Analytical Models for the Evaluation of Supplemental Aeration in Texas Estuaries

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ABSTRACT

In this study a one-dimensional dynamic mathematical model was developed for computer solution of estuarine dispersion problems. The math model was based on the one-dimensional mass transfer equation for the longitudinal distribution of a substance in a variable area estuary. Finite-difference approximations of the mass transfer equation were used to develop the numerical model. Several researchers have used similar modeling techniques, and their work has been summarized in the Literature Review.

The mathematical model described in this work was applied to three dispersion problems.

The first problem considered was the distribution of organic wastes in the Corpus Christi Harbor Channel. The goal of this study was to determine the effect of waste discharges on the organic loading in the Harbor Channel. Model results showed that wastes are adequately degraded in the channel and do not accumulate to excessive levels. These results were compared with field data obtained from sampling runs in the Harbor Channel.

Design of a surface aeration system for Vince Bayou was the second application of the math model. The results of this work showed that dispersion and dilution would greatly affect the oxygen transfer efficiency of a surface aeration system. Dissolved oxygen levels in the Houston Ship Channel, however, were determined to be the controlling factor affecting efficiency of aeration.

The third application of the computer program was to determine the effect of inchannel aeration on dissolved oxygen levels and organic waste concentrations in the Houston Ship Channel. Model results showed that inchannel aeration would not only improve oxygen levels in the channel, but greatly reduce the organic load discharged into Galveston Bay. Another conclusion was that tidal flows would have little influence on the efficiency of a surface aeration system operating on the Houston Ship Channel.

The one-dimensional dynamic mathematical model may be used effectively to solve such dispersion problems as considered here. However, it has other applications, particularly to dynamic modeling, that were outside the scope of the study.

CHAPTER I

INTRODUCTION AND GOALS

Texas estuaries play an important role in the economy, ecology, and esthetics of the Gulf Coast. Such estuaries as Galveston Bay provide the nursing area and home for numerous varieties of marine life. In fact, Galveston Bay is said to be the most biologically productive area in the Gulf of Mexico (8). The most important uses of our estuaries are transportation, oil production, fishing, and recreation. However, estuaries are also used to assimilate and transport vast quantities of liquid wastes from urban and industrial areas. It is this use that has created a threat to the ecology of Texas estuaries in recent years.

Urbanization in the Coastal Zone of Texas has stimulated dramatic increases in both quantity and variety of waste products discharged. In addition to the easily degraded organic wastes in sanitary sewage, estuaries receive hydrocarbons, metals, polymers, pesticides, and other pollutants. Some of these pollutants, such as cyanide, create an immediate hazard to living organisms; while others, for example the heavy metals, have known chronic effects (18). Other problems associated with the discharge of pollutants into estuaries are oxygen depletion, odors, and coloration.

The Style of the Journal of Sanitary Engineering (ASCE) has been followed.

Studies of estuarine systems have found them to be a complex blend of physical, chemical, biological, and hydraulic factors. To adequately predict the effect of waste loads on an estuarine system, extensive knowledge of the behavior of pollutants in estuaries is required. Mathematical models have been employed as an effective tool in this prediction process. Although math models may vary widely in their complexity and applicability, the one-dimensional dynamic model can provide the most efficient description of estuarine systems in many cases.

Goals

The primary goal of this study was to develop a one-dimensional water quality model suitable for solving estuarine dispersion problems.

A secondary goal was to apply the model to actual estuarine dispersion problems. The problems considered were:

1. Distribution of organic wastes in the Corpus Christi Harbor Channel.
2. Analysis of a surface aeration system for Vince Bayou.
3. Analysis of a surface aeration system for the Houston Ship Channel.

The first application of the computer program was to model biochemical oxygen demand (BOD) in the Corpus Christi Harbor Channel. The goal of this phase of the study was to calculate velocities and BOD distribution in the channel. The model results were to be compared with field studies of BOD distribution.

Vince Bayou was the second estuarine system chosen for study. This small bayou feeds the Houston Ship Channel about four miles downstream of the turning basin. Since its flow is maintained at a high level by a Houston Light and Power cooling water outfall, Vince Bayou has been considered as a site for a supplemental aeration system. The objective of this phase of the study was to model the performance of a surface aeration system in Vince Bayou. The results expected were to be the effects of dissolved oxygen (DO) levels, flow rates, and temperature on the rate of oxygen transferred by the proposed aerators.

The third application of the computer program was to model the effect of an aeration system on the Houston Ship Channel. The Houston Ship Channel has low dissolved oxygen levels much of the time due to excessive organic loads from cities, industries, and urban runoff (19). A supplemental aeration system would not only raise dissolved oxygen levels but provide a quality reserve to meet the demands created by urban runoff or treatment plant upsets (16). The goal of this final phase was to design an aeration system, based on low-flow summer conditions, for the Houston Ship Channel. Both BOD and oxygen distribution along the channel were to be determined. The results from the modeling work were to be the effect of aerator site location on oxygen levels in the Houston Ship Channel.

CHAPTER II

LITERATURE REVIEW

Recent One-Dimensional Dynamic Estuarine Models

Concern for the ability of estuaries to adequately assimilate imposed pollution loads has created a demand for increased efforts in the field of estuarine modeling. The use of mathematical models has provided much of the information needed to predict and explain estuarine response to waste loads. However, estuaries are intricate systems, governed by the complex interactions of tidal cycles, mixing forces, salinity stratification, temperature gradients, and fresh water flow rates. Since an exact mathematical representation of these physical processes is prohibitively complex, math models must employ varying degrees of simplification.

Many types of estuarine math models have been developed. They differ widely in both complexity and applicability. In terms of complexity and computer time requirements the hierarchy of estuarine math models is as follows (22):

1. Three-dimensional
2. Two-dimensional
3. One-dimensional, dynamic
4. One-dimensional, steady-state
5. Perfectly mixed.

Of these models the one-dimensional dynamic math model combines application to dynamic (nonsteady-state) studies with the ease in modeling one-dimensional variations.

The one-dimensional model is a significant simplification of the actual estuarine environment. It assumes that no gradients exist in the transverse or vertical planes. Thus, it considers only changes that occur along the length of the estuary. This type of approximation may yield reasonable results for some situations and is a good approximation in many cases, however significant errors would arise in application to stratified or wide, irregular estuaries.

An important aspect of any estuarine model is the data used to represent the prototype system. Particularly important in estuarine modeling are the values used for the dispersion, decay, and transportation of pollutants. In addition, an accurate physical representation of the prototype is required. Therefore, accurate mathematical representations of estuaries depend on the complexity of the system under study, the applicability of the math model, and the accuracy of the data used.

Many significant works have been published recently in the field of estuarine modeling (9)(22)(24)(30)(51). Several studies have employed one-dimensional dynamic models in solving estuarine dispersion problems. A brief review of these recent research efforts follows.

Modeling Work of Harleman

Harleman and several co-workers have made significant contributions to mathematical modeling of the behavior of actual estuaries (23)(24)(25). Although their studies have included many aspects of estuarine modeling, much of their efforts have involved applications of a one-dimensional dynamic model.

Harleman cites two advantages of one-dimensional models in comparison with their multi-dimensional counterparts. First, they have the advantage of mathematical tractability. Second, one-dimensional models utilize and predict information that is easily observed in actual estuaries (25).

A paper by Harleman, Lee, and Hall (24), describes the application of a one-dimensional dynamic model to a variable area estuary. It was the purpose of this paper "to develop a mathematical model which accurately describes the advective motion (including the tidal and fresh water flow) and the longitudinal dispersion term for a variable area estuary of arbitrary geometry." Model results were compared with field measurements of dye dispersion in the Potomac River.

The form of the one-dimensional equation used in this study was

$$\frac{\partial W}{\partial t} + U_{xt} \frac{\partial W}{\partial x} - U_{xt} \frac{W}{A} \frac{\partial A}{\partial x} = \frac{1}{A} \frac{\partial}{\partial x}\left(AE \frac{\partial W}{\partial x}\right) + P - K_1 W \quad . \ . \ . \ . \ . \ . \ . \ . \quad (2\text{-}1)$$

Where: W = weight of dye;

U_{xt} = instantaneous longitudinal velocity averaged over the section;

A = transverse cross-sectional area;

E = longitudinal dispersion coefficient;

P = weight of dye added per unit time; and

K_1 = the dye removal coefficient (24).

The term involving $\partial A/\partial x$ arises when considering a variable area estuary. Employing central-difference representations, the following finite-difference forms of the derivatives were used:

$$\frac{\partial W}{\partial t} = \frac{W_{i,t+1} - W_{i,t}}{\Delta t} \quad \text{(2-2)}$$

$$\frac{\partial W}{\partial x} = \frac{W_{i+1,t+1} - W_{i-1,t+1}}{2\Delta x} \quad \text{(2-3)}$$

$$\frac{\partial A}{\partial x} = \frac{A_{i+1} - A_{i-1}}{2\Delta x} \quad \text{(2-4)}$$

$$\frac{\partial}{\partial x}\left(AE\,\frac{\partial W}{\partial x}\right) = \frac{(A_{i+1}E_{i+1} + A_iE_i)(W_{i+1,t+1} - W_{i,t+1})}{2\Delta x^2} - \frac{(A_iE_i + A_{i-1}E_{i-1})(W_{i+1,t+1} - W_{i,t+1})}{2\Delta x^2} \quad \text{(2-5)}$$

This finite-difference model was applied to the fresh water tidal portion of the Potomac estuary. Velocities were approximated by combining the advective and tidal flows into the equation

$$U_{xt} = \frac{Qf}{A} + U_T \sin \sigma t \quad \ldots\ldots\ldots\ldots \quad (2\text{-}6)$$

Where: A = cross-sectional area;

Qf = fresh water discharge;

U_{xt} = instantaneous velocity;

U_T = the maximum value of the tidal velocity;

σ = tidal frequency; and

t = time.

For this study a distance increment of 1.72 miles and a time increment of 1/24 of a tidal cycle was used. The dispersion coefficient, assumed constant over the study area, was 0.1 square miles per day.

Good agreement between the prototype dye test and the mathematical model was obtained. However, comparisons were made only at high and low slack conditions since these were the concentrations reported by Hetling and O'Connell (26).

The conclusion resulting from this study was that a continuous, time-dependent solution of the mass balance equation for pollutant dispersion in estuaries could be obtained through numerical methods. However, an accurate description of the tidal advective motion is a significant factor in determining pollutant distribution (24).

Modeling Work of Hann and Young

A recent study by Hann and Young (22) dealt with mathematical modeling of water quality parameters in rivers and estuaries. The major goal of their research was to develop computer models which could calculate vertical and longitudinal mass transport in partially stratified estuaries. However, both one- and two-dimensional models were developed and comparisons were made between explicit and implicit one-dimensional finite-difference techniques.

Hann and Young found that good accuracy could be obtained with both explicit and implicit models when proper time and distance increments were used. A Crank-Nicolson implicit model, however, was found to be more accurate for a wider range of increment sizes. Although the Crank-Nicolson method required 1.75 times as much computer time as the explicit method, it was sometimes more economical because it remained stable and more accurate for larger time increments.

The one-dimensional model developed by Hann and Young was based on the one-dimensional mass transport equation with constant terms for cross-sectional area and velocity. The model was applied only to theoretical dispersion studies in uniform channels.

This one-dimensional implicit model was used to predict the distribution of slug loads and continuous sources of nonconservative substances in an ideal estuary. The results were compared with an exact solution for a constant width estuary. Excellent accuracy was obtained for a time increment of 0.01 days and a distance increment of 0.25 miles (22).

Modeling Method of Leendertse

J. J. Leendertse (36) has developed a finite-difference formulation based on a grid system that has the location of the flow situated between the grid point locations for the water levels and mass concentrations.

Leendertse used the following one-dimensional mass balance equation for the concentration of a dilute substance:

$$b\,\frac{\partial(HC)}{\partial t} + \frac{\partial(QC)}{\partial t} + b\,\frac{\partial\left(HD_x\,\frac{\partial C}{\partial x}\right)}{\partial x} + S = 0 \quad \ldots\ldots\ldots\ldots \quad (2\text{-}7)$$

Where: b = channel width;

H = temporal depth, averaged over the cross-section;

Q = transport;

C = mass concentration;

D_x = dispersion coefficient; and

S = the local source or sink per unit length.

The finite-difference equation at a location m for the mass concentration was given as

$$\frac{b}{\Delta t}\,[\{h_{m-1} + h_{m+1})/2 + \xi_m^{t+1}\}C_m^{t+1} - \{(h_{m-1} + h_{m+1})/2 + \xi_m^t\}C_m^t]$$

$$+ \frac{1}{8\Delta x}\,[Q_{m+1}^{t+1}\,(C_{m+2}^{t+1} + C_m^{t+1}) + Q_{m+1}^t\,(C_{m+2}^t + C_m^t) - Q_{m-1}^{t+1}\,(C_m^{t+1} + C_{m-2}^{t+1})$$

$$- Q_{m-1}\,(C_m^t + C_{m-2}^t)] + \frac{b}{8(\Delta x)^2}\,[\{h_{m+1} + \tfrac{1}{2}(\xi_m^{t+1} + \xi_m^{t+1})\}D_x(C_{m+2}^{t+1} - C_m^{t+1})$$

$$+ \{h_{m+1} + \tfrac{1}{2}(\xi^{t}_{m+2} + \xi^{t}_{m})\}D_x \ (C^{t}_{m+2} - C^{t}_{m}) - \{h_{m-1} + \tfrac{1}{2}(\xi^{t+1}_{m}$$

$$+ \xi^{t+1}_{m})\}D_x \ (C^{t+1}_{m} - C^{t+1}_{m-2}) - \{h_{m-1} + \tfrac{1}{2}(\xi^{t}_{m} + \xi^{t}_{m-2})\}D_x \ (C^{t}_{m} - C^{t}_{m-2})\}]$$

$$+ S_m = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2\text{-}8)$$

Where: h = water surface reference level;

t = tidal variation in the water level; and

ξ = variation from the reference water level.

In this equation three unknown values of the mass concentration appear, namely C^{t+1}_{m+2}, C^{t+1}_{m}, C^{t+1}_{m-2}. This equation applies for each segment of the estuary, thus forming a system of m equations, each with three unknowns. Once boundary conditions and initial concentrations are known this system of equations can be solved for the unknowns concentrations (36).

Modeling Efforts of Haag and Bedford

Haag and Bedford (17) have also developed an implicit one-dimensional dispersion model for application to estuarine studies. A unique characteristic of their work was the use of variable segment lengths along the estuary. This was done to allow model results to correspond spatially with field studies.

Haag and Bedford applied this model to a study of salinity distribution in the Hudson River estuary. A 65-mile reach of the Hudson from Rhinecliff to the Pioneer Boat Club was divided into

14 segments and modeled using a constant dispersion coefficient of 200 square meters per second (17).

Measured and calculated values of the salinity were compared over a 96-hour period on an hourly basis. Although there was good agreement in the magnitude of the salinity concentrations there was considerable scatter in the data. Haag and Bedford attributed this scatter to significant variations in the actual dispersion forces (17).

Modeling Methods of Bella, Dobbins, and Grenny

Bella and Dobbins (2) developed a one-dimensional model applicable to rivers and estuaries. Their modeling technique applies a mass balance of the stream directly in finite-difference terms. Therefore, a direct relationship between the model and physical process being modeled is established.

In this modeling method the estuary is divided into cells of equal length and a mass balance is made on each cell at successive time increments. During each time increment, the model performs the following steps for each cell, starting from the fresh water end of the channel:

1. Addition of pollutant,
2. Convection,
3. Dispersion, and
4. Decay.

After each cell has been treated in this manner the time increment is increased and the process is repeated (2).

Grenny and Bella (15) applied this model to a study of the slack water buildup of Rhodamine-B dye in the Yaquina River. The dye was injected at a constant rate of 68.1 g/hr (grams per hour) for 10 hours and over 400 samples were taken. Good comparisons between the model and the field test were obtained using a distance increment of 80.5 meters and a time increment of one-twelfth hour. A variable dispersion coefficient defined in terms of the velocity was found to give the best comparison between model and field results (15).

Grenny and Bella found that in this portion of the Yaquina River the magnitude of a pollutant concentration depends on the duration of the slack period, mixing depth of water at the outfall, and the strength of the polluting source. They concluded that failure to consider tidal flows can result in a considerable under-estimate of maximum pollutant concentrations (15).

Modeling Methods of Thomann

Thomann has also developed an estuarine model that has been widely used on the Delaware Estuary. Pence, et al. (42) have described an application of the Thomann model to predict annual variation of dissolved oxygen in the Delaware Estuary. The basic form of the model for BOD is as follows:

$$\frac{dL_k}{dt} = \frac{Q_{k-1,k}}{V_k} [\xi_{k-1,k}L_{k-1} + (1 - \xi_{k-1,k})L_k]$$

$$- \frac{Q_{k,k+1}}{V_k} [\xi_{k,k+1}L_k + (1 - \xi_{k,k+1})L_{k+1}]$$

$$+ \frac{E_{k-1,k}}{V_k} (L_{k-1} - L_k) + \frac{E_{k,k+1}}{V_k} (L_{k+1} - L_k)$$

$$- d_kL_k \pm J_k \quad \ldots \quad (2\text{-}9)$$

where: k = the section number;

L = the ultimate carbonaceous BOD concentration;

Q = net flow;

V = volume of the section;

ξ = an advective coefficient;

E = an eddy exchange coefficient;

d = the decay rate; and

J = sources of BOD.

Pence et al. (42) found that this model verified past records of dissolved oxygen concentrations with reasonable accuracy. The model can be used to analyze hypothetical pollution control measures with accurate results depending on the validity of assumptions made and data used (42).

Limitations to the Applicability of One-Dimensional Models

Several authors have criticised the use of the one-dimensional dispersion equation for estuaries. This criticism is based on the grounds that the one-dimensional approximation is an oversimplification of the actual processes involved.

Fischer and Holley (13) state that the most important limitation of the one-dimensional model is that the results describe dispersion only after complete mixing across the cross-section of the estuary is achieved. Ward and Fischer (52) have estimated that from four to forty days are required for transverse mixing in the Delaware Estuary. If such large initial mixing times exist, a nonconservative material may be substantially decayed before the one-dimensional approximation becomes valid.

A second limitation of the one-dimensional model is its restriction to the constant density portion of estuaries. The model assumes constant fluid density throughout the estuary reach being studied. Thus, significant longitudinal variations in density would create some error in the mass balance calculations. Even more important, however, are vertical density variations that are found in stratified estuaries. Significant errors would be expected if the one-dimensional model was applied to a partially or fully stratified estuary.

Another limitation (52) is using steady-state conditions to describe actual estuarine processes. Large estuaries may require days or even months to reach equilibrium conditions after a substantial change in the flow. Therefore, using constant flow rates and constant period tidal changes may be too much of a simplification to adequately describe an estuarine system.

Dispersion Considerations

The distribution of soluble or suspended materials in rivers and estuaries can be described mathematically in terms of dispersion coefficients. Dispersion, as used in this study, is the combined effect of all factors that contribute to the spread of materials in estuaries. Some of these factors are molecular and eddy diffusion, transverse and vertical velocity distributions, wind and boundary shear, density differences, flow rates, and tidal effects (47). Thus, the dispersion coefficient in the math model must approximate the combined effect of several complex forces occurring naturally in estuaries.

Prediction and measurement of dispersion coefficients in streams and estuaries has been the goal of many research efforts in recent years (3) (10) (11) (12) (14) (27) (41) (47) (48). Due to the number of complex factors involved, no universally applicable method has been developed to predict dispersion coefficients. In fact, there is considerable disagreement between researchers about the magnitude of the dispersion coefficient in specific cases (24) (27) (43). In general, dispersion studies have been made for three different flow regions; rivers and streams, the constant density tidal region of estuaries, and the salinity intrusion region of estuaries.

Dispersion in Rivers and Streams

Dispersion in rivers and streams has been shown to be created by variations in velocity across the width of the stream (10). Stream width, curvature, and flow rate effect velocity variations and thus are important factors in determining the magnitude of the dispersion coefficient.

Several researchers have developed numerical techniques for determining dispersion coefficients in streams. Fischer (12) has summarized the results of these studies.

Fischer (12) has also studied the effects of bends on stream dispersion. His findings were that bends cause changes in the velocity profile which increase dispersion, but concurrent increases in transverse mixing tend to reduce longitudinal dispersion. Therefore, no simple relationship between stream and curvature, and dispersion was found (12).

Dispersion in the Constant Density Tidal Portion of Estuaries

Dispersion in the constant density tidal portion of estuaries has been found to depend on the ratio of the period of oscillation to the time scale for cross-sectional mixing (27). In many estuaries the vertical velocity distribution is the primary factor in determining cross-sectional mixing. Several equations have been developed to calculate dispersion coefficients in tidal rivers. Some of these are discussed below.

Taylor (50) developed a dimensionless longitudinal dispersion coefficient for a pipe. This equation was later rewritten in terms of open channel flow as follows (23):

$$E_L = 77nUR_h^{5/6} \quad \text{(2-9)}$$

Where: E_L = dispersion coefficient in ft^2/sec;

n = Manning roughness coefficient;

U = average velocity; and

R_h = hydraulic radius.

Harleman (25) has suggested that the Taylor equation be modified in the following manner for application in estuaries:

$$E_L = 100nU_{max}R_h^{5/6} \quad \text{(2-10)}$$

Where: U_{max} = the maximum tidal velocity.

Dispersion coefficients estimated from this equation have been used to model dye studies on the Potomac River with good agreement (26). In this dye study Hetling and O'Connell (26) used the model developed by Thomann to study the dispersion of Rhodamine WT dye released for 13 days in the Potomac Estuary. They found that dispersion coefficients ranging from 65 to 195 ft^2/sec would adequately reproduce the results of the dye test.

Prych and Chidley (43) have made an error analysis of the modeling work of Harleman et al. and Hetling and O'Connell. They found that deficiencies in the finite-difference methods used account for the apparent agreement between dispersion coefficients calculated by Equation 2-10 and those used in the models. Their analysis shows that the average longitudinal dispersion coefficient for the reach of the Potomac River Estuary under investigation is about 320 ft^2/sec. Prych and Chidley concluded that agreement between Equation 2-10 and field data is obtained because of a compensating error caused by pseudo-dispersion that results from the finite-difference schemes used (43).

Holley et al. (27) have developed a method for estimating dispersion coefficients in the uniform density region of tidal waterways. Their method follows the same approach used by Taylor (50) but applied to a uniform, two-dimensional open channel flow with a linear, oscillating velocity distribution and a constant vertical diffusivity. Their analysis appears to give dispersion coefficients of the same order of magnitude obtained by using the modified Taylor equation. Furthermore, Holley et al. make a distinction between earlier math models that considered only fresh water advective flow and the models of Harleman et al. (24) and Bella and Dobbins (2) that consider the effects of tidal oscillations. Both types of models contain dispersion terms but the dispersion coefficients used in the models differ considerably both in magnitude and in the physical processes they represent.

Segall and Gidlunk (47) have also derived an equation to determine dispersion coefficients in tidal flows. Their dispersion equation defines the spread of a conservative substance discharged instantaneously into an oscillating turbulent flow system. The form of this equation indicates that dispersion is strongly influenced by tidal period and tidal amplitude.

Grenny and Bella (15) found that a variable dispersion coefficient defined by Equation 2-11 gave good results when applied to a dye study on the Yaquina River.

$$D = 16.0|\text{velocity}| \quad \ldots \quad (2\text{-}11)$$

Where: D = the dispersion coefficient in m^2/sec; and

Velocity = measured in m/sec (meters per second).

This equation gives dispersion coefficients similar in magnitude to those found by Harleman et al. (24) and Hetling and O'Connell (26). The model used by Grenny and Bella also accounted for the numerical error caused by pseudo-dispersion by subtracting the pseudo-dispersion coefficient from the dispersion coefficient defined by Equation 2-11 (15).

Summarizing, dispersion coefficients in the uniform density portion of estuaries may be estimated by the modified Taylor equation. However, the effects of pseudo-dispersion should be considered and the model verified by field studies if possible.

