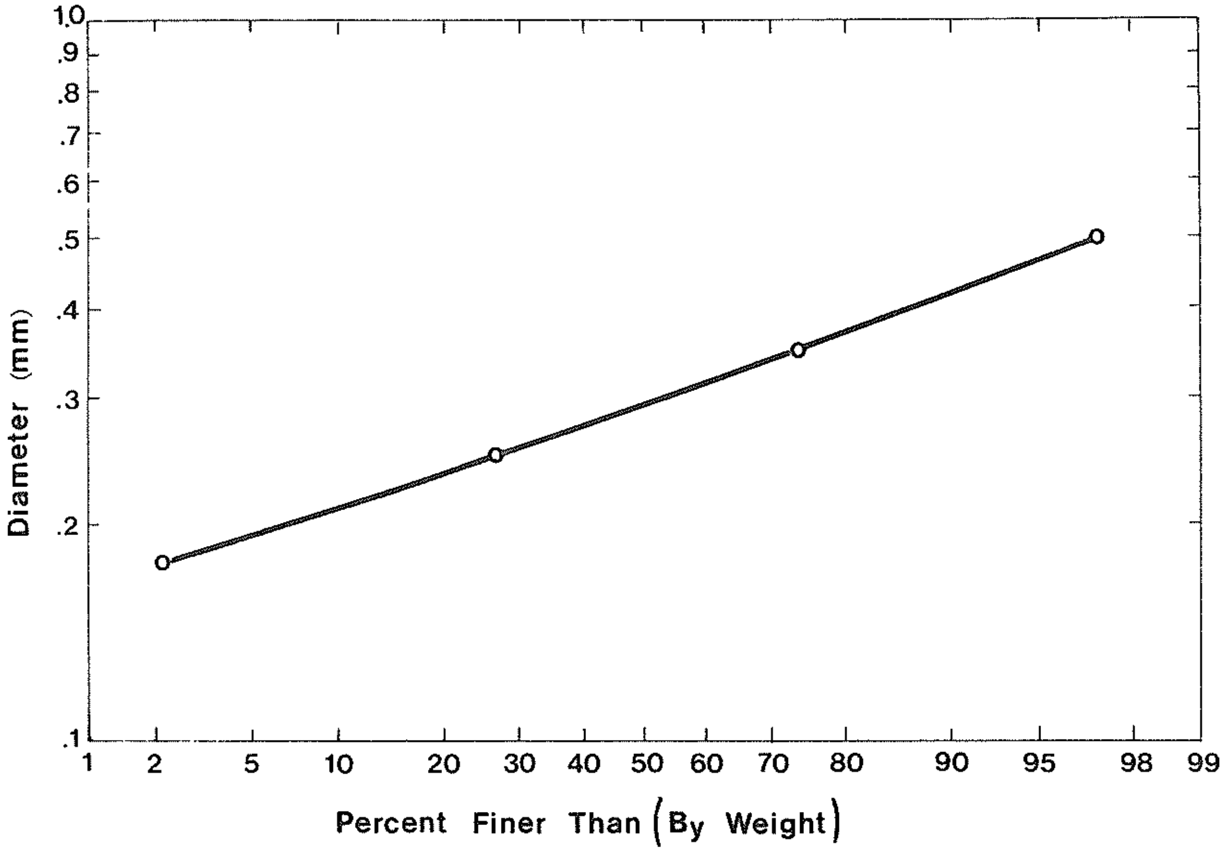


Figure 9. Sand Size Distribution

1:60 line, to a distance of 35 feet from the initial shoreline. Three tests were run for the first phase of the research. All three tests had identical wave conditions and, as nearly as could be reproduced, identical conditions such as initial slope, water depth, compaction of the bed and water temperature. Water level variations were not maintained with any type of device but were observed to stay within a 0.25 inch range. Water temperature was measured at the end of Test 3 to be $71^{O}F$ ($21.7^{O}C$), but was not maintained at this value. As all Phase I tests were run within a one-month period it was assumed that temperature variations within the indoor laboratory had negligible effects on sediment transport characteristics (of repeatability) of Phase I Tests 1, 2 and 3. For Test 1, only the final profile was recorded at t = 77 hours with initial wave parameters. Wave height, reflection, and profile measurements were made approximately every 3 to 4 hours for Test 2, for a test duration of 86 hours. Tests 1 and 2 were run to assess the repeatability of the equilibrium profile. Based on review of the literature, 77 hours was considered to be a long enough time to encompass most previous test duration times. Wave height, reflection and profile measurements were made at irregularly spaced intervals for a test duration of 400 hours. A number of observations were taken to coincide with those taken in Test 2. In addition, more frequent profile records were made (at one-hour intervals). Test 3 was designed to further address the repeatability problem, and primarily to study, in particular, the equilibrium and two-dimensionality assumptions.

All wave height and reflection readings were taken over the horizontal part of the tank in front of the beach toe. Reflection measurements were made using a movable probe method. The wave probe was mounted on a motorized carriage which moved on rails at the top of the wave tank. The probe was moved at a constant speed for a distance of at least 10-12 feet. This provided a record as indicated in Figure 10. Coefficient of reflection was then determined using an envelope method. Nodal and antinodal points are located at L/4 intervals, where L is the wavelength. Where the reflected wave crest meets an incident wave crest, an antinode is indicated, and where the reflected wave crest meets an incident wave trough, a node is formed in the wave record. The coefficient of reflection is then defined as the ratio of incident wave height, H_i, to reflected wave height, H_r

$$C_r = \frac{H_r}{H_i} = \frac{H_a - H_n}{H_a + H_n}$$

where C_r = coefficient of reflection, $H_r = H_a - H_n/2$, $H_i = H_a + H_n/2$, H_a = wave height of the antinode and H_n = wave height at the node. This method, however, assumes a sinusoidal wave, with constant amplitude. Repeated measurements indicated that the measurement was reproducible with an accuracy of approximately $\pm$ 0.015. Thus, measurements should give a relative indication of reflection, if not an absolute value. Most previous studies of equilibrium beaches used the variable probe envelope method described here. Wave period measurements were made by timing the wave generator with an electric timer for twenty oscillations and averaging the quantity.

Phase II testing was done similarly without changing either sand or wave channel, but with a different initial slope. Data obtained during these tests were designed to determine the effect of a pipeline

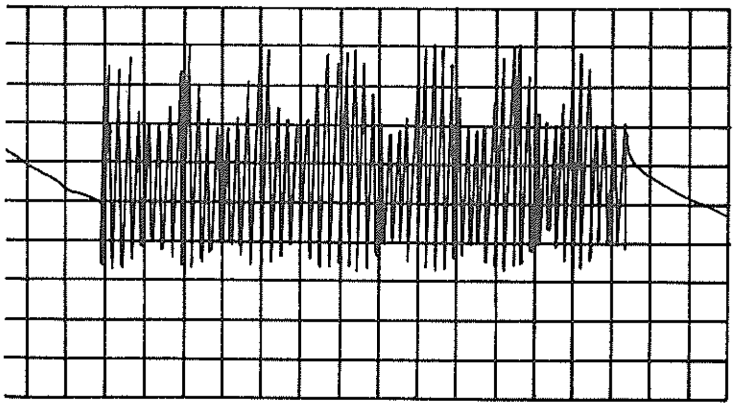


Figure 10. Reflection Record

H_a = 28 divisions

H_n = 14 divisions

$$C_r = \frac{28 - 14}{28 + 14} = 0.33$$

$$H_i = K \frac{28 + 14}{2} = 0.32 \text{ ft.}$$

where K = calibration coefficient

$$= \frac{0.4}{26.5} = 0.015 \frac{\text{feet}}{\text{division}}$$

on the equilibrium profile. Tests were run with a 1.75-inch diameter pipe, 20 feet long, placed at various depths relative to the initial slope surface. As indicated in Figure 11 these positions ranged from on top of the initial slope surface to one diameter below it. As no local scour was observed around the pipe, it was assumed that the effect of the pipeline on the equilibrium profile is negligible. The validity of this assumption may be questioned so that the evaluation of equilibrium profile characteristics was made only for those cases in which the pipe was buried below the surface (cases C, D and E, Fig. 11).

The procedures in Phase II testing were similar to those described for Phase I testing. The sand was initially smoothed to a constant slope indicated on the side of the tank. Wave height and reflection readings were taken only once, several hours after the initiation of each test. Duration of the tests was varied. The 1:20 initial slope tests were run a maximum of 13 hours. Unfortunately, the author of this report was not involved with this phase of study, but approximate equilibrium was reported to have been attained within this time interval.[1] The 1:30 initial slope tests generally ran a minimum of 30 hours. Profiles were recorded photographically and reproduced graphically for analysis. Profiles were generally recorded at 45 minutes, 5, 7, 13 and 30 hours. Data obtained in Phase II testing and used in this study are listed in Tables V, VI, and VII in Appendix A. Justification for how the data were selected is given in the Analysis section.

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[1] Personal communication with Dr. J.B. Herbich, 1975.

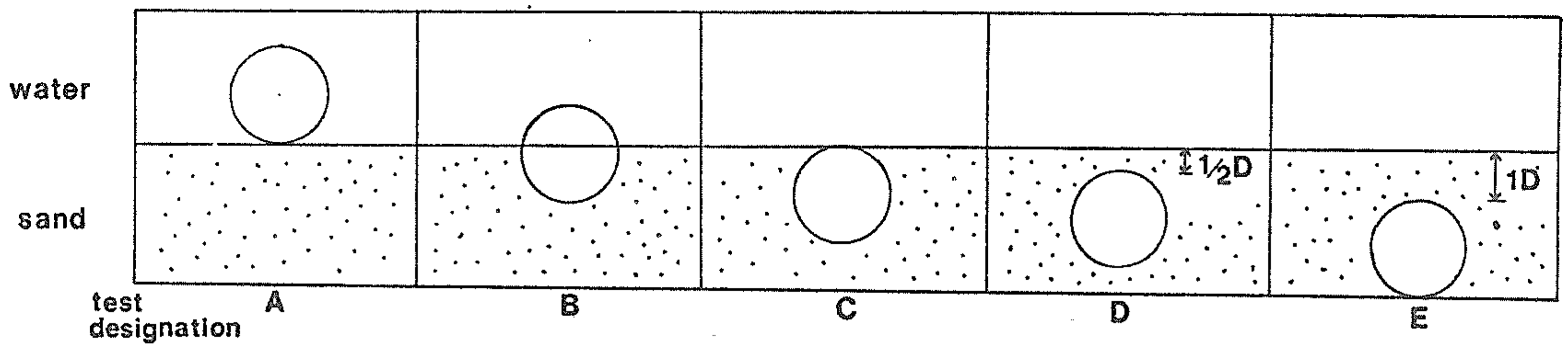

Figure 11. Location of Pipeline in Phase II Testing.

PRESENTATION AND DISCUSSION OF RESULTS

Phase I Tests

Phase I tests were designed to assess the validity of a number of laboratory assumptions generally made in small-scale studies of equilibrium beach profiles. These are the assumptions of two-dimensionality, a stable "equilibrium" form of the profile, and repeatability. These assumptions were treated here as hypotheses. The assumption that the dominant variables responsible for profile formation are known will be indicated. The condition of repeatability will be tested for two aspects: repeatability of the development sequence and repeatability of the stable beach form. Repeatability of the development sequence generally has not been assumed by previous investigators, but is a desirable property. Repeatability of the equilibrium form, generally assumed, requires that the other two assumptions tested in this research be valid. The requirement of two-dimensionality can actually be relaxed if, in fact, the lack of two-dimensionality is a repeatable phenomenon. This, however, seems doubtful, as will be shown. Thus, repeatability will be treated last.

The validity of these assumptions was considered for four features of the equilibrium profile represented by the following quantities: maximum height of the beach crest, maximum depth of the nearshore trough, maximum height of the offshore bar and sand level and the midpoint of the profile. The first three quantities were considered independent of location on the profile, but the fourth quantity was determined at a specific location in the tank. The midpoint of the profile was selected arbitrarily to represent an observed unstable area of the profile. Figure 12 indicates these quantities and the notation used in the following analysis. All quantities are designated as positive for their most frequent occurrence and all are measured relative to the original profile with exception of the maximum height of the beach crest, which is measured relative to the stillwater level.

Two-Dimensionality

The hypothesis that the equilibrium beach profile is two-dimensional was studied during the latter part of Test 3 (from 150-400 hours). This hypothesis requires that a profile taken at any point along the width of the tank will represent a profile taken at any other point along the width. Thus, the possibility of lack of equilibrium due to a side-to-side transport mechanism is ruled out, when the profile is two-dimensional. The hypothesis was tested using profile observations on opposite sides of the tank. Figures 13-16 present results graphically. The relative satisfaction of the two-dimensionality hypothesis by the individual features can be observed. The individual quantities, C_{max}, T_{max}, M and B_{max} are considered separately. In addition, a profile area indicator function is studied.

Figures 13-16 indicate the magnitudes of the four quantities, as observed on the opposite walls of the beach width, during Test 3. Qualitatively, the figures show that C_{max} tends to be a relatively two-dimensional quality. Comparative statements about the other quantities are difficult to make from these figures.

A measure of two-dimensionality was devised. For each time observation a spatial mean and a weighted deviation was determined.

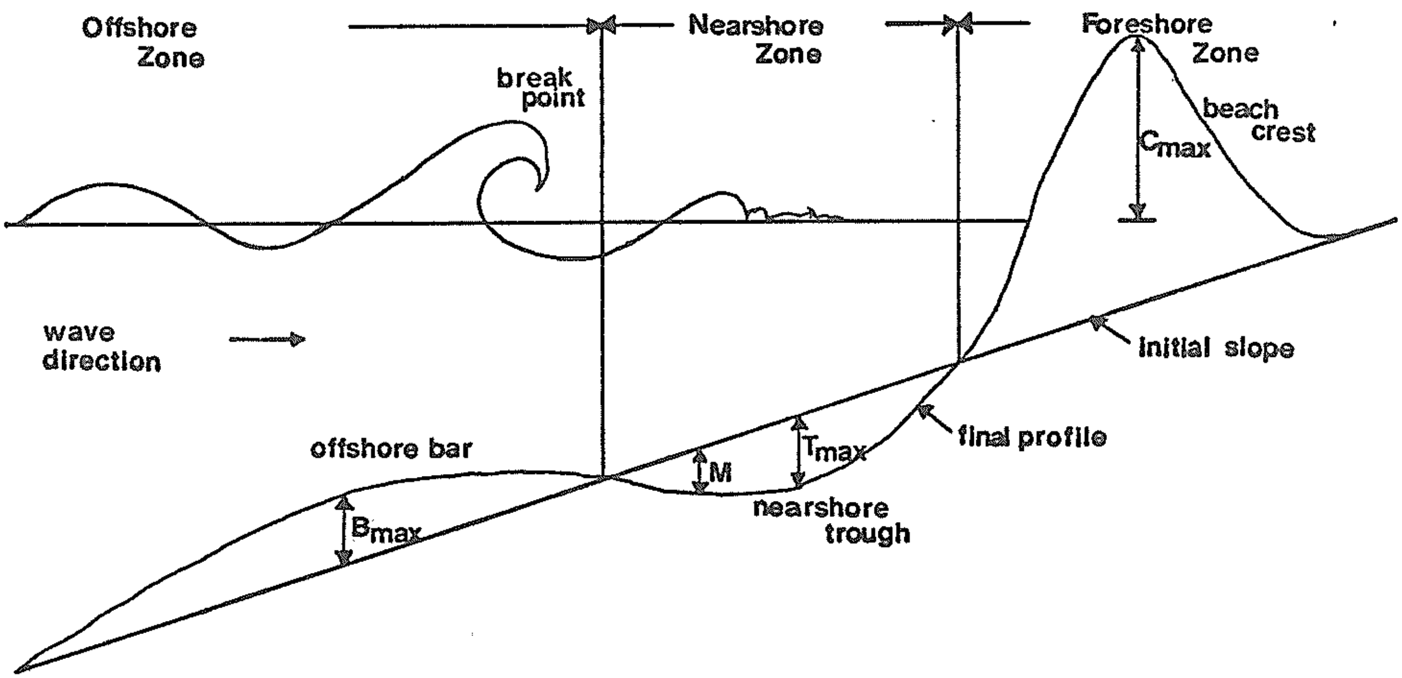

Figure 12. Notation Used in the Study

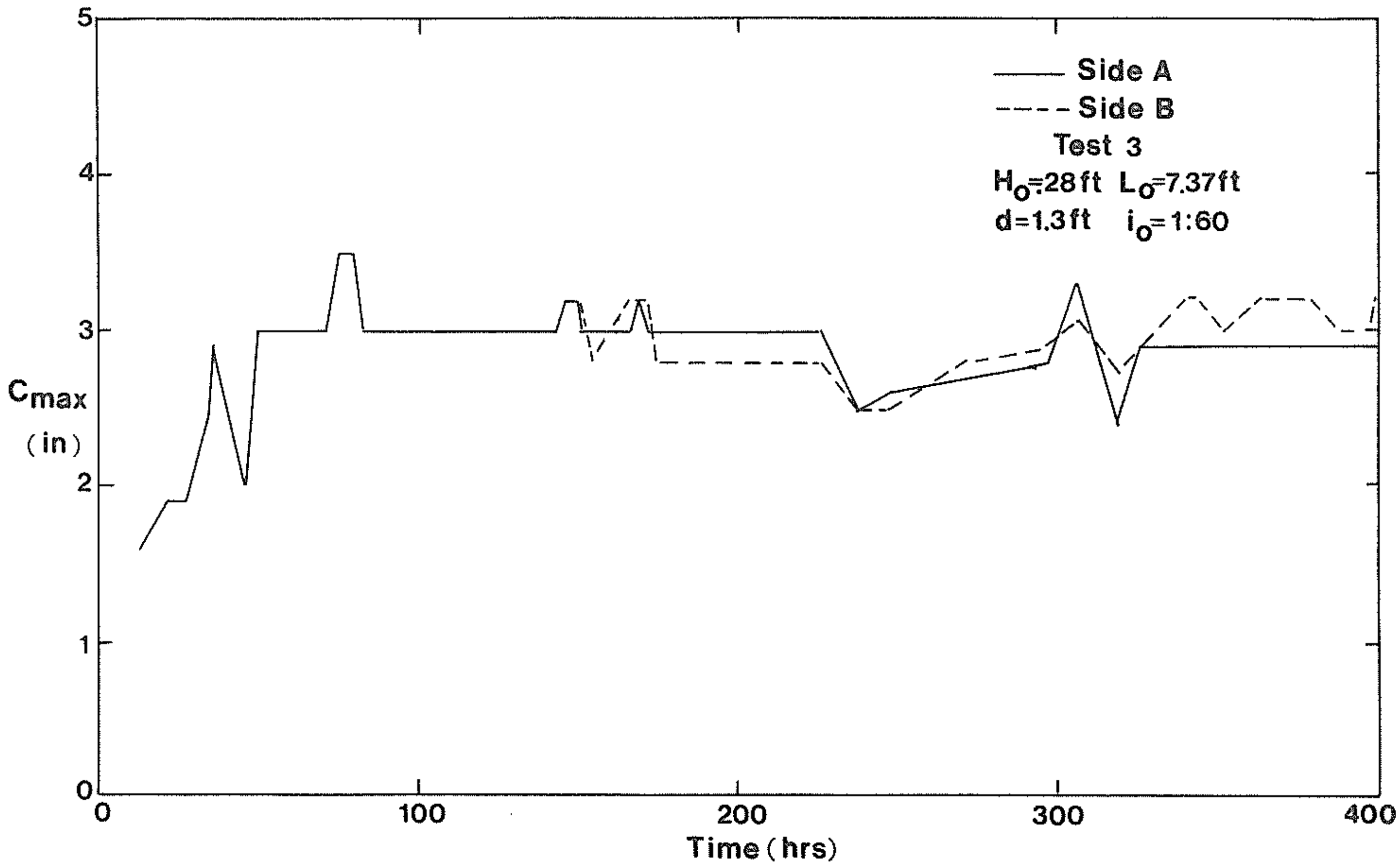

Figure 13. Two-Dimensionality and Stability of the Maximum Height of the Beach Crest.

