AN INVESTIGATION INTO THE PROPERTIES AND CHARACTERISTICS OF HOMOGENEOUS TAPERED CABLES

by

Russell O. deCastongrene and Richard F. Dominguez

Department of Civil Engineering
Coastal and Ocean Engineering Division
Texas A&M University

August, 1975

TAMU-SG-75-211

Partially supported through Institutional Grant 04-3-158-18
to Texas A&M University
by the National Oceanic and Atmospheric
Administration's Office of Sea Grants
Department of Commerce

Coastal and Ocean Engineering
Report No. C.O.E.-183

$3.00

Order from:

Department of Marine Resources Information
Center for Marine Resources
Texas A&M University
College Station, Texas 77843

ABSTRACT

The tapered cable concept is explored and developed. The introduction is followed by a review of historical background, examination of several types of taper from a geometrical viewpoint, and the derivation of the equilibrium equations pertaining to two cases of general interest: the axially suspended cable and the catenary cable configuration. These equations are then used to make comparisons between the optimum, tapered cable and the common cable. Advantages and disadvantages of the tapered cable are summarized and a set of tables is provided to assist in constant-stress catenary cable calculations.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF FIGURES (CONTINUED)

LIST OF TABLES

CHAPTER I

INTRODUCTION

This report has resulted from a consideration of deep ocean applications of cables and cable systems, specifically steel-wire rope. In such applications, where extremely long lengths of cable are required, a major problem arises. As the cable length increases, so does the cable weight, until a significant portion of the available strength of the cable is used in supporting itself, thereby reducing the permissible payload. A possible solution to this weight-load interdependence is to taper the cable from a large diameter at the top to a smaller diameter at the bottom. This results in a lighter weight cable with greater load-carrying capacity.

The objective of this investigation is to develop the equilibrium equations governing the static configuration to which a tapered cable conforms, and to use these equations to investigate the possible advantages that a tapered cable may have in common ocean applications.

The term "cable" is defined in the traditional mathematical sense as any inextensible, flexible string. A model based on these assumptions can reasonably approximate submarine communication cables, high tension electric power transmission lines, electro-mechanical cables for towed sonar applications, and mooring cables and chains, provided that the necessary assumptions are appropriate to the application. The term "common cable" will be used for all non-tapered, uniform-diameter cables.

Ocean applications typically fall into one of two major classifications; towing or mooring. The towing application occurs whenever an end of the cable is attached to a ship, or other moving object, and the other end to some payload, which is being pulled along. The mooring application occurs when one end is fixed to the ocean bottom with the cable being supported by a ship, buoy or other floating body and the moor holding the body in a relatively fixed position with respect to the bottom. Fig. 1 illustrates several such deep-ocean applications. Although this study will be largely concerned with mooring situations, many of the same mathematical relationships can be applied to towing problems. Fig. 2 presents a geometrical comparison of the two classifications, showing the relative direction of drag D, cable weight W, cable tension T and current. Cable applications can also be categorized as one-dimensional and two-dimensional.