Dispersion in the Salinity Intrusion Portion of Estuaries

Analytical methods have not yet been developed to predict dispersion coefficients in the salinity intrusion region of estuaries. However, the longitudinal variation of salinity, obtained from field studies, may be used to estimate dispersion coefficients. Several studies have shown that the longitudinal dispersion coefficient varies with distance along the estuary (23)(25)(26). Maximum values occur near the ocean entrance and approach values for the constant density region at the upstream limit of salinity (25).

Several empirical methods for determining dispersion coefficients in the salinity intrusion region have been used. Bella and Grenny (3) discussed several equations based on salinity distribution which have been used to estimate dispersion coefficients. One equation is based on a steady-state assumption and is represented by

$$D_L = \frac{QC}{A \frac{\Delta c}{\Delta x}} \quad \text{(2-12)}$$

Where: Q = the net seaward flow;

A = cross-sectional area;

C = the salinity concentration; and

$\Delta c/\Delta x$ = the salinity gradient at a given location.

Glenne and Selleck (14) used tracer studies to determine dispersion coefficients in San Francisco Bay. After considering several methods of predicting dispersion coefficients they found that the only relationship which appears to have merit is

$$E = \text{const}(\overline{U}\, U_m' D)^{3/4} \quad \text{(2-13)}$$

Where: E = the longitudinal dispersion coefficient;

$\overline{U}$ = the mean advective velocity;

U_m' = the mean tidal velocity; and

D = the hydraulic channel depth.

However, this equation is applicable only to regions where the advective velocity has a significant effect on the value of the longitudinal dispersion coefficient.

Another type of empirical equation has been used for salinity studies on the Rotterdam Waterway. The longitudinal dispersion coefficient was determined to be a function of the form

$$E = 13{,}000\ (1 - \frac{x}{L})^3 \quad \text{(2-14)}$$

Where: E = ft^2/sec units;

x = the longitudinal distance scale; and

L = the length of the entire tidal region of the estuary (25).

Table 2-1 shows dispersion coefficients determined for the salinity intrusion region of estuaries by several researchers.

Pseudo-dispersion

Pseudo-dispersion occurs as a result of using finite-difference techniques to represent continuous processes. The spread of slug loads in dispersion models occurs not only by the dispersion term used in the model but by pseudo-dispersion which results from assuming that channel segments are prefectly mixed. Thus, pseudo-dispersion is an inherent error in finite-difference models that increases the effective dispersion coefficients.

TABLE 2-1

DISPERSION COEFFICIENTS IN THE SALINITY

INTRUSION REGION OF ESTUARIES

Estuary	Longitudinal Dispersion Coefficient (ft^2/sec)	Researcher
San Francisco Bay	460 - 1,380	Selleck and Pearson (48)
San Francisco Bay	200 - 14,000	Owen (40)
South San Francisco Bay	100 - 4,000	Glenne and Selleck (14)
North San Francisco Bay	200 - 14,000	Glenne and Selleck (14)
Delaware Estuary	1,700 - 5,400	Kent (35)
Delaware Estuary	1,000 - 2,200	Paulson (41)
Hudson Estuary	320 - 3,200	O'Conner and Mancini (39)
Hudson Estuary	2150	Haag and Bedford (17)

Bella and Dobbins (2) were among the earliest researchers to consider the effect of pseudo-dispersion in their model. For their explicit finite-difference model they found that the total pseudo-dispersion could be approximated by

$$D_p = \frac{U}{2} \Delta X - U\Delta T \quad \text{(2-15)}$$

Where: D_p = the total pseudo-dispersion;

U = the advective velocity;

ΔX = the segment length; and

ΔT = the time increment.

The effect of pseudo-dispersion may be minimized by selecting ΔX and ΔT small enough that the pseudo-dispersion is much less than the actual dispersion or by calculating the pseudo-dispersion for each time interval and subtracting these values from the actual dispersion coefficients (2).

Prych and Chidley (43) calculated the pseudo-dispersion term for a central-difference implicit modeling scheme. They obtained

$$D_p = \frac{U^2 \Delta T}{2} \quad \text{(2-16)}$$

which unlike Equation 2-15 does not show any dependence on the segment length ΔX.

Mixing Times

Another consideration in applying one-dimensional models to estuarine dispersion problems is the time required for the one-dimensional approximation to become valid. In the math model, pollutants discharged into an estuary are assumed to be completely and instantaneously mixed throughout the segment volume. In actual practice the time required for complete mixing depends on the geometry of the estuary and the transverse dispersion rate.

Holley et al. (27) found that vertical and horizontal dispersion coefficients could be estimated for estuaries and used to calculate mixing times. Table 2-2 shows estimates of vertical and horizontal dispersion coefficients in several estuaries.

Fischer (11) has shown that the transverse mixing time for streams could be estimated from

$$T_m = 0.4\ell^2/e_t \qquad (2\text{-}17)$$

Where: T_m = transverse mixing time;

ℓ = characteristic width of the channel; and

e_t = the transverse dispersion coefficient.

Several researchers have pointed out that the one-dimensional model describes dispersion in estuaries only after well-mixed conditions have been reached (11)(13)(27).

TABLE 2-2*

VERTICAL AND TRANSVERSE DISPERSION

COEFFICIENTS

Location	Vertical Dispersion Coefficient (ft^2/sec)	Transverse Disp. Coeft. (ft^2/sec)
Hudson Estuary		
Hudson, N.Y.	22	120
Poughkeepsie, N.Y.	31	35
Potomac Estuary		
5 miles below Chain Bridge	3.1	0.5
20 miles below Chain Bridge	7.3	0.3
Delaware Estuary		
10 miles below Trenton	12	58
30 miles below Trenton	17	28
James River Estuary		
5 miles below Hopewell	13	31

*After Holley et al. (27)

CHAPTER III

MATHEMATICAL MODEL

Model Development

The one-dimensional mass transfer equation may be developed from the principles of conservation of mass. A general form of the mass balance equation for a particular component is:

Inflow = Outflow + Accumulation + Decay . . . . (3-1)

A rigorous derivation of the mass transfer equation has been given by Hann and Young (22). The most general form of the one-dimensional mass transfer equation for a nonconservation substance is shown below (25).

$$\frac{1}{A}\frac{\partial}{\partial t}(AC) + \frac{1}{A}\frac{\partial}{\partial x}(AUC) = \frac{1}{A}\frac{\partial}{\partial x}(AE_L \frac{\partial C}{\partial x}) + \frac{r_i}{\rho} + \frac{r_e}{\rho} \quad . . . . \quad (3\text{-}2)$$

Where: A = cross-sectional area;

C = concentration;

U = velocity;

E = longitudinal dispersion coefficient;

ρ = fluid density;

r_i = time rate of decay of a substance per unit volume;

r_e = time rate of increase of a substance per unit volume;

x = longitudinal distance measured along the axis of the estuary; and

t = time measured in any consistent units.

This equation is assumed to be valid for unsteady flows within the restrictions of the one-dimensional approximation (25).

Calculation Techniques

Finite-difference approximations of the partial differential equation may be made in several ways. Two possible choices are the explicit and implicit formulations. The explicit formulation expresses unknown concentrations in each segment of the estuary entirely in terms of known values. The implicit formulation expresses unknowns in terms of both known and unknown quantities. Therefore, since the implicit formulation has more than one unknown in each equation a set of simultaneous equations must be solved for each time step.

Of these two types of calculation techniques, the implicit formulation was chosen for this study. Although this method of calculation requires more calculation time than the explicit method it offers two advantages. First, the implicit method allows the user greater flexibility in setting time and distance increments because stability requirements are not as strict. Second, the accuracy of the implicit method is greater than the explicit method over a wider range of model conditions (22).

Formulation of the Finite-Difference Equations

The one-dimensional mass transfer equation is used in the following form for this study:

$$\frac{\partial C}{\partial t} + U\frac{\partial C}{\partial x} - U\frac{C}{A}\frac{\partial A}{\partial x} = \frac{1}{A}\frac{\partial}{\partial x}(AE\frac{\partial C}{\partial x}) + \text{BALANCE} \quad \ldots\ldots\ldots\ldots \quad (3\text{-}3)$$

In comparison with the most general form of the mass transfer equation, Equation 3-3 has three differences. First, the cross-sectional area is not considered to be a time-dependent variable. This is an acceptable approximation for deep estuaries with small tidal changes. The dredged estuaries in Texas fall in this category. The second difference involves neglecting the variation of the velocity in the longitudinal direction. This may be a good approximation in deep estuaries when small distance increments are used, since velocity changes are gradual along the longitudinal axis. The third difference is that all source and sink terms are combined in a BALANCE term that is calculated in explicit form. This change greatly simplifies the form of the solution equations.

The following central-difference implicit forms of the differentials in Equation 3-3 were used in this study:

$$\frac{\partial C}{\partial t} = \frac{C_m^{t+1} - C_m^t}{\Delta t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3\text{-}4)$$

$$U\frac{\partial C}{\partial x} = U_m^{t+1}\left(\frac{C_{m+1}^{t+1} - C_{m-1}^{t+1}}{2\Delta x}\right)\theta + U_m^t\left(\frac{C_{m+1}^t - C_{m-1}^t}{2\Delta x}\right)(1-\theta) \quad \ldots\ldots \quad (3\text{-}5)$$

$$\frac{UC}{A}\frac{\partial A}{\partial x} = \frac{U_m^{t+1}C_m^{t+1}}{A_m^{t+1}}\left(\frac{A_{m+1}^{t+1} - A_{m-1}^{t+1}}{2\Delta x}\right)\theta + \frac{U_m^t C_m^t}{A_m^t}\left(\frac{A_{m+1}^t - A_{m-1}^t}{2\Delta x}\right)(1-\theta) \quad . \qquad (3\text{-}6)$$

$$\frac{1}{A}\frac{\partial}{\partial x}(AE\frac{\partial C}{\partial x}) = \frac{1}{A_m^{t+1}}\left(\frac{A_{m+1}^{t+1}E_{m+1}^{t+1} + A_m^t E_m^t}{2\Delta x}\right)\left(\frac{C_{m+1}^{t+1} - C_m^{t+1}}{\Delta x}\right)\theta$$

$$+ \frac{1}{A_m^t}\left(\frac{A_{m+1}^t E_{m+1}^t + A_m^t E_m^t}{2\Delta x}\right)\left(\frac{C_{m+1}^t - C_m^t}{\Delta x}\right)(1-\theta)$$

$$- \frac{1}{A_m^{t+1}}\left(\frac{A_m^{t+1}E_m^{t+1} + A_{m-1}^{t+1}E_{m-1}^{t+1}}{2\Delta x}\right)\left(\frac{C_m^{t+1} - C_{m-1}^{t+1}}{\Delta x}\right)\theta$$

$$- \frac{1}{A^t}\left(\frac{A_m^t E_m^t + A_{m-1}^t E_{m-1}^t}{2\Delta x}\right)\left(\frac{C_m^t - C_{m-1}^t}{\Delta x}\right)(1-\theta) \quad . \; . \; . \; . \qquad (3\text{-}7)$$

Theta (θ), as used in these equations, determines the final form of the solution matrix for the set of simultaneous equations. For example, if θ = zero all of the unknown terms at t+1 drop out and an explicit set of m equations is formed. For θ = one-half a simultaneous set of equation known as the Crank-Nicolson implicit formulation is formed. When θ = one a fully implicit, backward formulation is obtained.

Equations 3-4 through 3-7 are substituted into Equation 3-3 and rearrangement yields the following form of the finite-difference equation:

$$A_m\ (C^{t+1}_{m-1}) + B_m\ (C^{t+1}_m) + G_m\ (C^{t+1}_{m+1}) = H_m \quad \ldots\ldots\ldots\ldots \quad (3\text{-}8)$$

The coefficients A_m, B_m, G_m, and H_m are defined as follows:

$$A_m = -\frac{U^{t+1}_m \Delta t\theta}{2\Delta x} - \frac{\Delta t\theta}{2\Delta x^2}\left(\frac{A^{t+1}_m E^{t+1}_m + A^{t+1}_{m-1}E^{t+1}_{m-1}}{A^{t+1}_m}\right) \quad \ldots\ldots\ldots \quad (3\text{-}9)$$

$$B_m = 1 + \frac{U^{t+1}_m \Delta t\theta}{2\Delta x}\left(\frac{A^{t+1}_{m+1} - A^{t+1}_{m-1}}{A^{t+1}_m}\right) + \frac{\Delta t\theta}{2\Delta x^2}\left(\frac{A^{t+1}_{m+1}E^{t+1}_{m+1} + A^{t+1}_m E^{t+1}_m}{A^{t+1}_m}\right)$$

$$+ \frac{\Delta t\theta}{2\Delta x^2}\left(\frac{A^{t+1}_m E^{t+1}_m + A^{t+1}_{m-1}E^{t+1}_{m-1}}{A^{t+1}_m}\right) \quad \ldots\ldots\ldots\ldots\ldots \quad (3\text{-}10)$$

$$G_m = \frac{U^{t+1}_m \Delta t\theta}{2\Delta x}\ \frac{\Delta t\theta}{2\Delta x^2}\left(\frac{A^{t+1}_{m+1}E^{t+1}_{m+1} + A^{t+1}_m E^{t+1}_m}{A^{t+1}_m}\right) \quad \ldots\ldots\ldots\ldots \quad (3\text{-}11)$$

$$H_m = C^t_m - \Delta t U^t_m (1-\theta)\left(\frac{C^t_{m+1} - C^t_{m-1}}{2\Delta x}\right) + \frac{\Delta t U^t_m C^t_m (1-\theta)}{A^t_m}\left(\frac{A^t_{m+1} - A^t_{m-1}}{2\Delta x}\right)$$

$$+ \frac{\Delta t(1-\theta)}{A^t_m}\left(\frac{A^t_{m+1}E^t_{m+1} + A^t_m E^t_m}{2\Delta x}\right)\left(\frac{C^t_{m+1} - C^t_m}{\Delta x}\right)$$

$$- \frac{\Delta t(1-\theta)}{A^t_m}\left(\frac{A^t_m E^t_m + A^t_{m-1}E^t_{m-1}}{2\Delta x}\right)\left(\frac{C^t_m - C^t_{m-1}}{\Delta x}\right) + \text{BALANCE} \quad \ldots\ldots \quad (3\text{-}12)$$

Figure 3-1 shows the matrix form of the set of m simultaneous equations formed by this procedure.

$$
\begin{bmatrix}
B_1 & G_1 & \cdot & \cdot & \cdot & \cdot & 0 \\
A_2 & B_2 & G_2 & \cdot & \cdot & \cdot & \cdot \\
\cdot & A_3 & B_3 & G_3 & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & A_{m-1} & B_{m-1} & G_{m-1} \\
0 & \cdot & \cdot & \cdot & \cdot & A_m & B_m
\end{bmatrix}
\begin{bmatrix}
C_1^{t+1} \\ C_2^{t+1} \\ C_3^{t+1} \\ \cdot \\ \cdot \\ C_{m-1}^{t+1} \\ C_m^{t+1}
\end{bmatrix}
=
\begin{bmatrix}
H_1 \\ H_2 \\ H_3 \\ \cdot \\ \cdot \\ H_{m-1} \\ H_m
\end{bmatrix}
$$

FIGURE 3-1. MATRIX FORMULATION

Solution of the System of Equations

This tridiagonal matrix can be solved by Gauss elimination for the unknown concentrations of the m cells. In the Gauss method the "A" coefficients in each equation except the first are eliminated by using the previous equation. This leaves the m^{th} equation with one unknown, C_m^{t+1}, which can be solved directly. The remaining unknowns are solved by back-substitution.

Stability Criteria and Boundary Conditions

Important considerations in formulating an effective one-dimensional finite-difference model are stability and appropriate boundary conditions. Calculation errors in finite-difference models are generated by the approximating techniques used in formulating the model. If errors introduced by computer round-off, inexact initial conditions, or inexact boundary conditions grow without limit the model results become unstable and inaccurate. Boundary conditions may also introduce significant errors into the calculations, and can distort the entire concentration profile if incorrectly formulated (22).

Stability Criteria

During numerical calculations random errors are introduced in several ways. If the magnitude of these errors decreases with successive time steps, calculation stability results. Since the physical and chemical components of the model are determined by the prototype system only time and distance increments may be adjusted to achieve stability.

Several equations have been used as a basis for establishing time and distance increments that will achieve calculation stability. The most commonly found equation for an explicit modeling technique is:

$$\frac{\Delta tE}{(\Delta x)^2} < \tfrac{1}{2} \quad \text{(3-13)}$$

Where: E = the dispersion coefficient;

Δt = the time increment; and

Δx = the distance increment.

The derivation of Equation 3-13 has been shown by Leendertse (36). However, this result is based on a simple linear case and is not necessarily applicable when systems of equations are involved.

Hann and Young (22) found that stability is assured when the coefficients of the solution matrix have values greater than zero. Using this as a basis they developed the following stability criteria for a one-dimensional implicit formulation:

$$\Delta x < \frac{2EX}{VX} \quad \text{(3-14)}$$

$$\Delta t < \frac{2(\Delta x)^2}{2EX + (\Delta x)^2 KD} \quad \text{(3-15)}$$

Where: EX = dispersion coefficient;

VX = velocity; and

KD = decay rate.

Equation 3-15 reduces to

$$\frac{\Delta t EX}{(\Delta x)^2} < 1 \quad \text{(3-16)}$$

when distance increments of a mile or less are used since the term $(\Delta x)^2KD$ is much smaller than 2EX. Comparison of Equations 3-13 and 3-16 indicates that the stability requirements for an implicit method allow the time increment to be twice that of an explicit method. This result is verified by the computational experience of Hann and Young (22).

Table 3-1 shows values of Equation 3-16 as determined from the studies of several researchers. These values are much less than one, indicating that considerations other than stability were the primary criteria for setting time and distance increments.

TABLE 3-1

STABILITY CRITERIA

Researcher	Value of $\frac{\Delta t E X}{(\Delta x)^2}$
Harleman et al. (24)	0.0014
Hann and Young (22)	0.025 - 0.25
Haag and Bedford (17)	0.008 - 0.12
Bella and Dobbins (2)	0.083 - 0.333

Boundary Conditions

The central-difference finite-difference equations developed in Chapter III require grid concentrations both upstream and downstream of the point being analyzed. Therefore, known boundary conditions must be applied at both ends of the estuary.

At the head of the estuary the condition of no mass transfer across the boundary may be required. This condition can be satisfied by using a reflection point at the upper boundary (See Figure 3-2). In this situation the end of the estuary is at $m = 2$ and the reflection concentration at $m = 1$ is set equal to the first downstream concentration at $m = 3$.

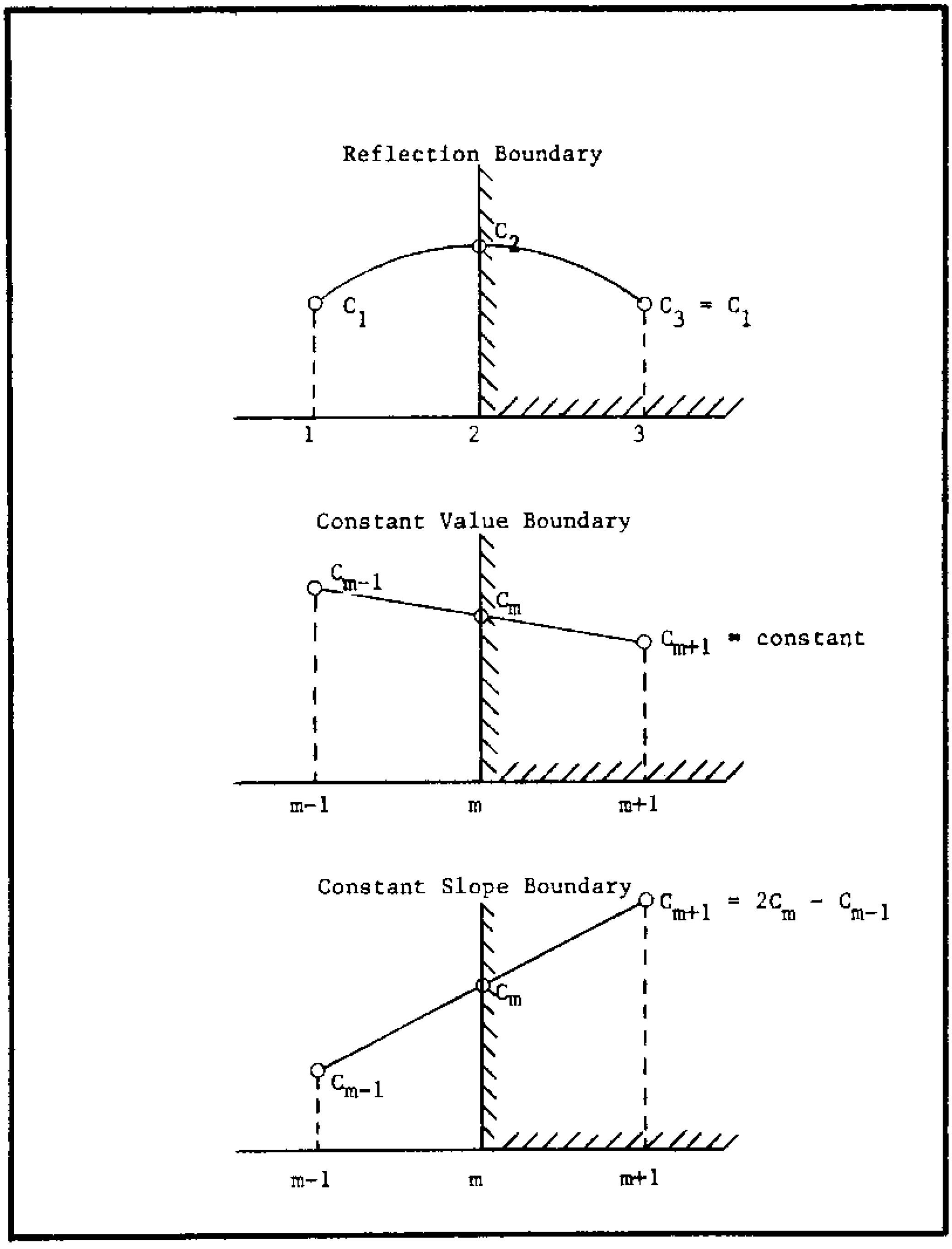

FIGURE 3-2. END BOUNDARY TREATMENT

At the open end of the estuary many different boundary conditions may be used, depending on the process being modeled. One method is the constant value boundary. This method assumes that the concentration beyond the last reach of the estuary has a constant value during the study period. This situation is approximated by constant salinity or dissolved oxygen levels at the ocean end of the estuary. A second method employs a constant slope extrapolation to determine the m+1 concentration at the end of the estuary. The assumption here is that a straight line can be extended from the last two concentrations nearest the boundary. Figure 3-2 illustrates these different boundary treatments.

Aspects of the Computer Model

This section describes the supporting programs and calculations used in the computer model.

Velocity Calculations

An important part of the one-dimensional estuarine dispersion model is the velocity term. This term includes both fresh water flows and tidal flows. Since these models were to be used primarily for predictive studies, constant advective velocities and a sinusoidal tidal velocity were used. These approximations represent a significant simplification of the actual estuarine system, thus eliminating the need for extensive tidal data.

The form chosen for the tidal computations was that of a damped cooscillating tide in a rectangular channel. The equations for tidal elevations and velocities have been given by Ippen and Harleman (31). They are:

$$\eta = a_o e^{-\mu x} \cos(\sigma t - kx) + e^{\mu} \cos(\sigma t + kx) \quad \ldots\ldots\ldots\ldots \quad (3\text{-}17)$$

$$u = \frac{a_o}{h} C_o \frac{k_o}{\sqrt{\mu^2 + k^2}} e^{-\mu x} \cos(\sigma t - kx + x)$$

$$- e^{\mu x} \sin(\sigma t + kx + \alpha) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3\text{-}18)$$

Where: η = instantaneous water level;

$2a_o$ = maximum tidal amplitude at $x = o$;

μ = damping coefficient;

σ = wave frequency;

k = wave number;

u = instantaneous velocity;

h = depth of flow;

C_o = wave propagation velocity;

k_o = wave number; and

α = wave phase angle.