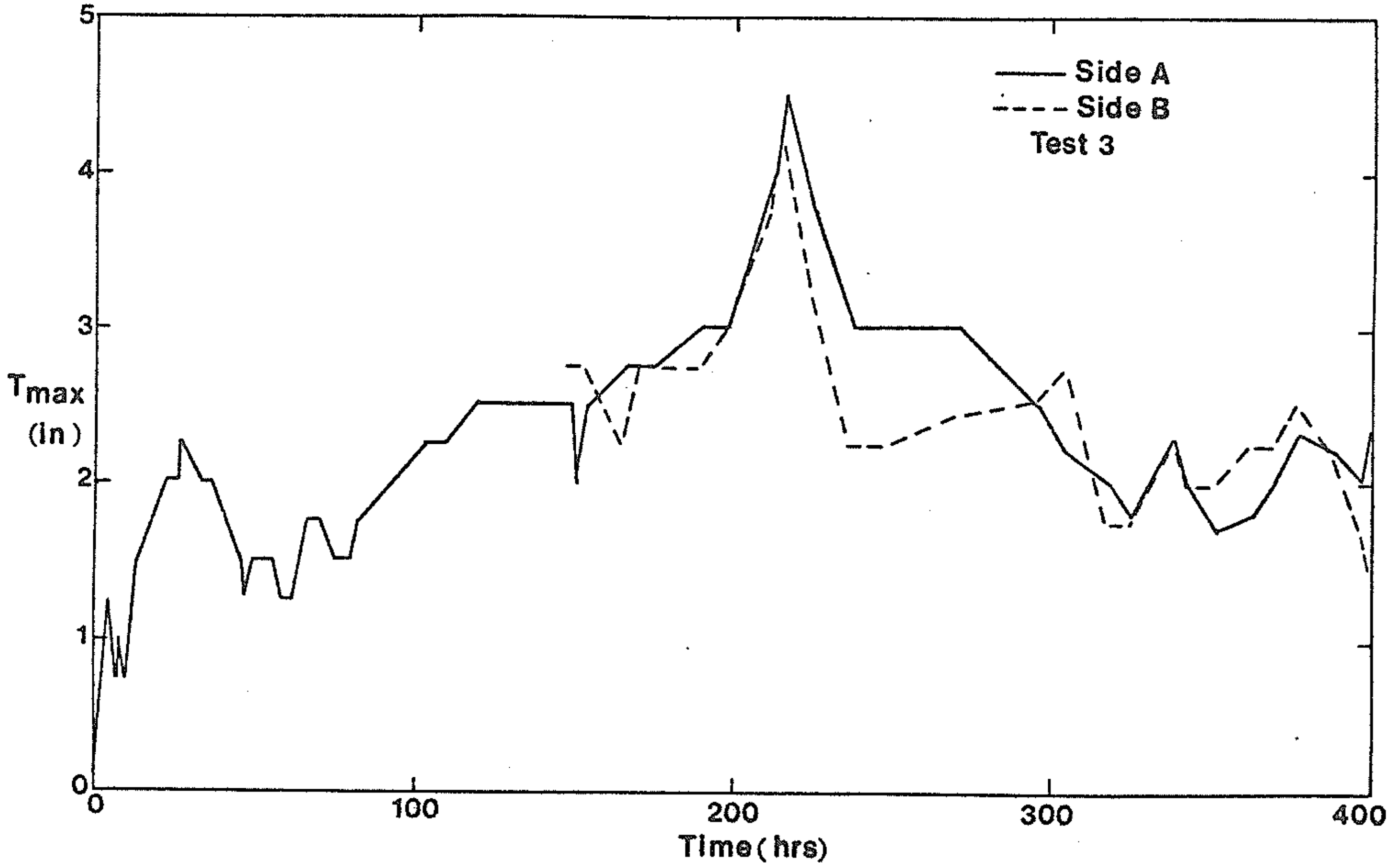

Figure 14. Two-Dimensionality and Stability of the Maximum Depth of the Nearshore Trough.

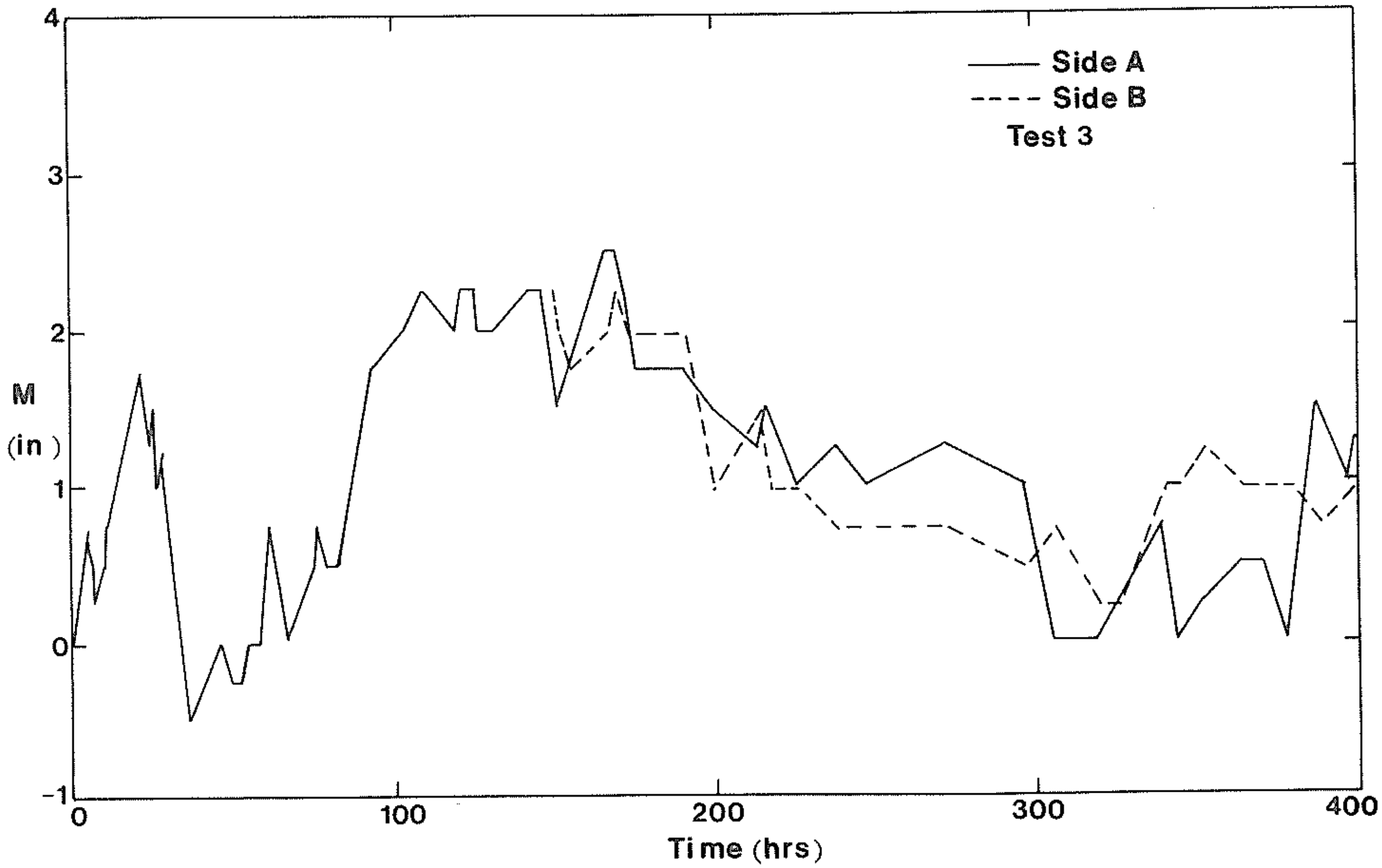

Figure 15. Two-Dimensionality and Stability of the Profile Midpoint.

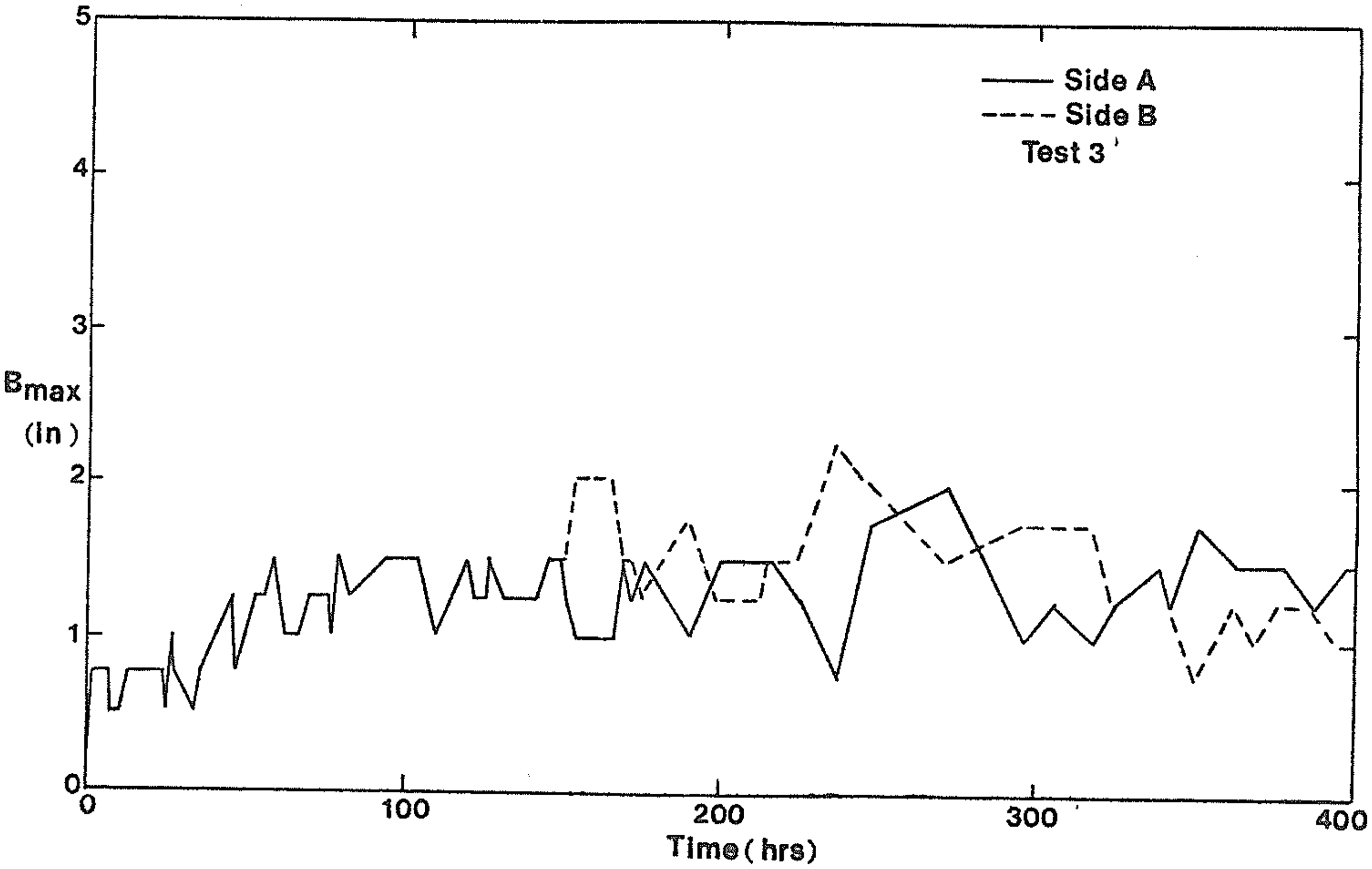

Figure 16. Two-Dimensionality and Stability of the Maximum Height of the Offshore Bar.

Given two points, x_1 and x_2, sampled at opposite sides of the tank (side A and side B) at a given time the weighted deviation was defined as $|x_2 - \overline{x}|/\overline{x} \cdot 100$ where $\overline{x}$ is the mean value of x_1 and x_2. This value was averaged over time to give a relative measure of two-dimensionality among the various quantities observed. High values of this average percent deviation from the mean indicate relative disagreement between side A and side B values.

Table I indicates the results obtained. In addition, maximum observed deviations from the mean are shown. As stated above, relative two-dimensionality of the defined maximum quantities decreases in a seaward direction along the profile. Again, it should be noted that these quantities are considered independently of their location on the profile. The two-dimensionality of the beach at a given point, represented by the profile midpoint, is relatively the poorest.

As is shown in Figures 13-16 (pp. 32-35), an oscillatory phenomenon was observed for all quantities considered. A profile area parameter was defined to determine the nature of the oscillation. The profile area indicator is defined as follows

$$A = \sum_{j=1}^{71} 0.5y_j$$

where y_j is the depth (feet) of the profile below the stillwater level at the j^{th} measurement station. As was previously noted, measurements were spaced at 0.5 feet intervals. This value represents the area of water (feet^2) over the profile, below the stillwater level, for a given side of the tank.

As indicated in Figure 17, mass conservation of sand was maintained

TABLE I

RELATIVE SATISFACTION OF HYPOTHESES BY TESTED QUANTITIES

	C_{max}	T_{max}	M	B_{max}
*Two-Dimensionality	4.0	8.4	28.6	14.6
	7.9	31.4	100.0	50.0
°Stability	7.6	26.7	49.2	24.0
†Two-Dimensional Mean Stability	8.2	27.0	57.2	22.5
*Sequence Repeatability	7.7	6.8	72.9	15.8
	36.2	33.3	400.0	58.2

*The upper figure denotes the average value of the weighted deviation and the lower figure denotes the maximum observed value.

°The figure denotes the coefficient of variation.

†The figure denotes the average value of the weighted deviation.

Note: Tested quantities are listed in order of occurrence progressing in a seaward direction.

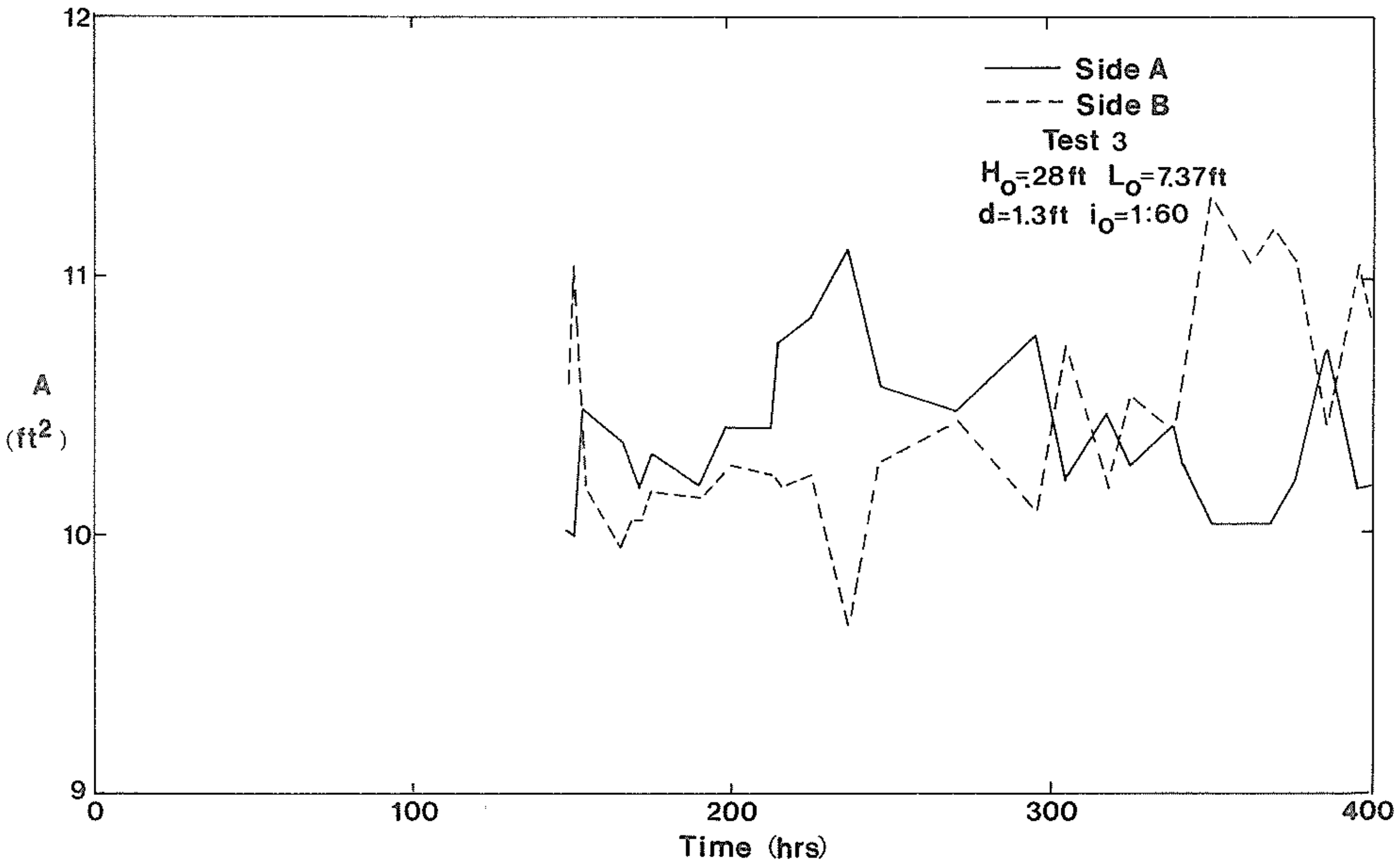

Figure 17. Two-Dimensionality and Stability of Profile Area Indicator.

in Test 3, through a side-to-side oscillation across the width of the tank. The period of oscillation appears to be approximately 100 hours. The phenomena governing the oscillation were not determined. Local wavelength was observed to be approximately 4 feet and tank width was 2 feet. The possibility of a cross-mode of wave oscillation exists. However, based on profile observations this seems to be an unsatisfactory explanation. Figure 18 shows observed profiles for opposite sides of the tank at t = 238 hours. For a cross-mode of oscillation, the bed crest and trough system would be expected to be out of phase 180° across the tank. However, at t = 238 as well as for other times at which poor two-dimensionality was observed, the bed undulations were in phase across the width of the tank.