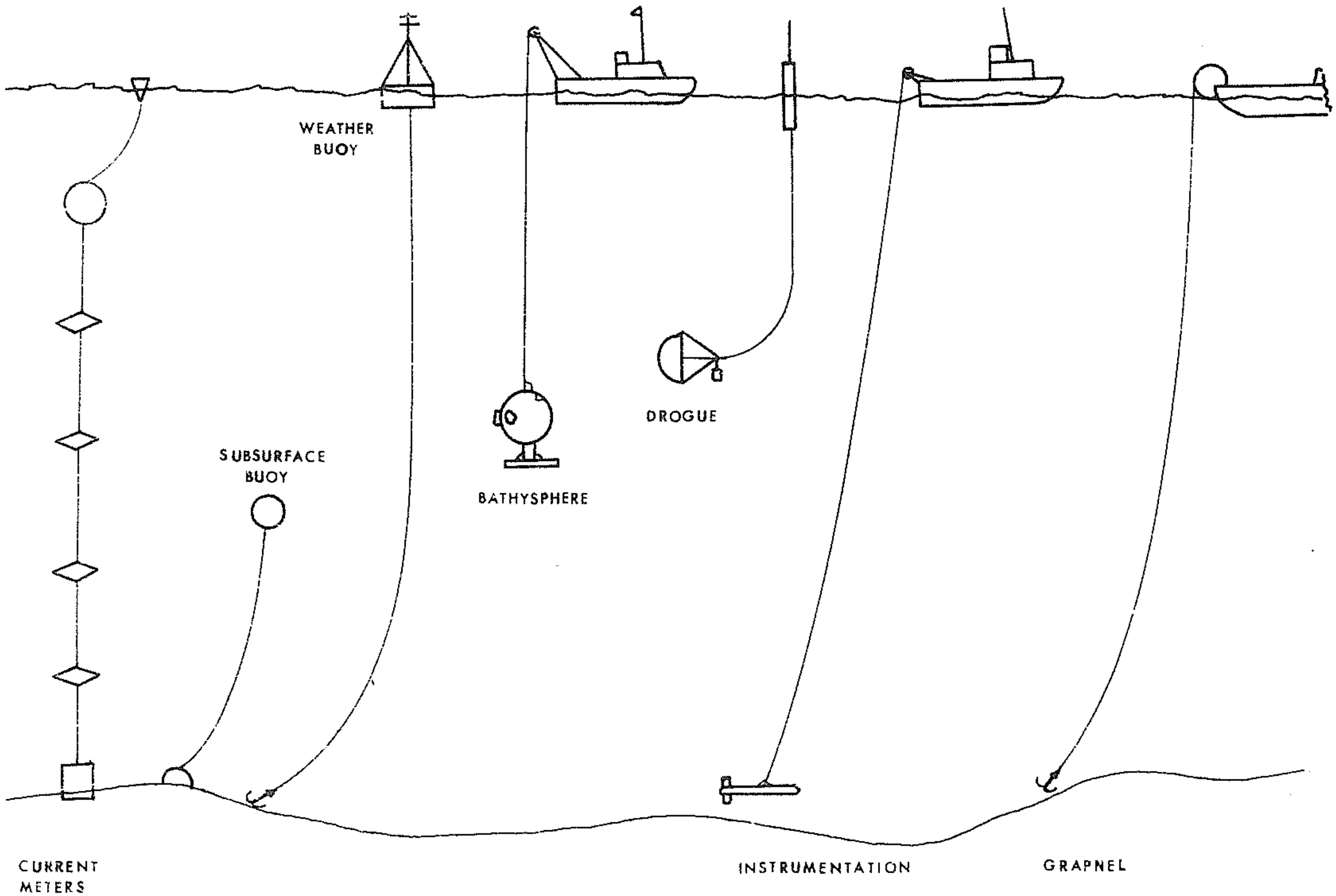

Fig. 1 Deep-Water Cable Applications

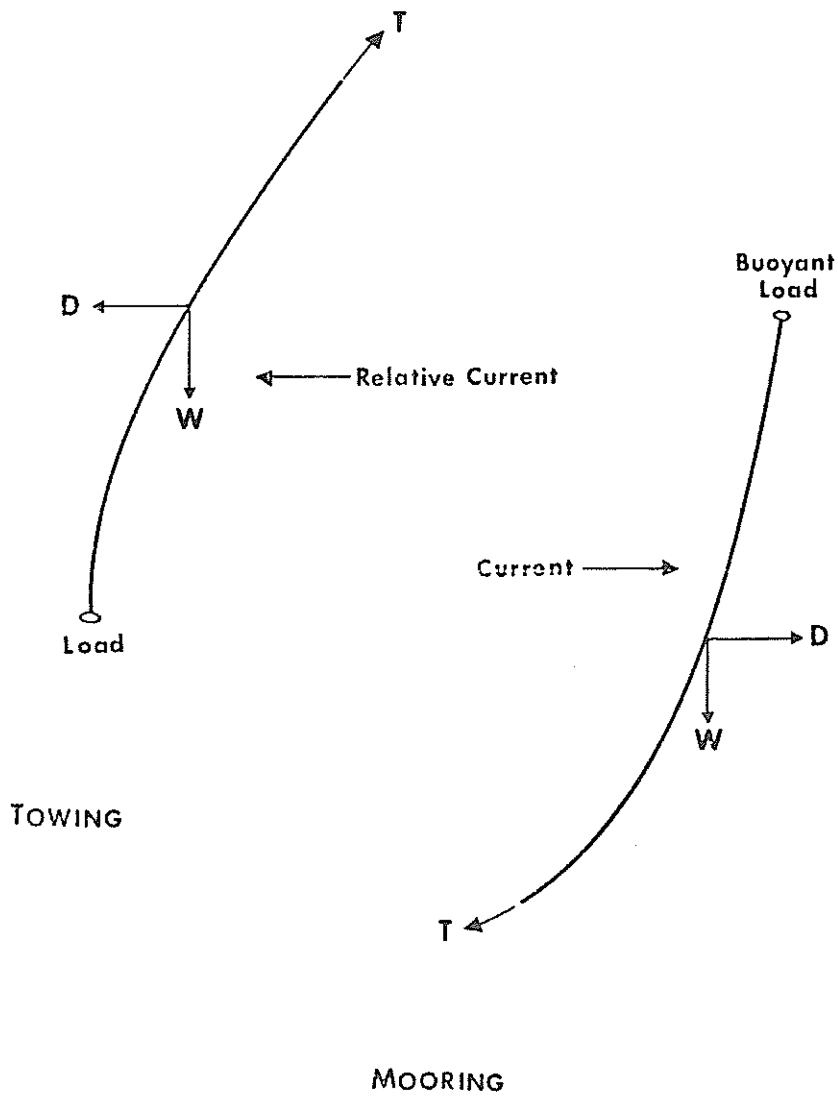

Fig. 2 Towing and Mooring Geometries

CHAPTER II

HISTORICAL BACKGROUND

The use of cables by man goes back before recorded history. Primitive bridges were frequently constructed of cables made from woven vines or native fibers. Existing examples of such suspension bridges can still be found in South America. These were originally designed by the Incas and have been continuously rebuilt until the present time. This has been shown by Jakkula (5) to be fairly typical of man's bridge building over much of the world, including Asia, Africa, Europe and Russia.

By about 65 A.D. the Chinese were constructing what would appear to be rather modern suspension bridges using iron chains as supporting cables with appropriate auxiliary structure to produce a nearly level walkway, many of which were still standing at the turn of the century.

Early applications also existed at sea. One can well imagine even the very early log or raft rider tying up his watercraft with a line to a tree on shore. The method of mooring had changed a great deal by Roman times, with the application of iron chains, to secure ship anchors. The use of iron chains was extremely advanced since, even up until the nineteenth century, rope cables were still used for anchors on most large sailing vessels.

Both suspension bridge and mooring cable applications are closely related mathematically. One of the first scientists to meditate upon the possible mathematics of cables may have been Galileo. Montucla (9) notes that Galileo considered the hanging chain to take the form of a parabola. Joachim Jurgins, the German geometrician, showed by several methods, in about 1669, that the hanging chain was neither a parabola nor a hyperbola. However, he was unable to shed any light on what the actual shape was. These were the years when calculus was being invented by Newton and Leibniz. Jakob (James) Bernoulli, through Leibniz's writings, had become familiar with calculus. He proceeded to apply it to various problems, one of which was the chain or hanging cable. Bernoulli proposed the problem and he, his brother Johann (Jean), Leibniz and Huygens all solved it. They each published their solutions in the Actes de Leipzig in 1691. A few years later, in 1697, David Gregory published a solution in the Philosophical Transactions in London. All of these solutions result in what is called the "catenary", from the Latin "catenarius", a chain.

The somewhat simpler problem of defining the approximate geometry of a suspension bridge support cable was not solved until 1794, when Nicholas Fuss developed the parabolic cable solution as a result of the proposal to construct a bridge across the Neva River in Russia.

In 1858, the Astronomer Royal of England, G. B. Airy, investigated the shape attained as a submarine cable was deposited. This study was initiated in response to problems encountered during the laying of the first trans-Atlantic cable. Airy demonstrated that a paying out cable assumes the form of a catenary traveling at the speed of the cable-laying ship, if it is payed out at the same speed as the ship moves and fluid resistance is ignored.

Applications such as the above involve the difficult problem of determining the hydrodynamic forces which act on the cable due to its relative motion through water. Landweber, from 1936 to about 1947, developed Tables of Cable Functions (7), by which the tension and loads on a cable immersed in a uniform current could be determined. This work was extended by Pode (13) and is applicable to both mooring and towing problems. The general availability of digital computers has led many recent investigators (3, 8) to use alternative procedures incorporating numerical techniques. The most commonly used of these is the finite element method, in which the cable is divided into a series of interconnected straight lengths with the solution being carried out in an iterative manner.

Tapered Cables

A review of the literature reveals a general lack of such analytical development for the tapered cable. Although Gilbert (4) investigated what he called "the catenary of equal strength" in 1826, and Routh (14) later referred to it in 1891, the idea was apparently dropped due to lack of distinct advantages in bridge applications.

A number of investigators have made casual mention of the tapered cable concept, with Terry (15) commenting on some of the limitations of present day cables (wire ropes) due to the cable's own weight at great depths. This was one of the limitations on Beebe and Barton's Bathysphere of 1934 and Benthoscope of 1950.

In the "Reports of the Swedish Deep-Sea Expedition", Kullenberg (6) describes the use of what he called a tapered cable aboard the Swedish oceanographic vessel ALBATROSS. This cable was composed of three sections with diameters of 20, 16 and 12 millimeters and lengths of 2,170, 2,820 and 3,000 meters, respectively.

Markula (8), in a study of aircraft target towlines, investigated reducing cable tension by use of a tapered cable, providing a reduction in both aerodynamic forces and weight. Markula went on to state that fabrication of the tapered cable was probably not feasible and suggested the use of a stepped diameter cable composed of varying lengths of constant diameter cables; adjacent lengths being of different diameters to roughly approximate the ideal tapered cable concept. The authors include a computer program to aid in design of such a stepped cable.

More recently, Notwatzki (11) considered the advantages to be accrued by a "perfectly tapered cable". He was primarily interested in electro-mechanical cable strength members and presented equations for the case of a simple vertical hanging cable with no horizontal tension components. Notwatzki conceded that it is difficult to construct a "perfectly tapered cable" but that it can be closely approximated by reducing the strength member area in several steps, i.e., the stepped cable as studied by Markula.

It then appears that while a number of people have actually considered the tapered cable idea, and in some cases attempted limited analysis, none have developed a complete analytical base which permits rational progress. Indeed, it appears that the individual investigators were generally not aware of any previous work in the area, so that little continuity of investigation has been observed.

CHAPTER III

ONE-DIMENSIONAL CABLE BEHAVIOR

A cable, if allowed to hang free under the influence of gravity only, conforms to a vertical straight line. This one-dimensional configuration can serve as a convenient medium for considering different types of tapered cables. Simply stated, a tapered cable is one in which the cross-sectional area varies in some specified manner from one end to the other. Table 1 indicates some possible variations with the meaning of symbols defined in Appendix IV.

The first two will be derived in later sections, and are based on constant stress throughout the length of cable for two separate design conditions. The next, the straight-taper "D", is a linear variation of diameter with length (Fig. 3), while the straight-taper "A" is a linear variation of cross-sectional area with length. The remaining types, the parabolic and the sine, are shown to indicate some of the other possibilities.

Two taper conditions will be considered in detail in this paper leading to configurations which have been labeled E-tapered and Constant-Stress Catenary. While these are optimized for their specific applications, they are not necessarily the best for deviations from design conditions, for design situations with currents present, or for other similar but not identical conditions.

The first case to be analyzed is the axially suspended cable. This is exemplified by the Bathysphere illustration in Fig. 1. The

Table 1. - Types of Tapers

Name	Equation		Remarks
E-Tapered	$A_s = A_o e^{\frac{\gamma s}{\sigma}}$	$D_s = D_o e^{\frac{\gamma s}{\sigma}}$	Constant stress hanging line
Constant-Stress Catenary	$A_s = \frac{Tx}{\sigma} \cosh \frac{\gamma s}{\sigma}$		Constant stress cable hanging between two points (Mooring Cable)
Straight-Taper "D"		$D_s = D_o + \frac{D_L - D_o}{L} s$	Based on linear diameter increase.
Straight-Taper "A"	$A_s = A_o + \frac{A_L - A_o}{L} s$		Based on linear area increase.
Parabolic Taper		$D_s = D_o + (D_L - D_o)(\frac{S}{L})^2$	
Sine Taper	$A_s = A_o + A_L \sin \frac{\pi s}{2L}$		
	$A_s = A_o + \frac{2}{\sqrt{2}} A_L \sin \frac{\pi s}{4L}$		Approximating a linear area increase.