These equations can be applied to real estuaries in finite-difference form. Each segment of the channel is treated separately by adding the flow obtained at the m-1st segment to the mth segment. This can be shown by:

$$QN_m = QN_{m-1} + W*D*(\text{Equation 3-18}) \quad \ldots\ldots\ldots\ldots\ldots \quad (3\text{-}19)$$

Where: QN_m = the flow at segment m;

QN_{m-1} = the flow at segment m-1;

W = the channel width; and

D = the channel depth.

However, this approximation to the tidal flow introduces a cumulative error. Table 3-2 shows the magnitude of this error for a 100-mile rectangular channel with 0.25 mile segment lengths. The error is less than five percent for estuaries which are less than 50 miles in length.

TABLE 3-2

COMPARISON OF MODEL AND THEORETICAL

VELOCITIES FOR A DAMPED COOSCILLATING TIDE

Distance From CLOSED END (miles)	Theoretical Velocity (ft/sec)	Model Velocity (ft/sec)	Percent Error (%)
0	0.000	0.000	0.0
5	0.046	0.044	4.9
10	0.093	0.090	2.3
15	0.139	0.137	1.3
20	0.184	0.183	0.5
25	0.229	0.229	0.1
30	0.274	0.276	0.8
35	0.318	0.322	1.5
40	0.360	0.369	2.3
45	0.402	0.415	3.2
50	0.443	0.461	4.1
55	0.482	0.508	5.2
60	0.521	0.554	6.4
65	0.557	0.600	7.8
70	0.592	0.647	9.2
75	0.626	0.693	10.8
80	0.657	0.740	12.6
85	0.686	0.786	14.5
90	0.714	0.832	16.6
95	0.736	0.879	19.0
100	0.761	0.925	21.5

Dispersion Coefficients

Dispersion coefficients may be entered into the math model either through the input list or by a suitable mathematical formulation (i.e. Equation 2-14). In this work the dispersion coefficients are considered to be constant with time. However, they are corrected for pseudo-dispersion at each time step in the following manner:

$$ELN_m = EL_m - UN_m*UN_m*DELT/2.0 \quad \text{. . . . .} \quad (3\text{-}20)$$

Where: ELN = the dispersion coefficient at segment m;

EL = the constant dispersion coefficient entered into the math model; and

$UN_m*UN_m*DELT/2.0$ = the model representation of Equation 2-16 for pseudo-dispersion.

Balance Term

Balance as used in Equation 3-3 includes all of the factors necessary to compute the mass balance of a substance except advection and dispersion. Thus, the balance term must account for mass changes due to inflow or outflows, decay, transfer across the upper and lower boundaries, and external sources or sinks. In the model, each of these factors that enter into the mass balance are calculated individually and combined into terms such as BODBAL (mass balance on BOD) and OXBAL (mass balance on oxygen).

CHAPTER IV

MODEL APPLICATION TO THE CORPUS CHRISTI HARBOR CHANNEL

The first application of the mathematical model was to determine the distribution of organic waste discharges in the Corpus Christi Harbor Channel.

Description of the Physical and Chemical System

The Corpus Christi Harbor Channel is a dredged canal serving the shipping needs of the Corpus Christi industrial and commercial area. The channel is closed at its upper end and no continuous fresh water flows, other than waste discharges, enter the system during dry periods. The channel is short, relatively straight, and uniform thus simplifying the modeling process.

Figure 4-1 presents a map of the Corpus Christi Harbor Channel showing mile designations and segment numbers assigned for the model inputs. Mile designations start with "0" at the bay end of the channel while segment numbers are assigned starting with "1" at the closed end of the channel. The Harbor channel is about 8.75 miles long and varies in dredged width from 200 feet at the upper end to 1000 feet at the lower end. The dredged depth is fairly uniform over the length of the channel averaging about 38 feet.

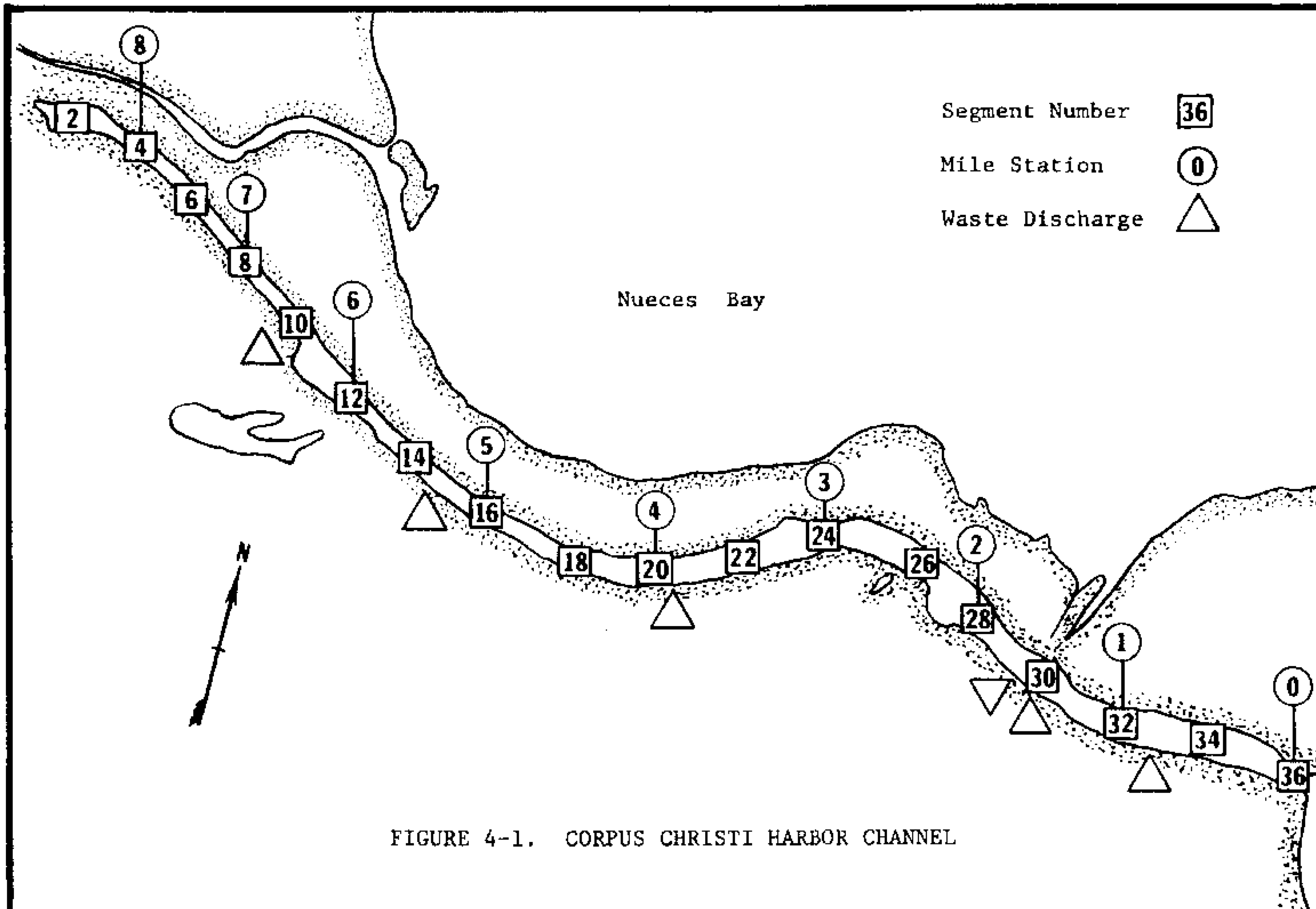

FIGURE 4-1. CORPUS CHRISTI HARBOR CHANNEL

Waste loads are discharged into the Harbor Channel from several industrial and domestic sources as shown in Figure 4-1. Table 4-1 shows the average and maximum BOD_u (ultimate biochemical oxygen demand) waste loads discharged into the channel. This table is based on self-reporting data for the period February 1972 through January 1973, as compiled by the Texas Water Quality Board. The BOD_5 (5-day biochemical oxygen demand) values from the self-reporting data were converted to BOD_u by assuming a first order decay rate of 0.23 per day.

The Environmental Engineering Division of Texas A&M University has monitored the water quality in the Corpus Christi Harbor Channel under a Texas Water Quality Board Interagency Contract. Table 4-2 summarizes the BOD_u concentrations found in the channel during recent sampling runs. The 1972 data has been reported by Hann et al. (21) and the 1973 data has been published as a continuation of this study.

The data presented in Table 4-2 indicates that there is considerable variability in the BOD_u concentrations found in the Corpus Christi Harbor Channel. The May data indicates that BOD concentrations are well mixed with channel depth. However, April and March data show stratification; higher BOD values occurring near the top in April but near the bottom in March. Since the mathematical model used in this study assumes well-mixed conditions exist, only the May 1973 data was used for comparing field results to model results.

TABLE 4-1

1972 WASTE LOADINGS* FOR THE CORPUS

CHRISTI HARBOR CHANNEL

Segment	Average Loading (lbs/day)	Maximum Loading (lbs/day)
9	4100	9720
14	10650	25370
18	40	230
20	290	570
25	470	550
28	220	630
30	150	220
32	2970	5670

*All loadings based on the BOD_u concentration of the effluent discharges.

TABLE 4-2

BOD_u* ANALYSIS IN THE CORPUS

CHRISTI HARBOR CHANNEL

Mile	May 1973		April 1972		March 1972	
	Top	Bottom	Top	Bottom	Top	Bottom
0	4.0	3.9			0.9	3.4
1					0.6	4.3
2	3.1	3.1	4.0	2.5	0.7	5.0
3			4.3	2.7		
4	3.5	4.2			1.6	5.8
5					1.0	7.3
6	6.0	6.4	7.3	2.6	1.0	8.6
7			8.0	4.3	2.3	6.6
8	3.8	3.5			0.9	9.5

*All results in ppm.

Modeling Considerations

The Corpus Christi Harbor Channel was divided into 36 quarter-mile segments for this study. For each segment the average surface width, cross-sectional area and volume were estimated using Coastal and Geodetic Survey Map Number 524 (June, 1973 revision).

Table 4-3 shows modeling conditions used in this study.

TABLE 4-3

MODEL CONDITIONS

Parameter	Range of Values
Time increment	1400 seconds
Distance increments	0.25 miles
Stability criteria	0.013 - 0.032
Pseudo-dispersion coefficient	0.0 - 7.5 ft^2/sec

Table 4-4 shows the input data to the math model of the Corpus Christi Harbor Channel. Dispersion coefficients were estimated by the modified Taylor equation for this constant density tidal region. Inflows and waste concentrations were determined from the self-reporting data. At segment 29 a power plant pumps cooling water out of the channel into Nueces Bay. This is modeled simply as a negative inflow to segment 29.

TABLE 4-4

CORPUS CHRISTI INNER HARBOR CHANNEL

TIDAL PERIOD = 24.84 HOURS. TIDAL AMPLITUDE = 2.0 FEET.
TEMPERATURE = 18.0 DEGREES CENTIGRADE.

SEG	WIDTH	AREA	VOLUME	DISP COEFF	INFLOW	WASTE CONC	INITIAL CONC
1	591.0	21000.0	23.4	16.0	0.00	0.00	0.0000
2	961.0	21000.0	38.1	16.0	0.00	0.00	0.0000
3	527.0	18000.0	20.9	18.0	0.00	0.00	0.0000
4	517.0	15000.0	20.5	18.0	0.00	0.00	0.0000
5	517.0	15000.0	20.5	19.0	0.00	0.00	0.0000
6	508.0	15000.0	20.1	18.0	0.00	0.00	0.0000
7	490.0	15000.0	19.4	20.0	0.00	0.00	0.0000
8	508.0	15000.0	20.1	20.0	0.00	0.00	0.0000
9	952.0	15000.0	37.7	20.0	1.90	400.00	0.0000
10	1644.0	20000.0	65.1	20.0	0.00	0.00	0.0000
11	1432.0	27000.0	56.7	18.0	0.00	0.00	0.0000
12	961.0	25000.0	38.1	18.0	0.00	0.00	0.0000
13	813.0	23000.0	32.2	20.0	0.00	0.00	0.0000
14	868.0	21000.0	34.4	20.0	2.90	680.00	0.0000
15	970.0	21000.0	38.4	22.0	0.00	0.00	0.0000

TABLE 4-4 (Continued)

SEG	WIDTH	AREA	VOLUME	DISP. COEFF.	INFLOW	WASTE CONC	INITIAL CONC
16	931.0	21000.0	32.9	22.0	0.00	0.00	0.0000
17	758.0	21000.0	30.0	24.0	0.00	0.00	0.0000
18	1007.0	22000.0	39.9	24.0	0.37	20.00	0.0000
19	942.0	23000.0	37.3	26.0	0.00	0.00	0.0000
20	887.0	23000.0	35.1	26.0	1.80	30.00	0.0000
21	1090.0	26000.0	43.2	28.0	0.00	0.00	0.0000
22	1635.0	30000.0	64.7	28.0	0.00	0.00	0.0000
23	1220.0	28000.0	48.3	28.0	0.00	0.00	0.0000
24	887.0	26000.0	35.1	30.0	0.00	0.00	0.0000
25	1146.0	26000.0	45.4	30.0	117.00	0.75	0.0000
26	1321.0	26000.0	52.3	30.0	0.00	0.00	0.0000
27	1192.0	26000.0	47.2	30.0	0.00	0.00	0.0000
28	822.0	24000.0	32.6	32.0	0.83	50.00	0.0000
29	841.0	24000.0	33.3	32.0	-840.00	0.00	0.0000
30	1100.0	26000.0	43.6	34.0	0.85	33.00	0.0000
31	1229.0	26000.0	48.7	34.0	0.00	0.00	0.0000
32	1238.0	30000.0	49.0	34.0	13.10	42.00	0.0000
33	1109.0	30000.0	43.9	36.0	0.00	0.00	0.0000
34	1053.0	30000.0	41.7	36.0	0.00	0.00	0.0000
35	831.0	30000.0	32.9	38.0	0.00	0.00	0.0000
36	1737.0	22000.0	68.8	40.0	0.00	0.00	0.0000

The boundary conditions used in this model were: no mass transfer at the closed end, and a constant BOD_u concentration of 2.2 ppm (parts per million) at the ocean end.

Model Results

Velocities

Table 4-5 shows the maximum ebb and flood velocities along the Corpus Christi Harbor Channel for tidal ranges of one, two and three feet. The velocities are small because the channel is short, deep and has very little fresh water inflow. The power plant intake at segment 29 provides the most significant factor affecting flow rates in the Harbor Channel.

Time Response

An important application of dynamic math models is to determine how conditions change with time. In estuary studies two time scales become important. When magnitude of the time scale exceeds that of the tidal cycle, the model describes changes due to non-tidal effects. Using this time scale concentrations will approach steady values with time. When the magnitude of the time scale is a fraction of the tidal cycle, tidal effects are considered. In this case, velocities and concentrations are always unsteady quantities.

Figure 4-2 shows the response of the Corpus Christi Harbor Channel to waste inputs using a time scale of one tidal cycle. The following model conditions were used:

1. No initial waste concentrations
2. Average 1972 waste inputs.

TABLE 4-5

MAXIMUM VELOCITIES* IN THE CORPUS CHRISTI

HARBOR CHANNEL

Mile	One-foot Diurnal Tide		Two-foot Diurnal Tide		Three-foot Diurnal Tide	
	Ebb	Flood	Ebb	Flood	Ebb	Flood
0.0	0.039	0.103	0.109	0.173	0.180	0.244
0.5	0.026	0.072	0.074	0.121	0.123	0.170
1.0	0.021	0.069	0.067	0.114	0.112	0.160
1.5	0.021	0.076	0.069	0.124	0.117	0.172
2.0	0.054	0.044	0.103	0.095	0.152	0.141
2.5	0.045	0.036	0.086	0.076	0.127	0.117
3.0	0.037	0.037	0.074	0.074	0.111	0.111
3.5	0.028	0.027	0.056	0.055	0.083	0.083
4.0	0.032	0.032	0.064	0.064	0.096	0.096
4.5	0.030	0.029	0.059	0.059	0.088	0.088
5.0	0.027	0.027	0.054	0.054	0.082	0.081
5.5	0.023	0.023	0.046	0.046	0.069	0.069
6.0	0.016	0.016	0.032	0.032	0.048	0.048
6.5	0.013	0.013	0.026	0.026	0.039	0.039
7.0	0.013	0.013	0.025	0.025	0.038	0.038
7.5	0.010	0.010	0.019	0.019	0.029	0.029
8.0	0.006	0.006	0.013	0.013	0.019	0.019
8.5	0.001	0.001	0.003	0.003	0.004	0.004

*All results in ft/sec.

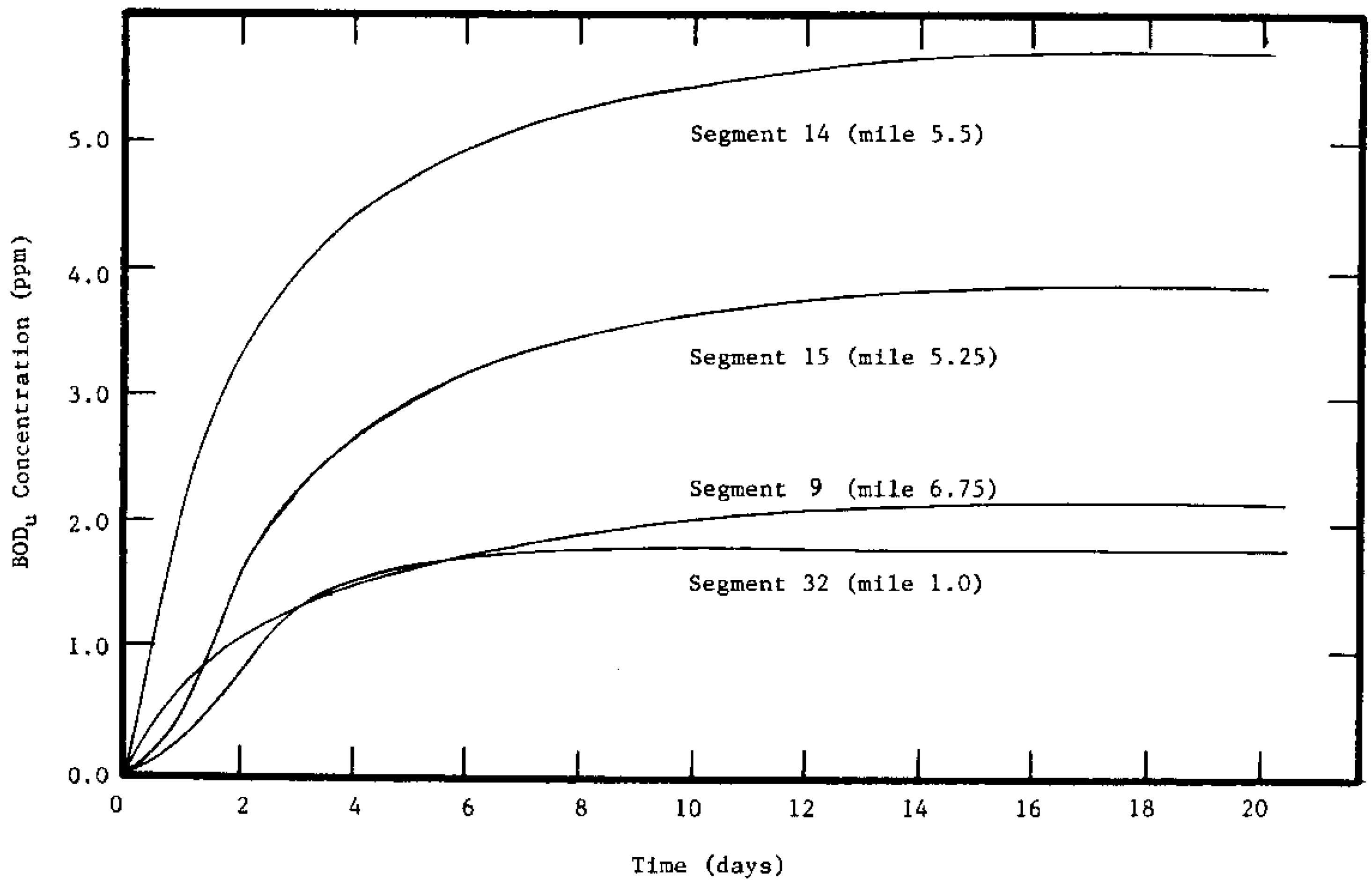

FIGURE 4-2. RESPONSE CURVES FOR WASTE INPUTS TO THE CORPUS CHRISTI HARBOR CHANNEL

3. Dispersion coefficients predicted by Equation 3-3.

4. One-foot diurnal tide.

5. Temperature = 18.0 °C.

6. Decay rate = 0.23 per day.

An interesting aspect of Figure 4-2 is the shape of the response curves. At segments 9 and 14 concentrations are dominated by waste inputs and the curves approximate a first order response. However, at segments 15 and 32 dispersion is the dominant factor and an S-shaped response curve develops. Another aspect of Figure 4-2 is the time required for steady-state conditions to develop. Approximately 10 days are required for concentrations to reach 95 percent of their steady-state values. This illustrates that considerable error can be expected when using constant conditions to model estuaries which may only rarely approach steady conditions.

Figure 4-3 shows the tidal variation of BOD_u concentrations at segments 13, 14, and 15. Segment 14 is the discharge point of the maximum waste load entering the Harbor Channel, while segments 13 and 15 receive waste loads only by the dispersion process. This figure illustrates the effect of flow direction and velocity on waste concentrations.