A possible explanation for the oscillatory phenomena is based on energy considerations. As a wave progresses from deep to shallow water a certain amount of energy is dissipated through a number of mechanisms: friction losses along the bottom and along the sidewalls before the wave breaks, turbulent losses in the breaker, friction losses in the uprush, potential energy in the form of increased inshore water level, friction losses in the backwash and reflected energy. In the wave channel the reflected energy may then be responsible for the lack of two-dimensionality. The lateral variation of reflection across the width of the tank could result in a similar response from the bed. A laterally uniform return flow of energy may be unstable, resulting in a concentration of return energy on one side of the tank, similar to a rip current in nature. This additional energy on one side of the tank could lower the bed in a self-governing process. As the bed is lowered a point is reached at which the inci-

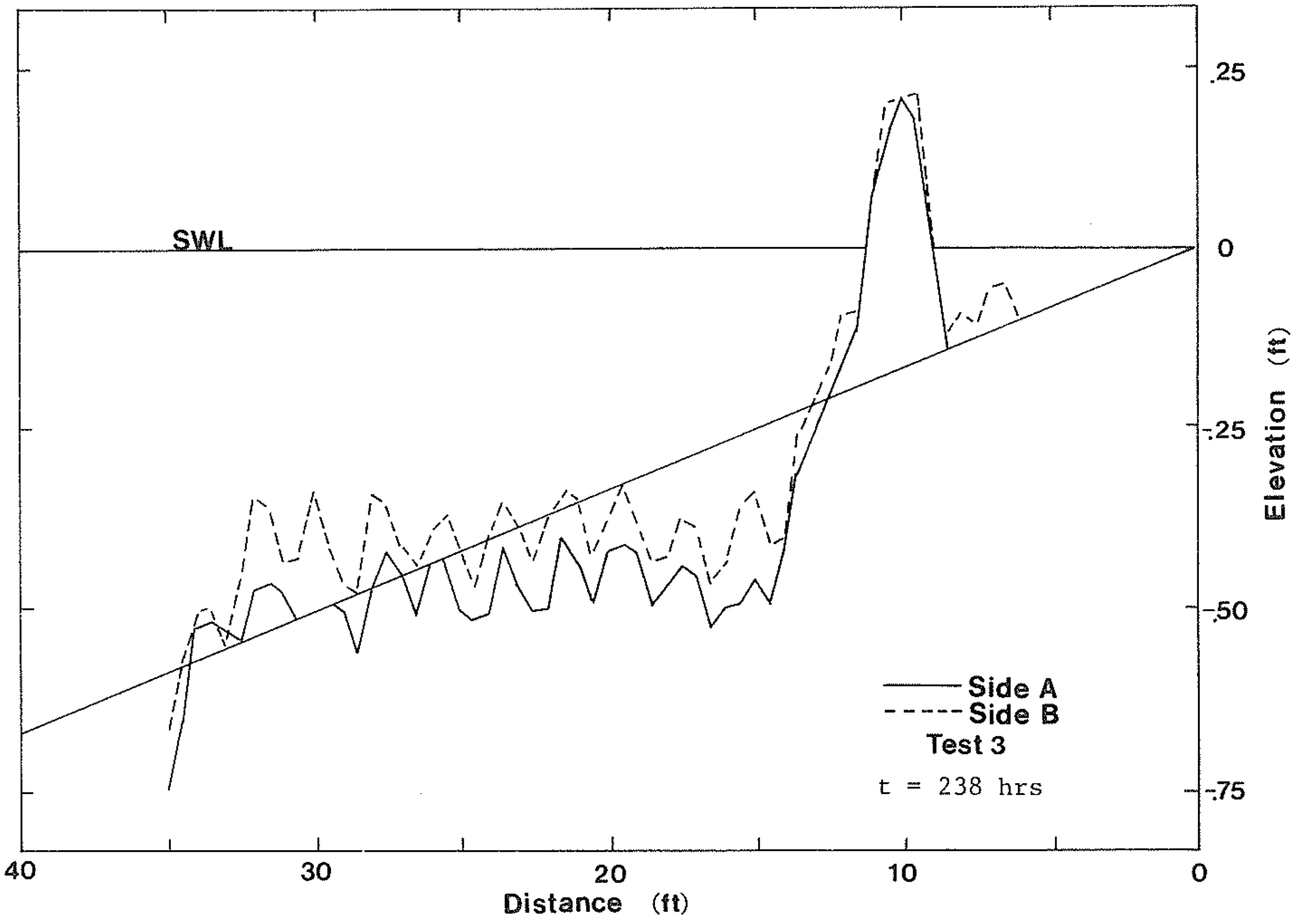

Figure 18. Two-Dimensionality of Test 3 at t = 238 Hrs.

dent wave no longer breaks. Thus, turbulent energy losses become minimal on this deep side of the tank and the energy reaching the beach is greater than that on the shallow side. As this incident energy reaches some critical value, the return flow will be forced outward where the incident energy is minimal, i.e., on the shallow side. At this point, the cycle repeats. The change of depth across the tank was observed to affect the breaker system, but no documentation of the above model was attempted.

Stability

The assumption that the beach profile reaches an equilibrium shape was tested using Test 3 data. The validity of this assumption is independent of the validity of the others, as a beach profile could feasibly reach a stable shape without being two-dimensional or repeatable. As already demonstrated, lack of two-dimensionality was found to be an oscillatory phenomenon for the given test conditions. Thus, for this particular case, as the profile area indicator for a given side did not reach a stable value [Figure 17, (p. 36)], the condition of stability requires two-dimensionality. Referring to Figures 13-16 (pp.32 -35), apparent equilibrium is reached by two of the four investigated beach quantities. C_{max} and B_{max} appear to oscillate about a mean value, whereas T_{max} and M do not.

To quantify the stability of the profile feature, the coefficient of variability was used. The coefficient of variability, defined as $(S/\overline{x}) \times 100$, gives a measure of variability of a number of observations about their total mean where S is the standard deviation and $\overline{x}$ is the mean. The mean here was taken as the time mean of all data, from 150-400 hours, for one side of the tank. Large values of this coefficient indicate a relatively unstable phenomenon. Results are presented in Table I (p. 37). As seen for the case of two-dimensionality, stability decreases in a seaward direction for those quantities defined as maxima. Again, the sand level at the given point in the nearshore zone was relatively least satisfactory in fulfilling the hypothesis. The coefficient of variation was also applied using data from both sides of the tank to determine if an improvement in equilibrium was obtained by using a two-dimensional mean. As seen in Table I (p.37), the improvement was not significant.

Repeatability

Tests 1, 2 and 3 of Phase I are used in testing the repeatability hypothesis. Two considerations were given: sequence repeatability and repeatable equilibrium. Repeatable equilibrium is the type generally assumed by investigators and requires that the previously considered hypotheses are valid. Sequence repeatability is a desirable quality, especially because there is widespread disagreement about the length of time necessary for development of an equilibrium profile. The length of time needed to attain equilibrium, if such a quantity exists, is a function of the wave and beach parameters. The large number of variables here implies a variable duration time. However, for sand beaches alone, the range of variability found in the literature is three orders of magnitude. If the possibility of comparing experimental results is to exist, sequence repeatability is highly desirable. For this reason it was tested as well as equilibrium repeatability.

Figures 19-22 indicate the repeatability of the four basic quanti-

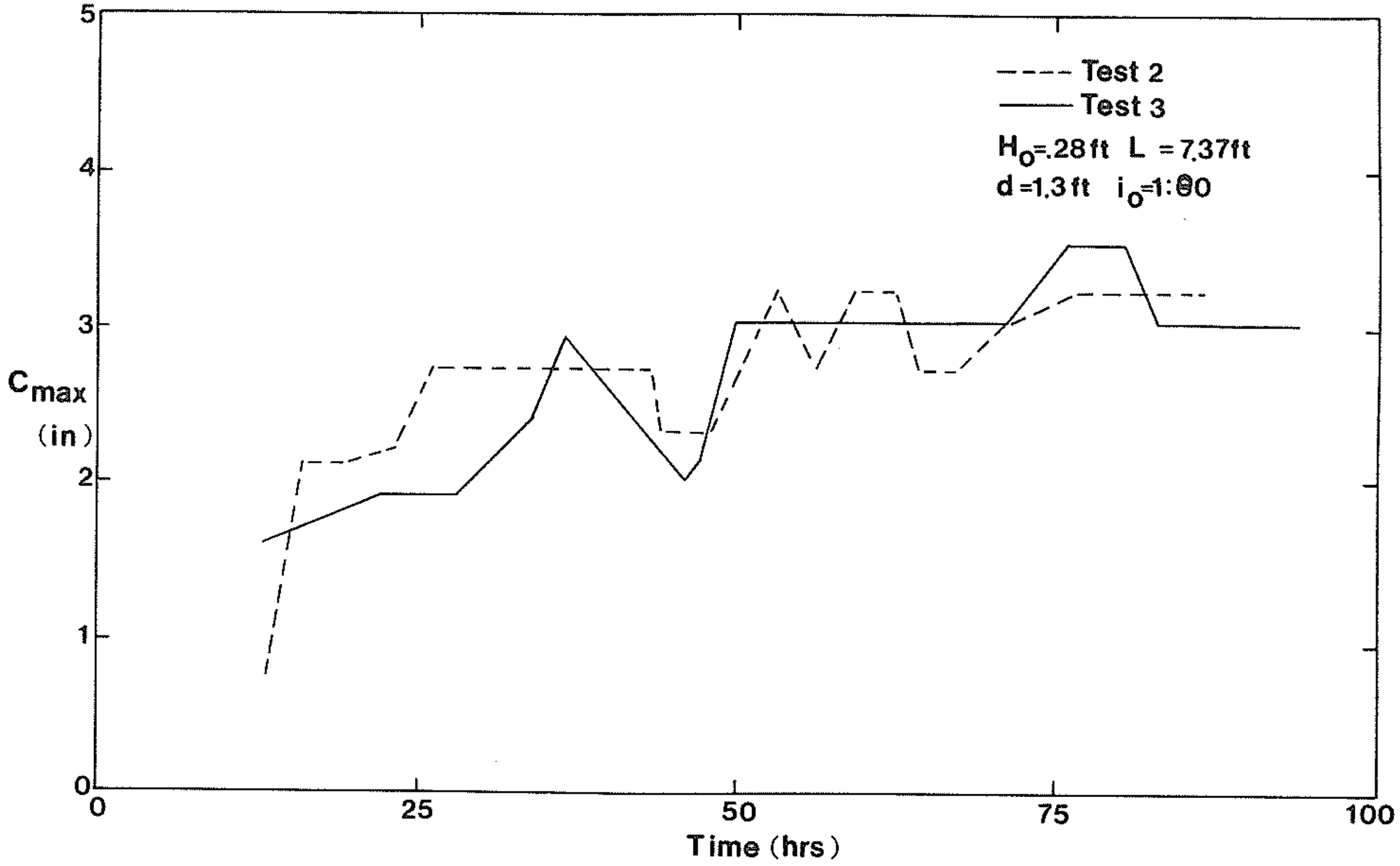

Figure 19. Sequence Repeatability of the Maximum Height of the Beach Crest.

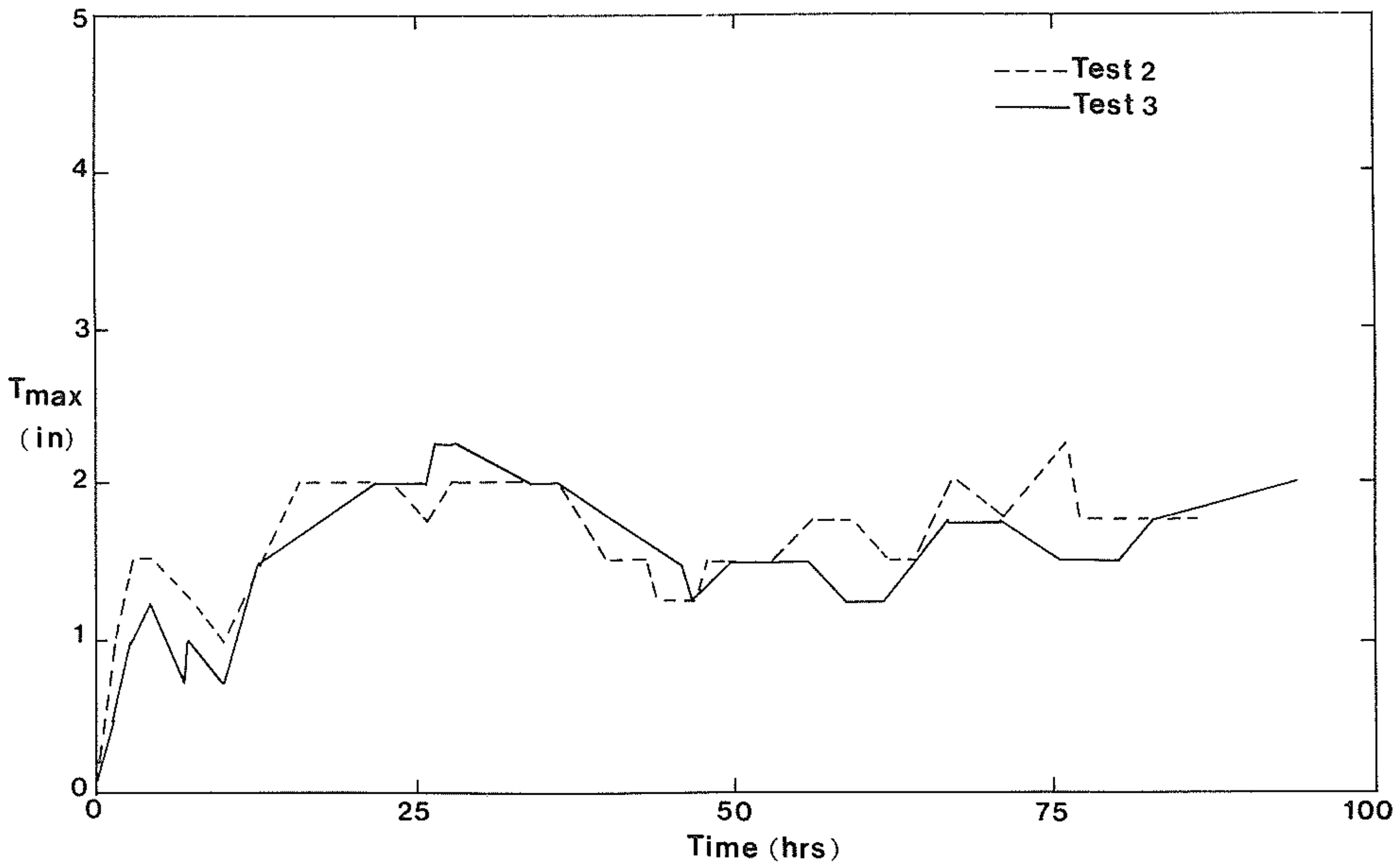

Figure 20. Sequence Repeatability of the Maximum Depth of the Nearshore Trough.

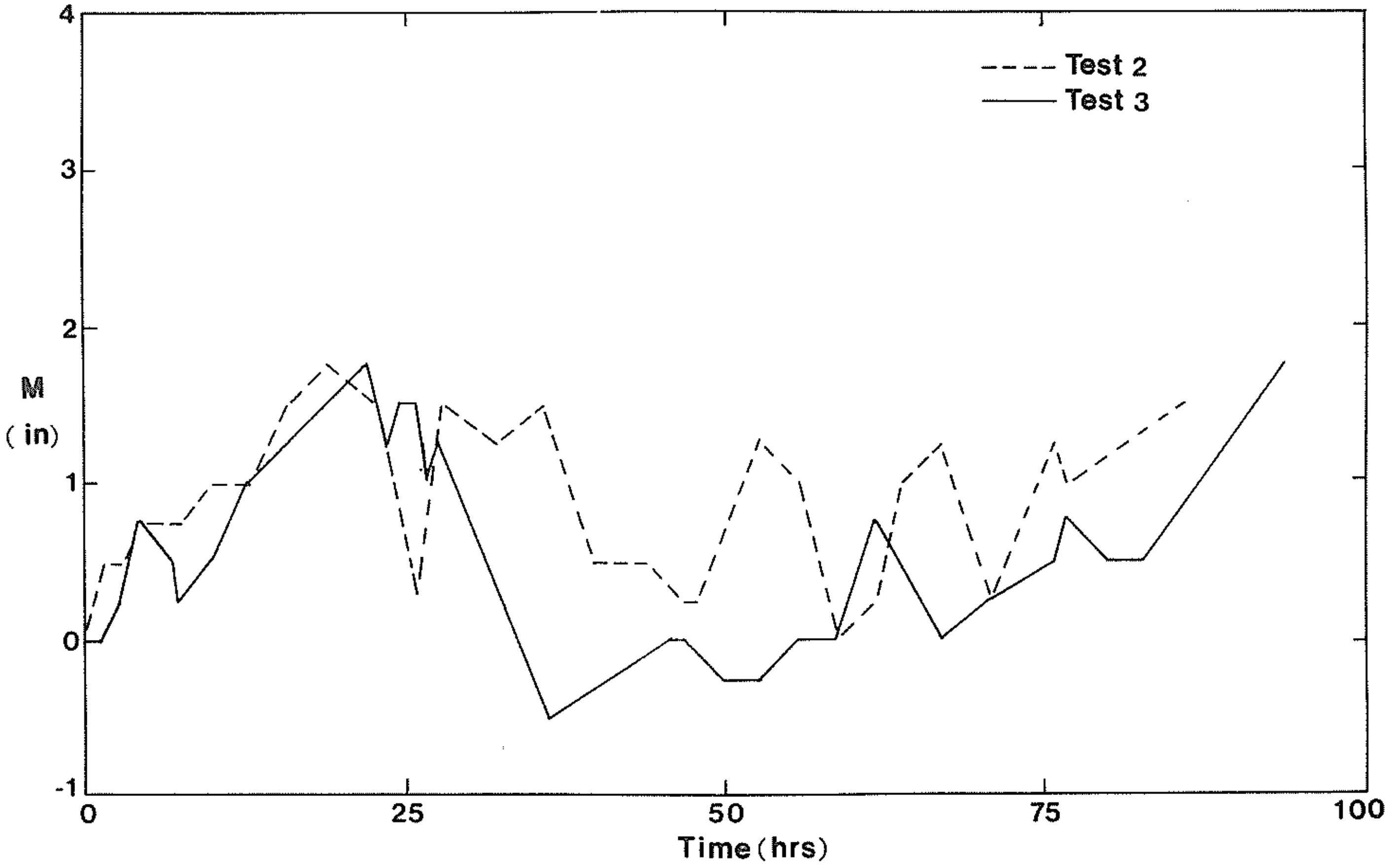

Figure 21. Sequence Repeatability of the Profile Midpoint.