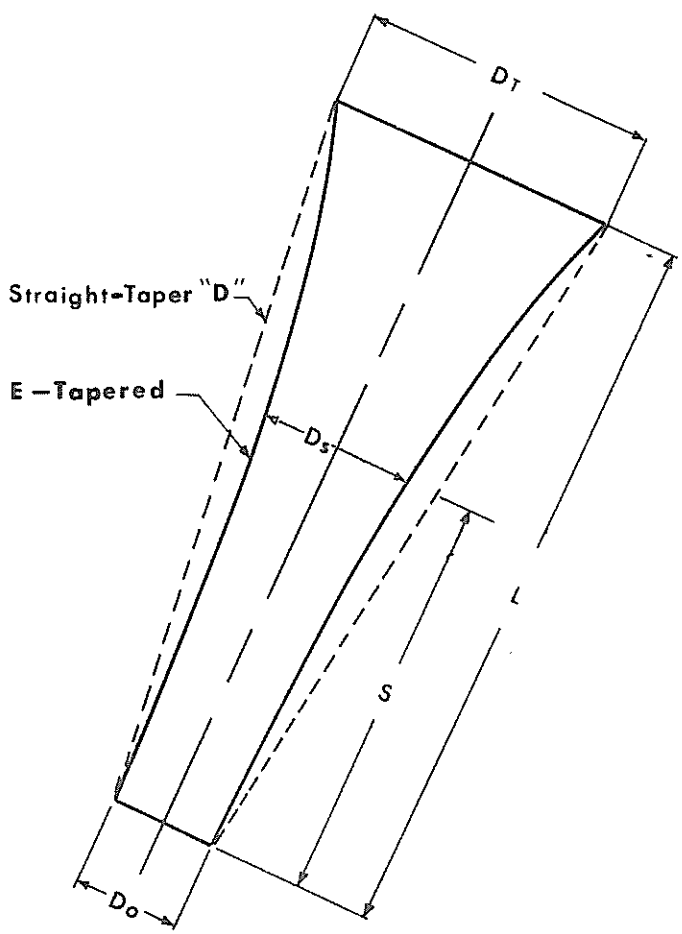

Fig. 3 Types of Taper Illustration

common constant diameter cable will be examined first and then the constant-stress (tapered) cable. These are then compared to illustrate the possible advantages that tapering may provide.

Common Axially Suspended Cables

The vertically hung cable assumes the simplest of cable geometries and is therefore an appropriate starting point for considering the effects of tapering. This configuration is also of widespread practical importance. Fig. 4 illustrates this, which can be thought of as a cable hanging from a ship with a load attached to its end. It is assumed that the only external forces that act on the cable are those produced by gravity.

It is apparent that

$$T = \int_0^s w\,ds + P$$

where: P = load supported by the cable

s = distance along the cable measured from the bottom end

T = cable tension

w = weight/unit length of cable

When the above expression is integrated, the tension at any point s along the cable is defined as

$$T = ws + P \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \quad (1)$$

Cable tension thus varies with the length and distributed weight of the cable. It is then possible to define a factor of safety, F_s,

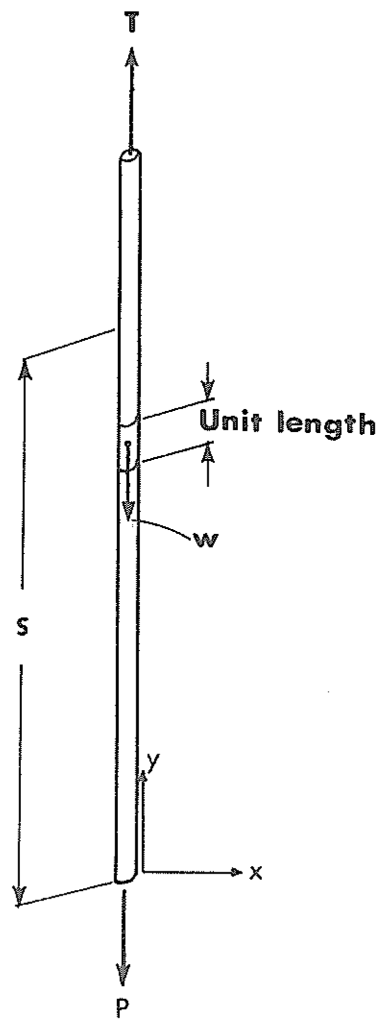

Fig. 4 The Hanging Cable

based on the ultimate static breaking strength, S_u, as

$$F_s = \frac{S_u}{T}$$

Where the allowable load P_a that a cable of length s can carry before exceeding its design condition is

$$P_a = \frac{S_u}{F_s} - ws \quad \ldots \ldots \ldots \ldots \ldots \ldots \quad (2)$$

Fig. 5 illustrates the relation that exists between P_a and the cable length, for four factor-of-safety values, applied to the tension at the top of the cable.

To use Fig. 5, find the cable length along the left-hand side, then move horizontally to the curve representing the factor of safety of interest and read the allowable load at the top scale. Note the cable-breaking-strength line. It is readily observable that the local factor of safety increases toward the lower end of the cable, pointedly illustrating the fact that the cable is much stronger near the bottom than it needs to be. By the same token, the weight of the cable, which is a major contribution to the low factor of safety at the top of the cable, is much greater than it needs to be. For the steel-wire rope illustrated, at about a length of 55,000 feet, the cable can just support its own weight with a Safety Factor of 1 (F_s = 1).

For other materials with greater strength-to-weight ratios, the curves would be displaced to the right, meaning that longer lengths can be supported, or a greater P_a for the same length.

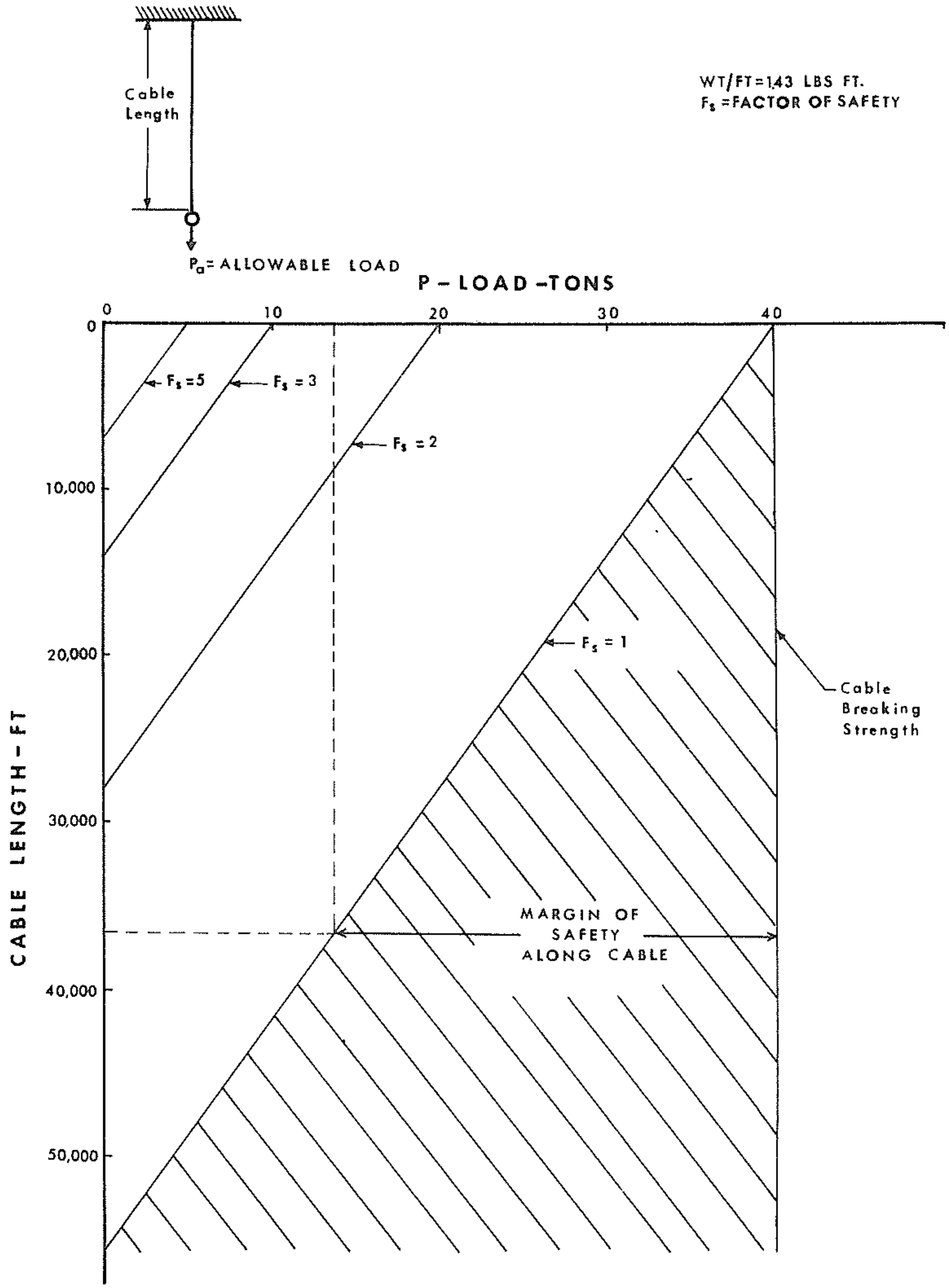

Fig. 5 Cable Load Capacity Characteristics for a 1" Diameter Steel-Wire Rope.