BOD Distribution

Figures 4-4 and 4-5 show low slack BOD_u concentrations along the Corpus Christi Harbor Channel as compared with field results obtained in May 1973. Model results are based on dispersion coefficients

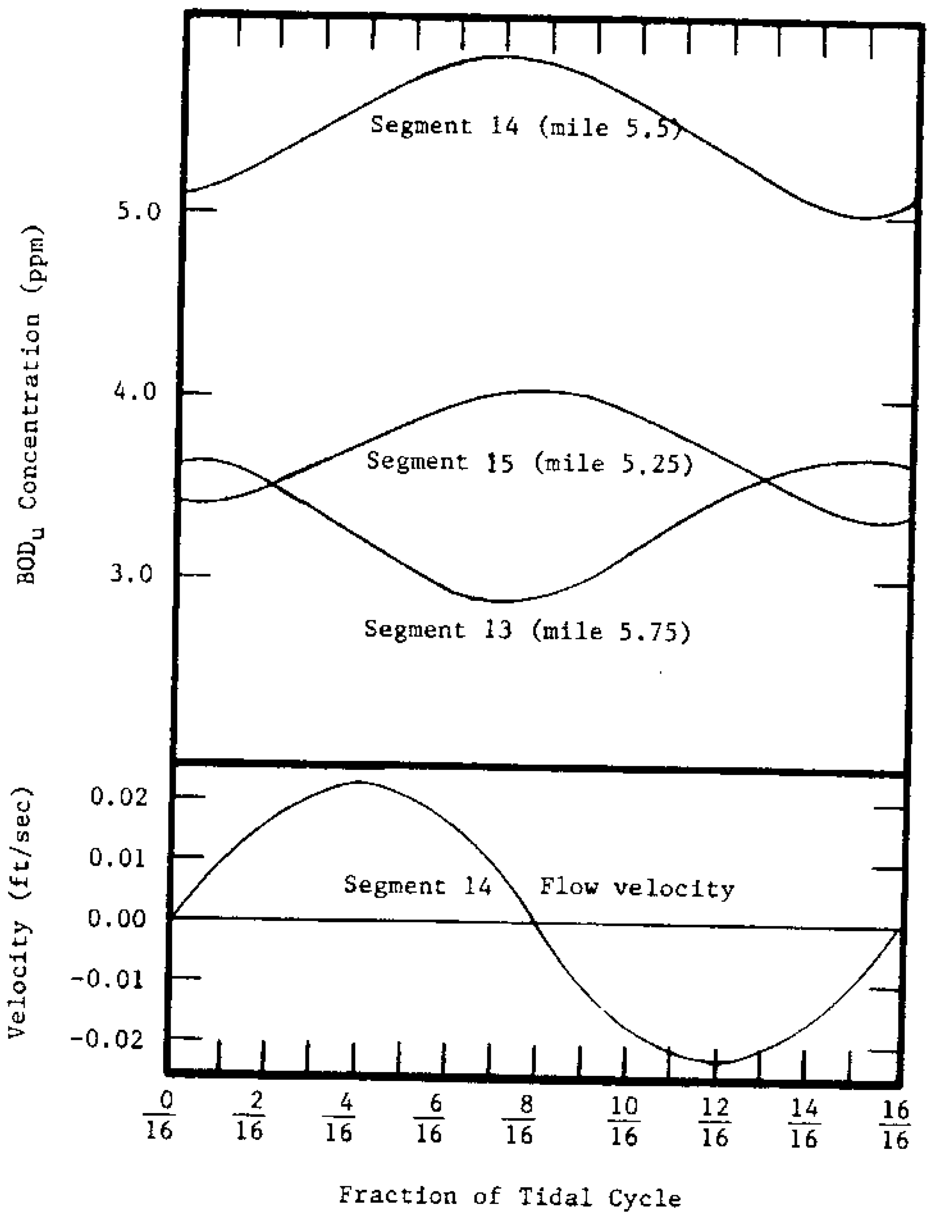

FIGURE 4-3. TIDAL VARIATION OF WASTE CONCENTRATIONS AND FLOW IN THE CORPUS CHRISTI HARBOR CHANNEL

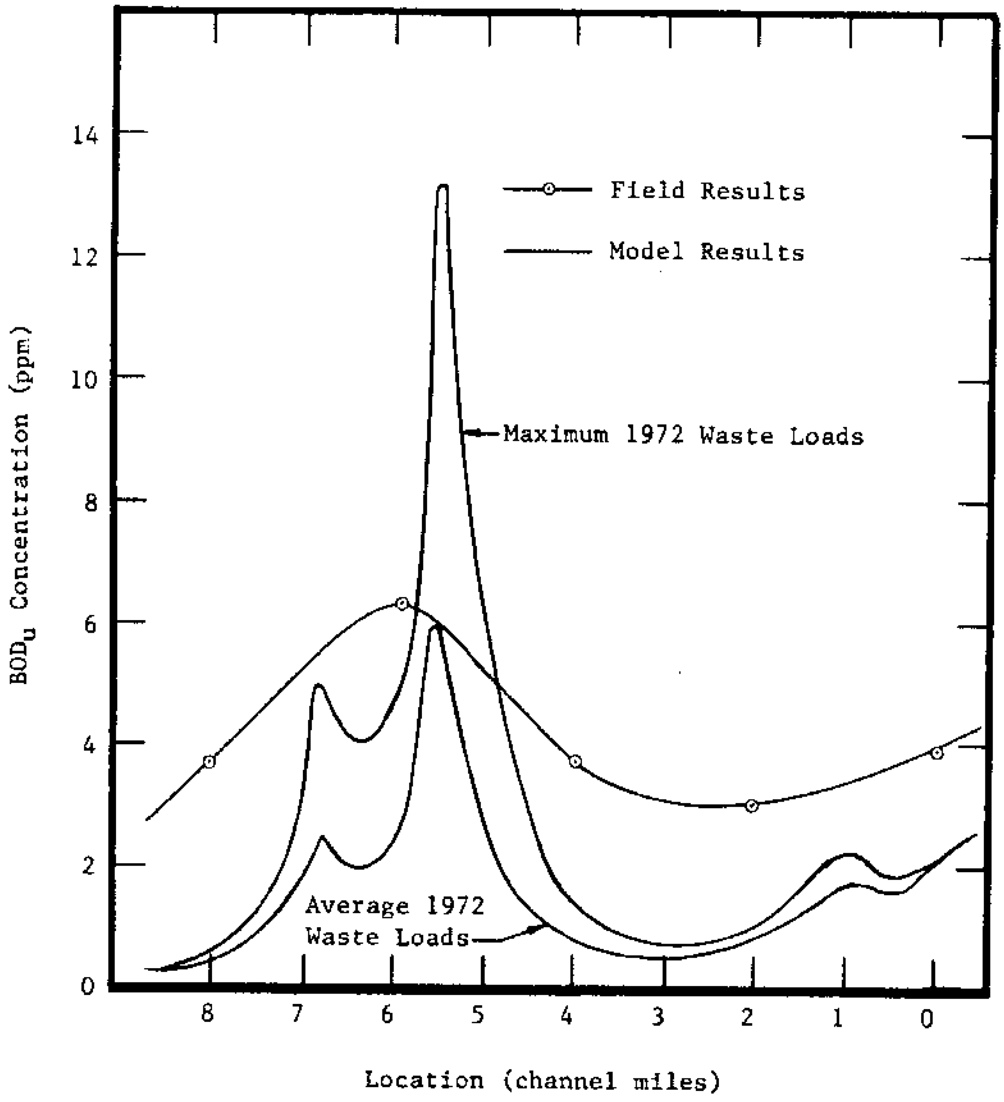

FIGURE 4-4. BOD_u DISTRIBUTION ALONG THE CORPUS CHRISTI HARBOR CHANNEL USING "TAYLOR E" DISPERSION COEFFICIENTS

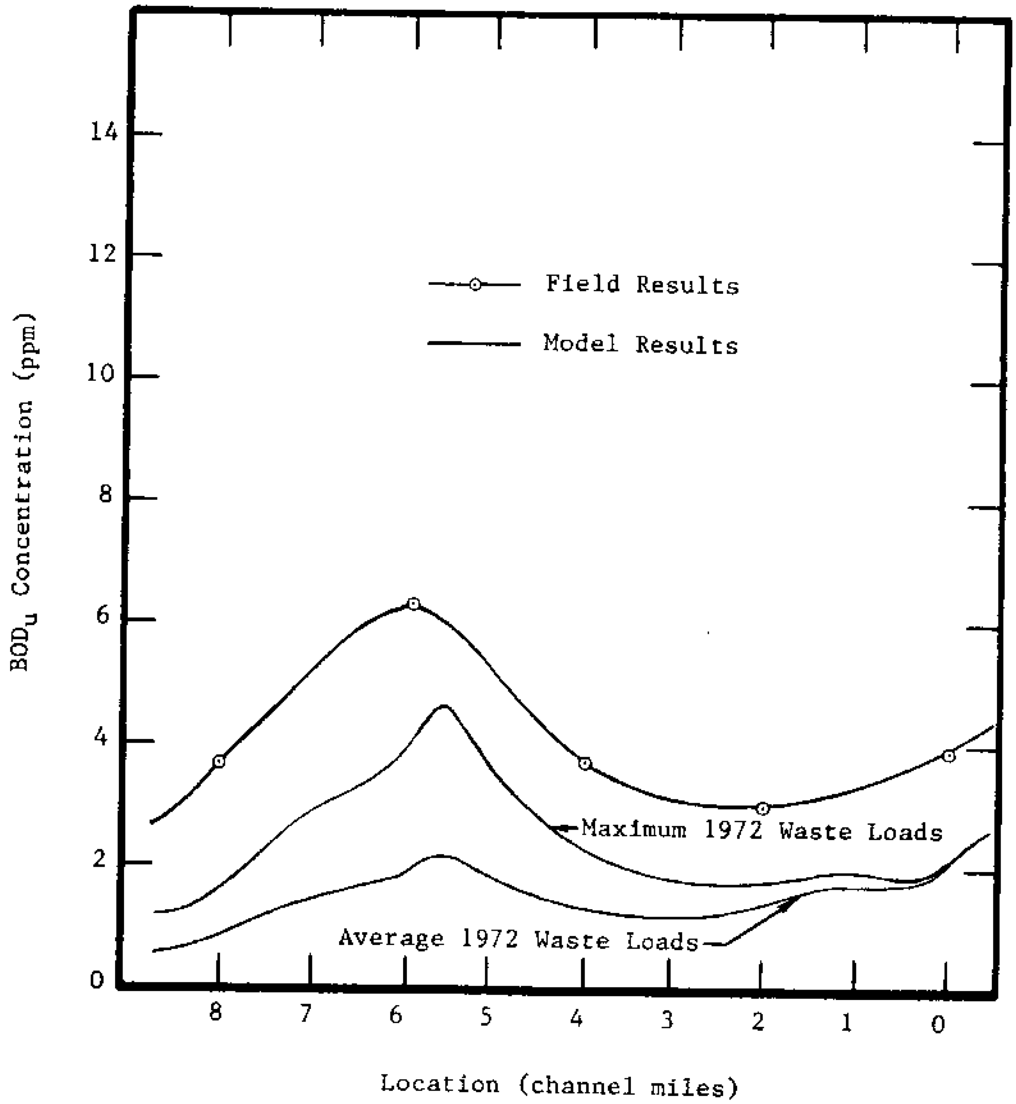

FIGURE 4-5. BOD$_u$ DISTRIBUTION ALONG THE CORPUS CHRISTI HARBOR CHANNEL USING DISPERSION COEFFICIENTS TEN TIMES LARGER THAN THOSE PREDICTED BY THE MODIFIED TAYLOR EQUATION

predicted by the modified Taylor equation. Figure 4-5 results are based on dispersion coefficients ten times larger than predicted by Equation 3-3. The shape of the BOD_u curves shown in Figure 4-5 compares well with field results but the magnitude of the model concentrations is about two ppm less than found in the channel. Although several reasons for this discrepency may be cited, one possibility is that algae in the channel water exert this additional demand. The math model did not consider this source of BOD_u.

Summary

Velocities in the Corpus Christi Harbor Channel are small. A net inflow to the channel created by the cooling water pump station at mile 1.5 is the dominant factor affecting dry weather flow.

Model results indicate that steady-state conditions are reached within about 10 days following significant changes in channel conditions. Tidal variations of BOD concentrations are generally less than one ppm thus tidal influence is not significant in the channel.

The best comparison of model and field results was obtained using maximum 1972 waste loads and dispersion coefficients ranging from 160 ft^2/sec at the closed end to 400 ft^2/sec at the ocean end of the channel.

CHAPTER V

EVALUATING A SURFACE AERATION SYSTEM FOR VINCE BAYOU

Introduction

The Houston Ship Channel has low dissolved oxygen levels much of the year due to high organic loads from municipal and industrial wastes. For this reason the Environmental Engineering Division of Texas A&M University has given consideration to using side stream aeration in Vince Bayou as a method of adding dissolved oxygen to the Houston Ship Channel. Inchannel aeration has been used effectively in similar situations. Appendix II discusses some of these applications of inchannel aeration.

Vince Bayou is a small tidal stream that enters the Houston Ship Channel about 4 miles downstream of the Turning Basin. The fresh water inflow to Vince Bayou from watershed drainage is normally less than 20 cfs (cubic feet per second). However, flow in the last 500 feet before the bayou enters the Houston Ship Channel may be as high as 700 cfs due to the cooling water discharge from a power plant operated by Houston Lighting and Power. This power generating station takes cooling water from the Houston Ship Channel about 0.25 miles upstream of Vince Bayou and discharges this water at a higher temperature back into the Houston Ship Channel via

Vince Bayou. Flow can be varied from 300 to 700 cfs depending on the cooling water requirements of the power plant.

Figure 5-1 shows the location of Vince Bayou and the Houston Lighting and Power generating station on the Houston Ship Channel. This location is near mile station 20 in the Ship Channel. Appendix III describes the physical characteristics of the Houston Ship Channel and the organic loading imposed on the Channel. Vince Bayou at this location ranges from 50 to 150 feet in width and from 4 to 18 feet deep. The bottom profile is very irregular.

The unique conditions at Vince Bayou offer several advantages for using this site to artificially add oxygen to the badly polluted Houston Ship Channel. Some of these advantages are:

1. High flow rate maintained at a constant level.
2. Minimal tidal influence.
3. No interference with navigation since the aerators would not be located on the Ship Channel.
4. Convenient supply of electrical power.

The purpose of this phase of the modeling work was to predict the oxygenation capacity of a hypothetical surface aeration system operating on Vince Bayou.

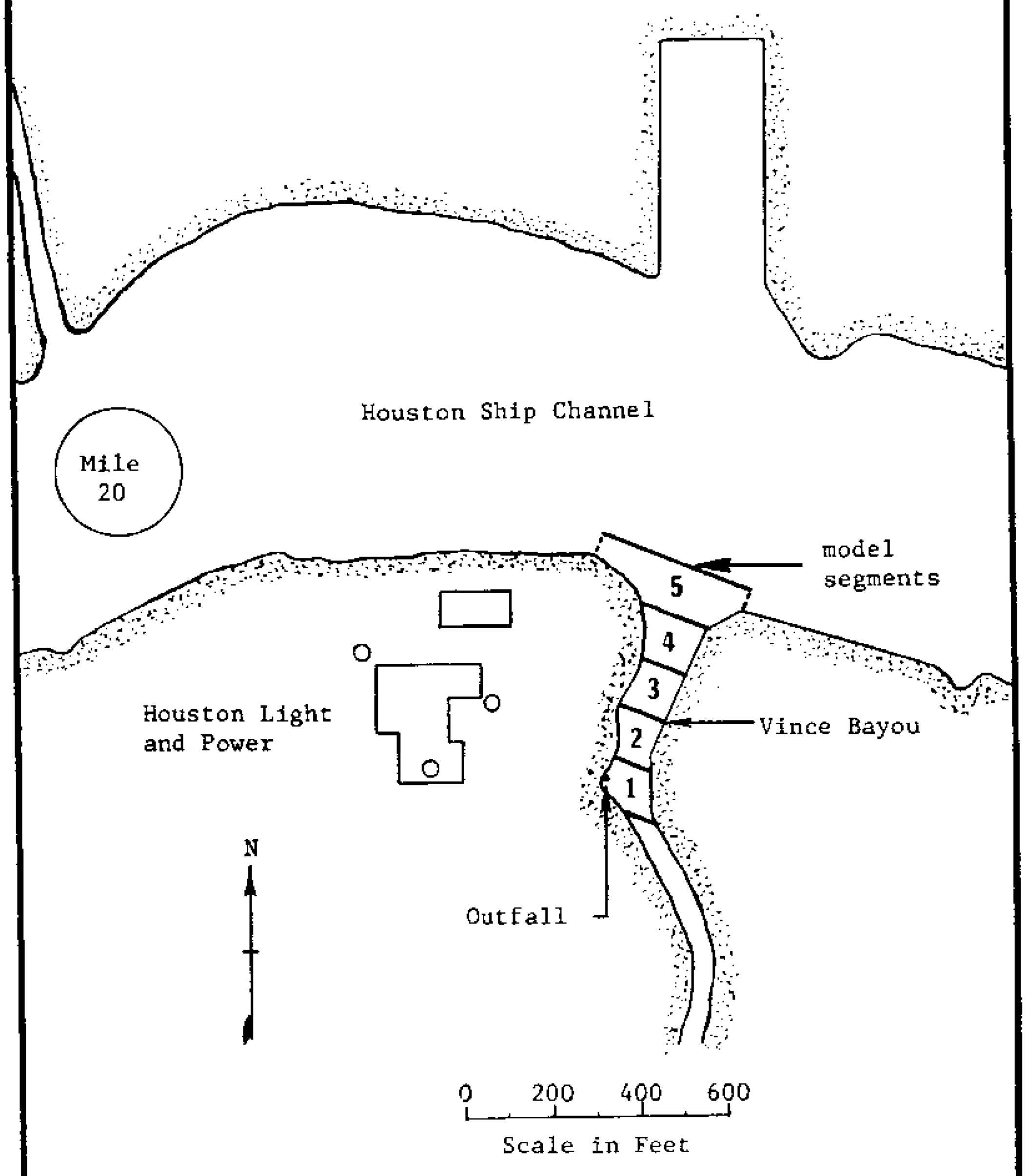

FIGURE 5-1. VINCE BAYOU

Model Considerations

Vince Bayou was divided into five 120-feet segments downstream of the cooling water outfall for this modeling work (See Figure 5-1). Only dissolved oxygen concentrations were modeled and atmospheric reaeration was not considered in the oxygen mass balance. Constant oxygen concentrations at both the upper and lower boundaries were used for boundary conditions. Table 5-1 shows other conditions used in the modeling work.

TABLE 5-1

MODEL CONDITIONS FOR VINCE BAYOU

Parameter	Range of Values
Time increment	124 seconds
Distance increment	120 feet
Dispersion Coefficients	150 - 230 ft^2/sec
Pseudo-dispersion coefficient	6 - 62 ft^2/sec
Stability Criteria	1.29 - 1.98
Theoretical aerator efficiency	3.0 lbs/hp·hr

Although the theoretical upper limit for stability (1.0) was exceeded in this study, no evidence of calculation instability was observed in the model results.

Model Results

The variables considered in this study of surface aeration in Vince Bayou were:

1. flow rate,
2. dissolved oxygen levels in the Houston Ship Channel,
3. temperature, and
4. installed horsepower.

Including such effects as dispersion, boundary conditions, and tidal influence, the analysis of surface aeration becomes a complex process. Figures 5-2 through 5-6 result from the application of the mathematical model to this complex physical system.

Table 5-2 shows the computer printout for a particular set of conditions on Vince Bayou. The model outputs cross-sectional areas, widths, tidal stage, installed horsepower, actual oxygen transfer rate, pounds of oxygen added per hour and oxygen concentrations in each segment. It also shows the number of time steps that have elapsed since the run was started.

Figure 5-2 shows dissolved oxygen concentrations in Vince Bayou downstream from the Houston Lighting and Power discharge point. The curves 2.5 and 0.0 ppm refer to the dissolved oxygen level in the Houston Ship Channel. Aerators were located at distances of 120, 240 and 360 feet downstream of the cooling water outfall, corresponding to segments 2, 3, and 4 as shows in Figure 5-1. Figure 5-2 illustrates

TABLE 5-2

TYPICAL MODEL RESULTS FOR VINCE BAYOU STUDY

POWER PLANT RELEASE = 500.0 CFS
DISSOLVED OXYGEN = 0.0 PPM
TEMPERATURE = 30.0 DEGREES C
THEORETICAL EFFICIENCY = 3.0 LBS PER HP-HR

SEG	AREA	WIDTH	STAGE	VEL	POWER	RATE	LBS IN	CONC
1	717.0	100.0	-1.000	0.720	0.0	0.000	0.0	0.000
2	716.0	105.0	-1.000	0.721	150.0	1.877	281.5	1.966
3	1010.0	105.0	-1.000	0.511	150.0	1.642	246.2	2.591
4	1106.0	135.0	-1.000	0.467	150.0	1.784	267.6	2.213
5	1312.0	230.0	-1.000	0.394	0.0	0.000	0.0	0.985

CYCLES = 1 STEPS = 90

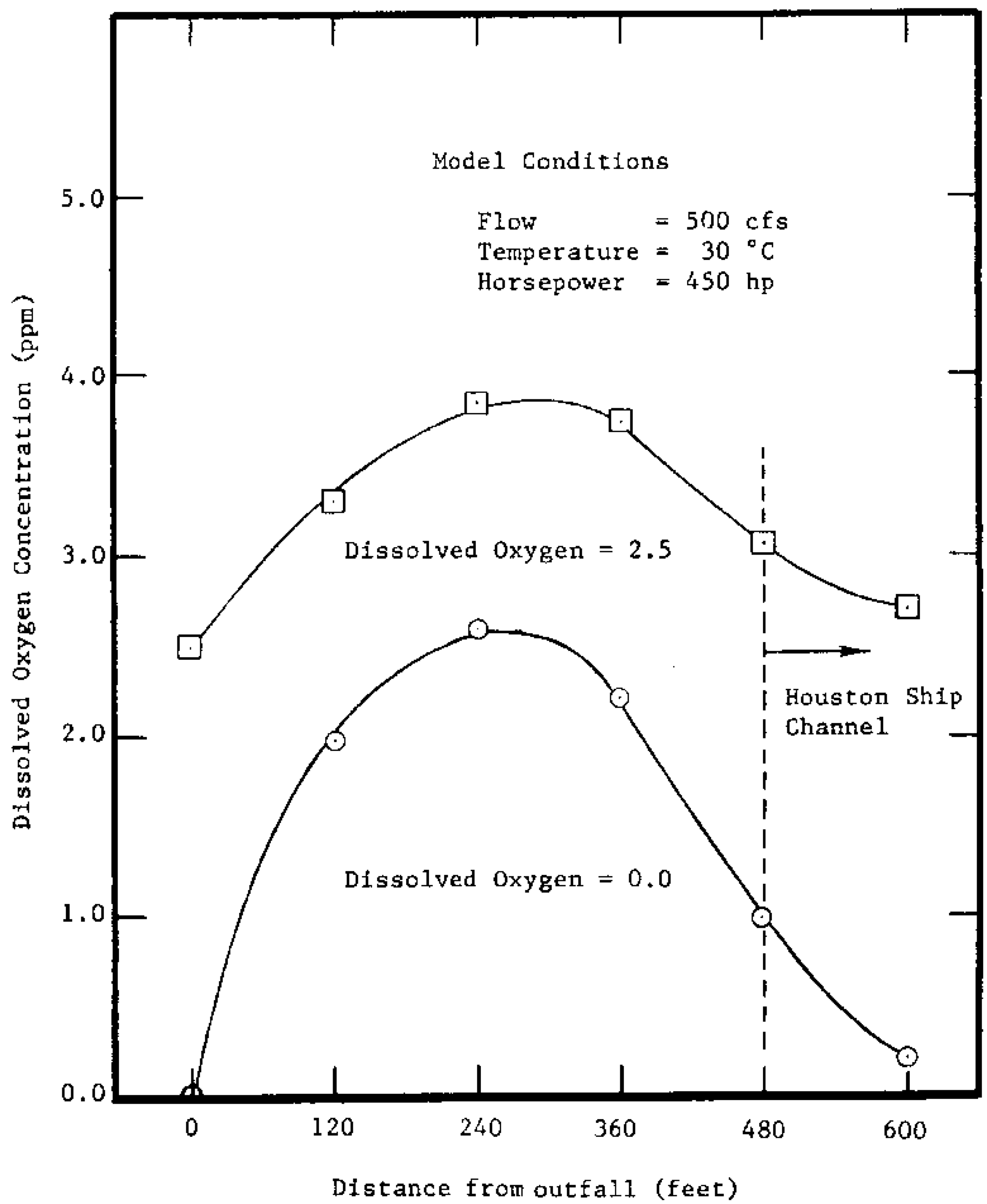

FIGURE 5-2. DISSOLVED OXYGEN CONCENTRATIONS IN VINCE BAYOU

that dilution and dispersion into the Houston Ship Channel quickly reduces dissolved oxygen concentrations after the flow has passed the aerator sites. This dispersion effect greatly increases the oxygen transfer rate of the aerators by keeping the dissolved oxygen deficit high. However, the shape of the curves in Figure 5-2 is dependent on the boundary conditions used in the model. In this case the boundary influence is significant and the accuracy of the results depends on the validity of assuming a constant DO level in the Houston Ship Channel.

In contrast to the dispersion effect, flow rate has little influence on the aerator transfer rate as shown by Figure 5-3. In the range of flows shown, 300-900 cfs, the effect of flow rate is dominated by the dispersion effect. Figure 5-3 indicates that normal changes in the cooling water discharge rate from the power plant would not significantly affect the efficiency of a surface aerator system in Vince Bayou.