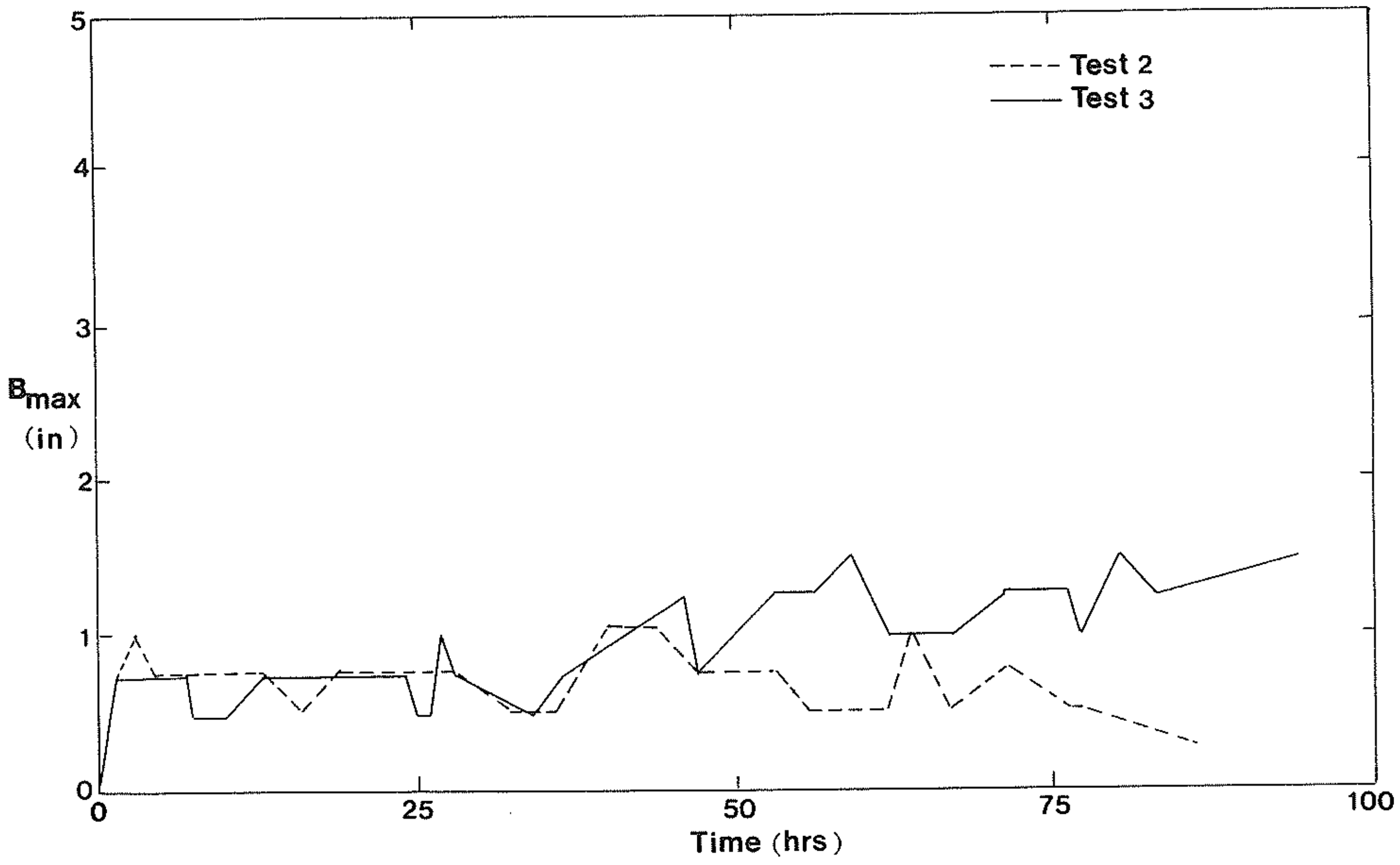

Figure 22. Sequence Repeatability of the Maximum Height of the Offshore Bar.

ties, for two identical tests (Tests 2 and 3). Due to lack of control of water temperature and possible variations in compaction of the initial slope for the two tests, it would seem that the transition from initial to final profile would not be a repeatable sequence. In addition, an oscillating lack of two-dimensionality as observed for the latter part of Test 3 would destroy the repeatability of a sequence. The effect of poor two-dimensionality on the transition profile was, however, not studied. From Figures 19-22, rather close agreement between development sequences is seen to occur initially for all features considered. Also displayed are rather significant differences which had been found to occur for several features later in the tests.

Maximum height of the beach crest and maximum depth of the nearshore trough appear to develop in a similar sequence and to come to a relatively constant value. As was mentioned previously C_{max} maintained this value whereas T_{max} did not. It was observed that C_{max} maintained a relatively stable position, but that the location of T_{max} varied radically with time. Quantities defined by B_{max} and M after an initially similar sequence of repeatability became significantly different. The fact that any of the quantities showed relative satisfaction of sequence repeatability indicates that either the two-dimensionality hypothesis was valid or that the lack of two-dimensionality occurred in a repeatable sequence. The latter appears to be highly unlikely, based on the previously proposed mechanism.

The average percent deviation from the mean, as previously defined, was applied as before. The mean, however, was computed at each given time for each test. High values of this parameter indicate relative disagreement between Test 2 and Test 3 values, and hence poor sequence repeatability. Table I (p. 37) indicates the results. As discussed qualitatively the sequence repeatability of C_{max} and T_{max} is relatively greater. The maximum height of the offshore bar is of low repeatability. The sand level at the profile midpoint showed the poorest behavior relative to satisfying the tested assumption. M was observed to deviate by as much as $\pm$ 200% about the mean at a given time.

The final analysis of Phase I is to assess equilibrium repeatability. As already noted, equilibrium repeatability demands that the hypotheses of two-dimensionality and stability be valid. As several of the quantities already tested have failed these assumptions, these quantities are not equilibrium-repeatable.

Table II summarizes the analysis of the previous sections. The beach quantities tested are ranked in order of relative satisfaction of the tested hypotheses. Rank 1 indicates relative validity of the hypotheses whereas Rank 4 indicates relative invalidity. These data indicate that the maximum height of the beach crest is the only feature which demonstrates a repeatable equilibrium. The maximum height of the offshore bar, while hardly of two-dimensional nature, reached a two-dimensional mean equilibrium. Maximum depth of the nearshore trough and the sand level at a given location were poor with respect to equilibrium repeatability. As the measurement error associated with recording the profile is not the same for the entire profile, the values obtained in the previous analysis and presented in Table V (p. 82) are probably amplified for the nearshore and offshore zone quantities. It is felt, however, that the relative nature of these quantities is correct and that the ranking as performed in Table II is representative of the actual case.

TABLE II

RANK OF TEST QUANTITIES

	C_{max}	T_{max}	M	B_{max}
Stability	1	3	4	2
Two-Dimensionality	1	3	4	2
Sequence Repeatability	1	2	4	3
Equilibrium Repeatability	yes	no	no	no

Note: Tested quantities are listed in order of occurrence progressing in a seaward direction.

To evaluate the repeatable equilibrium hypothesis for the profile as a whole, three identical tests (Tests 1, 2 and 3) were compared for t = 77 hours. This test duration was selected rather arbitrarily to represent a time period longer than that generally found in the literature. Figure 23 shows the results. It was found that minor water level deviations (0.25 inch range) resulted in a significant horizontal range of observed shoreline positions (1.5 feet). This can be seen by considering the initial slope. For a 1:60 initial slope, a change in the vertical of 1 inch results in a 5-foot change in the horizontal. As a result, the three profiles were found not to agree at the shoreline. As the water level in the tank could not be controlled with greater accuracy the resulting profiles were shifted graphically to agree at the stillwater level. Figure 24 shows the shifted profiles.

Figure 24 shows that the general shape of the profiles produced by three identical tests is the same, indicating that the type of profile which is produced is repeatable. However, the quantitative agreement between the three profiles is rather poor. As already demonstrated, certain features are relatively repeatable. The beach crest and the slope of the foreshore, for example, appear to be relatively the same. Repeatability of the sand level at any given location in the nearshore and offshore zones is not demonstrated. As the failure of nearshore and offshore features to satisfy certain laboratory assumptions has already been demonstrated, this result is to be expected.

A mean profile was constructed for the three identical tests and is shown in Figure 25. Error bars, about each sampling point denote the observed deviations, plus and minus about the computed mean for

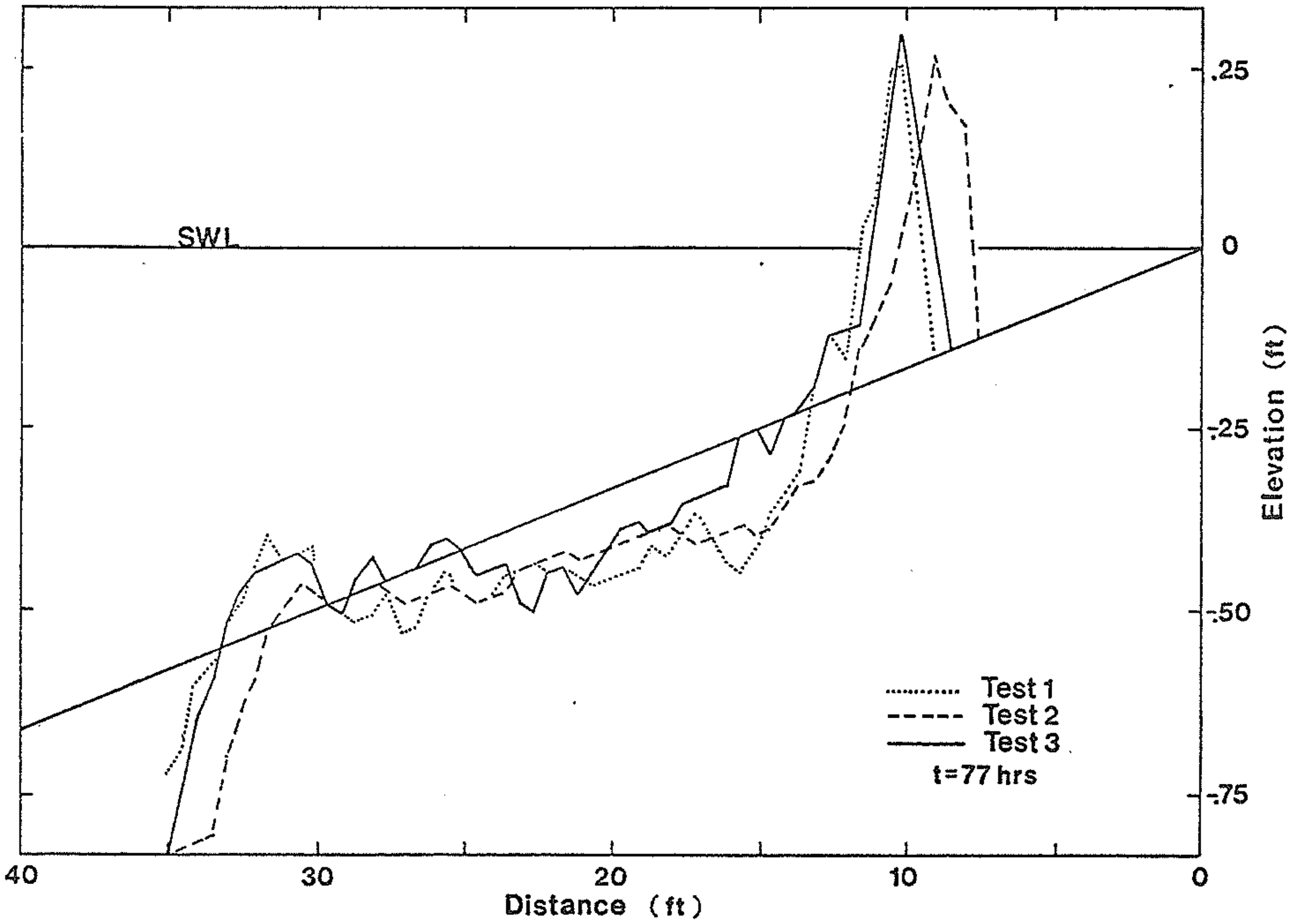

Figure 23. Comparison of Three Identically Repeated Tests at t = 77 Hrs.

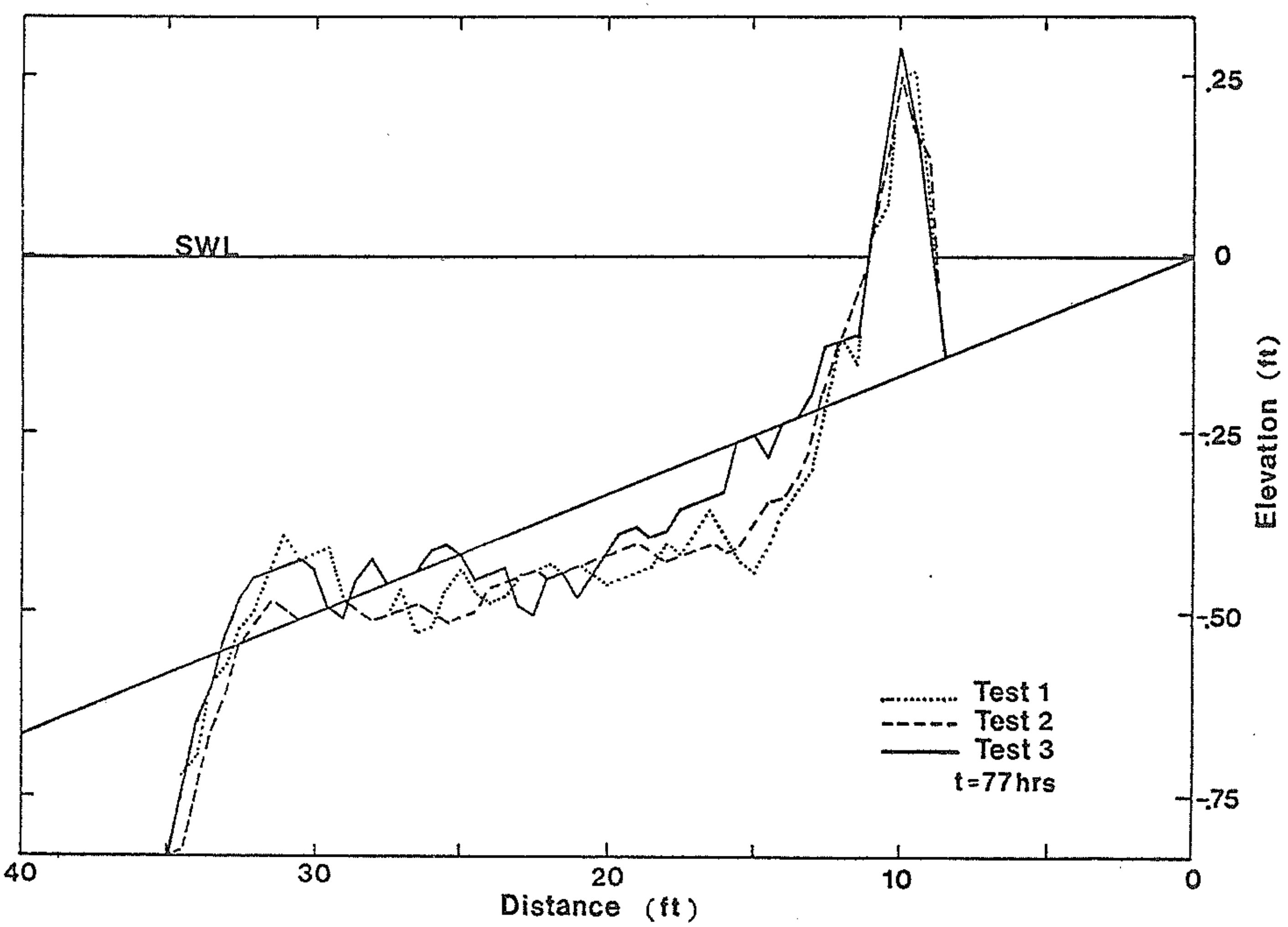

Figure 24. Comparison of Profiles From Three Identical Tests Shifted to Agree at the Final Shoreline.

