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The U. S. Standard Atmosphere on the HP67/HP97

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General

When one of pressure, height, temperature, or potential temperature is input, the program outputs the other three. The program is on a magnetic card, one side containing the program itself, the other side the constants.

Input and output

To operate the program, first read in both sides of the magnetic card. Then key in the input parameter and press the key shown in the table. Outputs will be stored in the registers as shown, including the input parameter.

Mathematical symbol	Input parameter	Key	Output register
p	Pressure (mb)	A	A
z	Height (m)	B	B
T	Temperature (°A)	C	C
θ	Potential temperature (°K)	D	D

On the HP97 output will also be printed in the order: pressure, height, temperature, potential temperature. On the HP67 output will be flashed on the display in the same order.

Symbols

g	gravity
R	gas constant
γ	lapse rate of temperature in troposphere
$\gamma_a = g/R$	autoconvective lapse rate
$c = \gamma/\gamma_a$	
c_p	specific heat at constant pressure
$k = R/c_p$	

Parameter	Symbol	Mean sea level	tropopause
pressure	p	p_o	p^*
height	z	$z_o = 0$	z^*
temperature	T	T_o	T^*
potential temperature	θ	θ_o	θ^*

Constants

The basic constants are taken from List, R. J., 1951: Smithsonian Meteorological Tables, 6th rev. ed., Smithsonian Institution, Washington, pp. 265, 266, 289, 308.

Basic constant	Value
γ	.0065 °A/m
c	0.190 284
p_o	1013.25 mb
T_o	288 °A
T^*	218 °A
k	2/7
P	1000 mb

The constants stored are

Register	Symbol	Value
0	p* (mb)	234.510 0006
1	z* (m)	10 769.230 76
2	T* (°A)	218.
3	θ* (°K)	329.921 3257
4	c	0.190 284
5	$-T^*/\gamma$ (m)	-33 538.461 54
6	k	0.285 714 2857

Formulas

The constants, p*, z*, T*, θ*, were calculated from formulas numerically consistent with the programmed calculations for p, z, T, θ:

$$p^* = p_o\left(\frac{T^*}{T_o}\right)^{1/c}$$

$$z^* = \frac{T^*}{\gamma}\left[\left(\frac{p_o}{p^*}\right)^{c} - 1\right]$$

$$\theta_o = T_o\left(\frac{P}{p_o}\right)^{k}$$

$$\theta^* = \theta_o \exp\left[\left(c - k\right)\ln\frac{p^*}{p_o}\right]$$

Outputs, p, z, T, θ, are calculated from the following formulas:

Output	Troposphere	Stratosphere
p	$p^* \exp\left[\frac{1}{c}\ln\left(\frac{-\gamma}{T^*}(z-z^*)+1\right)\right]$	$p^* \exp\left[\frac{1}{c}\frac{-\gamma}{T^*}(z-z^*)\right]$
p	$p^*\left(\frac{T}{T^*}\right)^{1/c}$	p^* (see below)
p	$p^* \exp\frac{\ln(\theta/\theta^*)}{c-k}$	$p^* \exp\frac{\ln(\theta/\theta^*)}{-k}$
z	$z^* + \frac{T^*}{-\gamma}\left[\left(\frac{p}{p^*}\right)^c - 1\right]$	$z^* + \frac{T^*}{-\gamma}\ln\left(\frac{p}{p^*}\right)^c$
T	$T^*\left(\frac{p}{p^*}\right)^c$	T^*
θ	$\theta^* \exp\left[-(c-k)\ln\frac{p^*}{p}\right]$	$\theta^* \exp\left(k \ln\frac{p^*}{p}\right)$

The U. S. Standard Atmosphere has no temperature below T*. If input $T \leq T^*$, the program will yield values of p, z, θ at the tropopause. T as input will be output, however.

Note also that z, T, and θ are calculated from p. If z, T, or θ are input, then output p is calculated first from one of the first three formulas, and the other outputs from two of the last three formulas.

Use and status of calculator features.

The parts of the calculator that are used are

Registers 0 - 6, A - D
Labels A - D, 0 - 9
Program memory steps 001 - 112

No flags are used. Display status is FIX DSP 9. Trig status is DEG, but is immaterial to the operation of the program.

Only two returns are held pending for nested subroutines, so you may program the unused parts of the calculator, using this program as a set of

subroutines. However, you must not write a GSB A, GSB B, GSB C, or GSB D within one of your subroutines, otherwise you will violate the limit of three pending returns.

Parts of the calculator available for your program are

Registers 7 - 9, S0 - S9, E, I
Labels E, a - e
Program memory steps 113 - 224
All flags
Trig status

Program listing

```
001  *LBL A      21 11
002   STO A      35 11
003   GSB 3      23 03
004   STO 0      22 00
005  *LBL B      21 12
006   STO B      35 12
007   RCL 1      36 01
008     -          -45
009   RCL B      36 05
010     ÷          -24
011   X≠0 1      16-45
012   GTO 3      22 03
013     1          01
014     +          -55
015     LN         32
016  *LBL C      21 06
017   RCL 4      36 04
018     ÷          -24
019     eˣ         33
020   RCL C      36 06
021     ×          -35
022   STO 4      35 11
023  *LBL 3      21 03
024   GSB 3      23 03
025   GTO 1      22 01
026  *LBL 1      21 13
027   STO C      35 13
028   RCL 2      36 02
029   X≠Y?       16-34
030   ENT↑       -21
031     ÷          -24
032   RCL 4      36 04
033     1/x        52
034     √x         51
035   RCL 0      36 00
036     ×          -35
037   STO 4      35 11
038   GSB 3      23 03
039  *LBL 1      21 01
040   RCL 3      36 00
041   RCL 4      36 11
042     ÷          -24
043   GSB 4      23 04
044     ×          -35
045     CHS        -22
046     eˣ         33
047   RCL 3      36 03
048     -          -45
049   STO 2      35 14
050   GTO 2      22 02
051  *LBL 2      21 14
052   STO 3      35 14
053   RCL 3      36 03
054     ÷          -24
055   GSB 4      23 04
056     ÷          -24
057     eˣ         33
058   RCL 3      36 00
059     ×          -35
060   STO 4      35 11
061   GSB 3      23 03
062   GSB 3      23 03
063  *LBL 2      21 02
064   RCL 4      36 11
065   RCL B      36 12
066   RCL C      36 13
067   RCL D      36 14
068   FRST       16-14
069   RTN          24
070  *LBL 3      21 03
071   GSB 9      23 09
072     1          01
073   X≠Y?       16-35
074   X≠Y          -41
075   RCL 2      36 02
076     ×          -35
077   STO C      35 13
078   RTN          24
079  *LBL 5      21 05
080   GSB 9      23 09
081     1          01
082     -          -45
083   X≥0?       16-44
084   GTO 6      22 06
085     1          01
086     -          -55
087     LN         32
088  *LBL 6      21 06
089   RCL 3      36 05
090     ×          -35
091   RCL 1      36 01
092     +          -55
093   STO E      35 12
094   RTN          24
095  *LBL 4      21 04
096     LN         32
097     0          00
098   X≠Y?       16-35
099   GTO 7      22 07
100     R↓         -31
101   RCL 4      36 04
102  *LBL 7      21 07
103   RCL 6      36 06
104     -          -45
105   RTN          24
106  *LBL 9      21 09
107   RCL A      36 11
108   RCL 0      36 00
109     ÷          -24
110   RCL 4      36 04
111     Yˣ         31
112   RTN          24
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