The most dominant factor affecting oxygen transfer rate in Vince Bayou, as determined by model results, is the dissolved oxygen concentration of the cooling water discharge. Figure 5-4 shows the effect of DO levels on the aerator transfer rate. Based on a surface aeration system consisting of six 75 hp surface aerators operating with an theoretical efficiency of 3.0 lbs/hp·hr, the actual oxygen transfer rate would be about 18 lbs/hp·hr with zero DO in the cooling water and about 0.7 lbs/hp·hr with 5 ppm DO in the cooling water. Figure 5-4 indicates that while the aeration system could

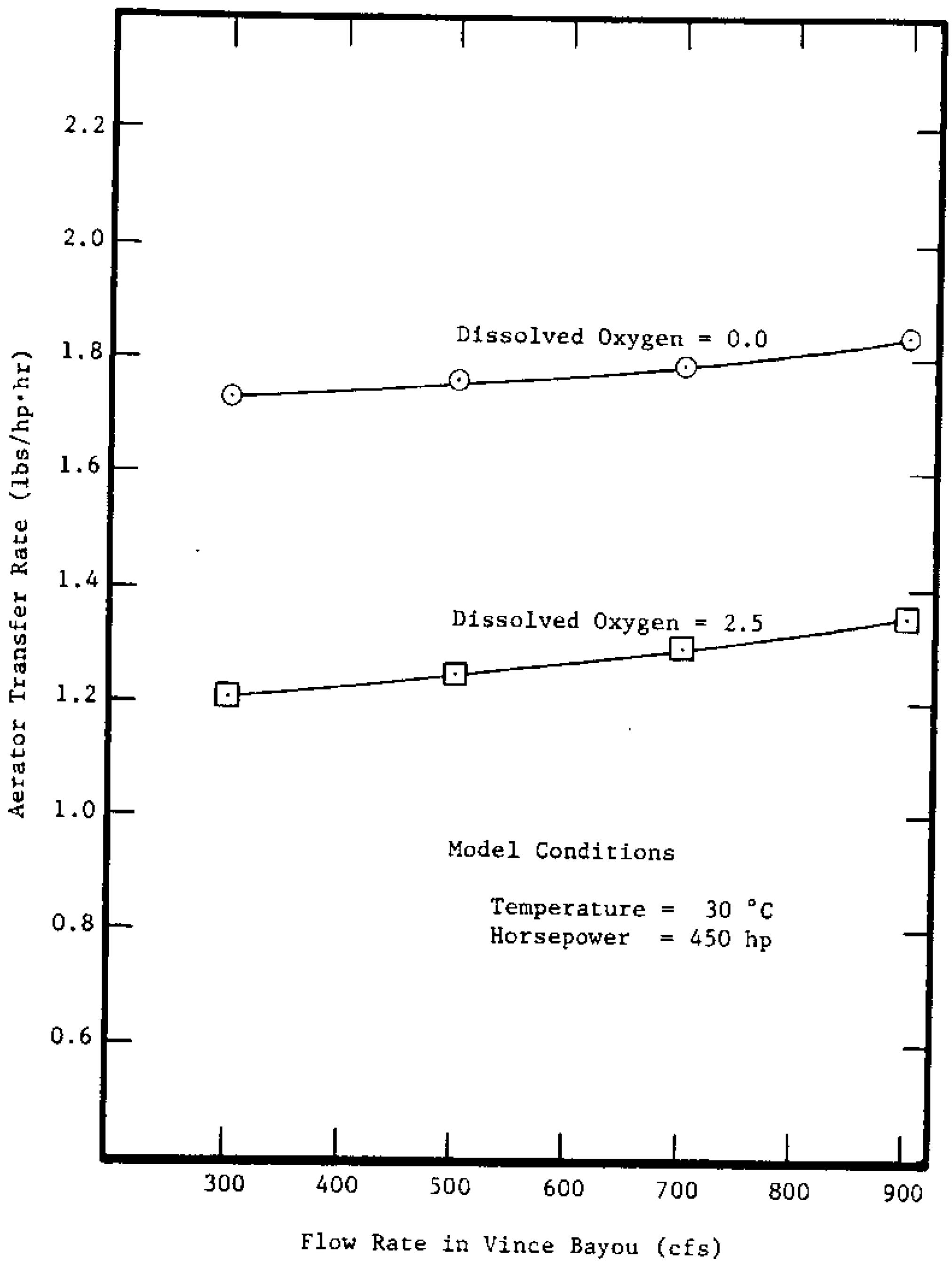

FIGURE 5-3. EFFECT OF FLOW RATE ON AERATOR TRANSFER RATE IN VINCE BAYOU

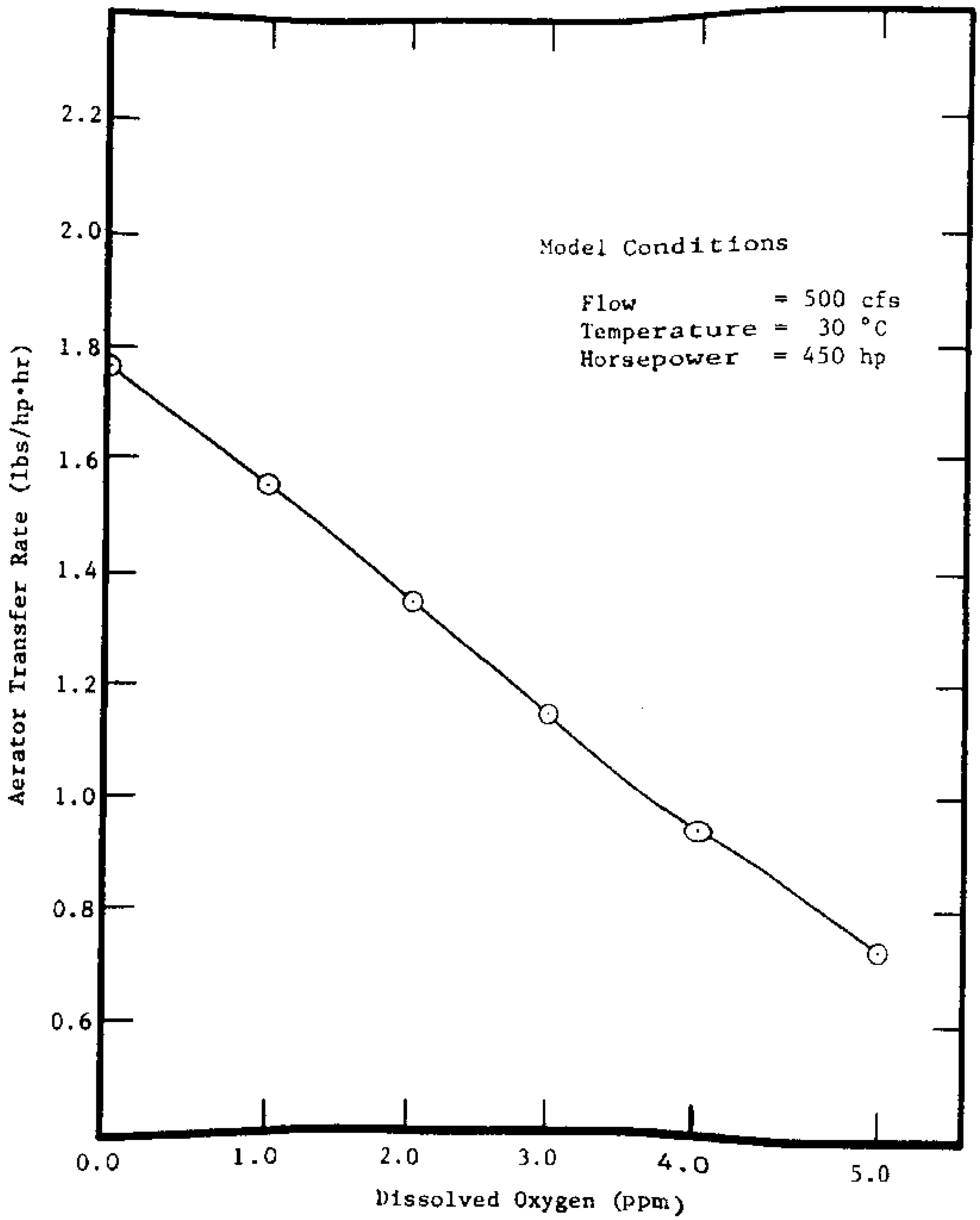

FIGURE 5-4. EFFECT OF DISSOLVED OXYGEN ON AERATOR TRANSFER RATE IN VINCE BAYOU

be expected to add about 900 lbs/hr (pounds per hour) oxygen to the Houston Ship Channel under optimum conditions, when the DO level in the channel is 3.0 ppm only about 600 lbs/hr oxygen would be added and at 5.0 ppm only 300 lbs/hr.

The effect of temperature on aerator transfer rate is shown in Figure 5-5. As with the flow rate, the effects of temperature on oxygen transfer efficiency do not appear to be significant.

Figure 5-6 can be used to estimate the optimum aerator horsepower for an aeration system on Vince Bayou. It shows the changes in the overall aerator transfer rate which might result if the number of 75 hp aerators installed was varied from two to nine. The sharp slope of the curve between 300 hp (horsepower) and 450 hp indicates that the optimum installed horsepower may be about 300 hp. Therefore, for Vince Bayou, an aeration system consisting of four 75 hp aerators may produce the most efficient use of aerator horsepower.

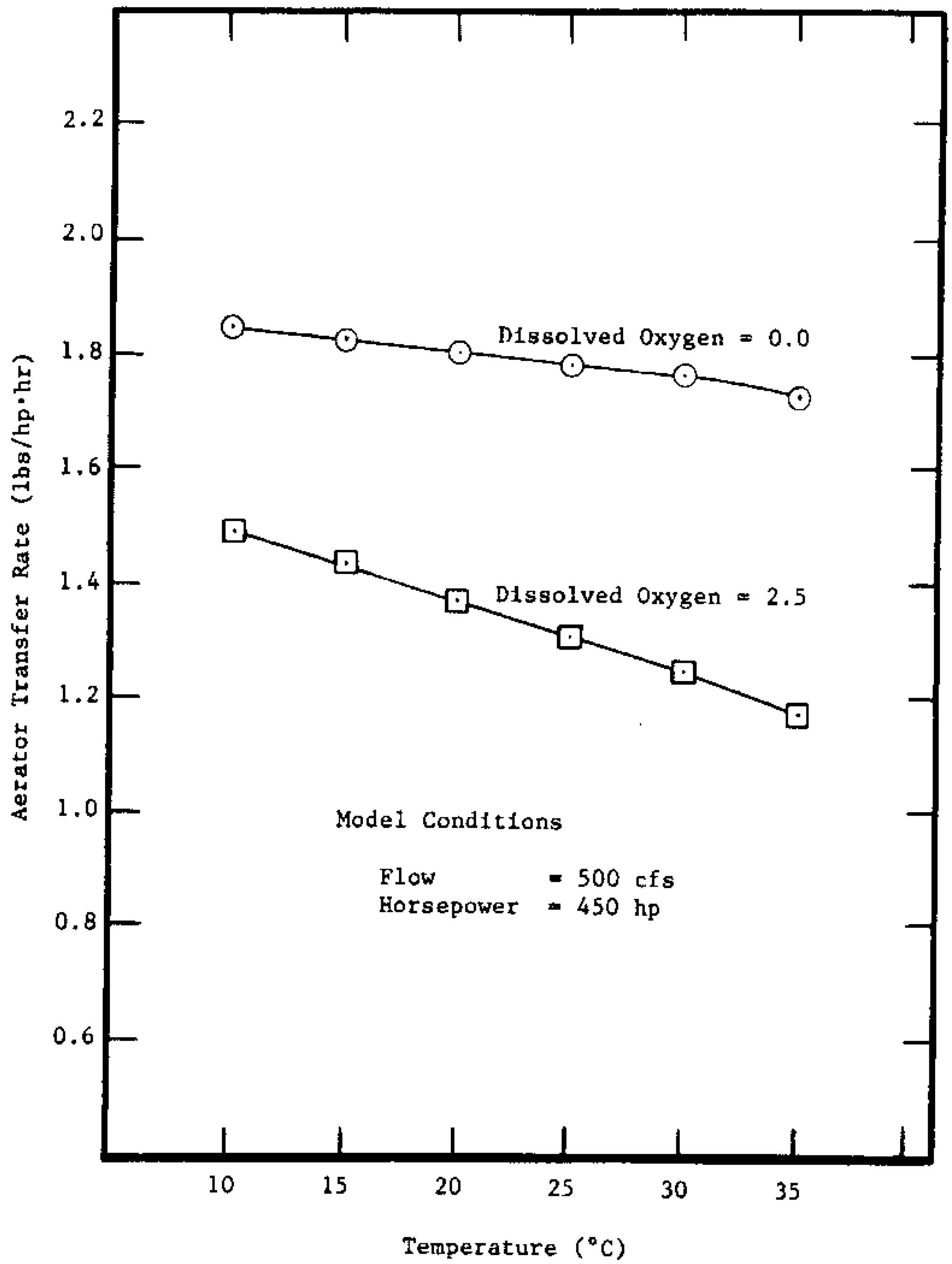

FIGURE 5-5. EFFECT OF TEMPERATURE ON AERATOR TRANSFER RATE IN VINCE BAYOU

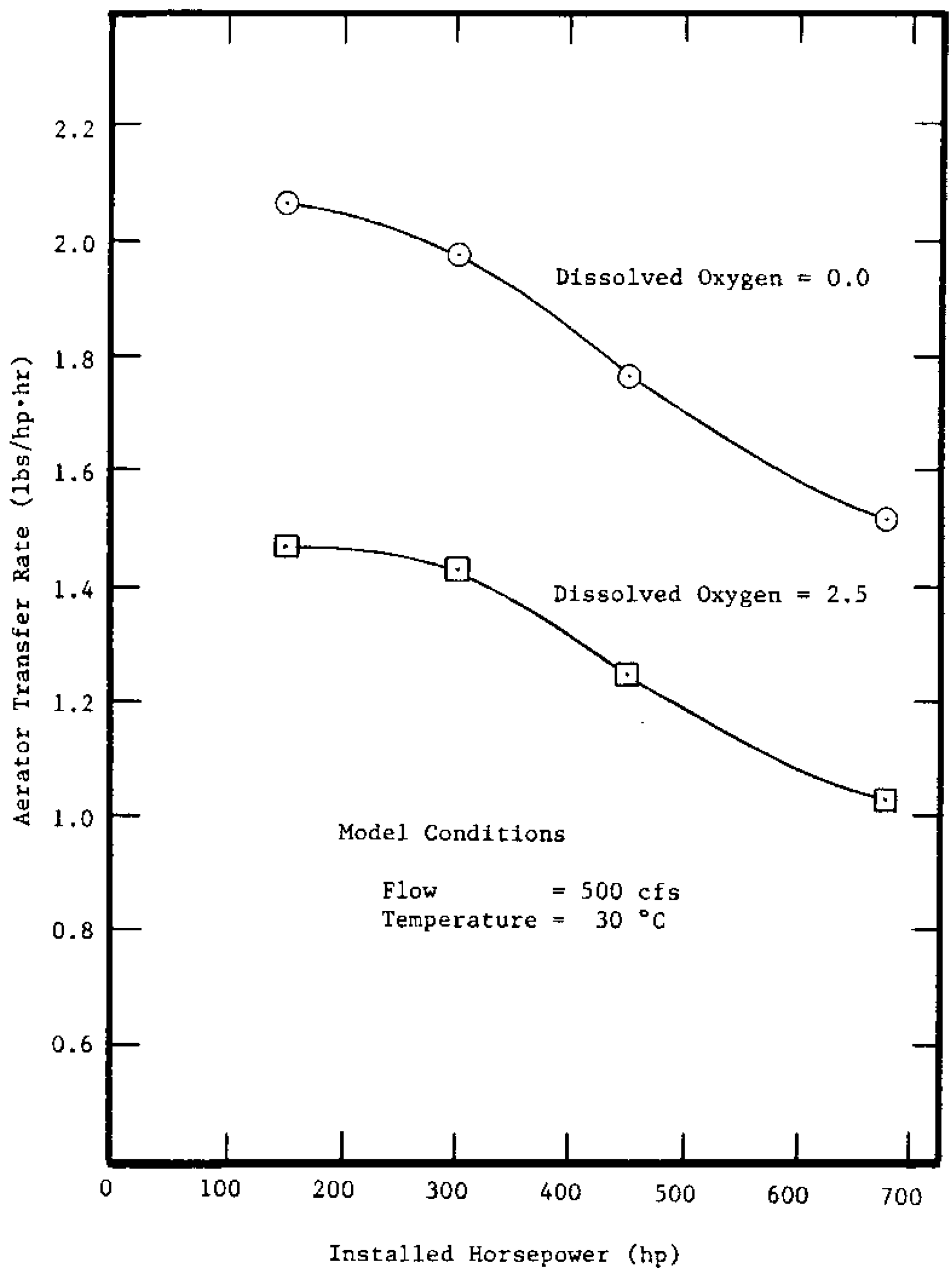

FIGURE 5-6. EFFECT OF INSTALLED HORSEPOWER ON AERATOR TRANSFER RATE IN VINCE BAYOU

Conclusions

The actual aerator transfer rate of a surface aeration system operating on Vince Bayou may range from one to two lbs/hp·hr depending on conditions at the aerator site. Model results indicate that the most significant factors that would effect the transfer rate are dissolved oxygen concentration in the Houston Ship Channel and the installed horsepower at the aeration site. Flow rate and temperature would apparently have little effect on the oxygen transfer rate. Based on model results, the optimum surface aeration system for Vince Bayou would consist of four 75 hp aerators operating only when dissolved oxygen levels are less than about three ppm in the Houston Ship Channel.

The mathematical model used in this theoretical study could be employed effectively in conjunction with an actual aeration system in Vince Bayou. Field data could be used to determine dispersion effects and boundary conditions while model results could be used to verify oxygen transfer rates determined in field studies.

Improvements to this modeling effort could be made in several ways. First, the equations used to model aerator performance could be verified by applying the model to actual inchannel aeration data such as that given by Yu (57). Second, smaller time and distance increments could be used. And third, an analysis of the effects of boundary conditions on the model results could be made.

CHAPTER VI

SUPPLEMENTAL AERATION OF THE HOUSTON SHIP CHANNEL

Introduction

The Houston Ship Channel is economically important because it carries commercial shipping to the Port of Houston. It is also ecologically important as the primary recipient of liquid waste discharges in the Houston area. Partially treated waste streams from industries and sewage treatment plants, as well as storm runoff, are the principal sources of pollution. As a result of these pollutants, the ship channel generally has low dissolved oxygen levels and high concentrations of nutrients and toxic compounds (16)(45). Unless degraded in route, wastes discharged into the Houston Ship Channel eventually reach Galveston Bay which is a primary commercial fishery, recreational area, and nursery ground for aquatic life (8).

Because of its importance to the ecology of Galveston Bay, the Houston Ship Channel has been the subject of much environmental concern in recent years. Studies have shown that the ship channel's natural reaeration capacity is not adequate to assimilate the cumulative oxygen demand of storm water runoff, bottom deposits, and untreated waste streams (16).

Although higher levels of waste treatment will significantly reduce the oxygen demand on the Houston Ship Channel, it has been projected that the nitrogenous, benthal, and storm runoff demands will continue to exceed the natural assimilative capacity (16). For this reason, supplemental aeration has been given serious consideration as a means to achieve and maintain required dissolved oxygen levels in the Houston Ship Channel.

The Gulf Coast Waste Disposal Authority and Texas A&M Research Foundation (16) have proposed that a supplemental aeration system be constructed and operated on the Houston Ship Channel. Two objectives of the proposed aeration system are "to demonstrate the feasibility and effectiveness of inchannel aeration as a means of providing failsafe quality management for the Houston Ship Channel" and "to develop the scientific and engineering technology needed to implement inchannel aeration for the Houston Ship Channel."

The Environmental Engineering Division of Texas A&M University has the primary responsibility for the technical evaluation and research phase of the proposed aeration system. Preliminary investigations by the Environmental Division concerning this project have been directed toward the following goals (16):

1. Determining dispersion and mixing characteristics in the Houston Ship Channel.
2. Determining flow patterns in the ship channel due to tidal effects and stratification.

3. Recording ship traffic and its effect on mixing.

4. Measuring chemical and physical parameters such as dissolved oxygen, salinity, and temperature.

Several studies have shown that inchannel aeration can be a practical and economical method of replenishing the oxygen of polluted streams (See Appendix II). In this case, supplemental aeration would provide a quality reserve in the Houston Ship Channel to overcome treatment plant upsets, spills, residual demand of treated waste streams, sediment demand, and storm runoff demand.

The goal of this final phase of the model application was to provide information concerning the design of an aeration system for the Houston Ship Channel. The modeling work was based on low-flow, summer conditions for two reasons. First, well-mixed conditions in the Houston Ship Channel occur during low flow, therefore the one-dimensional approximation becomes valid. Second, summer temperatures and low flows produce the lowest dissolved oxygen levels in the channel. An aeration system designed for these extreme conditions should be adequate on a year-round basis.

Model Inputs

One of the most important aspects of estuarine modeling is an adequate description of the prototype system. For a large, complex estuary such as the Houston Ship Channel extensive data is needed to fit the math model to the prototype. The information used in this modeling effort has been taken from several sources and describes as nearly as possible conditions existing in the Houston Ship Channel during summer low flows.

For modeling purposes, the ship channel was divided into 60 quarter-mile segments numbering from the Turning Basin to the San Jacinto State Park. Average widths, depths, surface areas, cross-sectional areas, and volumes were determined for each segment using Coastal and Geodetic Survey Map Number 590 (November, 1970, revision).

Figure 6-1 shows the temperature, salinity, and benthal decay rate values used in the model. The temperature data was taken from a Houston Ship Channel Data Summary by Hann et al. (19). The salinity data was reported by Hann (20) and the benthic oxygen demands were reported by Reynolds et al. (46).

Longitudinal dispersion coefficients for the low-flow conditions used in the model were assumed to vary from 500 ft^2/sec at mile 24 to 1000 ft^2/sec at mile 10. These values are less than those estimated from the salinity profile shown in Figure 6-1.

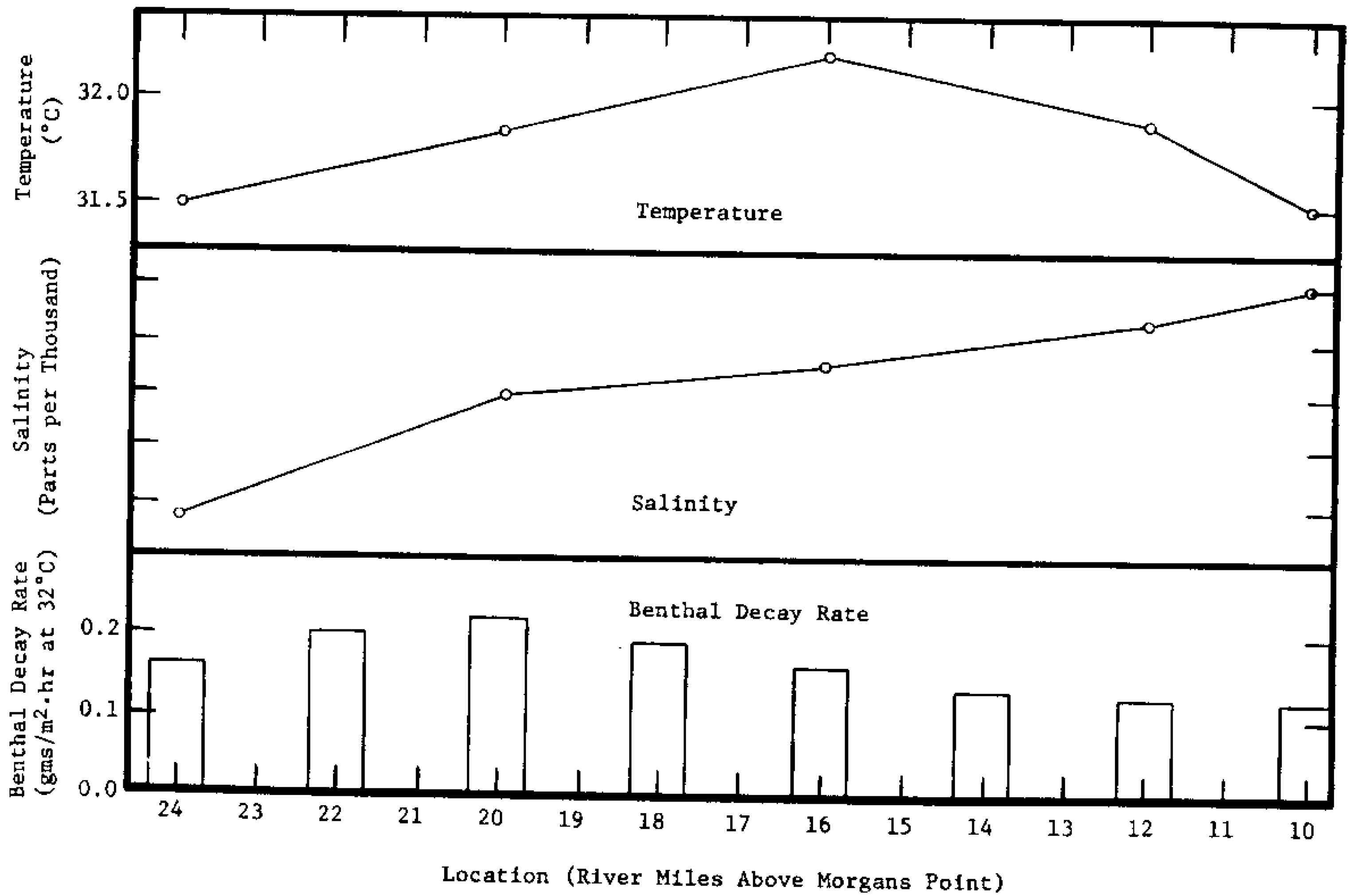

FIGURE 6-1. INPUT DATA FOR THE HOUSTON SHIP CHANNEL MODEL

Stream discharges and corresponding BOD_u and DO concentrations were estimated from United States Geological Survey (USGS) data. Flows were based on the median July discharge during the period 1699 to 1970 as determined from USGS gauging station data. Table 6-1 shows stream flow information for tributaries to the upper 15 miles of the Houston Ship Channel. Table 6-2 shows the BOD_u loading in lbs/day (pounds per day) for the upper 15 miles of the ship channel from all sources. This table is based on self-reporting municipal and industrial effluent data for the period December 1971 through March 1972.