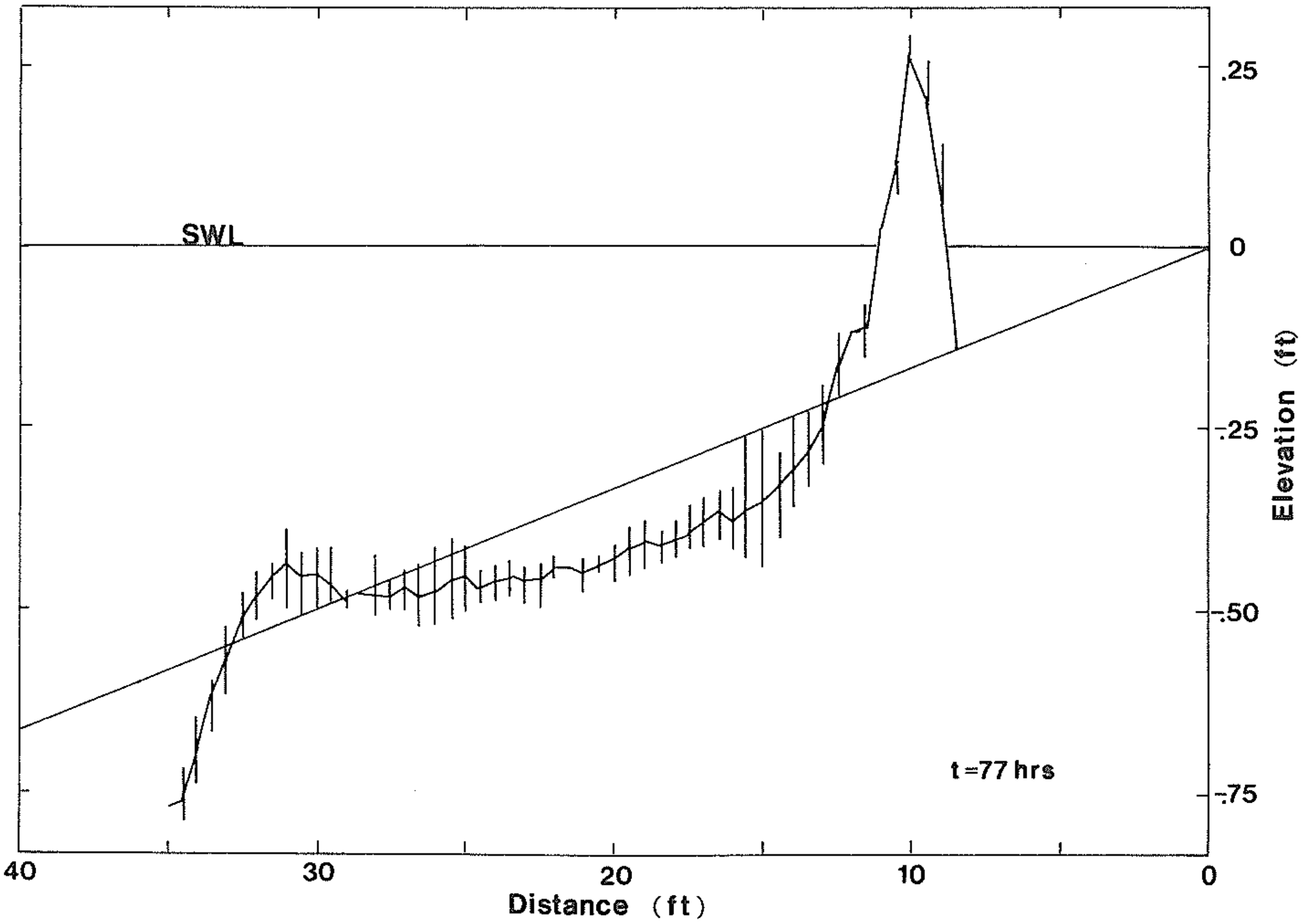

Figure 25. Mean Profile With Observed Deviation for Three Identical Tests.

that location. This, of course, assumes no experimental error in recording the profile. As this assumption is not valid, Figure 25 is designed only to indicate relative deviations from the mean at a given location. Figure 25 supports the conclusion that the beach crest has the most repeatable equilibrium.

For comparison of Tests 1, 2 and 3 at t = 77 hours it was assumed that sufficient time had elapsed for an equilibrium profile to develop (based on literature review). For the given test conditions, further methods were used to determine if equilibrium had been reached: reflection readings from the beach and observation of depth contours.

From the data obtained in the long-term test (Test 3) locations of certain depth contours were obtained and observed as a function of time. The contours plotted for their most seaward occurrence only, would come to a stable value with time for an equilibrium profile. Figure 26 shows this type of analysis as presented by Chestnutt and Galvin (1974) for a Type I beach. As can be seen from the figure the profile did not come to equilibrium. Figure 27 shows depth contours for the Type II profile (Test 3) of this research. As indicated by fluctuations in the -0.3 foot and -0.4 foot seaward contours, the profile did not reach a stable shape in as much as 400 hours.

Minor water level variations had a significant effect on shoreline position for the 1:60 initial slope. At approximately 250 hours a shoreline recession occurred after the beach crest location was stable for the preceding 150 hours. This can be observed in Figure 27 by the shoreward movement of the 0.1 and 0.2 foot contours shortly after t = 200 hours. Water level variations for this period remained within the 1.3 ± 0.02 foot range as for the preceding test period. No

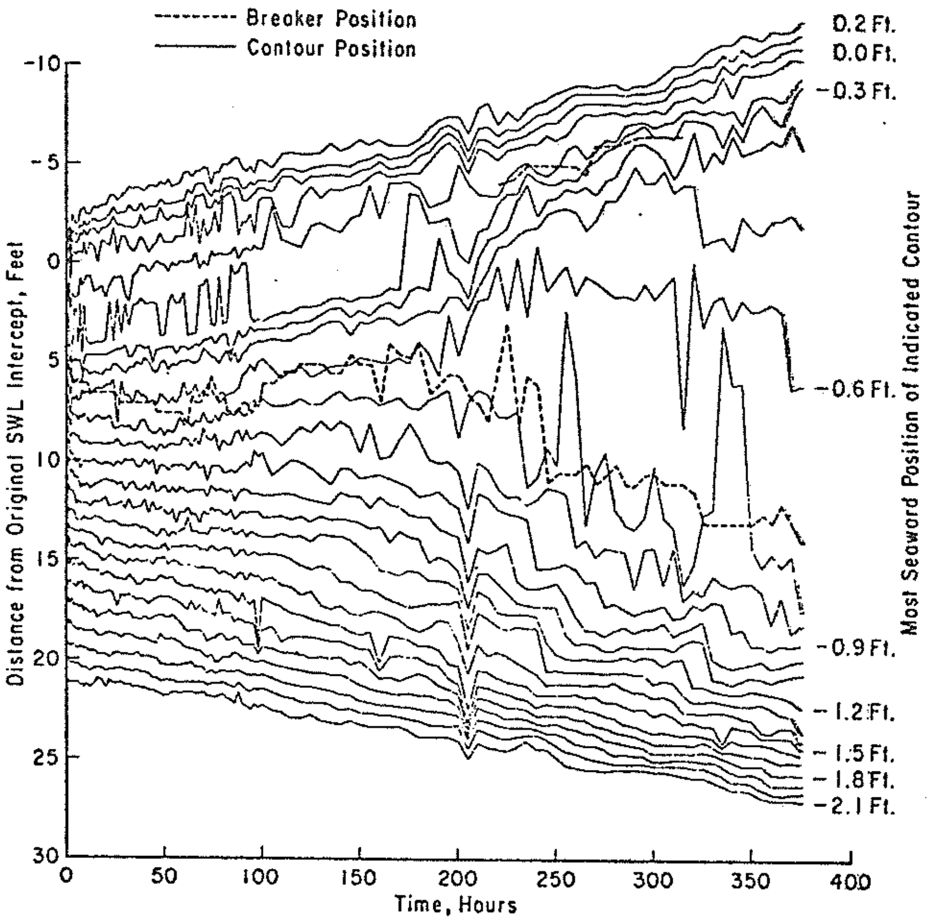

Figure 26. Most Seaward Occurrence of Depth Contours for a Type I Profile.
After Chestnutt and Galvin (1974).

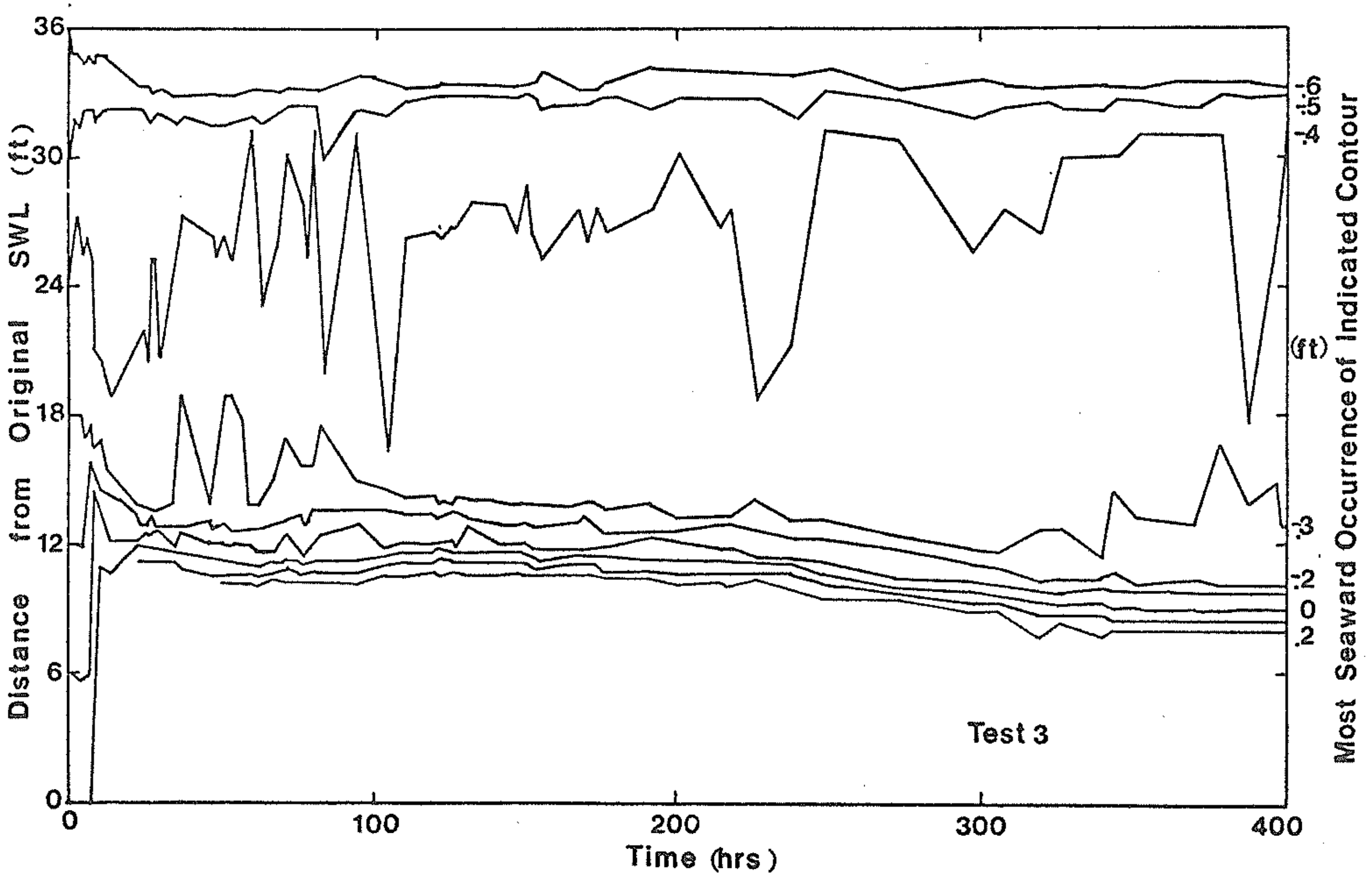

Figure 27. Most Seaward Occurrence of Depth Contours for a Type II Profile.

explanation for the change is evident in the data. Most probably, an increase in water level from the lower limit of the depth range to the upper limit occurred. Unfortunately, no record of the water level within the range was maintained. From Figures 13 (p. 32) and 28 it can be seen that the period of adjustment had a lower C_{max} and an increasing coefficient of reflection. Also, from Figure 14 (p. 33) T_{max} had just completed a sudden decrease. A sudden decrease in T_{max} would probably allow a more energetic uprush. However, the mechanism which caused the decrease in T_{max} is not obvious. No concrete evidence exists for explaining this sequence of events.

As an additional attempt to determine the occurrence of equilibrium, reflection records were taken throughout Tests 2 and 3. It was felt that if the bed reached a stable shape, the reflection would reach a stable value. This was not the case, as indicated in Figure 28. The coefficient of reflection K_r varied radically even through 400 hours. Figure 29 shows a comparison between the two tests for the first 77 hours. Experimental error for the recording procedure was found to be approximately $\pm$ 0.015. A rather similar trend in reflection coefficient was observed for the two tests. Large variations in the coefficient were not found to correlate well with changes of any of the beach features, however.

Chestnutt and Galvin (1974) found reflection variations to correlate with foreshore and offshore steepening. For a movable bed, it was observed that K_r varied from 0.05 to 0.30, whereas for a constant 1:10 concrete slope it varied only 0.03 to 0.07. He found variations in the -0.7 foot contour related to variations in the reflection

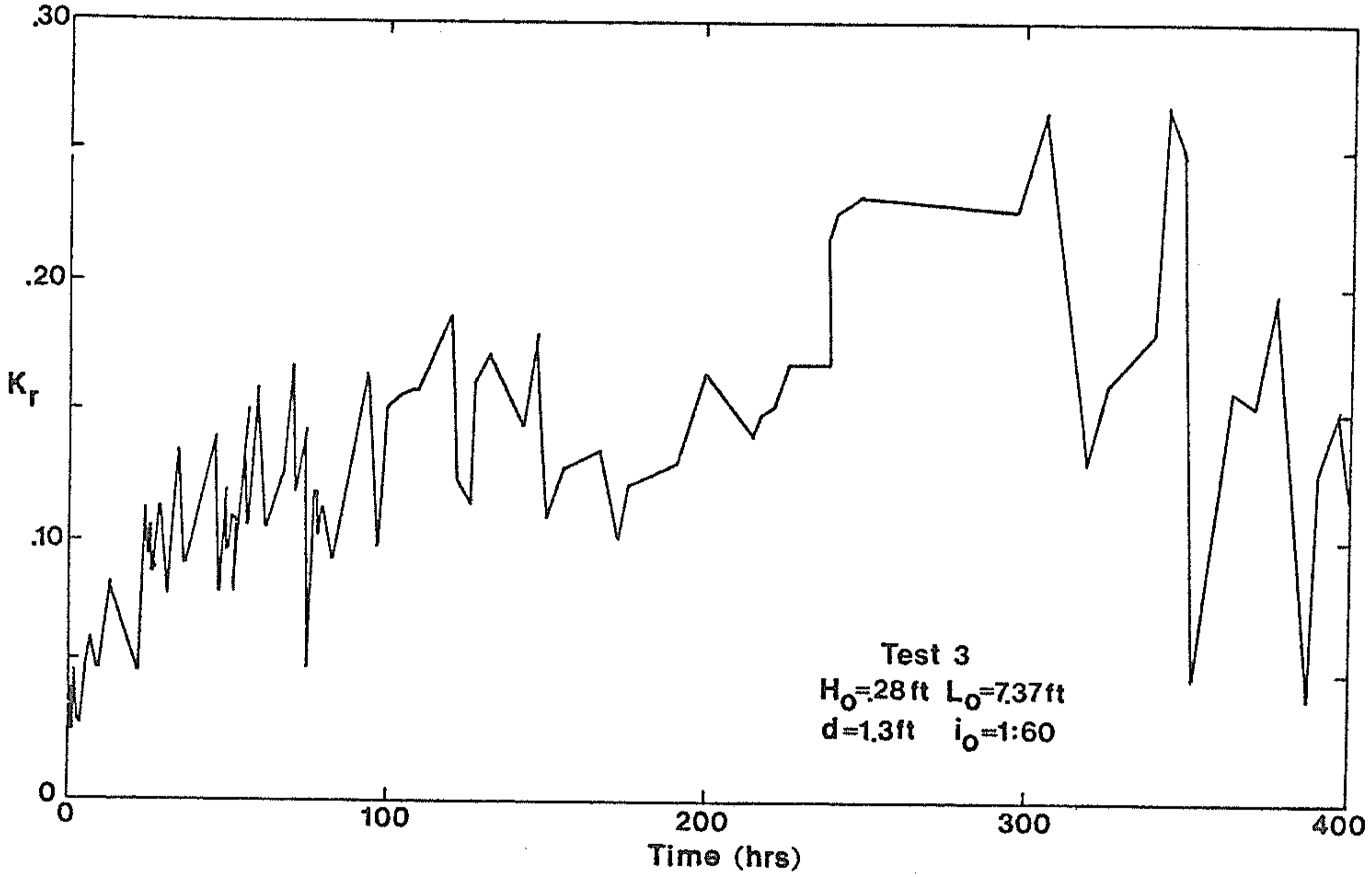

Figure 28. Coefficient of Reflection for the Long-term Test.

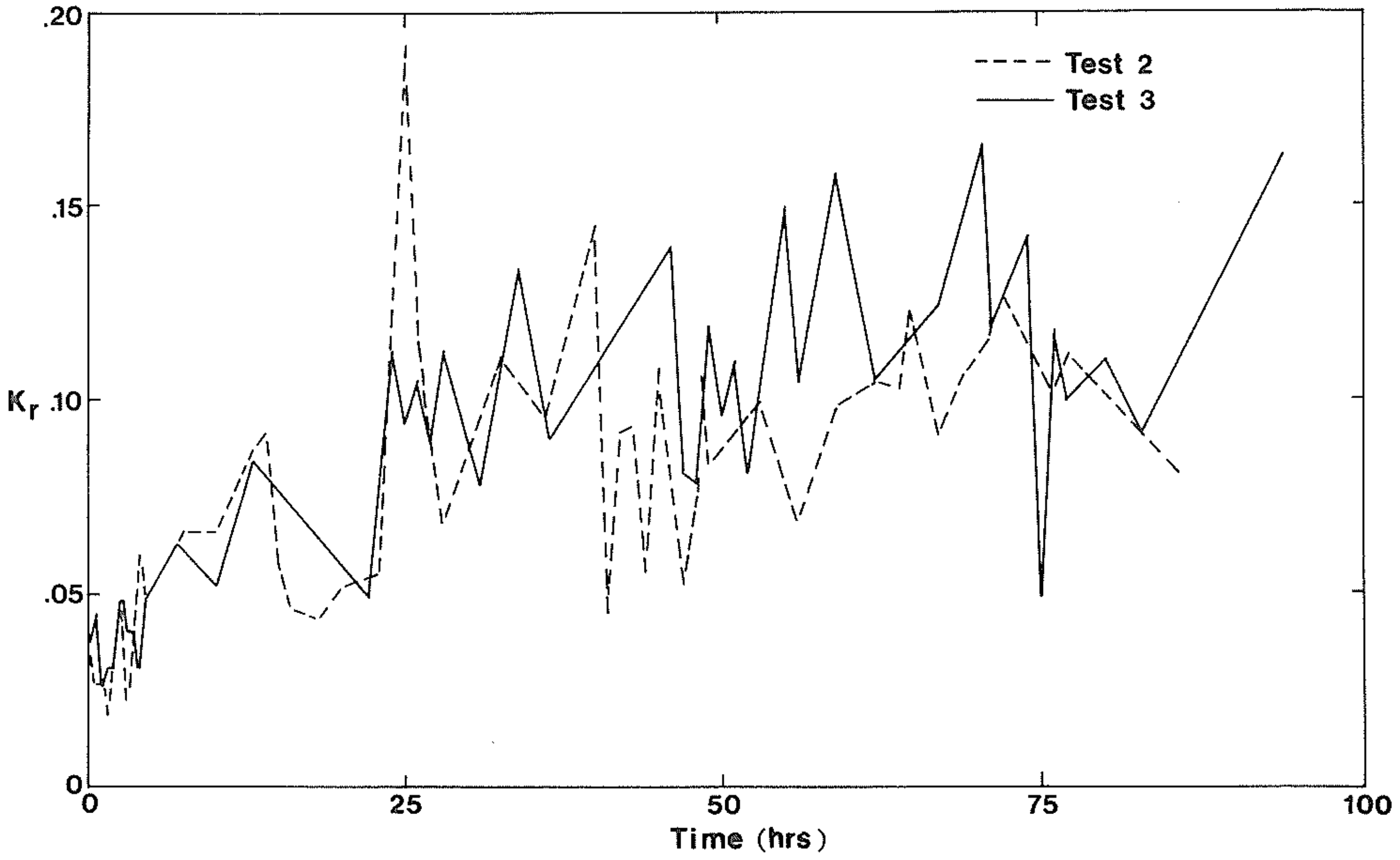

Figure 29. Repeatability of Reflection for the Long-term Test.

coefficient after 175 hours. He suggested that reflection is very sensitive to depth changes near the offshore bar. Changes in this depth would result in changes in reflected energy from the offshore slope and variability in the amount of energy trapped in the inshore shelf. Figure 30 shows this to be only a partially valid explanation. Observation of sequences of profiles indicated that the occurrence of multiple bars on the profile coincided with periods of high reflected energy. Figures 31 and 32 give two representative examples. Attempts to correlate K_r [Figure 28 (p. 58)] with the profile area indicator [Figure 17 (p. 38)] were unsuccessful. A detailed study of the reflection phenomena was not within the scope of this research.