Conversely, for reduced strength-to-weight ratios, only shorter cables or reduced allowable loads are possible. Since safety factors recommended by the wire rope industry are in the order of five, it is readily apparent that deep-water applications of wire rope are highly limited. This is particularly true when additional allowance must be made for dynamic loads being imposed on the cable. It is apparent that if excess material could be removed from the lower end of the cable, where it is not required for supporting the load, and replaced on the upper end, a better cable design would result.

This is what the tapered cable design attempts to do. It may be noted that for materials of lower specific weight, the slope of the $F_s = 1$ line tends to increase until, for a neutrally buoyant cable, it becomes superimposed over the breaking strength line indicating a constant load-carrying capacity at any cable length.

Tapered Axially Suspended Cables

Although several types of tapers are possible, it is highly desirable to devise an optimum taper configuration so as to maximize the load carrying capacity by minimizing the total weight of the cable. Minimal cable weight not only enables a maximum load to be carried by the cable, but reduces the size and required power of a ship's winch. In buoy-mooring applications, a decrease in cable weight may permit reducing the size of the buoy. The optimum use of cable material is achieved only when the stress is constant throughout the cable length.

Fig. 6 depicts a typical section of cable. T defines the tension at the lower end of the cable, and ΔT defines the change in tension due to the weight of the section, W. The segment weight may then be defined as

$$W = \gamma \hat{A} \Delta s$$

with

γ = unit weight of cable material

$\hat{A}$ = average cross-sectional area of cable

Δs = small increment of cable length

If the cable segment is assumed to be in static equilibrium, a summation of forces in the vertical direction gives

$$\frac{\Delta T}{\Delta s} = \gamma \hat{A}$$

Taking the limit as $\Delta s \rightarrow 0$ results in the following differential equation:

$$dT = \gamma\, A ds \quad \text{. . . . . (3)}$$

The stress σ , at any point can be expressed as a function of the cable tension and cable area A, at that point

$$\sigma = \frac{T}{A} \quad \text{. . . . .(4)}$$

The substitution of Eq. 4 into Eq. 3 yields

$$dT = \gamma \frac{T}{\sigma} ds \quad \text{. . . . .(5)}$$

When integrated, assuming γ and σ are constant, Eq. 5 provides the following relationship

$$T = T_o e^{\frac{\gamma s}{\sigma}} \quad \text{. . . . .(6)}$$

where T_o represents the tension or load at the bottom of the

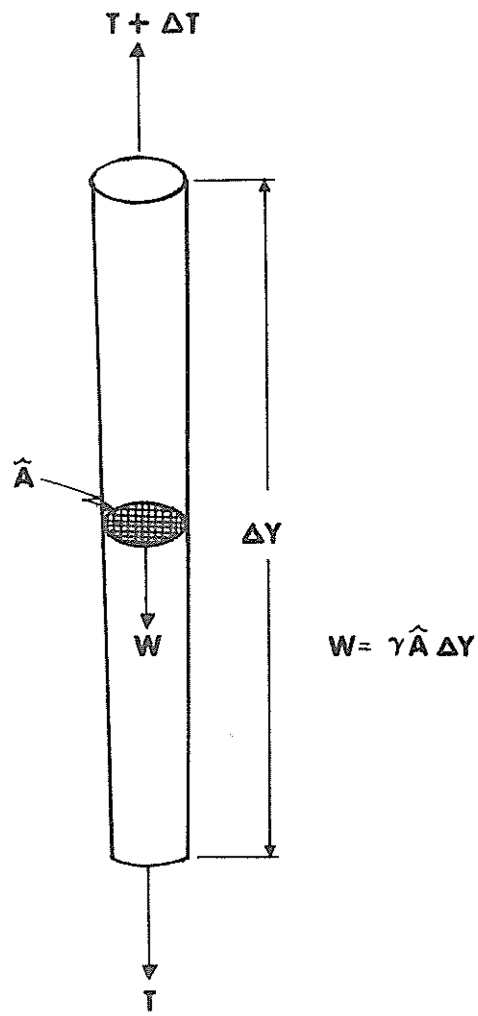

Fig. 6 Forces on a Tapered Cable Segment

cable corresponding to s = o. The substitution of Eq. 4 into Eq. 6 allows for a given area at the bottom of cable, A_o, when σ is expressly defined as being constant throughout the cable.

$$T = A_o \sigma e^{\frac{\gamma s}{\sigma}} \quad \ldots\ldots (7)$$

and to provide the area at any distance s along the cable

$$A = A_o e^{\frac{\gamma s}{\sigma}} \quad \ldots\ldots (8)$$

or to reduce to a ratio

$$\frac{T}{T_o} = \frac{A}{A_o} = e^{\frac{\gamma s}{\sigma}} \quad \ldots\ldots (9)$$

Eqs. 5 and 8 constitute the descriptive equations for what Nowatzki called a "perfectly tapered cable". Herein, this will be referred to as an E-tapered cable.

In Fig. 7, Eq. 9 is plotted using a unit weight, γ , typical of steel-wire rope and several representative values of σ. To use, enter the chart on the length of cable scale and move to the right to the line representing the operating stress level desired. Then the ratio of T/T_o can be read from the abscissa and T calculated for the given depth.

It is also possible to show that the total cable weight is given by

$$W = T_o \left(e^{\frac{\gamma s}{\sigma}} - 1\right) \quad \ldots\ldots (10)$$

$$= A_o \sigma \left(e^{\frac{\gamma s}{\sigma}} - 1\right) \quad \ldots\ldots (11)$$

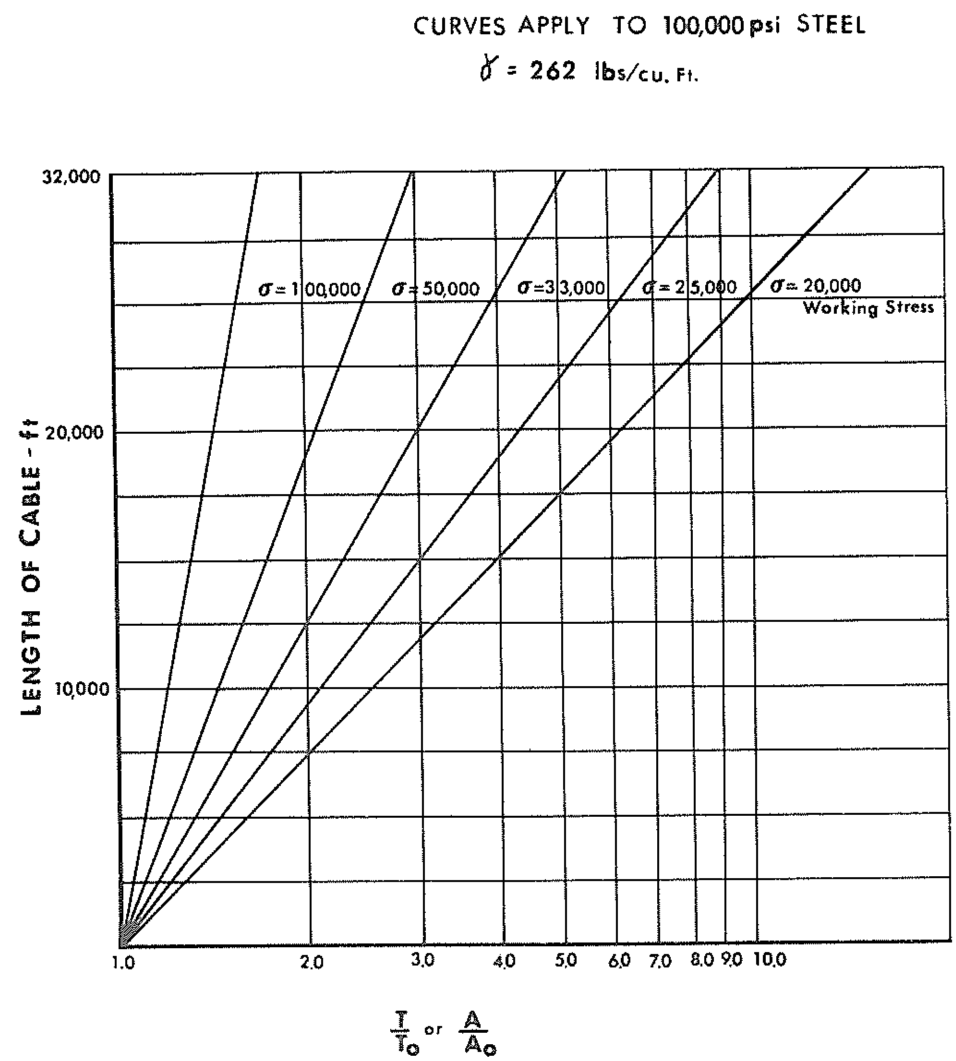

Fig. 7 Tapered Cable Area and Tension Ratios

Comparison

It is now possible to compare the tapered with the common cable for the axially suspended condition. Parameters of interest to engineering design include cable diameter or area, load carrying capacity, cable weight and potential current effects. Fig. 8 presents a comparison of the common cable cross-sectional area to that of an E-tapered cable with the same top diameter of 1 inch. The tapered cable stress σ was taken as 100,000 psi. This illustrates the major difference in cross-sectional area between the tapered and common cables even for a modest length cable. Two other taper designs, based on the same top and bottom areas have been plotted to show the small variation from the E-tapered configuration that occurs.