Aspects of the Houston Ship Channel Model

The Houston Ship Channel Model considers mass balances on both dissolved oxygen and BOD_u. The oxygen balance term (OXBAL) in the math model for segment K is as follows:

$$\mathrm{OXBAL(K) = ROXA(K) + AAER(K) + REARN(K) - DECAY(K) - BENTH(K)} \quad \ldots\ldots\ldots\ldots (6\text{-}1)$$

TABLE 6-1

STREAM FLOW DATA

Bayou	Flow (CFS)	BOD_u (PPM)	DO (PPM)
Buffalo	140	10.1	5.5
White Oak	20	16.1	7.0
Brays	40	7.0	8.0
Sims	25	14.7	5.5
Vince	10	12.0	7.0
Hunting	5	24.5	6.5
Halls	5	17.7	7.0
Greens	5	9.4	8.0
Carpenters	20	11.6	7.0

TABLE 6-2

HOUSTON SHIP CHANNEL BOD_u LOADINGS

Segment	Flow (CFS)	BOD_u Loading (lbs/day)
1	250	48,930
8	44	1,780
14	2	700
17	110	23,520
18	12	6,230
19	7	810
20	55	13,300
23	10	650
26	5	4,220
29	10	1.130
32	3	120
33	34	2,520
36	21	3,260
37	16	2,780
38	10	1,120
44	2	770
50	10	930
53	175	6,040
55	5	4,500
59	20	1.250
		124,560

where: ROXA(K) = oxygen added to the segment by inflow;

AAER(K) = artificial source of oxygen;

REARN(K) = atmospheric reaeration;

DECAY(K) = oxygen lost by the decay of dissolved organic matter (BOD_u); and

BENTH(K) = oxygen lost by sediment oxygen uptake.

Equation 6-1 units are in ppm/sec (parts per million per second).

Atmospheric reaeration is modeled in the following form:

AR(K) = (0.06 + 3.3*UN(K)/(DEPTHN(K)**1.33))*CF . . . . . . (6-2)

REARN(K) = AR(K)*(CSAT(K) - CONC) . . . . (6-3)

where: AR(K) = the atmospheric reaeration coefficient in units of 1/sec (liter per second) at segment K;

UN(K) = the velocity in ft/sec;

DEPTHN(K) = the depth of segment K;

CF = a correction factor for temperature and salinity;

REARN(K) = atmospheric reaeration for segment K in ppm/sec;

CSAT(K) = saturation oxygen concentration for segment K; and

CONC = actual oxygen concentration in segment K.

The benthal oxygen uptake rate is calculated as:

BENTH(K) = BRC(K)*W(K)*DELX . . . . (6-4)

where: BENTH(K) = the benthal oxygen uptake rate at segment K in ppm/sec;

BRC(K) = the benthal demand in ppm/ft^2·sec (parts per million per feet squared per second);

W(K) = the segment width; and

DELX = the segment length.

The mass balance on BOD_u (BODBAL) is the same as used in the Corpus Christi Harbor Channel study.

BODBAL(K) = RBODA(A) - DECAY(K) . . . . (6-5)

where: RBODA(K) = BOD_u added to the segment by inflow; and

DECAY(K) = BOD_u lost by decay.

Loss of BOD_u by sedimentation was not considered in the Houston Ship Channel model although it could be included as an additional term in the BOD_u mass balance.

Specific model conditions are shown in Table 6-3. Although the stability criteria exceed 1.0 for some of the model segments no evidence of instability was observed in the actual modeling work.

TABLE 6-3

MODEL CONDITIONS FOR THE HOUSTON SHIP CHANNEL

Parameter	Value
Time increment	2800 seconds
Distance increment	0.25 miles
Stability criteria	0.80 - 1.60
Pseudo-dispersion	0 - 112 ft^2/sec
Dispersion coefficients	500 - 1000 ft^2/sec

The following boundary conditions were employed in the math model.

1. No mass transfer of either BOD_u or DO across the upper boundary of the model region (Turning Basin).
2. Constant slope extrapolation of BOD_u concentrations below mile 10.
3. Constant DO concentration of 2.0 ppm at mile 9.5 (San Jacinto River influence).

All model results shown are based on a two-foot diurnal tide.

Results

The results presented in this section are based on a numerical analysis of inchannel aeration in the Houston Ship Channel. Aeration sites were modeled as 600 horsepower of surface aeration with a theoretical efficiency at standard conditions of 3.0 lbs/hp•hr. Oxygen added by surface aeration was assumed to be mixed uniformly within the quarter-mile channel segments.

The effects of a surface aeration site at river mile 20 on dissolved oxygen and biochemical oxygen demand levels in the Houston Ship Channel are shown in Figure 6-2. One set of curves shows concentrations that may exist during low-flow summer conditions without artificial aeration. Under these conditions the channel has no DO from mile 12 to mile 23 and BOD_u concentrations range from 33 ppm at mile 20 to 11 ppm at mile 10. The other set of curves shows concentrations that may exist with an aerator site at mile 20. The primary effect of this aeration scheme would be to reduce BOD_u concentrations along the channel and particularly at mile 20 where peak concentrations occur. An aeration site at mile 20 would not significantly shift DO concentrations upstream of mile 10 and would raise DO levels only slightly at mile 20.

Figure 6-3 illustrates the possible impact of full scale aeration systems operating in the Ship Channel. In comparison with no inchannel aeration, supplemental aeration sites at miles 13 and

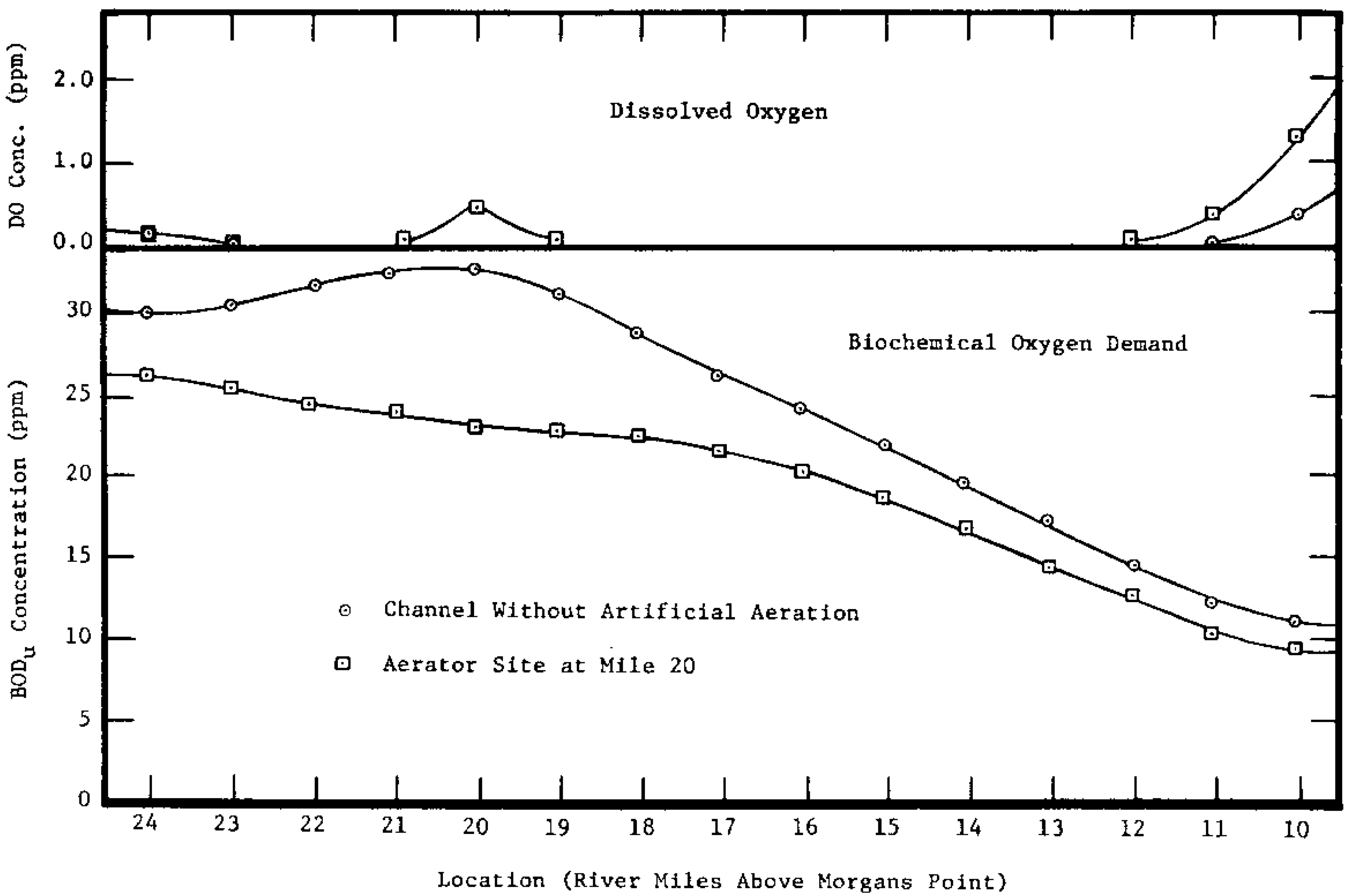

FIGURE 6-2. EFFECT OF A SURFACE AERATOR SITE AT MILE 20

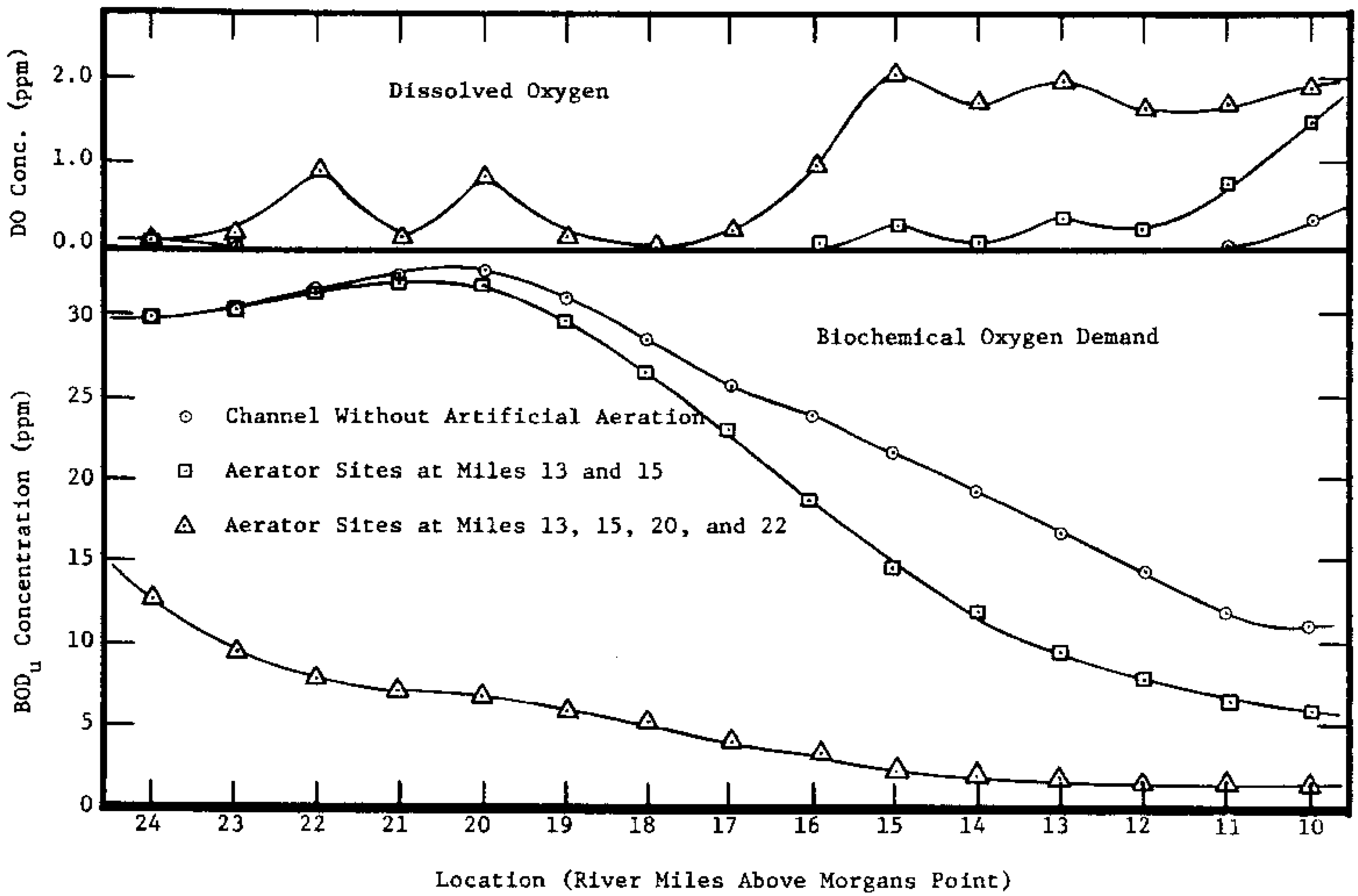

FIGURE 6-3. EFFECTS OF SURFACE AERATION SYSTEMS IN THE HOUSTON SHIP CHANNEL

15 would shift the zero oxygen level from mile 10 to mile 16 while BOD_u concentrations would be reduced at mile 10 from 11 ppm to 6 ppm. The remaining set of curves shows the effect of an aeration system consisting of four 600 hp aeration sites at miles 13, 15, 20, and 22. This system would substantially reduce BOD_u levels throughout the channel. There would be measurable dissolved oxygen from mile 10 to the Turning Basin even under low-flow summer conditions. The two ppm DO level would be moved upstream as far as mile 15, creating a quality reserve that would be adequate to handle demands created by storm water runoff or treatment plant upsets. These model results show that inchannel aeration would significantly improve water quality in the Houston Ship Channel and minimize the effects of uncontrolled waste discharges.

Figure 6-4 indicates the time period required for changes, such as supplemental aeration, to be felt throughout the channel. Changes in the BOD_u and dissolved oxygen concentrations at mile 10 are shown over a 30-day period after supplemental aeration at miles 13, 15, 20, and 22 has begun. For a change of this magnitude, Figure 6-4 indicates that about a month is required for equilibrium conditions to become reestablished. This long response period implies that steady-state conditions are probably never obtained in a physical system as large as the Houston Ship Channel and that significant errors may be introduced by applying a steady-state analysis to actual channel conditions.

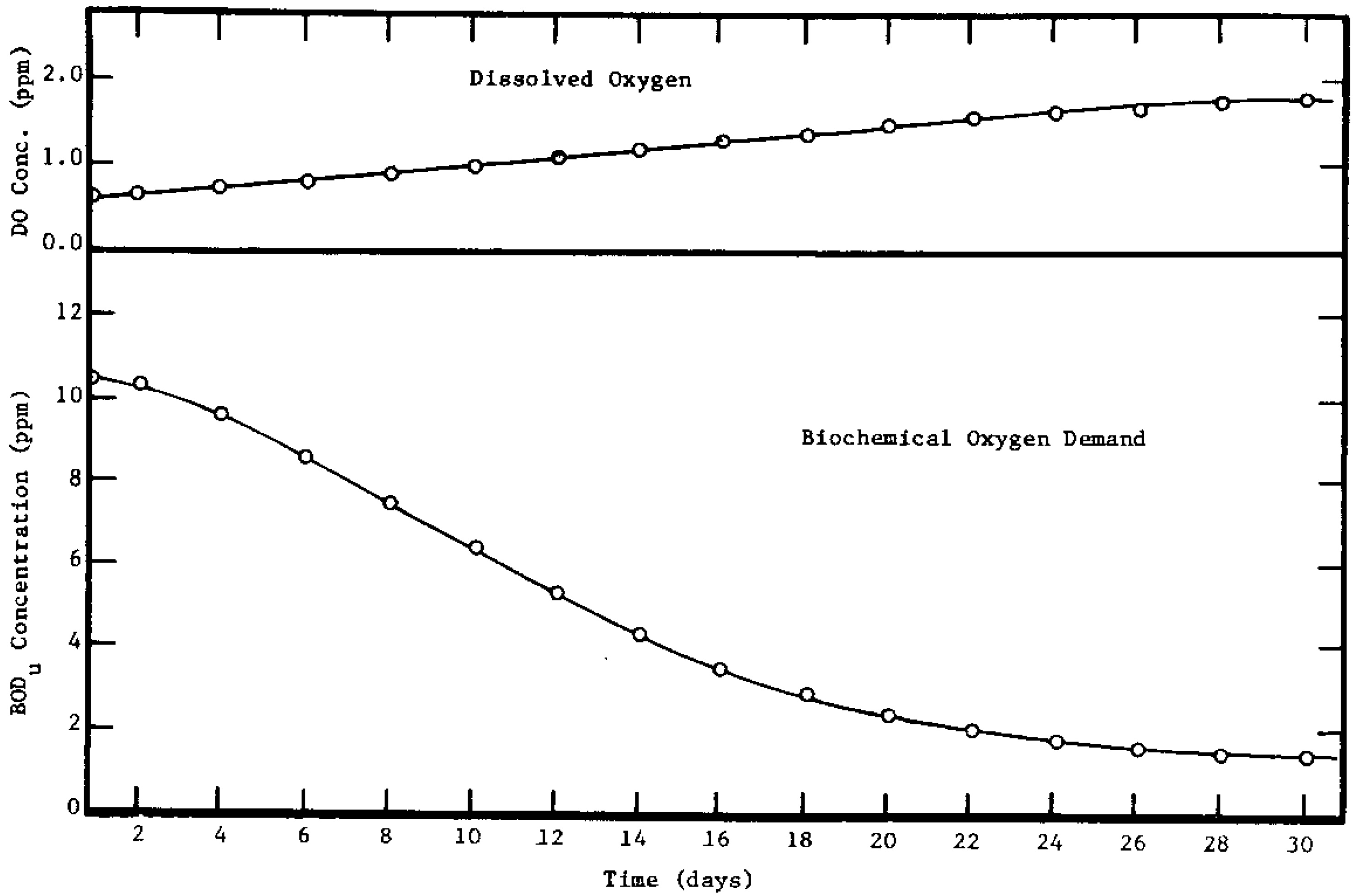

FIGURE 6-4. TIME RESPONSE AT MILE TEN IN THE HOUSTON SHIP CHANNEL

Figure 6-5 illustrates the effect of tidal movement on concentrations and velocities in the Houston Ship Channel. The tidal response curve is for mile 10 with the theoretical aeration site at mile 20. Figure 6-5 shows that the BOD_u concentration at mile 10 reaches a maximum about five hours after the peak ebb (downstream) velocity while the DO concentration reaches a maximum about two hours after the peak flood (upstream) velocity. This effect is caused by tidal movement of better quality water below mile 10 past the observation point at mile 10.

One of the most important questions to be answered by this study was: "What effect does tidal flow have on aerator performance in the Houston Ship Channel?". Figure 6-6 shows the tidal variation of dissolved oxygen concentrations and aerator efficiency at mile 13. These model results show that flow direction and velocity may have very little effect on aerator efficiency. There is some justification for these results because dispersion has a more dominant effect on concentrations in the Houston Ship Channel during low-flow conditions than do the tidal velocities. However, the one-dimensional assumption and relatively large increments used in the model, probably override actual factors affecting aerator efficiency and bias the results. Thus, the inadequacies of the mathematical model may be the major factor in the results shown by Figure 6-6.