As pointed out by Chestnutt and Galvin (1974), the variability in reflection observed in movable bed studies can result in a wide range of measured wave height. For this research, K_r varied from 0.04 to 0.26 with a variation in measured wave height from 0.22 to 0.32 feet. This wide range of measured wave heights indicates that wave height is not exactly known from a given spatial or temporal measurement. This requires that the wave height used must be a time-and-space average of a number of observations spaced throughout the test and along the tank. This, in part, points to the third hypothesis mentioned in the scope of the study. If, in fact, wave height for a given test is determined by a single measurement, then attempts to correlate beach changes with wave height should prove unsuccessful. An assumption made in determining linear regressions is that the independent variable, in this case, wave height, is known exactly. Wave period, generally obtained by timing the wave generator, should not vary. Confidence in

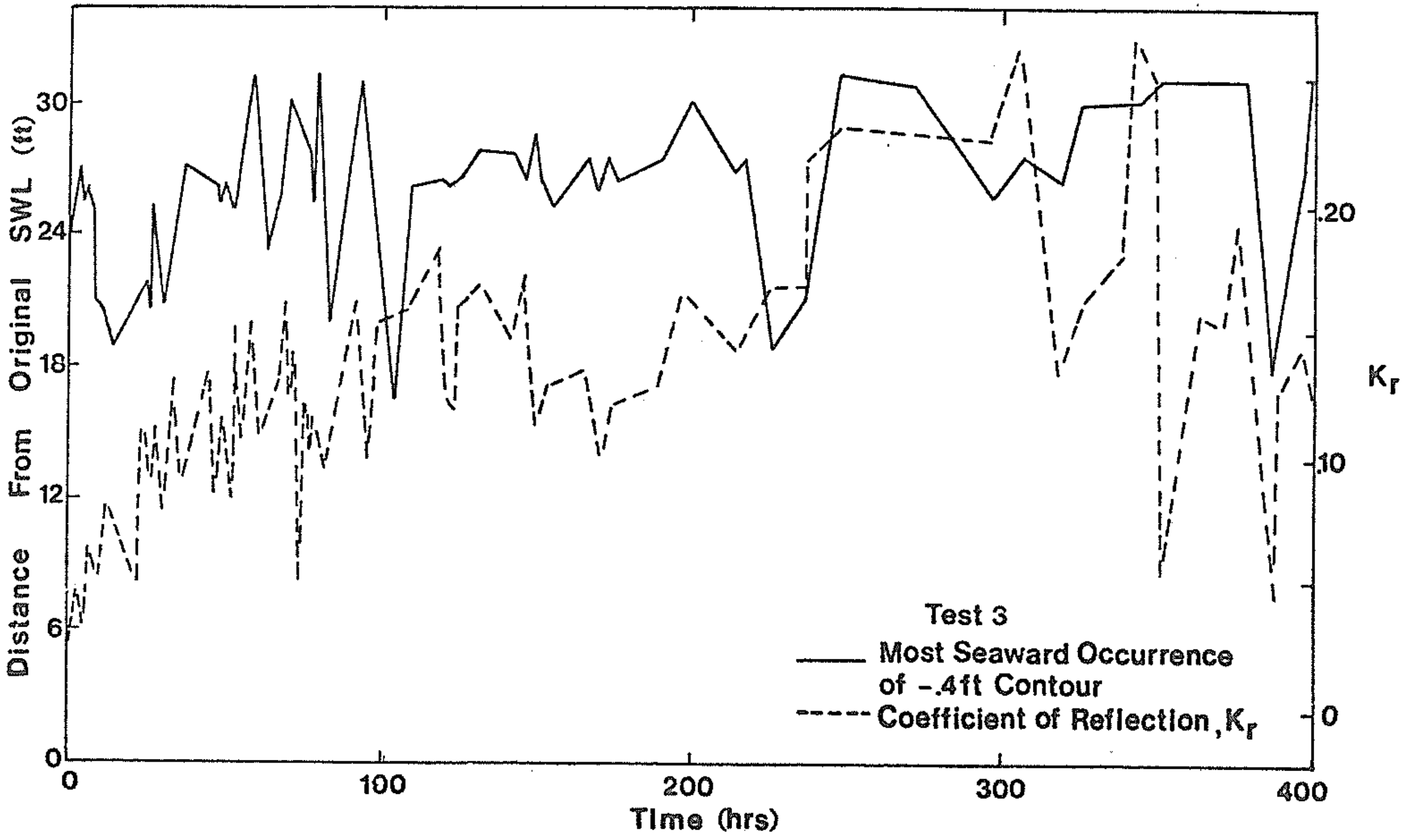

Figure 30. Comparison of Coefficient of Reflection Variations with Variations in the -0.4 ft. Contours.

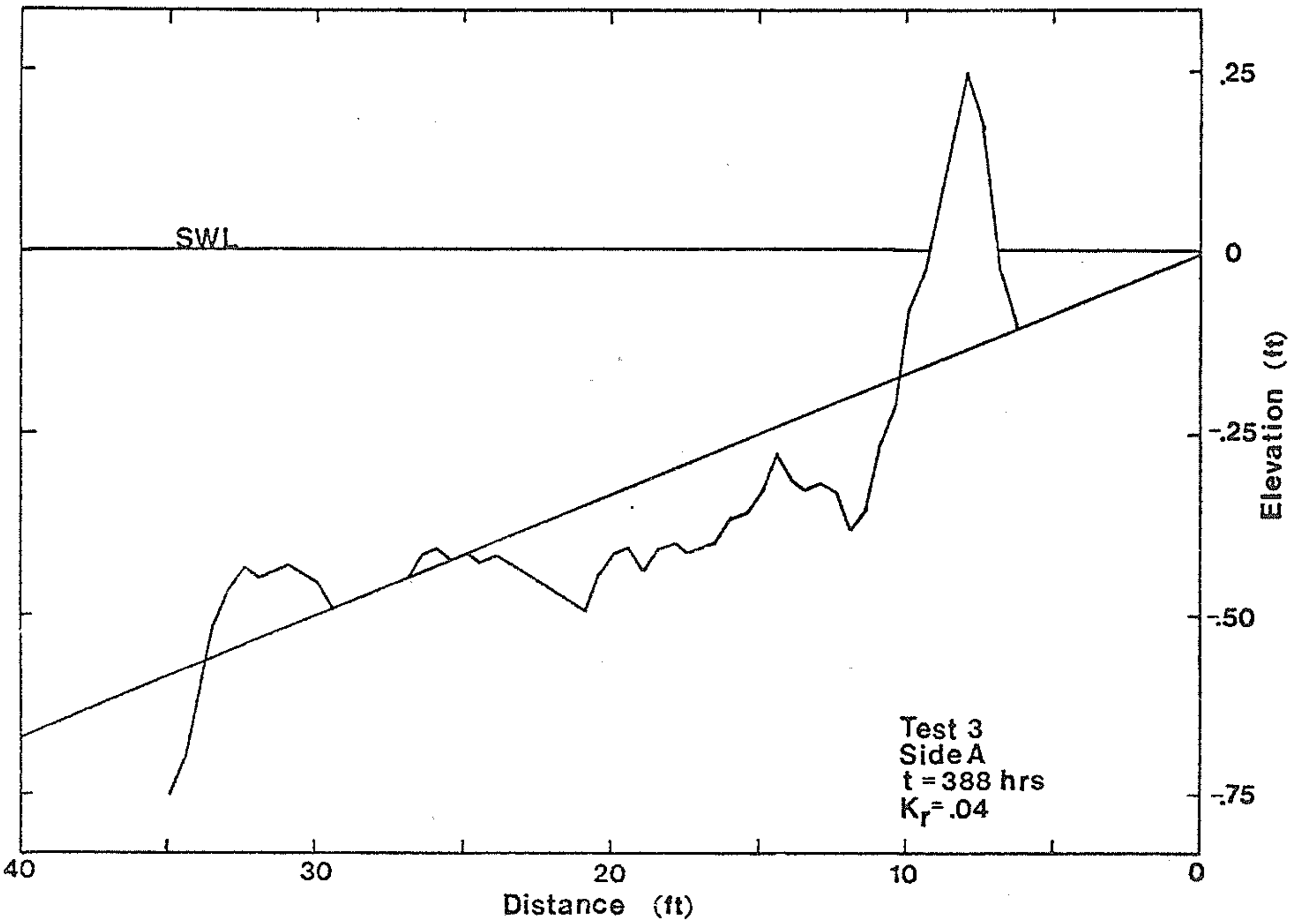

Figure 31. Representative Beach Profile for Low Coefficient of Reflection.

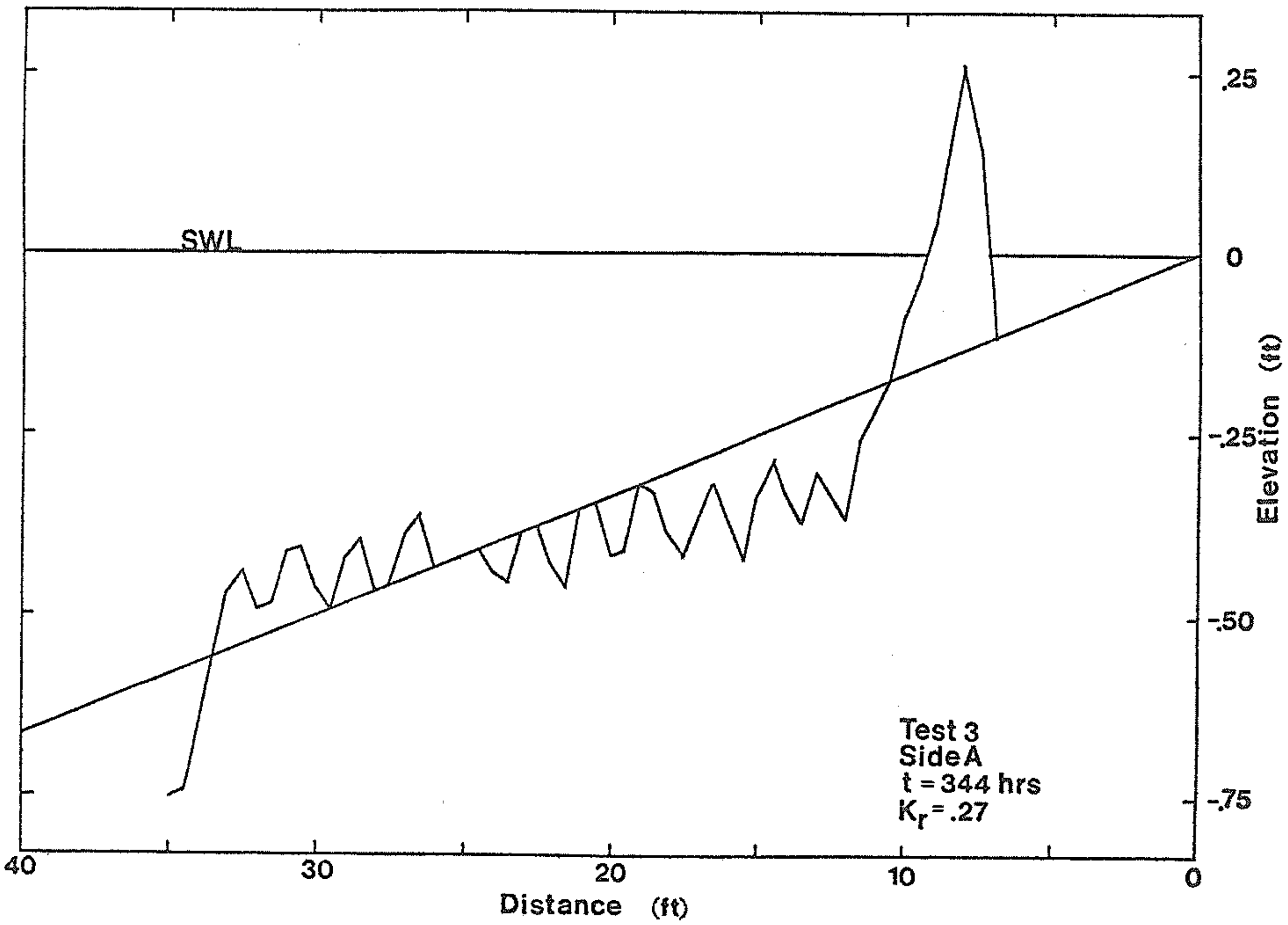

Figure 32. Representative Beach Profile for High Coefficient of Reflection.

determining this variable is thus much greater than that for wave height.

As previously mentioned, other variables such as initial slope, initial length, water temperature, grain shape and size distribution may be important in determining profile quantities. These are the variables generally not considered in the literature. Collins and Chestnutt (1975) found that a uniform sand size with narrow grain size distribution and smooth spherical shapes produced a profile with an unstable offshore bar. From Figure 16 (p. 35) this trend was not observed in this research, whereas the sand used fits the description of narrow grain size distribution and smooth spherical shapes. The growth and disappearance of multiple bars, however, was observed to occur. Collins and Chestnutt (1975) also observed multiple bars on beaches with strongly bi-modal grain size distributions and waves of small steepness.

Phase I Conclusions

The beach profile for the given wave, beach and tank characteristics did not come to equilibrium. The effect of minor water level variations was found to be an important variable in determining the location of the shoreline. Its effect on the prevention of equilibrium of the long duration test is difficult to ascertain. The beach crest feature of the beach does appear to reach a repeatable two-dimensional equilibrium form. The offshore bar feature may be described in terms of a time mean quantity. The number of variables which may have a significant effect on the beach formation process is large. Even the more obvious ones, such as wave height, are not exactly known from a given measurement.

Phase II Tests

Based on Phase I conclusions, an analysis of previous tests was executed. As noted in Phase I conclusions, wave height is variable throughout a given test due to variable reflection. Thus, for the purposes of correlating beach quantities with wave parameters, the deep water wave length was selected. Deep water wave length, calculated using linear wave theory is dependent on wave period, a variable which is more likely to remain constant throughout a given test. Also, as previously mentioned, the beach crest height appears to come to a stable repeatable form. Hence, for the existing data (Appendix A), C_{max} was correlated with deep water wavelength.

The data used in Phase II analysis are for three initial slopes: 1:20, 1:30, and 1:60. These are listed in Tables V, VI, and VII of Appendix A. Extrapolation of Phase I results to different slopes is questionable. As such, Phase II analysis is limited. Phase II data are not available at closely spaced time intervals, but observation of sequences of profiles for a given test indicate stability within shorter times than those observed for stable Phase I features. Additionally, considering the initial slopes, minor water level variations may not have been as influential on shoreline fluctuations. Figures 33 and 34 indicate that the beach crest feature may come to a stable dimension for the 1:20 and 1:30 tests as well as in the Phase I tests.

The profiles observed and recorded in Phase II testing were Type II profiles with an offshore bar and an accreted shoreline. Figure 35 shows that the observed profiles are in agreement with the

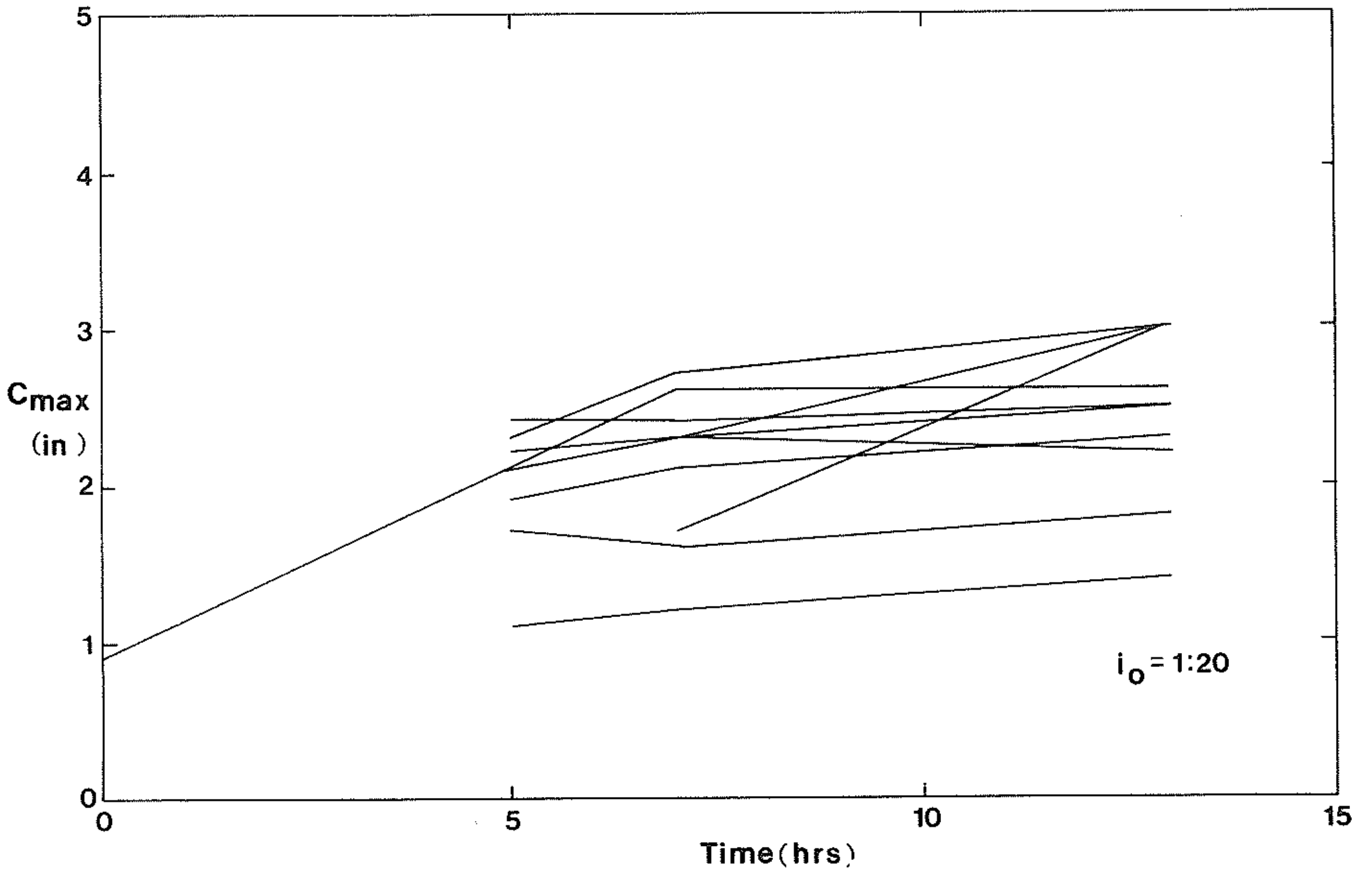

Figure 33. Beach Crest Development for All 1:20 Tests.