Fig. 9 compares the load-carrying capacity of tapered and common cables. In the cases chosen for illustration, the cables are all of 1 inch diameter at the top and are loaded to a maximum stress level of 100,000 psi ($F_s = 1$), or 50,000 psi ($F_s = 2$). It is apparent that the load-carrying capacity of the tapered cable exceeds that of the common cable for the same length. This excess strength is amplified at the higher factor of safety. Fig. 10 illustrates the total weight relationship that exists for cables of the same length. The tapered cable is shown to be lighter in weight at all lengths.

Many cable applications involve towing a sonar or instrumenta-

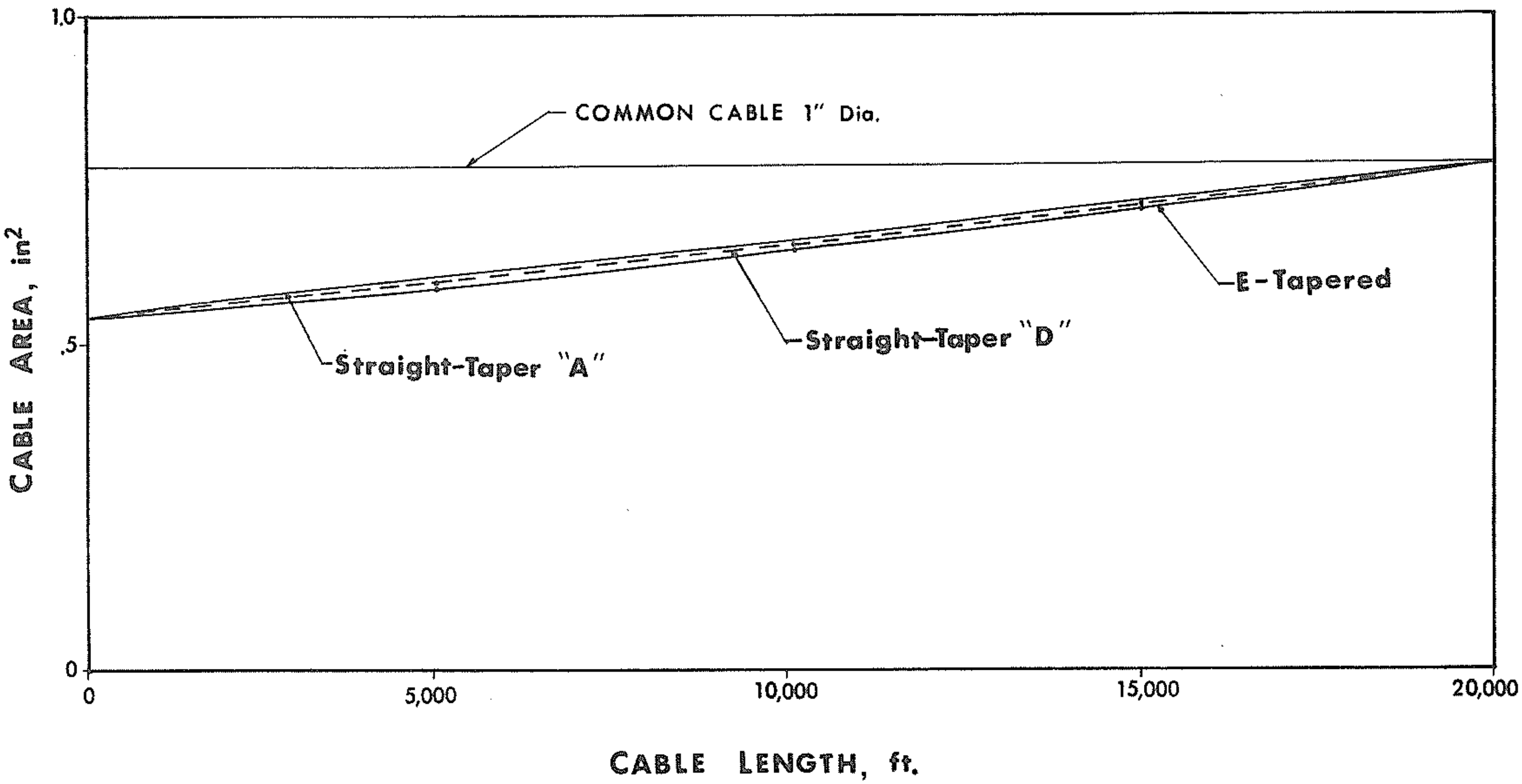

Fig. 8 Cable Area Variation for Four Types of Cables

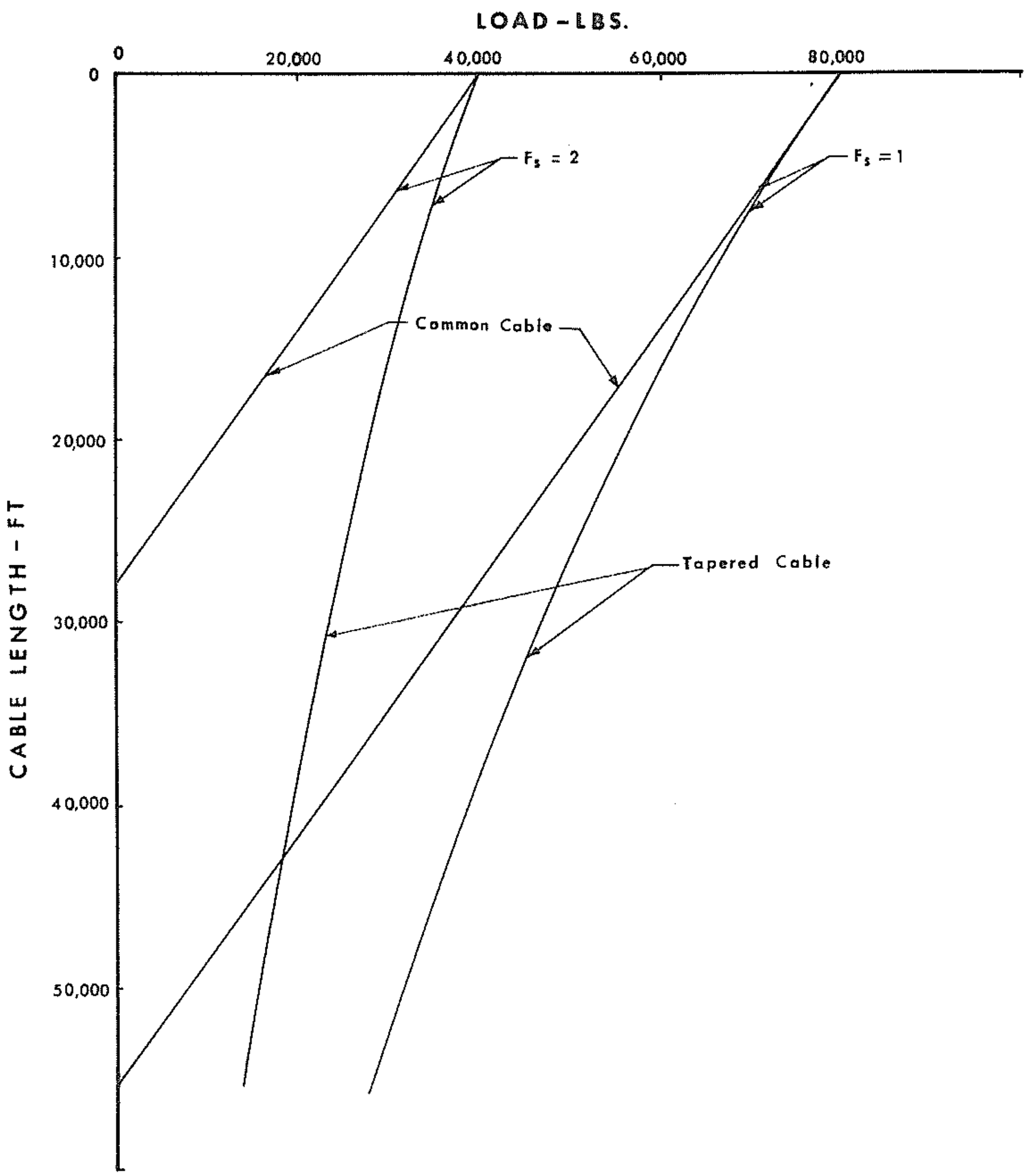

Fig. 9 Load-Capacity Comparisons for Cables with a 1" Top Diameter and a Maximum Tensile Stress of 100,000 psi.