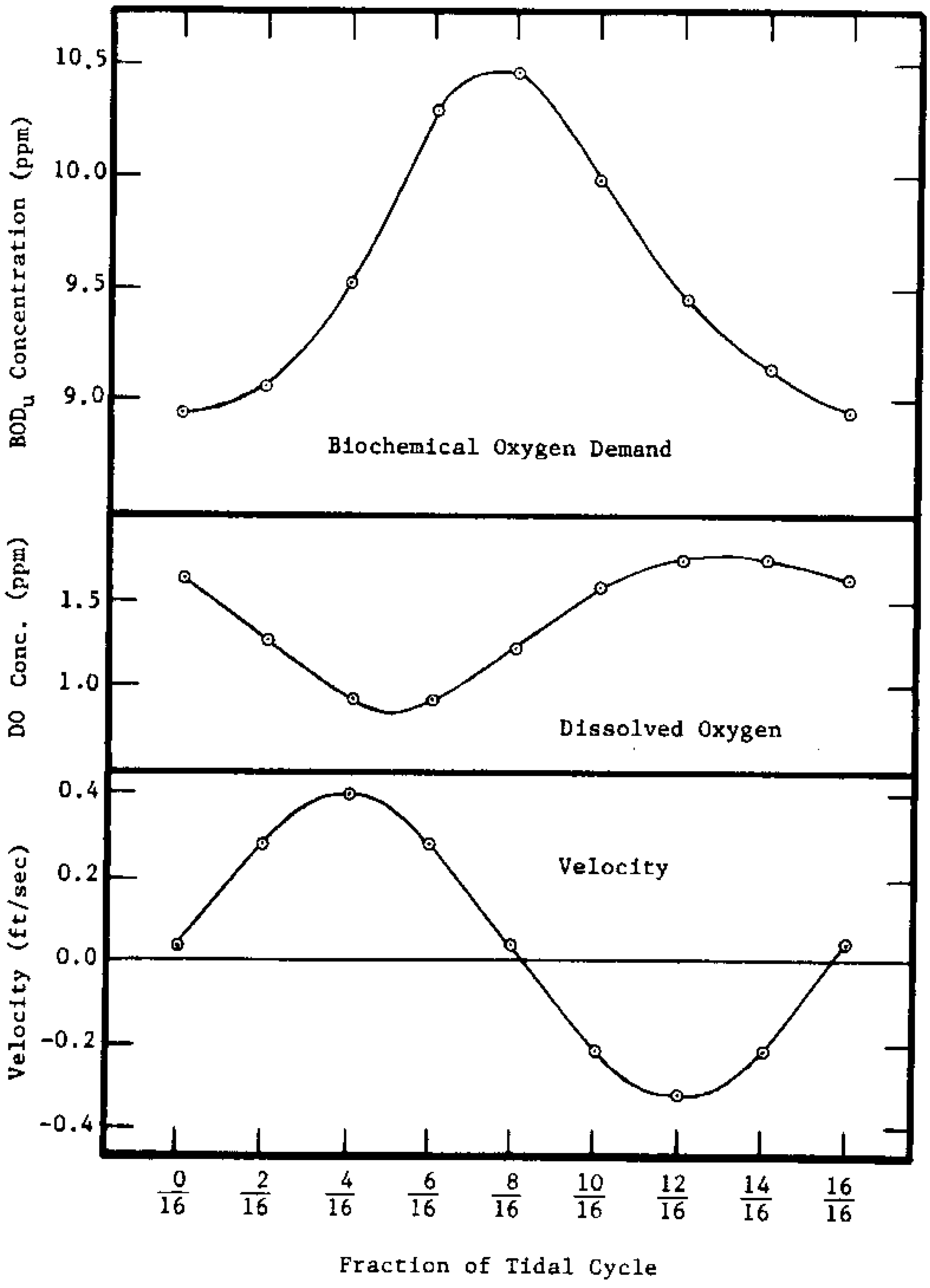

FIGURE 6-5. TIDAL RESPONSE AT MILE 10 WITH AN AERATOR SITE AT MILE 20

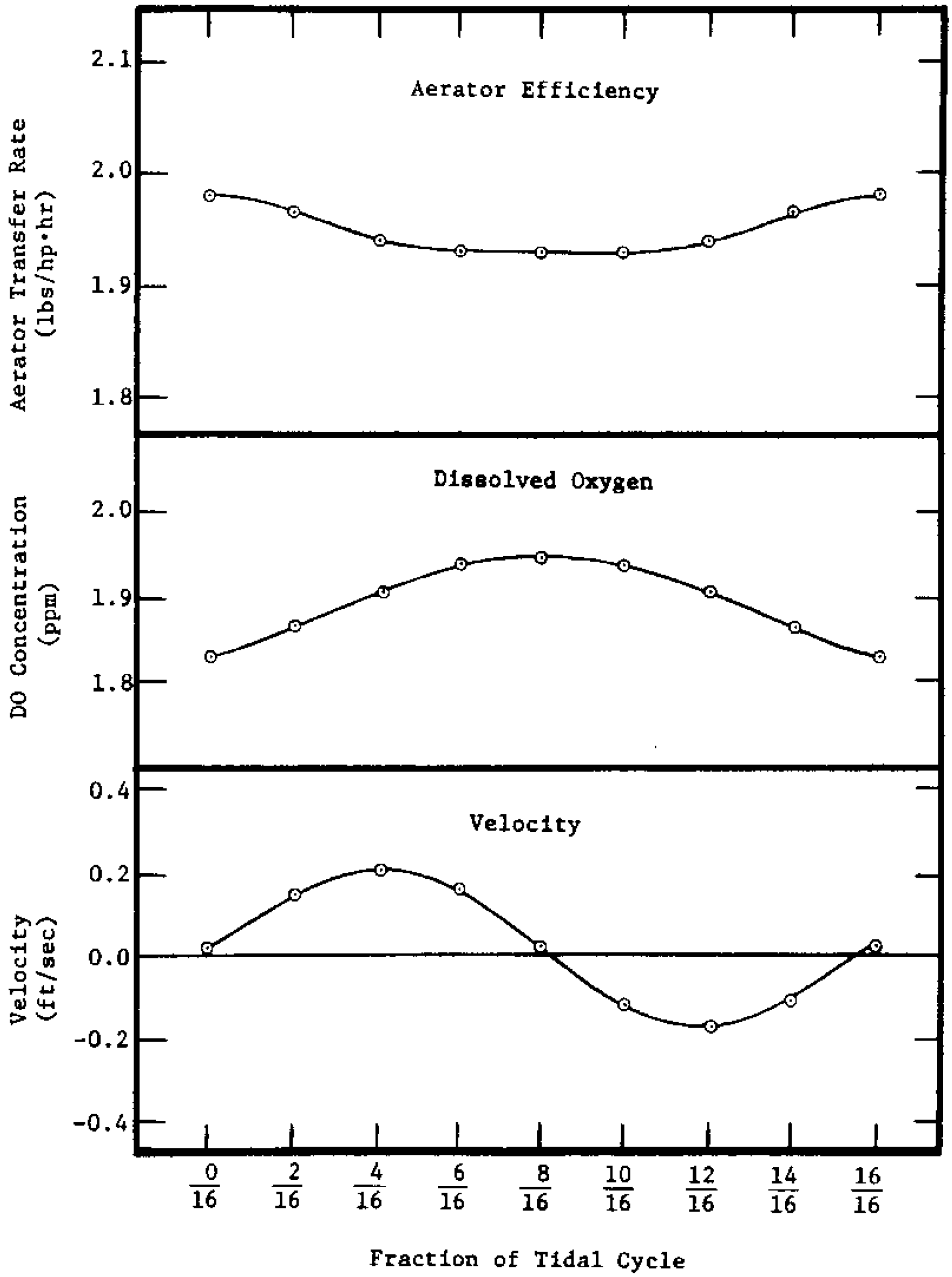

FIGURE 6-6. TIDAL EFFECTS ON AERATOR TRANSFER RATE

Summary

Model results indicate that artificial aeration could be an effective method of supplementing the natural reaeration capacity of the Houston Ship Channel. Based on low-flow summer conditions, an aeration system consisting of four aerator sites with 600 horsepower at each site would significantly improve DO levels in the channel. Although this aeration system would not be completely adequate during extreme conditions, it would reduce BOD_u values to a level which would not degrade Galveston Bay waters.

The Houston Ship Channel was shown to respond slowly to significant changes, such as artificial aeration, with steady-state conditions being reached only after about 30 days. Therefore, the full effect of an artificial aeration system would require over one month to be felt throughout the channel and a full appraisal could not be made before this response period.

Model results have also shown that tidal effects may have little influence on the efficiency of a surface aeration system in the ship channel.

This study represents an initial effort to determine quantitatively the benefits of supplemental aeration of the Houston Ship Channel. It has significant limitations to its applicability due primarily to the complex and dynamic nature of organic loadings and flows in the ship channel.

Several improvements in the techniques and data used in this study could be made to improve the accuracy of the results. One improvement would be to make a more concise estimation of the magnitude and source of biodegradable organics entering the Houston Ship Channel. This estimation should include storm runoff demand, nitrogenous demands, and an update of organics contributed by waste treatment plants. A second improvement would be to consider changes in flow rates, tide heights, and temperatures over a longer study period. Consideration of these changes would require significantly more data but would be more representative of the dynamic nature of the Houston Ship Channel.

CHAPTER VII

SUMMARY AND CONCLUSIONS

The goals accomplished in this study were the development of a one-dimensional dynamic mathematical model suitable for use in estuaries and the application of this math model to estuarine dispersion problems.

The mathematical model developed was based on the one-dimensional mass transport equation written in finite-difference terms. An implicit, central-difference form of the finite-difference equations was used since it offered greater stability and more flexibility in setting time and distance increments. This model was applied to three dispersion problems using a wide range of model conditions.

The dispersion problems considered were: (1) distribtion of organic wastes in the Corpus Christi Harbor Channel, (2) design of an aeration system for Vince Bayou, and (3) aerator performance in the Houston Ship Channel. Based on model results, the following conclusions may be made concerning these dispersion problems:

1. Corpus Christi Harbor Channel
    a. Tidal velocities are small and the hydraulic characteristics of the Harbor Channel are dominated by power plant intake at mile 1.5.
    b. Tidal variations in BOD_u concentrations are generally

less than one ppm.

c. The channel response time to major changes is about 10 days.

d. The magnitude of the dispersion coefficients in the Corpus Christi Harbor Channel is approximately 250 ft^2/sec.

2. Aeration system for Vince Bayou

a. The two most important factors affecting oxygen transfer rates of the proposed surface aeration system are the installed horsepower and DO levels in the Houston Ship Channel.

b. Dispersion may also have a significant effect on the oxygen transfer rate because DO concentrations will be kept low by dispersion into Houston Ship Channel waters.

3. Aeration systems for the Houston Ship Channel

a. Tidal effects may have little influence on proposed surface aeration systems in the Houston Ship Channel.

b. Oxygen levels in the ship channel will be increased by artificial aeration, but a more significant effect may be the reduction of waste loads carried into Galveston Bay.

c. The response time to a major change in Houston Ship Channel conditions is about 30 days.

The one-dimensional dynamic mathematical model has been shown to be an effective tool in analyzing these dispersion problems. The model is also applicable to dynamic estuarine situations that have not been considered in this study, for instance:

1. Modeling estuarine response to large-scale changes such as seasonal temperature and flow variations.
2. Modeling diurnal oxygen variations due to photosynthesis and temperature changes.

However, one-dimensional models are limited to use in well-mixed systems and adequate results can be expected only when the prototype system corresponds to the one-dimensional approximation.

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APPENDIX II

INCHANNEL AERATION

Upgrading the water quality of rivers and estuaries is a complex problem facing engineers and scientists today. Probably the most important factor in maintaining satisfactory water quality is the degree of treatment of waste effluents from municipal and industrial facilities. Secondary treatment of waste effluents is now required and tertiary treatment has received consideration in several situations. However, higher treatment standards produce much higher treatment costs and alternate ways to upgrade water quality are being sought.

Whipple (53) has reviewed some of the methods suggested for upgrading water quality such as:

1. Storing and treating stormwater runoff,
2. Identifying and treating now-overlooked waste sources,
3. Low-flow augmentation,
4. Diverting polluted waters around rivers, and
5. Instream aeration.

Of these methods Whipple says that "instream areation probably holds the greatest promise as a supplementary method for upgrading water quality"(53). In addition, instream aeration will probably be less costly than tertiary treatment for improving the quality of receiving waters.

Inchannel Aeration Studies

Several studies have involved the use of inchannel aeration as a means of increasing oxygen levels in polluted waters. A brief summary of some of these studies follows.

Chicago River Study

In 1962 and 1963 an experimental aeration study was carried out on the Chicago River by the Metropolitan Sanitary District of Greater Chicago (44). The aeration system consisted of two 75 horsepower (hp) Yeoman surface aerators mounted on a catamaran. Oxygen transfer rates were determined at several locations, with dissolved oxygen levels recorded upstream and downstream of the aerator site. Flows were calculated from hourly velocity readings taken at the Chicago Avenue bridge.

Kaplovsky et al. (34) summarized the results of these tests. They found that the oxygen transfer rate at standard conditions (20°C, zero DO, and clean water) ranged from 1.49 to 4.56 pounds per horsepower per hour (lbs/hp•hr). However, the transfer rate was found to increase dramatically with a change in flow rate from 2800 to 4400 cfs. No explanation was given for this dependence of transfer rate of flow.

Jackson River Study

As part of the pollution abatement program for the Covington Mill of West Virginia Pulp and Paper, a pilot mechanical aeration study (4) was conducted in 1963 on the Jackson River. During low flow and high temperature summer conditions, DO levels were not sufficient to prevent nuisance conditions in the river. Two 15 hp mechanical aerators were installed on the Jackson River about 1.8 miles below the Mill outfall. The experimental study consisted of taking velocity and DO readings over the cross-section of flow above and below the aerator site. The results of this study showed that the transfer efficiency at standard conditions ranged from 1.36 to 3.27 lbs/hp·hr. Also a correlation was seen to exist between transfer efficiency and stream discharge, with efficiency increasing with increasing flow rate (4).

Ruhr River Study

Imhoff (29) has reported on artificial methods used to influence the oxygen balance of the lower Ruhr River in West Germany. Oxygen and BOD are critical in this river because the treated waste water load of some three million population equivalents are discharged into the river and part of the river flow is diverted to recharge the ground water supply. An oxygen level of 3.5 mg/l must be maintained in this diverted stream to prevent quality problems in the public water supply (29).

Three aeration devices were installed and operated on the lower Ruhr River: a compressed air system, a floating surface aerator, and a power house turbine aerator. Each system was studied for its reaeration capacity and total cost. The surface aerator system consisted of two Simcar aeration cones driven by a 150 hp engine. The oxygen transfer rate achieved by the surface aerators was about 1.5 lbs/hp·hr at standard conditions. In terms of total costs the surface aeration system was half as expensive as the compressed air system but about 3.5 times as expensive as the turbine system.

Imhoff concluded from his study that it is more feasible to use artificial aeration than to provide tertiary treatment or secondary treatment beyond 85 percent removal (29).

<u>Passaic River Study</u>

The potential of instream aeration was recently investigated by a group of engineers and scientists at Rutgers University (54). Their work was designed to provide cost and effectiveness criteria for instream aeration systems. Tests on the Passaic River in 1967 and 1968 combined the use of commercially-available equipment on a small polluted river, with scientific investigation techniques and modern systems analysis (54).

The aeration devices which were tested by Whipple et al. (54) on the Upper Passaic River, consisted of a 75 hp electrically driven surface aerator and a diffuser system. The aerator site was excavated to eight feet and was about 100 feet wide. Oxygen transfer rates determined for the surface aerator ranged from 1.2 to 3.1 lbs/hp·hr at standard conditions. Results obtained also showed some correlation between flow rate and transfer efficiency. Whipple et al. suggested that higher stream velocities breakup recirculation tendencies of aerators thus increasing the amount of oxygen transferred (54).

Hunter and Whipple (28) used modeling techniques and analog simulation to describe the BOD-DO relationships in the Passaic River. Their analysis included the effects of stream reoxygenation, photosynthesis, benthal oxygen demand, biochemical oxygen demand by both carbonaceous and nitrogenous compounds, and hydrologic considerations. Hunter and Whipple also considered optimal spacing of aerators in the Passaic River but could not complete this aspect of the study until a complete analysis of the biochemical oxygen demand was made.

Delaware Estuary Study

An extensive study of an artificial aeration system in an estuary was conducted by the New Jersey Department of Conservation and Economic Development. The results of this study, supported by an Environmental Protection Agency (EPA) grant, have been published as a Water Pollution Control Research Series (55) and by Whipple and Yu (56).

Two years of aeration field tests (1969 and 1970) were conducted at Camden, New Jersey on the Delaware River, where oxygen deficiency presents a major problem over a 40-mile stretch of the Delaware River during the summer months. Whipple and Yu found that to maintain oxygen levels at the adopted standards of 3.5 mg/l by improving waste treatment would cost millions of dollars, while the same results could be achieved by artificial aeration at one-third this cost (56). Therefore, the economic incentive of using instream aeration to provide part of the desired oxygen level was a primary consideration of this study (56).

The Delaware River at the test site is about one-half mile wide, 40 feet deep, and has a normal tidal range of six feet. Two aeartion devices, a diffuser aerator and a 75 hp surface aerator were installed at the test site. This equipment had previously been used for the Passaic River study. Results of the oxygen transfer study for the surface aerator showed a transfer rate of 3.06 lbs/hp·hr. This was higher than the 2.16 average rate obtained in the Passaic River tests. Good agreement for the diffuser transfer rate were found between the two test sites (56).

The Delaware Estuary study also included dispersion tests at the aerator site. It was found that the aerators were very effective devices for increasing river turbulence. Whipple and Yu concluded that optimum aerator site spacing would require an analysis of dispersion characteristics as well as BOD considerations.

Aeration of the Thames

A practical application of artificial aeration has recently begun on the Thames River Estuary in England. A 200 hp Simplex Cone Aerator has been installed on the Thames River at the outfall at Thames Board Mills, Ltd. to supply part of the oxygen demand required by the company's effluent. In this case, treatment facilities were impractical because of limited land and high pumping costs. The aerator has shown an efficiency rating of 2.5 lbs/hp•hr and plans have been made to add a second 300 hp unit to fully compensate for the mill's oxygen demand (1).

Limitations to the Use of Surface Aerators in Rivers and Estuaries

Artificial aeration has been shown to be an economical and effective method of raising dissolved oxygen levels in overloaded rivers and estuaries (32)(33)(38)(55). However, there are several drawbacks limiting the widespread use of artificial aeration. First, artificial aeration cannot replace effective waste treatment as the most practical means of reducing waste loads. It can, however, provide a quality reserve to meet the oxygen demands of storm runoff, untreated sources, and organic materials not reduced by secondary treatment. Second, aeration systems for large, polluted rivers would require substantial construction and operating costs. Third, surface aerators would limit the use of rivers and estuaries for navigation, water sports, and dredging. Finally, aesthetic problems such as noise, foaming, and odors would need to be resolved. These limitations, however, may be a minor disadvantage in cases where artificial aeration is the only practical means of maintaining dissolved oxygen levels (38).

Aerator Performance

Performance of surface aerators in streams and estuaries depends on such variables as aerator transfer efficiency, mixing characteristics of the aerator, stream flow rate, dispersion in the stream, and the oxygen absorption capacity of the stream. Because of variables the oxygenation capacity of surface aerators in streams is not well understood (49).

<u>Aerator Transfer Efficiency</u>

The oxygen transfer efficiency of surface aerators is usually expressed in pound per horsepower per hour (lbs/hp·hr) at standard conditions. Most commercially-available surface aerators are rated between 3.5 and 4.5 lbs/hp·hr (5). However, practical applications have shown that the transfer efficiency may range from 1.0 to 4.5 lbs/hp·hr (5)(34)(54)(56). The average transfer efficiency can be taken as 3.0 lbs/hp·hr for most applications in streams and estuaries.

The basic equation used to calculate the actual transfer rate has been given by Eckenfelder (6) as:

$$N = N_o\left(\frac{C_{sw} - C_L}{C_s}\right)(1.02)\alpha^{(t-20)} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad \text{(II-1)}$$

where: N = oxygen transfer rate at operating conditions;

N_o = oxygen transfer rate at standard conditions;

t = lagoon temperature, °C;

C_{sw} = saturation DO level at operating conditions;

C_L = actual DO level;

C_s = saturation DO of clean water at 20°C; and

α = oxygen transfer ratio.

Susag et al. (49) have investigated the oxygenation capacity of mechanical surface aeration units under flow-through conditions. They have suggested two equations for determining the theoretical oxygen transfer rate in flowing systems. The first, based on the upstream DO is as follows:

$$N_o = \frac{16.77\ Q[Cd - Cu]\ Cs}{PE[C_{sw}(\frac{\rho}{29.92}) - Cu][1.018]^{(t-20)}\alpha} \quad \text{(II-2)}$$

Where: P = the power drawn in kilowatts;

E = the mechanical efficiency of the aerator;

Q = the flow rate in cubic feet per second;

ρ = barometric pressure in inches of H_g;

C_u = upstream DO concentrations;

C_d = downstream DO concentrations;

C_s = saturation DO of clean water at 20°C;

C_{sw} = saturation DO level at operating conditions;

t = water temperature; and

α = oxygen transfer ratio.

The second, based on the arithmetic average of the upstream and downstream DO concentrations is given by:

$$N_o = \frac{16.77\ Q[Cd - Cu]\ Cs}{PE[C_{sw}(\frac{\rho}{29.92}) - (\frac{Cu + Cd}{2})][1.018]^{(t-20)}\alpha} \quad . . . . (II-3)$$

Aerator Mixing Characteristics

McKeown and Buckley (37) have made an extensive study of the mixing characteristics of surface aerators in basins. One aspect of their work considered the zone of outward influence exerted by surface aerators. Table II-1 summarizes their results.

TABLE II-1

AERATOR ZONE OF INFLUENCE

Name Plate Horsepower	Outward Zone of Influence (ft)
10	90
25	60
40 - 60	75 - 125
75	100 - 125
100	150 - 200

Table II-1 may be used to estimate aerator spacing within particular aerator site in a river or estuary. The work of McKeown and Buckley (37) also showed that the circulation pattern for surface aerators is generally outward bound in the top three feet of the water level and inward bound below three feet.

<u>Oxygen Absorption Capacity</u>

The oxygen absorption capacity of an estuary depends on two factors. One of these factors is the dissolved oxygen deficit. The DO deficit is defined as the saturation value of the water at existing conditions (C_{sw}) less the actual DO level (C_L).

$$\text{Deficit} = C_{sw} - C_L \quad \text{(II-4)}$$

The saturated DO concentrations depends on the temperature, salinity, and partial pressure of oxygen contacting the liquid. The second factor effecting absorption capacity is the relative transfer rate of the aerated liquid. Waste waters have been found to have lower rates of oxygen transfer than clean water. This can be accounted for by including the factor α in the transfer equations. Alpha (α) can be defined as:

$$\alpha = \frac{K_{LA}(\text{waste})}{K_{LA}(\text{water})} \quad \text{(II-5)}$$

Where: K_{LA} = the oxygen transfer rate.

K_{LA} is a function of the liquid film coefficient and the interfacial surface area generated per unit volume of liquid. Thus α depends not only on the waste concentration but on the turbulence generated by the aerator (7).

APPENDIX III

CHARACTERISTICS OF THE HOUSTON SHIP CHANNEL

The Houston Ship Channel is a dredged channel extending from the Gulf of Mexico to the Turning Basin in downtown Houston. Houston is one of the largest and fastest growing areas along the Texas Coast in terms of both industrial expansion and population growth. The Houston Ship Channel serves not only as the main artery to the Port of Houston but also as the primary recepient of waste effluents from the Houston area. These two uses have superceded the recreational, esthetic, and ecological functions of the ship channel, and made this waterway one of the most heavily polluted on the Gulf Coast (45).

Physical Characteristics

Extending from the Gulf of Mexico to Houston, the Houston Ship Channel is about 50 miles long with an average depth of 40 feet and minimum dredged width of 300 feet. From the Gulf to the San Jacinto River, a distance of about 35 miles, the ship channel crosses the relatively open waters of Galveston and smaller bays. However, in the 15 miles from the San Jacinto River to the Turning Basin, the ship channel is an inland waterway, depending on tidal action and fresh water flow to move pollutants into the bays (Figure B-1).

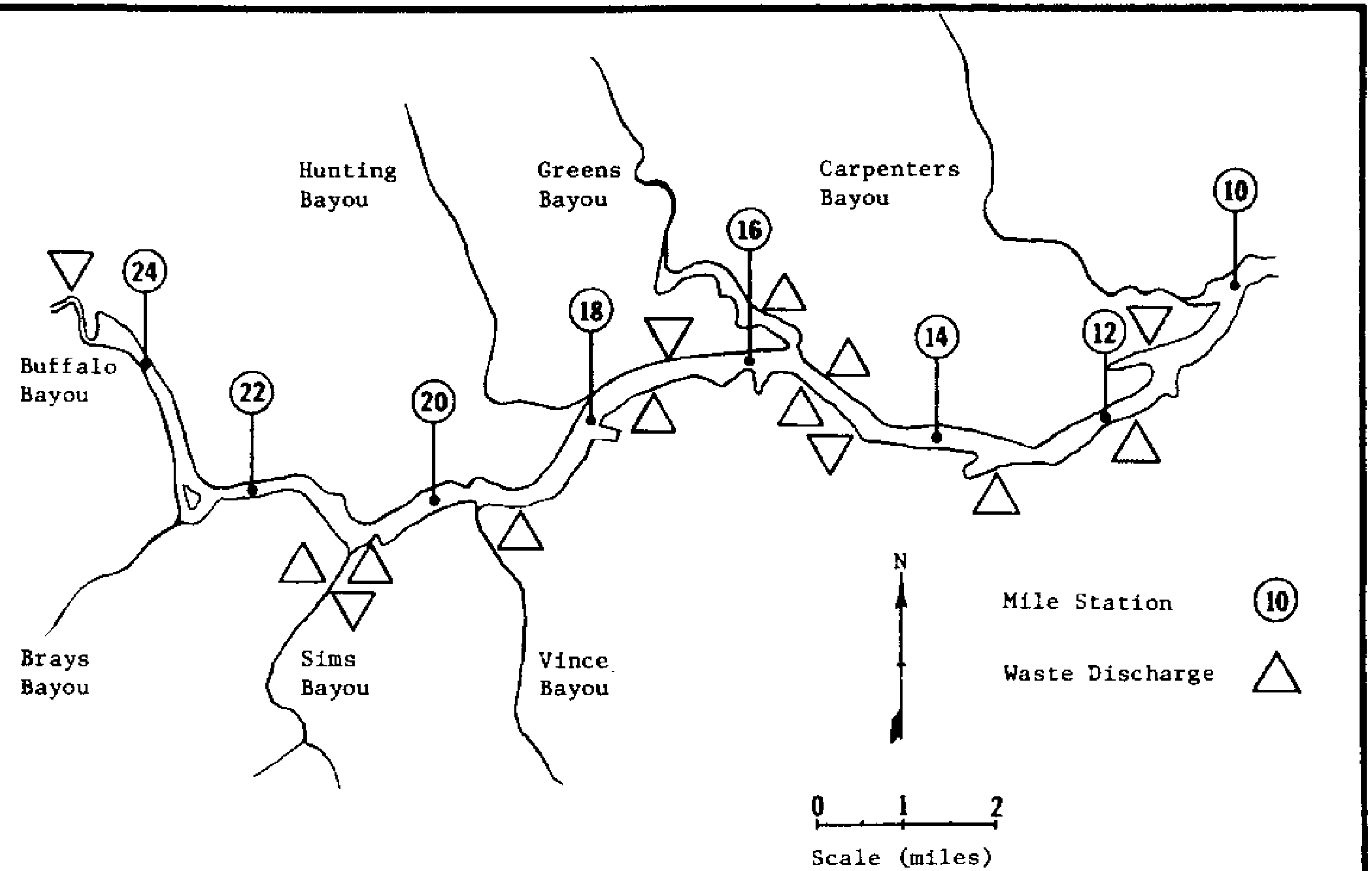

FIGURE III-3. HOUSTON SHIP CHANNEL

The Houston Ship Channel has been classified as a "partially stratified" estuary by Hann and Young (22). During periods of low flow the channel may be well mixed vertically by tidal forces, but during periods of heavy rainfall salinity stratification may develop. This variability in the salinity structure creates conditions that cannot be adequately modeled by a one-dimensional approximation. Therefore, the one-dimensional math model used in this study is limited to low-flow conditions when stratification is minimal.