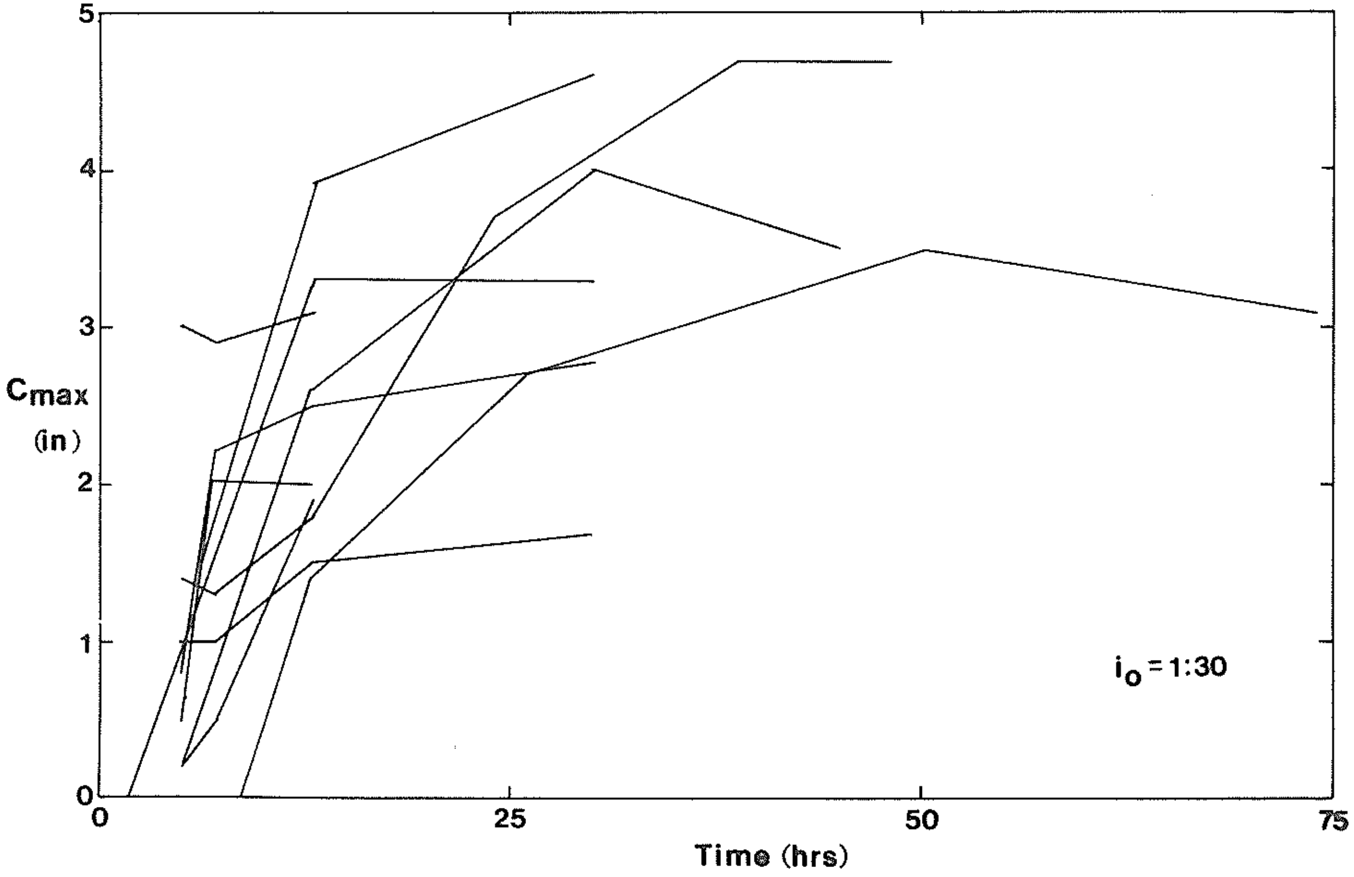

Figure 34. Beach Crest Development for All 1:30 Tests.

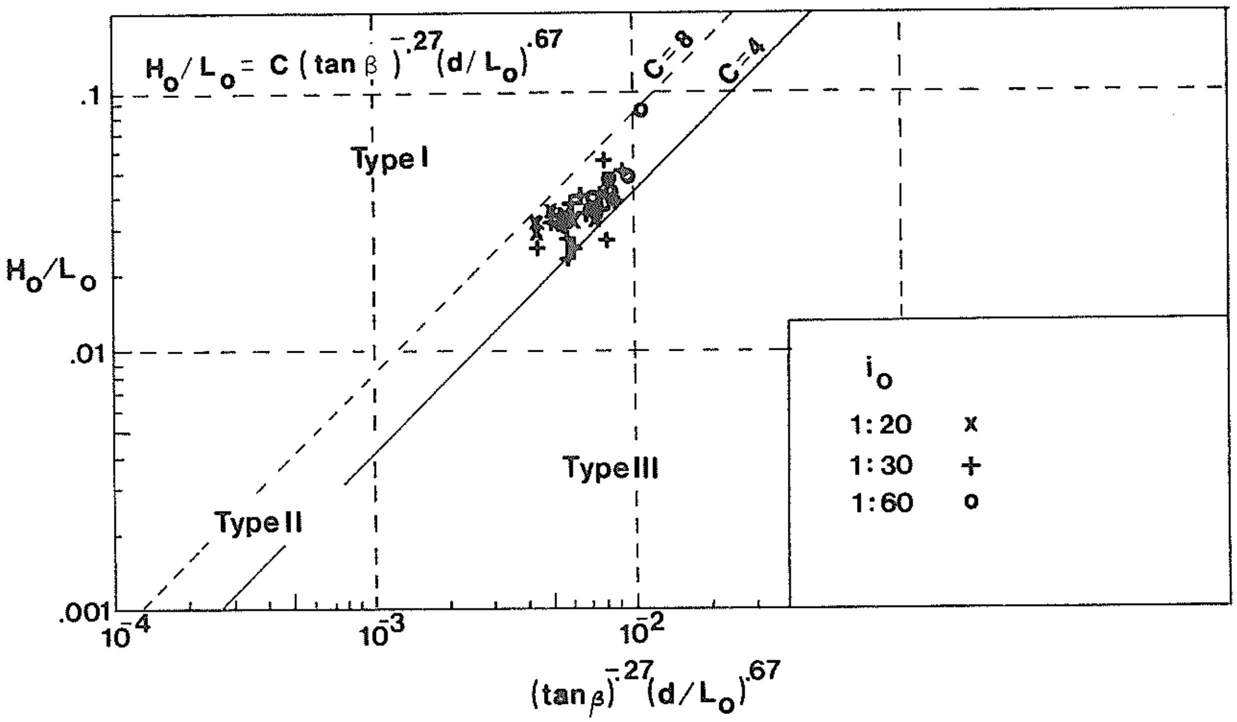

Figure 35. Classification of Observed Phase II Beach Profiles. (As Compared with Figure 6, p. 14).

classification scheme proposed by Sunamura and Horikawa (1974). Hence, any conclusions made in this analysis will be restricted to the Type II beach. Also, as the problem of scale effects is not evaluated in this research, the extrapolation of these conclusions to prototype dimensions is not intended.

Figures 36 and 37 show the results of linear regression application. C_{max} is the dependent variable and deepwater wavelength, L_0, is the independent variable. As observed, rather significant relationships were found. Table III gives the statistical parameters obtained in the regressions. The correlation coefficients were tested for significance at the 5% level.

Figures 38 and 39 show linear regressions for dependency of C_{max} on H_0. Surprisingly, one of the correlations was determined to be significant, as indicated in Table III. As H_0, for Phase II was determined in spot measurements the lack of correlation is understandable. If H_0 was based on time-averaged measurements the correlation would hopefully improve.

For the 1:60 data, no significant relationships were found. The 1:60 testing of Phase II was of a limited extent. Also, of possible influence on the regression, is the fact that the majority of the 1:60 data was of a different nature from the 1:20 and 1:30 tests. The pipeline located in the tank was placed directly on the surface of the 1:60 initial profile, whereas for the steeper slopes, the pipeline was located at either one-half or one diameter below the initial profile surface. The influence of the surface position pipeline on C_{max} would seem negligible but no proof exists based on the data. Conclu-

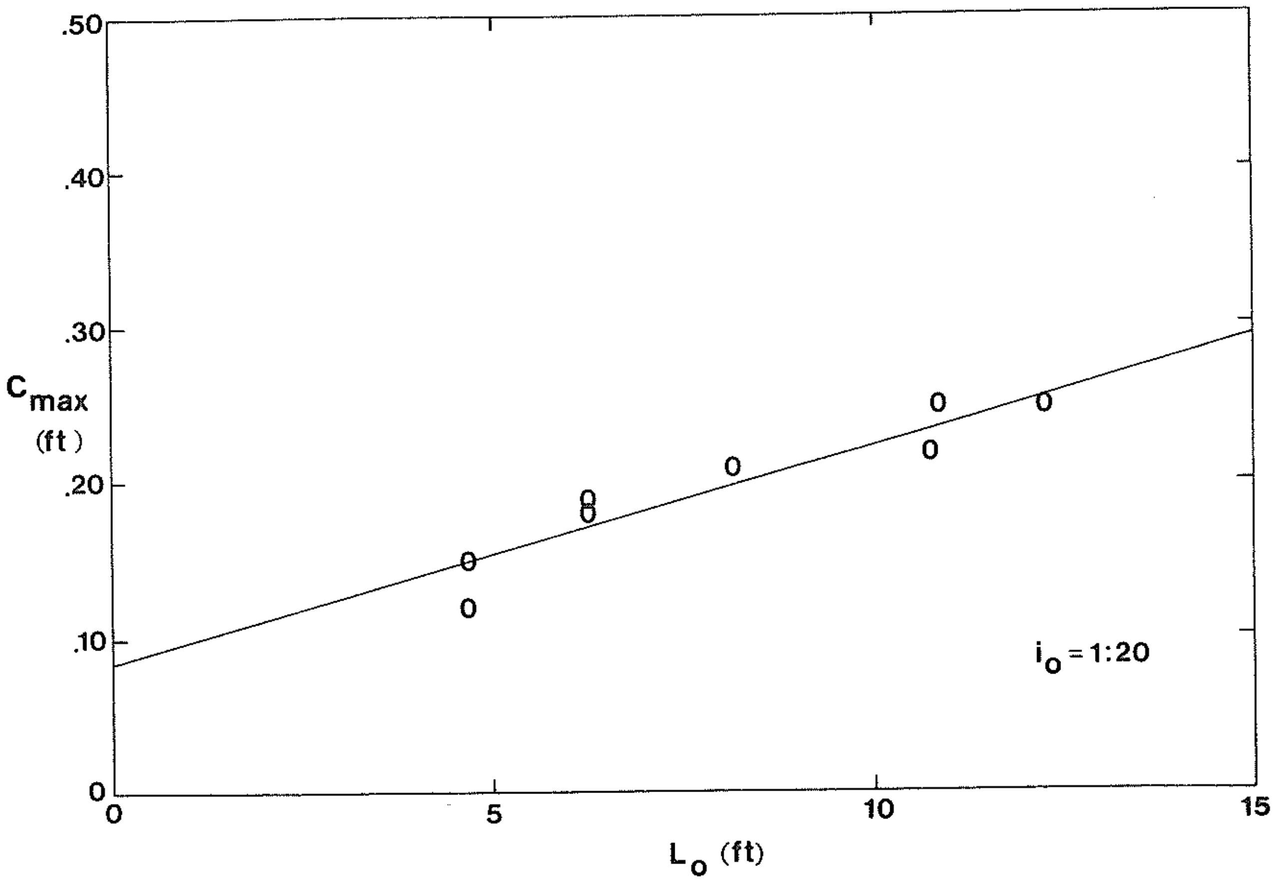

Figure 36. Linear Regression of Maximum Height of Beach Crest on Deepwater Wavelength for All 1:20 Tests.

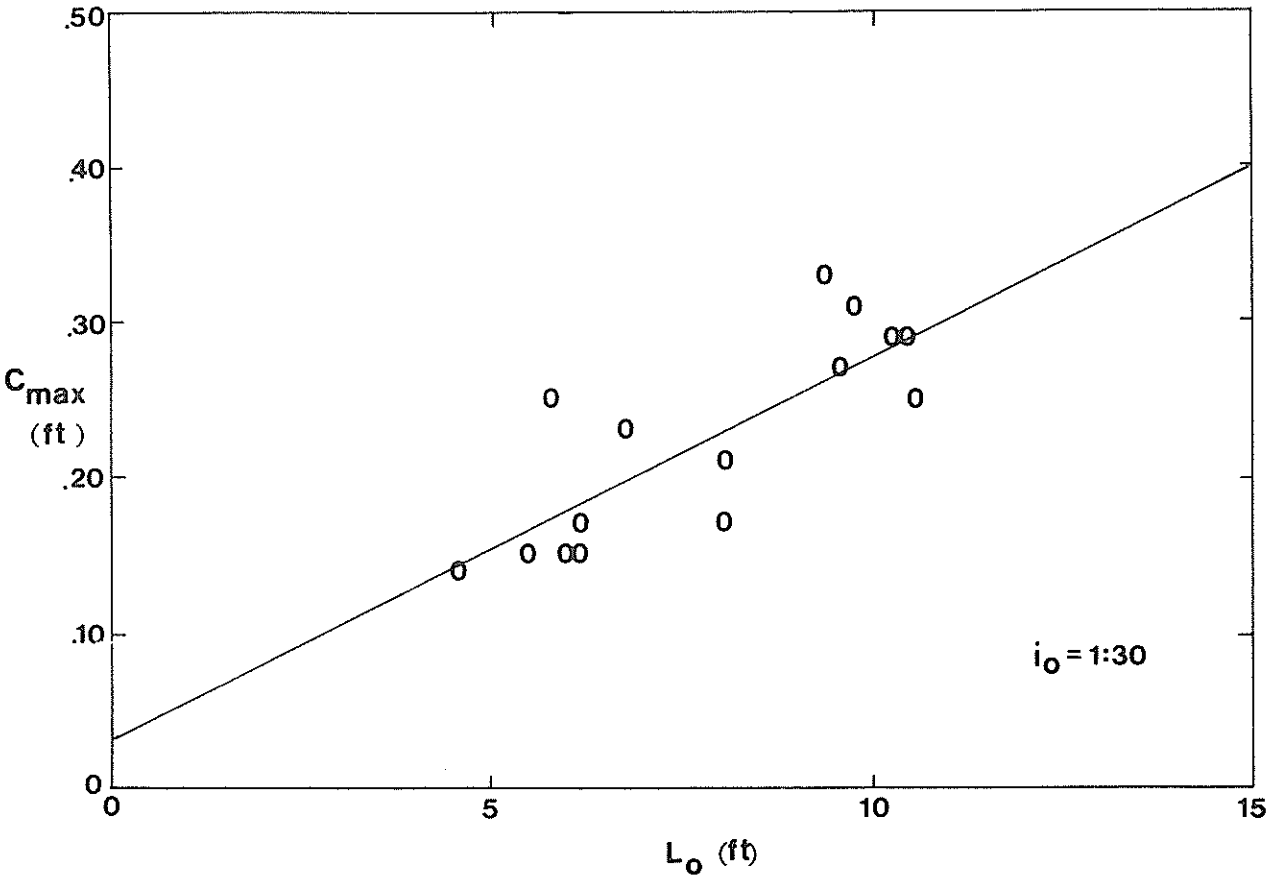

Figure 37. Linear Regression of Maximum Height of Beach Crest on Deepwater Wavelength for All 1:30 Tests.