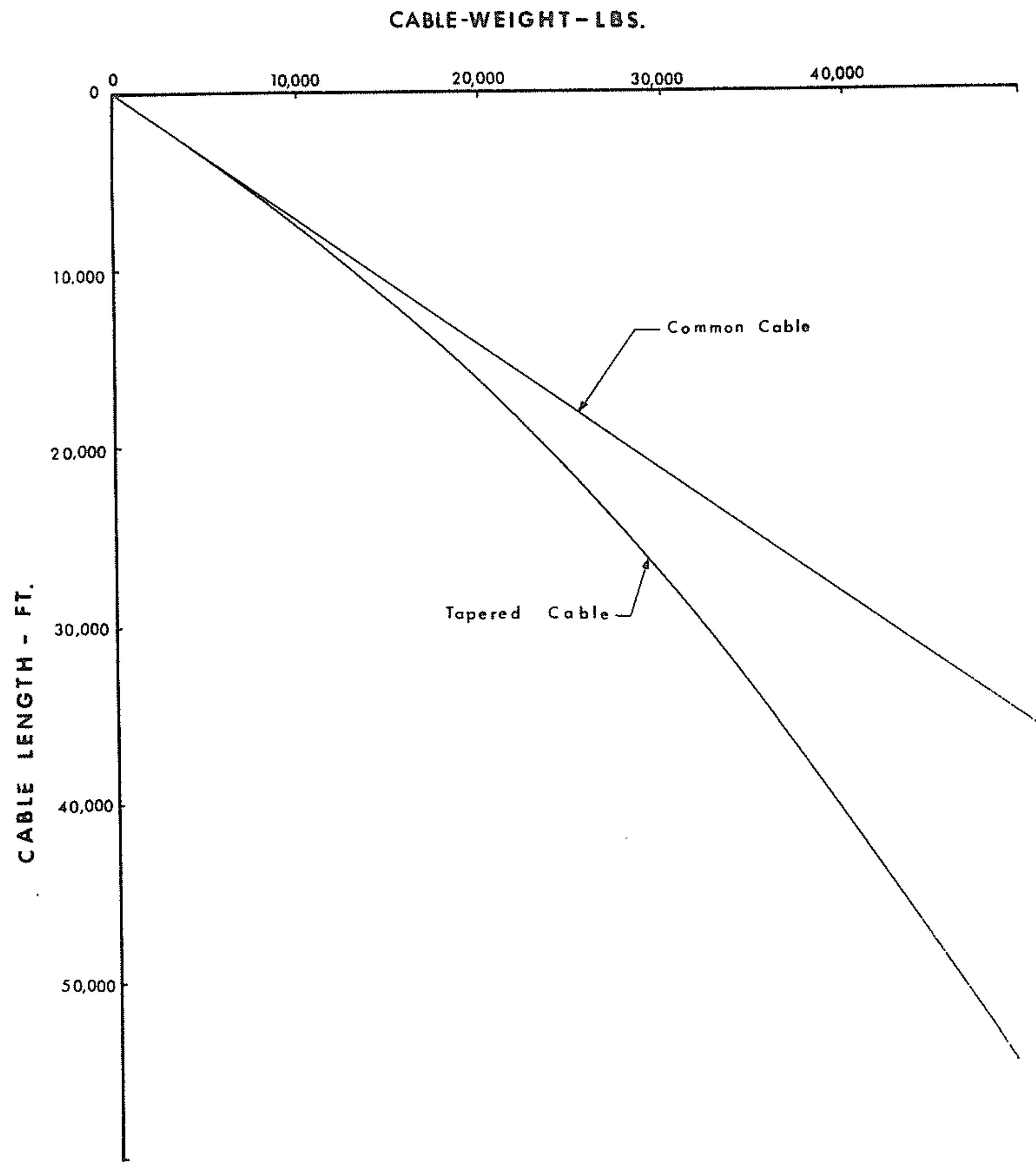

Fig. 10 Weight Comparison for 1" Top Diameter Cables

tion device behind a slowly moving ship. Tapering should reduce the drag on such a cable and thereby produce a lower cable tension for the same load, and reduced length to reach the same operational level. To enable a comparison of current effects upon cables, a finite element computer program was used as described in Appendix VI. This program makes it possible to incorporate cable position hydrodynamic forces into the equilibrium analysis. Fig. 11 depicts the effects of 1-knot, 2-knot and 4-knot currents on the cables, each having a 1 inch top diameter and a 8,000 pound load suspended from the lower end. The cables were selected to represent a typical configuration for a towed instrumentation package with the drag of the load being simulated by use of a 1,000 pound constant horizontal force in the direction of the current. This force is maintained constant for all currents so that the effect of the currents on the cable will not be obscured. The difference in drag for this set of conditions is shown to be very small as illustrated by the small separation between the common cables and the tapered cables at all currents.

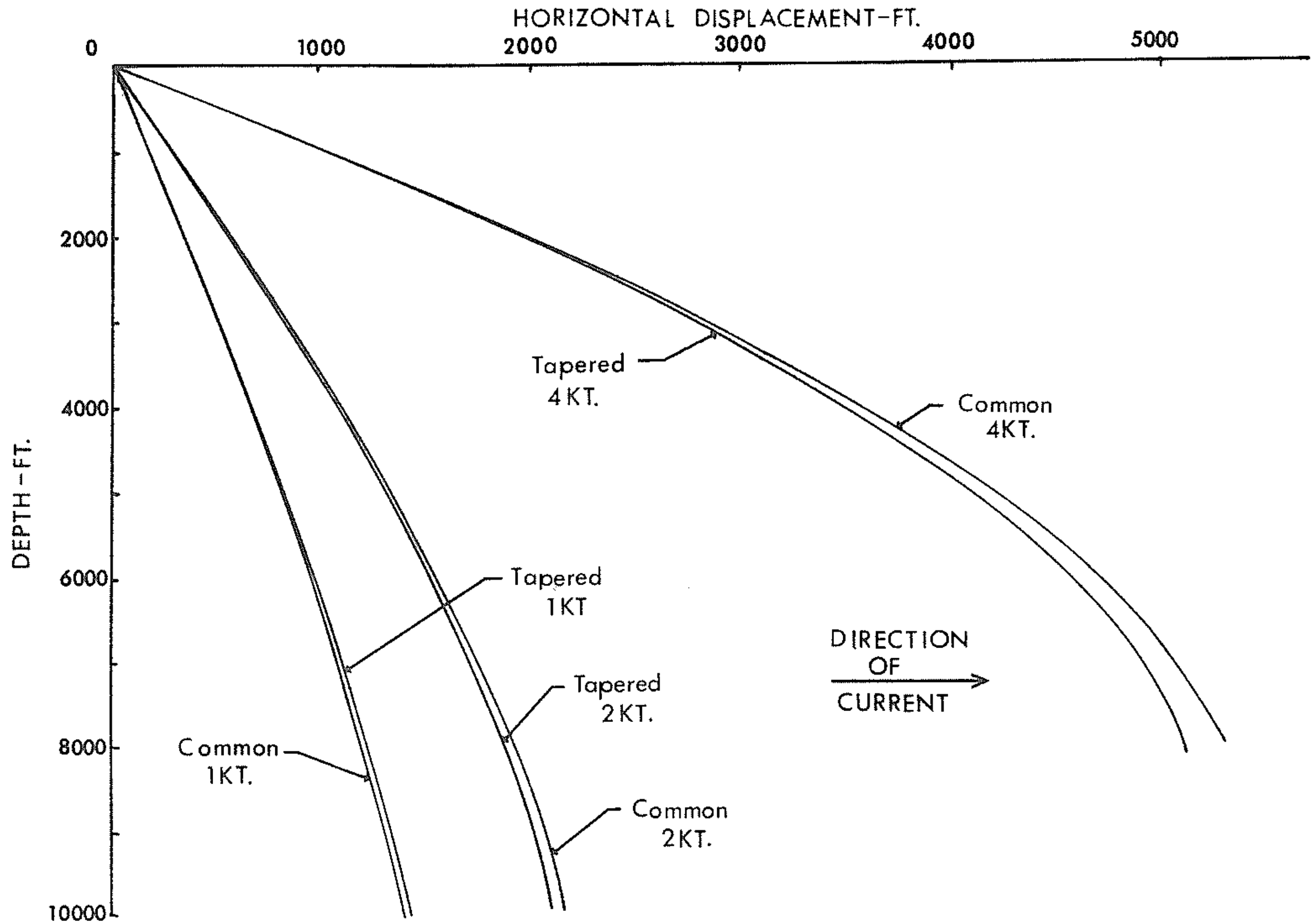

Fig. 11 Current Effect on Cables

CHAPTER IV

TWO-DIMENSIONAL CABLE BEHAVIOR

Many cable applications in the ocean are of a two- and three-dimensional nature. Consequently, the one-dimensional theory presented must be expanded. A good example of this is the sub-surface buoy with a multi-legged moor illustrated in Fig. 1. Each of these mooring legs conform to a segment of a catenary if currents are not present. Analysis of these mooring cables can usually be performed in a two-dimensional framework. It is often desirable, when using mechanical anchors, to have a length of chain laying on the sea floor to insure that the shank of the anchor is not lifted by the motion of the moored vessel and to absorb some of the shock brought about by the action of waves (Fig. 12).