Organic Loading and Dissolved Oxygen Levels

The Houston Ship Channel receives waste loads primarily from municipal and industrial waste discharges. Figure III-2 shows the history of BOD waste loadings on the ship channel (19). The present decline is an indication of increased efforts to adequately treat waste discharges. Although the waste loadings are expected to decrease even further in coming years, there is a practical limit to the reduction that can be achieved through waste treatment. In addition to municipal and industrial waste discharges, storm runoff from urban areas contributes a significant oxygen demand to the Houston Ship Channel. A third source of oxygen demand arises from the decay of bottom sediments. This benthal decay rate alone may exceed the natural reoxygenation capacity of the ship channel during summer months.

As a result of these oxygen demanding wastes, oxygen levels in the upper 15 miles of the Houston Ship Channel are generally very low. During the summer months, no DO is found from mile 10 to mile 24 and rarely do dissolved oxygen levels reach two ppm even with ideal conditions. Figure III-3 shows dissolved oxygen levels in the Houston Ship Channel during the summer of 1970 (19).

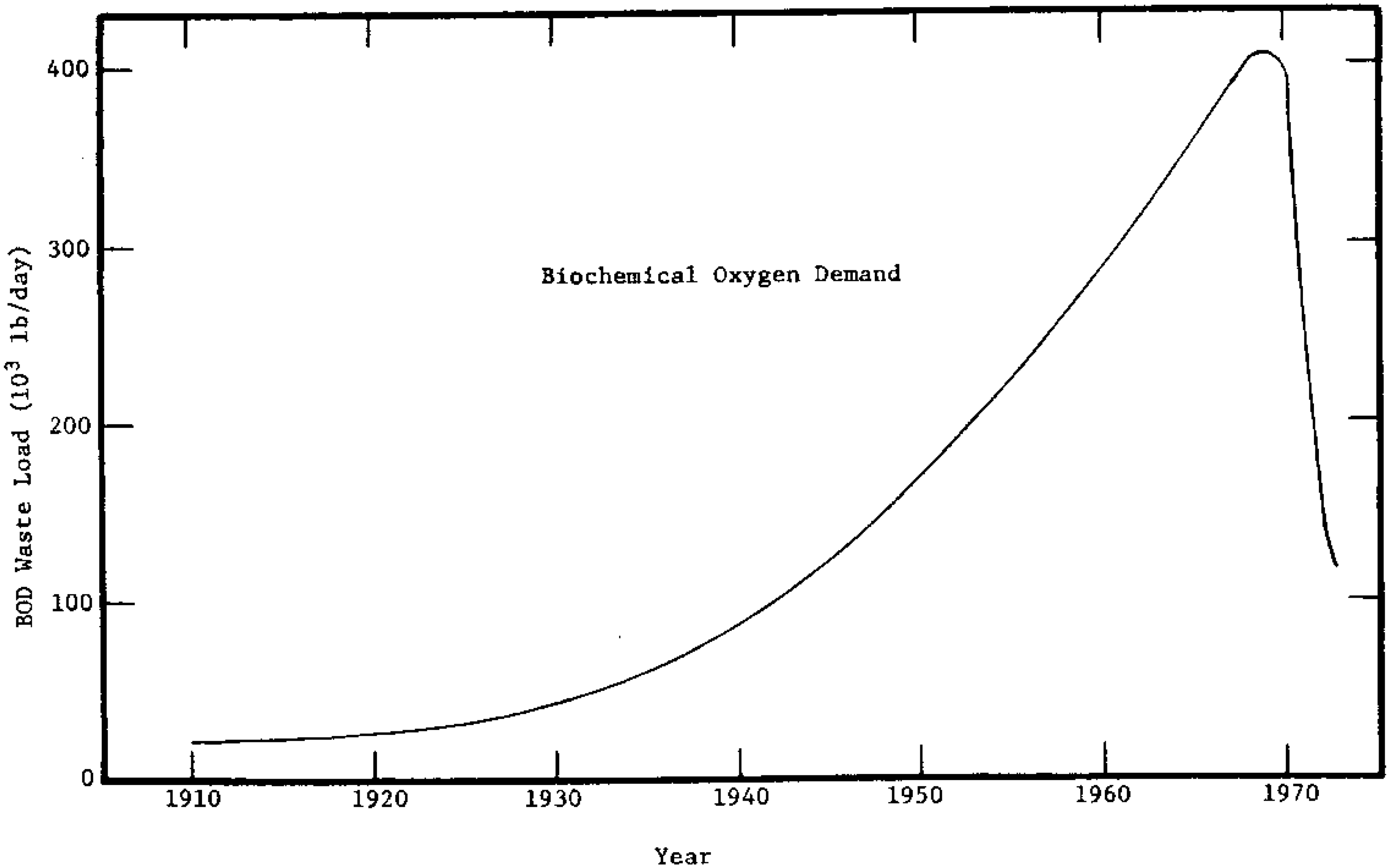

FIGURE III-2. HISTORY OF BOD_5 WASTE LOADINGS ON THE HOUSTON SHIP CHANNEL

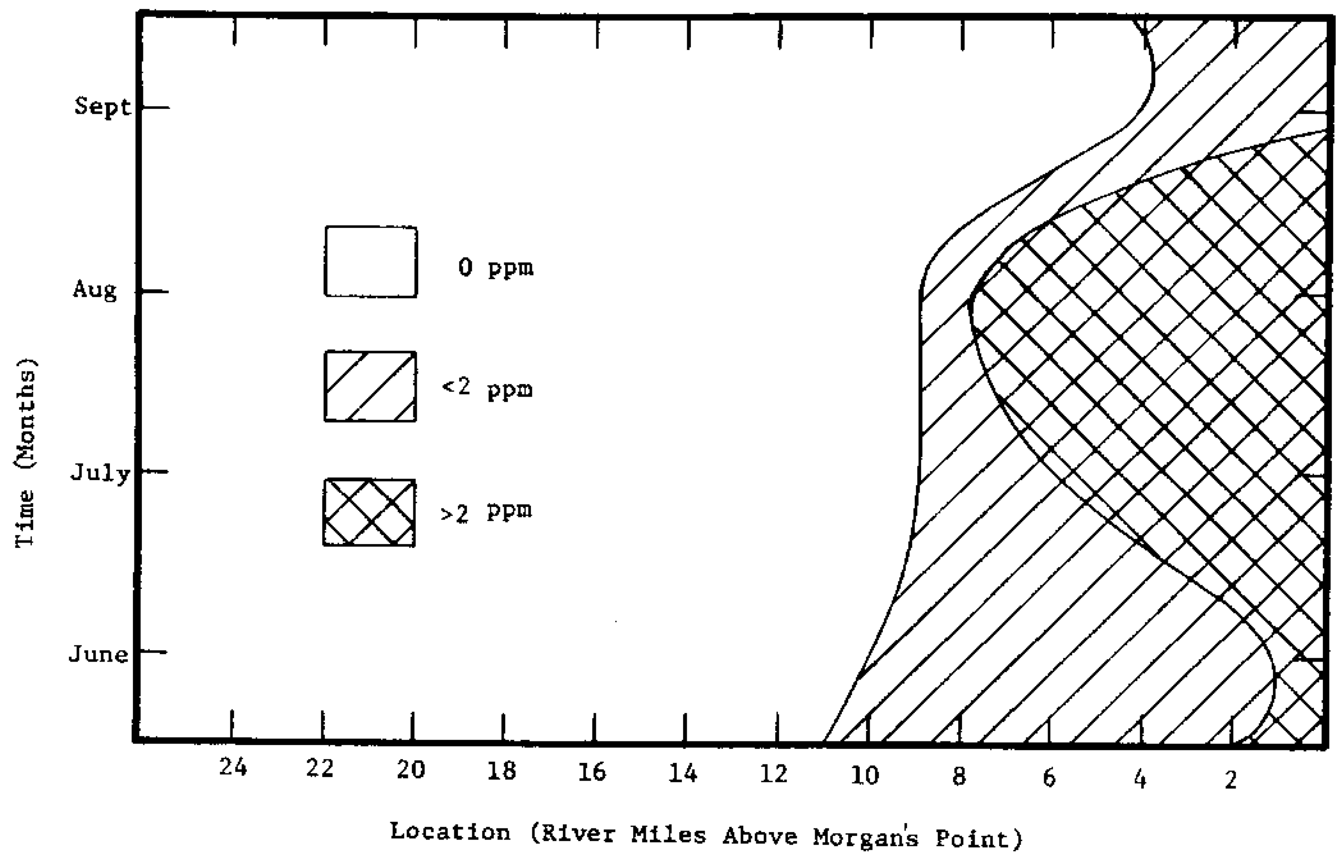

FIGURE III-3. SUMMER DISSOLVED OXYGEN LEVELS IN THE HOUSTON SHIP CHANNEL

APPENDIX IV

COMPUTER LISTING FOR THE HOUSTON SHIP CHANNEL MODEL

The mathematical models used in this study for the Corpus Christi Harbor Channel, Vince Bayou, and the Houston Ship Channel are basically similar. Therefore, only the Houston Ship Channel model is presented here.

```
C       ****************************************************************
C       *                                                              *
C       *                                                              *
C       *                ONE - DIMENSIONAL  DYNAMIC                    *
C       *                                                              *
C       *                   MATHEMATICAL  MODEL                        *
C       *                                                              *
C       *                           OF                                 *
C       *                                                              *
C       *               THE  HOUSTON  SHIP  CHANNEL                    *
C       *                                                              *
C       *                                                              *
C       ****************************************************************
        COMMON      NCELLS,NM,FA,FB,FC,FC
        COMMON      A(99),B(99),C(99),H(99),S(99),UC(99),UN(99),
       1   ARO(99),ARN(99),ELO(99),ELN(99)
        DIMENSION   RBODA(99),ROXA(99),PPDBOD(99),PPDOX(99),AAER(99),
       1   OXADA(99),PPH(99),AERC(99),BENTH(99),DECAY(99),BODBAL(99),
       2   OXLOSS(99),REARN(99),OXBAL(99),OXADN(99),VOL(99),VOLN(99),
       3   BAL(99),W(99),POW(99),OXO(99),OXN(99),BODO(99),BODN(99),CO(99)
        DIMENSION   CN(99),Q(99),QN(99),TEMP(99),SAL(99),DR(99),CSAT(99),
       1   AREA(99),ETA(99),EL(99),WC(99),OXC(99),BR(99),AR(99),AAC(99),
       2   BRC(99),SAREA(99),DEPTH(99),DEPTHN(99)
        READ(5,1001) NCELLS,NCYCLE,NTSPC
 1001   FORMAT(3I10)
        NP = NCELLS+1
        NM = NCELLS-1
```

```
C        INPUT DATA
      DO  1 I=1,NP
      READ(5,1020) J,AREA(J),VOL(J),W(J),DEPTH(J),SAREA(J)
    1 CONTINUE
      DO 101 I=1,NP
      READ(5,1020) J,TEMP(J),SAL(J),BODO(J),OXO(J),EL(J),BR(J)
  101 CONTINUE
 1020 FORMAT(I10,7F10.0)
      READ(5,1002) D,AO,T,DELX,CR,THETA,OXOUT,BODOUT,DC,BETA,AC
 1002 FORMAT(13F6.0)
C        INITIALIZE AREAS AND DISPERSION COEFFICIENTS
      DO  2 I=1,NP
      Q(I) = 0.0
      WC(I) = 0.0
      OXC(I) = 0.0
      PPH(I) = 0.0
      OXADA(I) = 0.0
      RBODA(I) = 0.0
      PPDBOD(I) = 0.0
      ROXA(I) = 0.0
      PPDOX(I) = 0.0
      AAER(I) = 0.0
      AAC(I) = 0.0
      AERC(I) = 0.0
      POW(I) = 0.0
      SAREA(I) = SAREA(I)*1000000.0
      DR(I) = DC*(1.047**(TEMP(I)-20.0))/86400.
      CSAT(I) = 14.652-0.41022*TEMP(I) + 0.007991*TEMP(I)*TEMP(I) -
     1   0.000077774*TEMP(I)*TEMP(I)*TEMP(I)-SAL(I)*(0.0841-0.00256*
```

```
      2   TEMP(I)+0.0000374*TEMP(I)*TEMP(I))
       BRC(I) = BR(I)*(1.055**(TEMP(I)-32.0))/17578751.0
       ARO(I) = AREA(I)
       ELO(I) = EL(I)
    2 CONTINUE
      READ(5,1020)  NIF
      DO  3 I=1,NIF
      READ(5,1020) J,Q(J),WC(J),OXC(J)
    3 CONTINUE
      READ(5,1020)  NAS
      DO  4 I=1,NAS
      READ(5,1020) J,POW(J)
      AAC(J) = AC*POW(J)*(1.025**(TEMP(J)-20.0))*BETA/(9.17*3600.)
    4 CONTINUE
      READ(5,1030) (BODO(I), I=1,NP )
 1030 FORMAT ( 16F5.0 )
C         CALCULATE CONSTANTS FOR VELOCITY CALCULATIONS
      T = T*3600.
      AO = AO/4.0
      SPC = NTSPC
      DELT = T/SPC
      PS = SQRT(32.17*D)
      WL = PS*T
      WNO = 6.2832/WL
      WN = WNO/CR
      GNU = WNO*(SQRT(1.0/CR/CR - 1.0))
      ALPHA = ATAN(GNU/WN)
      SIGMA = 6.2832/T
      F1 = SQRT(GNU*GNU + WN*WN)
      F1 = AO*PS*WNO/F1
      F2 = EXP(-GNU*DELX)
      F3 = EXP( GNU*DELX)
```

```
C          CALCULATE CONSTANTS FOR DISPERSICN EQUATION
       FA = DELT*THETA/( DELX*2.0 )
       FB = DELT*THETA/(DELX*DELX*2.0)
       FC = DELT*(1.0-THETA)/(DELX*2.0)
       FD = DELT*(1.0-THETA)/(DELX*DELX*2.0)
C          CALCULATE INITIAL VELOCITIES
       QN(1) = Q(1)
       UO(1) = QN(1)/ARO(1)
       DO 5 I=2,NP
       QN(I) = QN(I-1) + Q(I)
       UO(I) = QN(I)/ARO(I)
     5 CONTINUE
C          START TIME STEPS FOR CALCULATICNS
       DO 16 I=1,NCYCLE
       DO 15 J=1,NTSPC
       E = J
       TIME = DELT*E
       FE = SIGMA*TIME
C          CALCULATE TIDE HEIGHTS AND AREAS
       DO 6 K=1,NP
       EE = K
       X = DELX*EE - DELX
       ETA(K) = AO*(EXP(-GNU*X)*COS(FE-WN*X) + EXP(GNU*X)*COS(FE+WN*X))
       ARN(K) = AREA(K) + W(K)*ETA(K)
       DEPTHN(K) = DEPTH(K) + ETA(K)
     6 CONTINUE
C          CALCULATE VELOCITIES
C          CALCULATE DISPERSION COEFFICIENTS
```

```
      F4 = FE-WN*DELX+ALPHA
      F5 = FE+WN*DELX+ALPHA
      QN(1) = Q(1)+(SAREA(1)/DELX)*(F1*(F2*COS(F4)-F3*COS(F5)))
      UN(1) = QN(1)/ARN(1)
      ELN(1) = EL(1) - (UN(1)*UN(1)*DELT/2.0)
      DO 7 K=2,NP
      QN(K) = QN(K-1)+Q(K)+(SAREA(K)/DELX)*(F1*(F2*COS(F4)-F3*COS(F5)))
      UN(K) = QN(K)/ARN(K)
      ELN(K) = EL(K) - (UN(K)*UN(K)*DELT/2.0)
    7 CONTINUE
C        CALCULATE RATE OF ADDITION OF CONTAMINANTS AND BOD BALANCE
      DO 9 K=1,NCELLS
      CONC = (OXO(K)+OXO(K+1))/2.0
      VOLN(K) = VOL(K)*1000000.0 + W(K)*ETA(K)*DELX
      AR(K) = (0.06+3.3*UN(K)/(DEPTHN(K)**1.33))*(1.029**(TEMP(K)-20.0)
     1   -0.059*SQRT(SAL(K)))/86400.
      REARN(K) = AR(K)*(CSAT(K)-CONC)
      AAER(K) = AAC(K)*(CSAT(K)-CONC)*1000000./(VOLN(K)*62.4)
      ROXA(K) = Q(K)*OXC(K)/VOLN(K)
      BENTH(K) = BRC(K)*W(K)*DELX*1000000./(VOLN(K)*62.4)
      RBODA(K) = Q(K)*WC(K)/VOLN(K)
      DECAY(K) = DR(K)*(BODO(K)+BODO(K+1))/2.0
      TEST = DECAY(K)*DELT
      IF(TEST .LT. CONC) GOTO 99
      DECAY(K) = CONC/DELT
   99 OXLOSS(K) = (DECAY(K) + BENTH(K))*86400.0*VOLN(K)*0.0000624
      PPDBOD(K) = Q(K)*WC(K)*5.4
      PPDOX(K) = Q(K)*OXC(K)*5.4
      BODBAL(K) = (RBODA(K)-DECAY(K))*DELT
```

```
      OXBAL(K) = (ROXA(K)+AAER(K)+REARN(K)-DECAY(K)-BENTH(K))*DELT
      OXADA(K) = AAER(K)*86400.*VCLN(K)*0.0000624
      OXADN(K) = REARN(K)*86400.*VCLN(K)*0.0000624
      PPH(K) = OXADA(K)/24.0
      IF(POW(K) .EQ. 0.0 ) GOTO 9
      AERC(K) = PPH(K)/POW(K)
    9 CONTINUE
      BODOUT = BODO(NCELLS)*2.0 - BODO(NCELLS-1)
      CALL SOLVE (BODBAL,BODO,BCDN,BODOUT)
      CALL SOLVE (OXBAL,OXO,GXN,OXOUT)
      IF ( I .EQ. 30 .AND. J .EQ. 4 ) GOTO 11
      IF ( I .EQ. 30 .AND. J .EC. 8 ) GOTO 11
      IF ( I .EQ. 30 .AND. J .EQ. 12) GOTO 11
      IF ( I .EQ. 30 .AND. J .EQ. 16) GOTO 11
      IF ( I .EQ. 30 .AND. J .EQ. 20) GOTO 11
      IF ( I .EQ. 30 .AND. J .EQ. 24) GOTO 11
      IF ( I .EQ. 30 .AND. J .EQ. 28) GOTO 11
      IF(J .EQ. 32) GOTO 11
      GOTO 13
   11 DO 12 K=1,NCELLS
      WRITE(6,1009) K,ETA(K),UN(K),PPDOX(K),OXADN(K),AERC(K),PPH(K),
     1   OXN(K),PPOBOD(K),OXLOSS(K),BODN(K)
 1009 FORMAT ( 10X, I2,11F10.2 )
   12 CONTINUE
      WRITE(6,1050) I,J
 1050 FORMAT ( 2I10 )
   13 DO 14 K=1,NP
      BODO(K) = BODN(K)
      IF(BODO(K) .LT. 0.0 ) BODO(K) = 0.0
      OXO(K) = OXN(K)
      IF( OXO(K) .LT. 0.0 )  OXO(K) = 0.0
```

```
      UO(K) = UN(K)
      ELO(K) = ELN(K)
      ARO(K) = ARN(K)
   14 CONTINUE
   15 CONTINUE
   16 CONTINUE
      STOP
      END
```

SUBROUTINE SOLVE

```
      SUBROUTINE SOLVE (BAL,CO,CN,CLAST)
      COMMON      NCELLS,NM,FA,FB,FC,FD
      COMMON      A(99),B(99),C(99),H(99),S(99),UO(99),UN(99),
     1   ARO(99),ARN(99),ELO(99),ELN(99)
      DIMENSION BAL(99),CO(99),CN(99)
C        CALCULATE COEFFICIENTS FOR THE DISPERSION EQUATION
      A(1) = -FA*UN(1) - FB*ELN(1)
      B(1) = 1.0 + FA*UN(1)*ARN(2)/ARN(1) + FB*(ARN(2)*ELN(2)
     1   + 2.0*ARN(1)*ELN(1))/ARN(1)
      C(1) = FA*UN(1) -FB*(ARN(2)*ELN(2) + ARN(1)*ELN(1))/ARN(1)
      H(1) = CO(1)-FC*UO(1)*CO(2)+FC*UO(1)*CO(1)*ARO(2)/ARO(1)
     1   +FD*(ARO(2)*ELO(2)+ARO(1)*ELO(1))*(CO(2)-CO(1))/ARO(1)
     2   -FD*ELO(1)*CO(1) + BAL(1)
      DO 1 K=2,NCELLS
      A(K) = -FA*UN(K)-FB*(ARN(K)*ELN(K)+ARN(K-1)*ELN(K-1))/ARN(K)
      B(K) = 1.0+FA*UN(K)*(ARN(K+1)-ARN(K-1))/ARN(K) + FB*
     1   (ARN(K+1)*ELN(K+1)+2.0*ARN(K)*ELN(K)+ARN(K-1)*ELN(K-1))/ARN(K)
      C(K) = FA*UN(K)-FB*(ARN(K+1)*ELN(K+1)+ARN(K)*ELN(K))/ARN(K)
      H(K) = CO(K)-FC*UO(K)*(CO(K+1)-CO(K-1))+FC*UO(K)*CO(K)*(ARO(K+1)
     1   -ARO(K-1))/ARO(K) + FD*(ARO(K+1)*ELO(K+1)+ARO(K)*ELO(K))*
     2   (CO(K+1)-CO(K))/ARO(K) - FD*(ARO(K)*ELC(K)+ARO(K-1)*ELO(K-1))*
     3   (CO(K)-CO(K-1))/ARO(K) + BAL(K)
    1 CONTINUE
```

```
      NP = NCELLS + 1
      CO(NP) = CLAST
      CN(NP) = CLAST
C        SET UP SOLUTION MATRIX COEFFICIENTS
      DO 2 K=2,NCELLS
      S(K) = B(K-1)/A(K)
      H(K) = H(K)*S(K)-H(K-1)
      B(K) = B(K)*S(K)-C(K-1)
      C(K) = C(K)*S(K)
    2 CONTINUE
C        SOLVE FOR NEW CONCENTRATIONS
      DO 3  K=1,NM
      CN(NP-K) = (H(NP-K)-CN(NP-K+1)*C(NP-K))/B(NP-K)
    3 CONTINUE
      CN(1) = (H(1)-CN(3)*(C(1)+A(1)))/B(1)
      RETURN
      END

//$DATA
```