TABLE III

REGRESSION RESULTS

L_o vs. C_{max}

	r	r^2	ν	$r(0.05, \nu)$	Significant
1:20	0.94	0.88	8	0.632	yes
1:30	0.86	0.75	14	0.497	yes
1:60	0.60	0.37	4	0.811	no

H_o vs. C_{max}

	r	r^2	ν	$r(0.05, \nu)$	Significant
1:20	0.88	0.78	8	0.632	yes
1:30	0.43	0.19	14	0.497	no
1:60	-0.18	0.03	4	0.811	no

Where

r = correlation coefficient

ν = degrees of freedom = number of observations - 2

$r(0.05, \nu)$ = tabulated value of r at the 5% significance level

r^2 = percent variance explained by the linear relationship of C_{max} and L_o

C_{max} = maximum height of the beach crest above the still-water level

L_o = deepwater wavelength

H_o = deepwater wave height

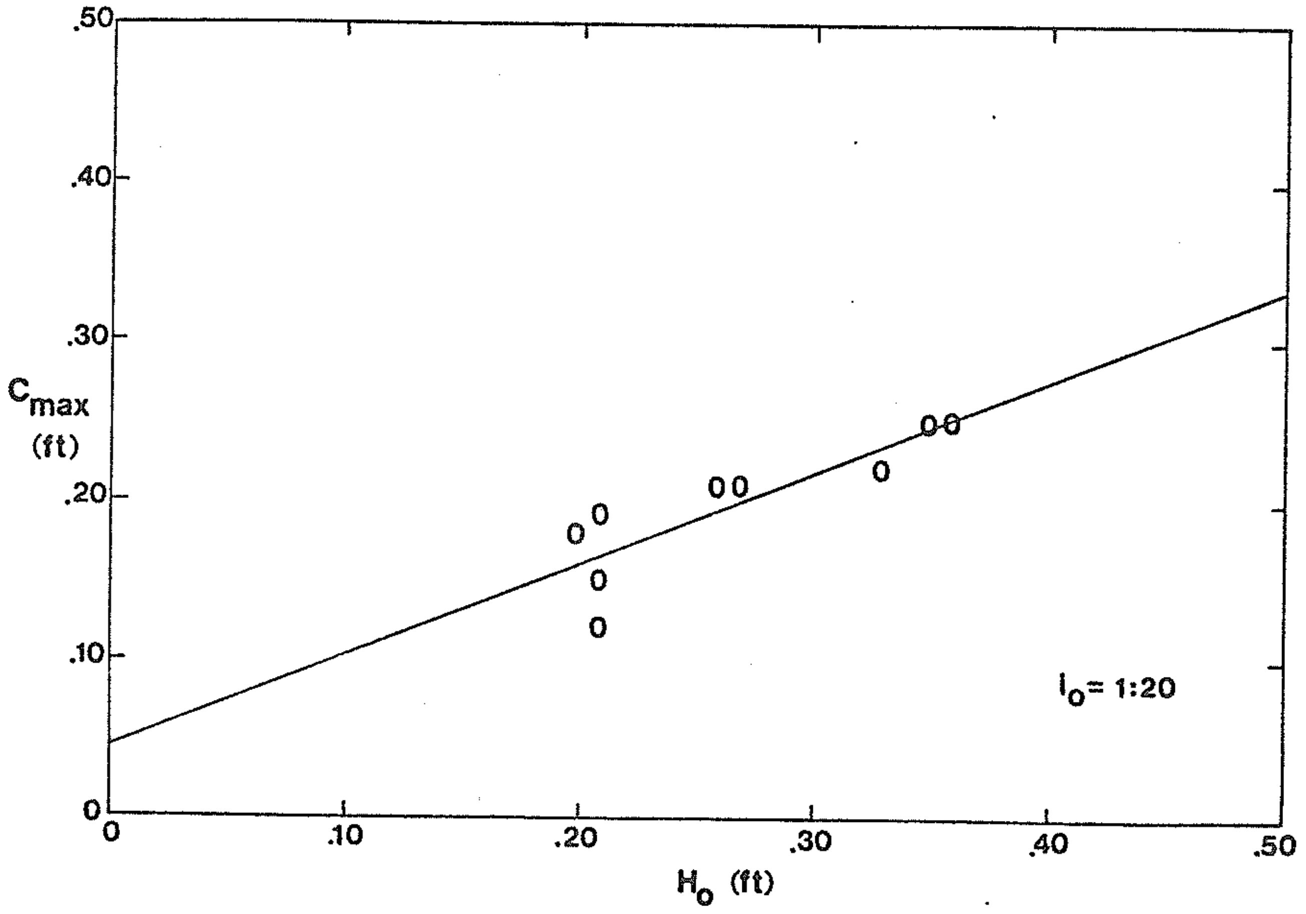

Figure 38. Linear Regression of Maximum Height of Beach Crest on Deepwater Wave Height For All 1:20 Tests.

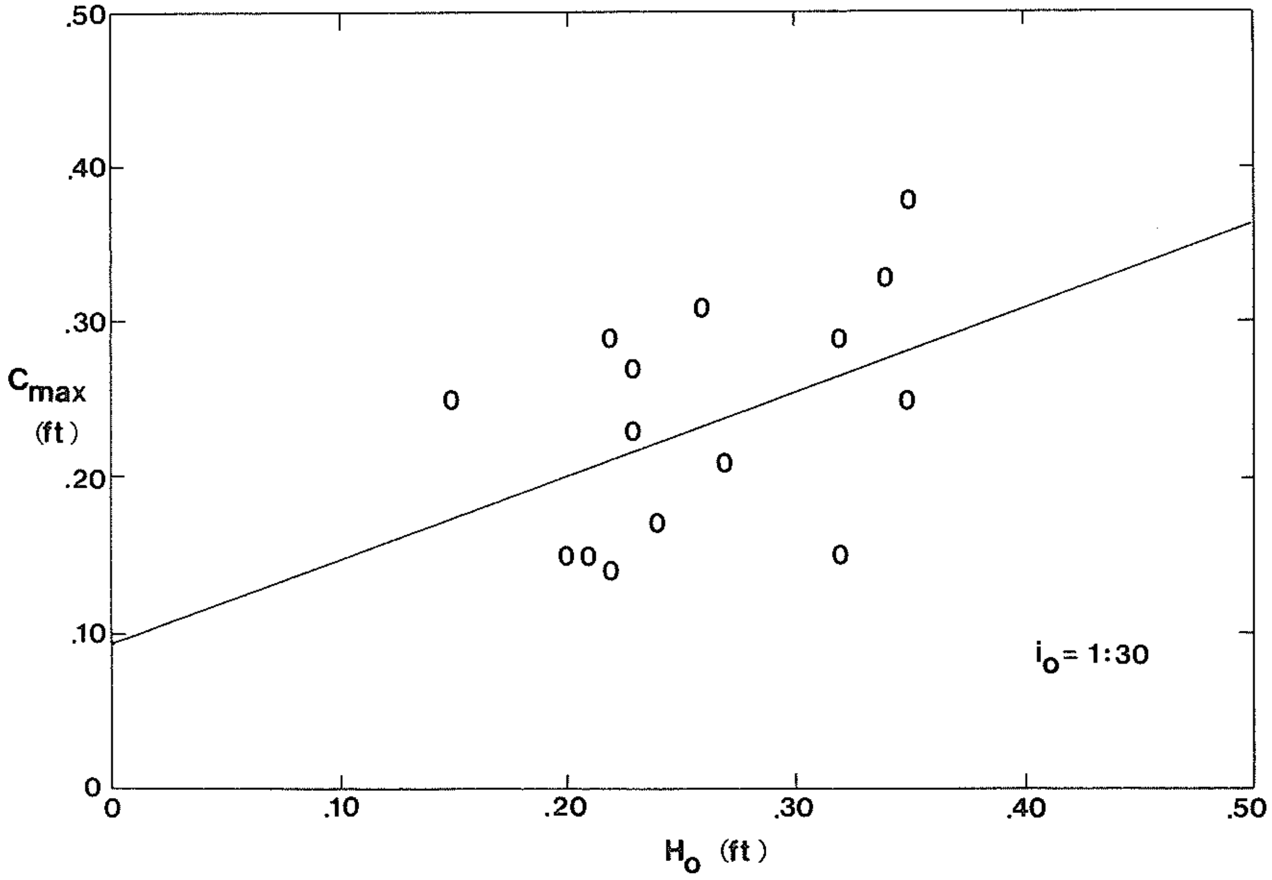

Figure 39. Linear Regression of Maximum Height of Beach Crest on Deepwater Wave Height for All 1:30 Tests.

sions as to the dependency of C_{max} on L_o will be restricted to the steeper slopes.

Statistical tests were performed to determine if the regression lines obtained for the 1:20 and 1:30 data were significantly different. If not, this would mean that they represent the same population and hence, initial slope of the model beach has no influence on the relationship between C_{max} and L_o. Confidence intervals were determined for the regression slopes and intercepts (Till, 1974). Appendix B presents the statistical concepts. The 95% confidence intervals for the slope and intercept of the 1:20 data are

$$b_1 = 0.014 \pm 0.003$$

$$a_1 = 0.085 \pm 0.028$$

Similarly for the 1:30 data

$$b_2 = 0.025 \pm 0.006$$

$$a_2 = 0.030 \pm 0.061$$

Based on a two-tailed t-test, it was determined that at the 95% confidence level the populations represented by the 1:20 and 1:30 data are significantly different. Thus, for the given range of test variables, the dependence of C_{max} on L_o is not independent of initial slope. Thus, for the 1:20 initial slope the equation of the regression line is

$$C_{max} = 0.085 \pm 0.014L_o$$

and for the 1:30 initial slope

$$C_{max} = 0.030 \pm 0.025L_0.$$

These conclusions are valid only for the range of tested variables and usefulness of results in larger natural dimensions is not intended.

Other investigators have examined the dimensions of the beach crest. Rector (1954) found the following relation:

$$C_{max} = 0.024\ L_0 \text{ for } H_0/L_0 > 0.018.$$

He used laboratory beaches with initial slopes 1:15, 1:20, 1:30 and a combination slope of 1:10 foreshore and 1:20 offshore. His result is then considered independent of slope, a result not in agreement with this study. This regression, however, does fall within the 95% confidence interval for this study, a remarkable fact considering the different tank and beach variables used here. Bagnold (1940) also studied the beach crest. He found

$$C_{max} = 1.3\ H_0$$

for beaches with initial slope approximately 1:7. This is in marked disagreement with the significant relationship found here; $C_{max} = 0.04 \pm 0.58\ H_0$. This supports the conclusion that comparison of results between individual investigations is difficult due to the large number of variables generally not accounted for.

Silvester (1974) explains beach crest development as follows. Long-period, swell waves (low H_0/L_0) deposit material on the beach face and short-period, storm waves (high H_0/L_0) erode material from the beach face in a manner related to porosity of the beach and the water table level. When long-period waves persist, sufficient time is available for percolation of the water into the beach face and the backwash is not as strong as for short-period waves. For short-period waves saturation of the beach face is much more attainable as the water table coincides with the beach face. In the saturated state, the beach face prohibits percolation and deposition of material and thus a strong backwash and resulting erosive force exists. As the water table and porosity are not reproduced to scale in the model, it is doubtful that the relationships obtained in the study for the height of the beach crest are similar to those for natural prototype scales. The results do agree qualitatively in that shorter period waves create smaller beach crests.

Phase II Conclusions

The maximum height of the beach crest above the stillwater level is dependent on the deepwater wavelength. In addition, initial slope has a significant effect on the relationship. Similar relationships may exist between the maximum height of the beach crest and the deepwater wave height, if the deepwater wave height is known in terms of space and time averages.

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

For a 1:60 initial slope, model beach equilibrium was not observed in as much as 400 hours. Repeatability of beach profiles under identical wave conditions was found to be rather poor, in conjunction with lack of two-dimensionality in the two-foot-wide tank. Minor water level variations in the range of 1.3 ± 0.02 feet had a significant effect on shoreline position in the tank and may have been a factor in preventing an equilibrium, although absence of two-dimensionality is also responsible.

The maximum height of the beach crest came to a repeatable equilibrium that was independent of minor water level variations. The maximum height of the offshore bar reached an oscillatory equilibrium, and may possibly be defined as a time mean quantity. Other observed features of the beach exhibited poor laboratory characteristics. Maximum depth of the nearshore trough and sand level at the profile midpoint showed absence of both stability and two-dimensionality and, hence, non-repeatability. All quantities defined as maxima, independent of location, resulted in better relative verification of the tested assumptions than fixed location quantities.

The coefficient of reflection varied radically even after 400 hours and was found to be not suitable as an indicator of beach profile equilibrium. Variation in wave height due to variable reflected energy was observed. This requires that wave height determination for movable bed studies be in terms of time and space averages. Wavelengths determined from spot measurements of wave periods are representative of wavelengths for the entire test. The stable repeatable feature observed for a 1:60 initial slope, maximum height of the beach crest above the stillwater level, was found to correlate well with the deep water wavelength. It was determined that initial slope has significant effect on the relationship. For the range of variables tested the following relationships were determined:

For a 1:20 initial slope

$$C_{max} = 0.085 + 0.014L_o$$

For a 1:30 initial slope

$$C_{max} = 0.030 + 0.025L_o.$$

Recommendations

The conclusion that minor water level variations prevent a repeatable equilibrium may be significant if, in fact, equilibrium is attainable for exactly constant water depth. Since a 1:60 initial slope is desirable to minimize the necessary scale distortions, its laboratory behavior needs further evaluation.

Certain variables, generally not accounted for, may have a significant influence on model beach profile studies. The influence of tank variables such as initial length from generator to beach and tank width; beach variables such as grain shape and size distribution and initial slope; and the water temperature need to be evaluated. As these variables are generally not accounted for, comparisons among existing model results are difficult.

The time necessary to reach equilibrium, if in fact equilibrium exists, needs further study. The wide range of values found in the literature also makes model comparisons difficult. A standardized method for determining equilibrium time as a function of beach, tank, and wave characteristics would be of great value to beach modellers. In addition, the concept of "equilibrium" needs further investigation.

Scale effects and the relation between model and tank beaches need to be studied. The calibration of the model requires numerous prototype data and trial-and-error model scale distortion procedures. However, these recommendations assume that the laboratory behavior of the model beach is well understood, an assumption of questionable validity.

APPENDIX A - TEST DATA

TABLE IV

CHESTNUTT (1975) DATA COMPARED WITH PHASE I DATA

	Chestnutt (1974)	Phase I Data		
Parameters	Test 1-06	Test 1	Test 2	Test 3
Tank Width (ft)	6	2	2	2
Initial Length (ft)	93	98	98	98
Initial Slope	1:10	1:60	1:60	1:60
Final Profile Type	I	II	II	II
T(sec)	1.90	1.20	1.20	1.20
L_0(ft)	14.3	7.37	7.37	7.37
D_{50}(mm)	0.2	0.3	0.3	0.3
d(ft)	2.33	1.30	1.30	1.30
H_0(ave)	0.36	0.33*	0.28	0.28
t_{max}	375.00	77.00	86.00	400.00

* Determined from a spot measurement (not an average quantity).

TABLE V

1:20 Test Data

	Test									
Parameter	1E	1C	2E	2C	3E	3C	4E	4C	5E	5C
T(sec)	1.55	1.55	1.45	1.46	1.26	1.27	1.11	1.11	0.96	0.96
L_0(ft)	12.30	12.30	10.79	10.85	8.18	8.22	6.33	6.33	4.73	4.73
H_0(ft)	0.36	0.35	0.33	0.35	0.26	0.27	0.21	0.20	0.21	0.21
d(ft)	1.75	1.75	1.75	1.75	1.75	1.75	1.75	1.75	1.75	1.75
t_{max}(hrs)	13.00	13.00	13.00	13.00	13.00	13.00	13.00	13.00	13.00	13.00
B_{max}(ft)	0.25	0.25	0.22	0.25	0.21	0.21	0.19	0.18	0.12	0.15

TABLE VI

1:30 Test Data

Parameter	1C	2C	2E	3C	3E	4C	4E	5C	6E	7E
	Test									
T(sec)	1.55	1.45	1.44	1.26	1.26	1.10	1.10	0.95	1.42	1.69
L_o(ft)	12.30	10.76	10.58	8.13	8.13	6.20	6.20	4.61	10.32	14.54
H_o(ft)	0.38	0.35	0.35	0.27	0.27	0.21	0.24	0.22	0.22	0.35
d(ft)	1.75	1.75	1.75	1.75	1.75	1.75	1.75	1.75	1.75	1.75
t_{max}(hrs)	13.00	13.00	13.00	13.00	13.00	13.00	13.00	13.00	13.00	13.00
B_{max}(ft)	*	*	0.25	0.17	0.21	0.15	0.17	0.14	0.29	0.38

Parameter	8E	9E	10E	11E	12E	13E	14E	15E
	Test							
T(sec)	1.04	1.15	1.39	1.07	1.36	1.37	1.43	1.32
L_o(ft)	5.54	6.77	9.82	5.81	9.40	9.61	10.47	5.99
H_o(ft)	0.20	0.23	0.26	0.15	0.34	0.23	0.32	0.32
d(ft)	1.75	1.75	1.75	1.75	1.75	1.75	1.75	1.75
t_{max}(hrs)	30.00	65.00	45.00	13.00	45.00	30.00	30.00	45.00
B_{max}(in.)	0.15	0.23	0.31	0.25	0.33	0.27	0.29	0.15

*Beach crest equilibrium not indicated by profile sequences.

TABLE VII

1:60 Test Data

Parameter	2D	4A	5A	6A	7A
T(sec)	0.99	1.07	1.19	1.29	1.22
L_o(ft)	4.99	5.81	7.25	8.52	7.56
H_o(ft)	0.42	0.27	0.30	0.31	0.30
d(ft)	1.30	1.30	1.30	1.30	1.30
t_{max}(hrs)	145.00	30.00	96.00	45.00	30.00
B_{max}(ft)	0.17	0.19	0.21	0.25	0.17

APPENDIX B

STATISTICAL CONCEPTS

The populations are assumed to have normal distributions Π_1, Π_2, with means $\overline{x}_1$, $\overline{x}_2$; standard deviations, s_1, s_2' number of observations, n_1, n_2; linear regression slopes b_1, b_2; intercepts a_1, a_2; and correlation coefficients r_1, r_2.

The standard deviation of the intercept is

$$s_a = s_y \left(\frac{1-r^2}{n} \left[1 + \frac{\overline{x}^2}{s_x{}^2}\right]\right).$$

The standard deviation of the slope is

$$s_b = b \left(\frac{1-r^2}{n}\right).$$

The 95% confidence levels ($\alpha = 0.05$) for the slope and intercept are $b \neq 1.96s_b$ and $a \neq 1.96s_a$, respectively.

For comparison of the regression lines a critical t-value was calculated

$$t_o = \frac{|b_1 - b_2|}{s_{b_1}{}^2 + s_{b_2}{}^2}$$

The hypothesis that the two populations are significantly different, based on their regression line slopes, is accepted for $t_o \leq -t$ or $t_o \geq t$, where $t_{(\alpha,\ n_1 + n_2 - 2)}$ is obtained from standard tables. Otherwise the hypothesis is rejected.

APPENDIX C

NOTATION

B_{max} - maximum height of the offshore bar above the original profile

C_{max} - maximum height of the beach crest above stillwater level

D - pipeline diameter

d - water depth

D_{50} - median grain size

H_a - wave height at the antinode

H_i - incident wave height

H_n - wave height at the node

H_o - deepwater wave height

H_r - reflected wave height

K_r - coefficient of reflection

i_o - initial slope

L_o - deepwater wave height

M - sand level at the profile midpoint

r - correlation coefficient

S - specific gravity

SWL - stillwater level

t_{max} - maximum duration of the model test

T_{max} - maximum depth of the nearshore trough below the original profile

α - significance level

ν - degrees of freedom

μ - vertical scale (y_m/y_p)

λ - horizontal scale (x_m/x_p)

γ' - relative specific weight of bed material

γ - specific gravity

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