In this type of situation, the cable or chain is tangent to the sea floor at the lowest point of the catenary. The cases to follow will all be limited to cables which are tangent to the sea floor.

The Common Catenary

A homogeneous constant diameter cable takes the form of a catenary when supported between the points separated by a horizontal distance. It is useful to examine the catenary in some detail since it is a basic cable shape occurring when hydrodynamic forces are not present. Fig. 13 illustrates a portion of such a cable

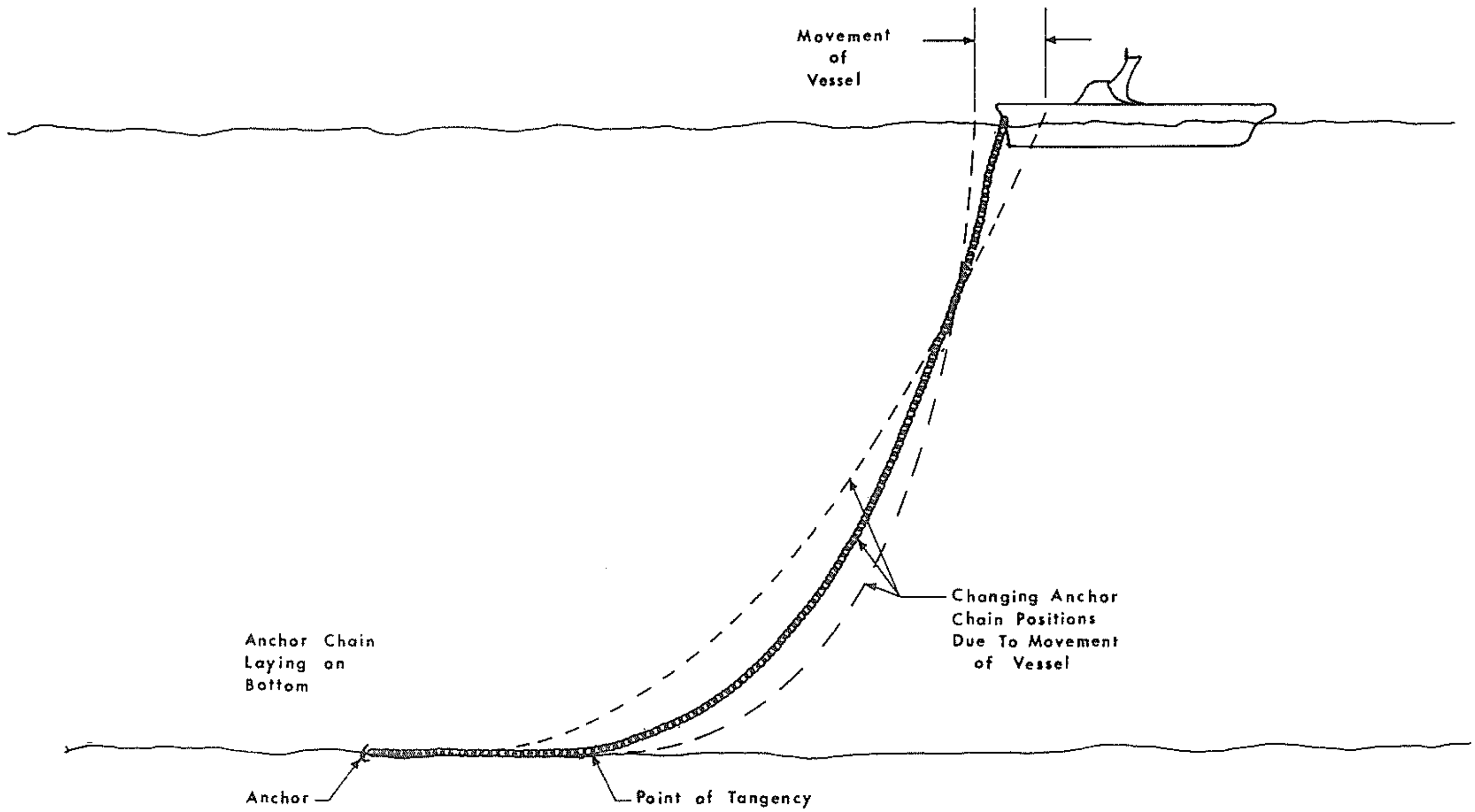

Fig. 12 Anchor Cable Configuration

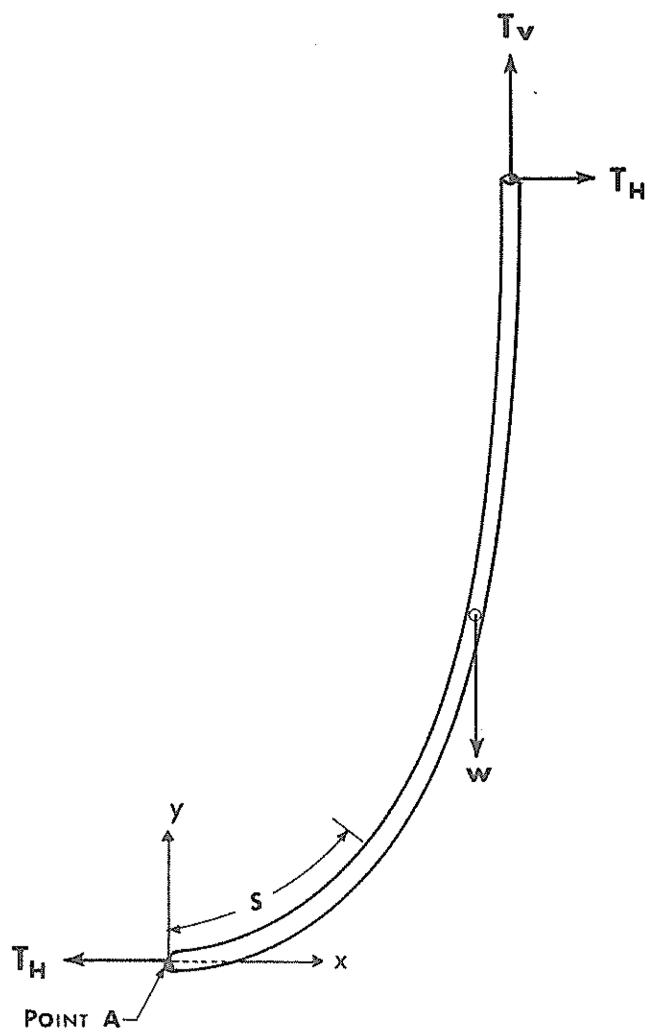

Fig. 13 Catenary Nomenclature and Coordinate System

as it might appear in a mooring application. The tension at any point along the cable is composed of a horizontal and a vertical component. The horizontal component T_H is the same everywhere along the cable, while the vertical component, T_V, is equal to the weight of the cable between point A, where the slope of the cable is 0, and the point of interest. At point A, the point where the cable becomes tangent to the ocean floor, $T = T_H$, and the tension at any point is defined as

$$T = \sqrt{T_H^2 + T_V^2} \quad \text{. . . . . (12)}$$

where

T_H = constant horizontal tension component

T_V = W = ws

W = total weight of cable length s

Due to the space required, the complete derivation of the catenary has been relegated to Appendix II. There the assumptions are made and the formulation of the basic cable differential equations are provided. This has the intended purpose of introducing the reader to the basic method of solving the cable equations used later to expressly solve for the constant-stress catenary relations. From this development, the following two-dimensional equations are determined which describe the geometric shape of the catenary, and give the length of cable, s, between the origin point, A, and the point (x, y) on the cable (Fig. 13).

$$y = a \cosh \frac{x}{a} \quad \text{. . . . .(13)}$$

$$s = a \sinh \frac{x}{a} \quad (14)$$

where

$$a = \frac{T_H}{w} \quad (15)$$

Fig. 14 depicts a family of catenaries. The only distinguishing variable involved in these curves is the parameter a . This parameter is uniquely determinable for any combination of x, y and L and reflects a change in either the horizontal tension component or cable weight. The catenary can be considered as a fundamental shape for any uniform cable hanging under its own weight.

The Constant-Stress Catenary

Next to be considered is a tapered cable suspended in a two-dimensional manner. The constant-stress condition is again imposed. Thus Eq. 5, derived for a tapered axially suspended cable, still holds.

In this case, however, if the condition

$$dT = dT_y$$

as in the catenary and

$$T = \sqrt{T_H^{\,2} + T_V^{\,2}}$$

are incorporated, Eq. 5 becomes

$$dT_y = \gamma \frac{\sqrt{T_H^2 + T_V^{\,2}}}{\sigma} \, ds \quad (16)$$

Since γ will be a constant, it is possible to proceed in a manner similar to that of the catenary derivation. The complete derivation of the constant-stress catenary is presented in Appendix III. The

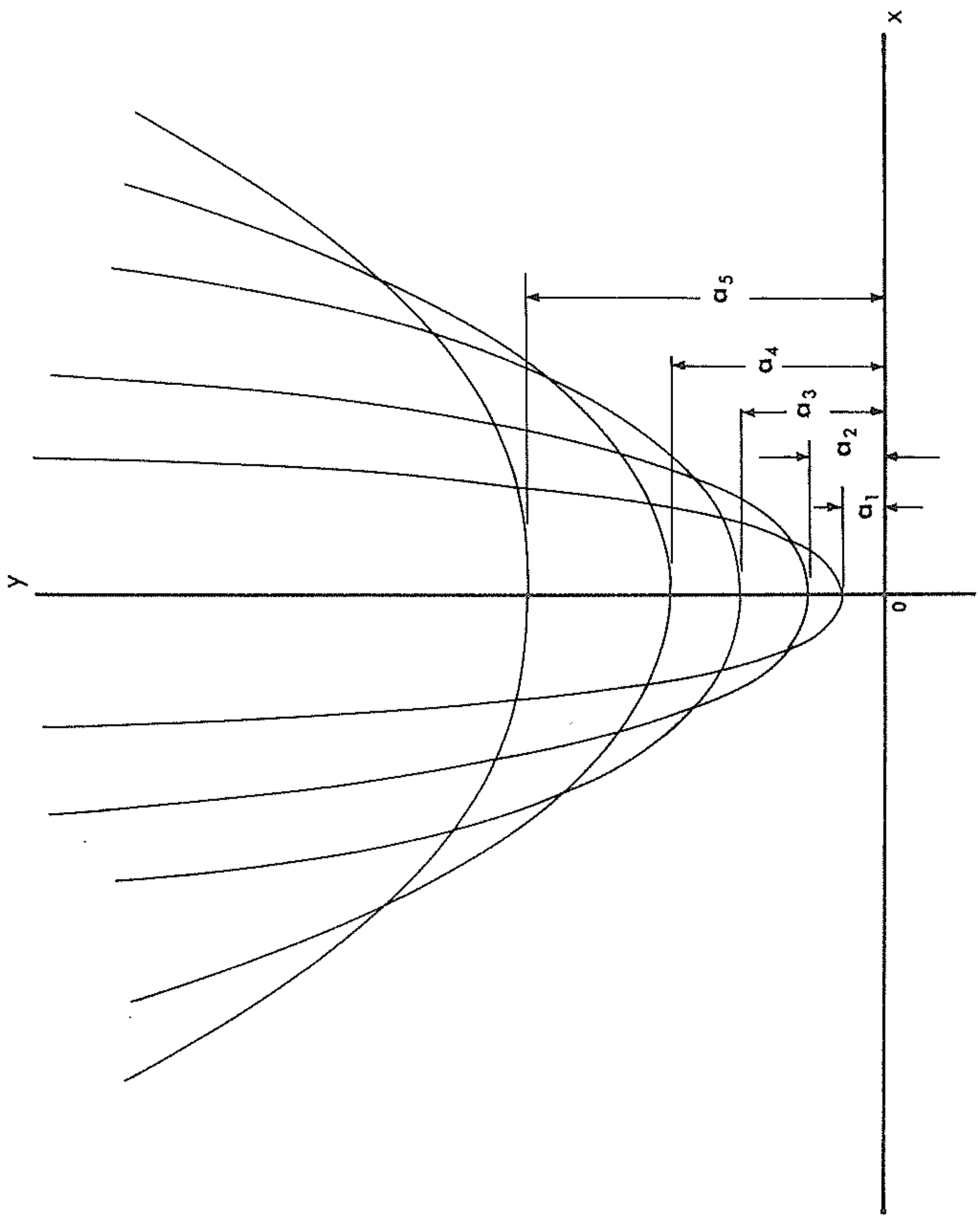

Fig. 14 Family of Catenaries

important geometrical relationships that are determined and will be required subsequently are

$$y = c \log_e \sec \frac{x}{c} \quad \ldots\ldots (17)$$

$$s = c \cosh^{-1} e^{\frac{y}{c}} \quad \ldots\ldots (18)$$

and

$$A = \frac{T_H}{\sigma} \cosh \frac{s}{c} \quad \ldots\ldots (19)$$

where

$$c = \frac{\sigma}{\gamma} \quad \ldots\ldots (20)$$

Eq. 17 defines the geometrical form of the constant-stress catenary. It has the general mathematical form $f(x) = \log_e \sec x$ (when $c = 1$). To assist the reader in visualizing the curve shape, this function has been plotted in Fig. 15.

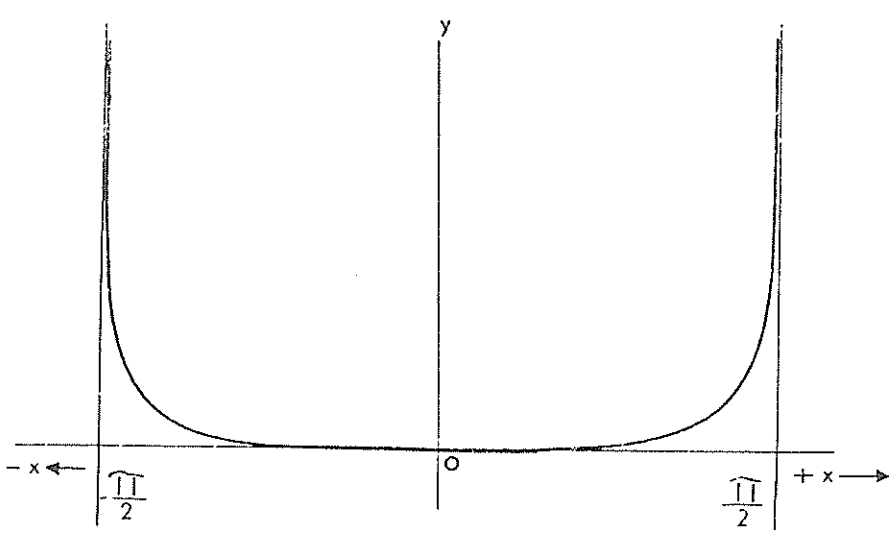

Fig. 15 Log (sec x)Function

It should be noted that as x approaches asymptotes of $\pm \frac{\pi}{2}$, y approaches ∞. Therefore, the values of $\left| \frac{\gamma}{\sigma} x \right|$ are always less than $\left| \frac{\pi}{2} \right|$. For the convenience of the reader, in applying the constant-stress catenary relations, a table of log(sec x) functions has been computed and is included as Appendix V.

Eq. 18 defines the length of the cable, as measured from the tangency point A as shown in Fig. 13.

Eq. 19 defines the required cross-sectional area variation such that the stress is constant along the length of the cable. The term $\frac{\sigma}{\gamma}$ can be considered a parameter for the constant stress catenary, similar to the catenary parameter α.

Eqs. 17 and 18 can be rearranged as follows:

$$\frac{\gamma y}{\sigma} = \log_e \sec \frac{\gamma x}{\sigma}$$

$$e^{\frac{\gamma y}{\sigma}} = \sec \frac{\gamma}{\sigma} x$$

and

$$e^{\frac{\gamma y}{\sigma}} = \cosh \frac{\gamma s}{\sigma}$$

The term, $e^{\frac{\gamma y}{\sigma}}$ is seen to be the function describing the area ratio from the tapered axially suspended cable. Therefore, Eqs. 17 and 18 are very closely related to Eqs. 8 and 11, as might be expected. The left-hand term indicates the effect of depth on the taper, while the right-hand term controls the cable-length relationship. Thus $e^{\frac{\gamma}{\sigma} y}$ holds for both types of constant stress cables that have been discussed, while the right-hand term represents the modification based on length of cable and geometry. It can then be seen that for a given parameter c, A_o and depth, the area of the cable is the same, at any given depth, for an axially suspended constant-stress cable as for a constant-stress catenary. Fig. 7 thus applies in two-dimensional cases also.

Fig. 16 illustrates what might be considered to be typical of ocean-mooring geometries using the constant-stress catenary, with γ and σ selected to represent appropriate values for steel wire rope. Note that if A_o is held constant, then varying σ has an effect similar to varying T_H in the simple catenary since $T_o = T_H$ and $\frac{T_H}{A_o} = \sigma$ when s = 0. Therefore, these curves can be thought of as representing three different values of T_H, assuming constant A_o, or three different values of A_o assuming constant T_H. Both alternatives assume a constant γ.

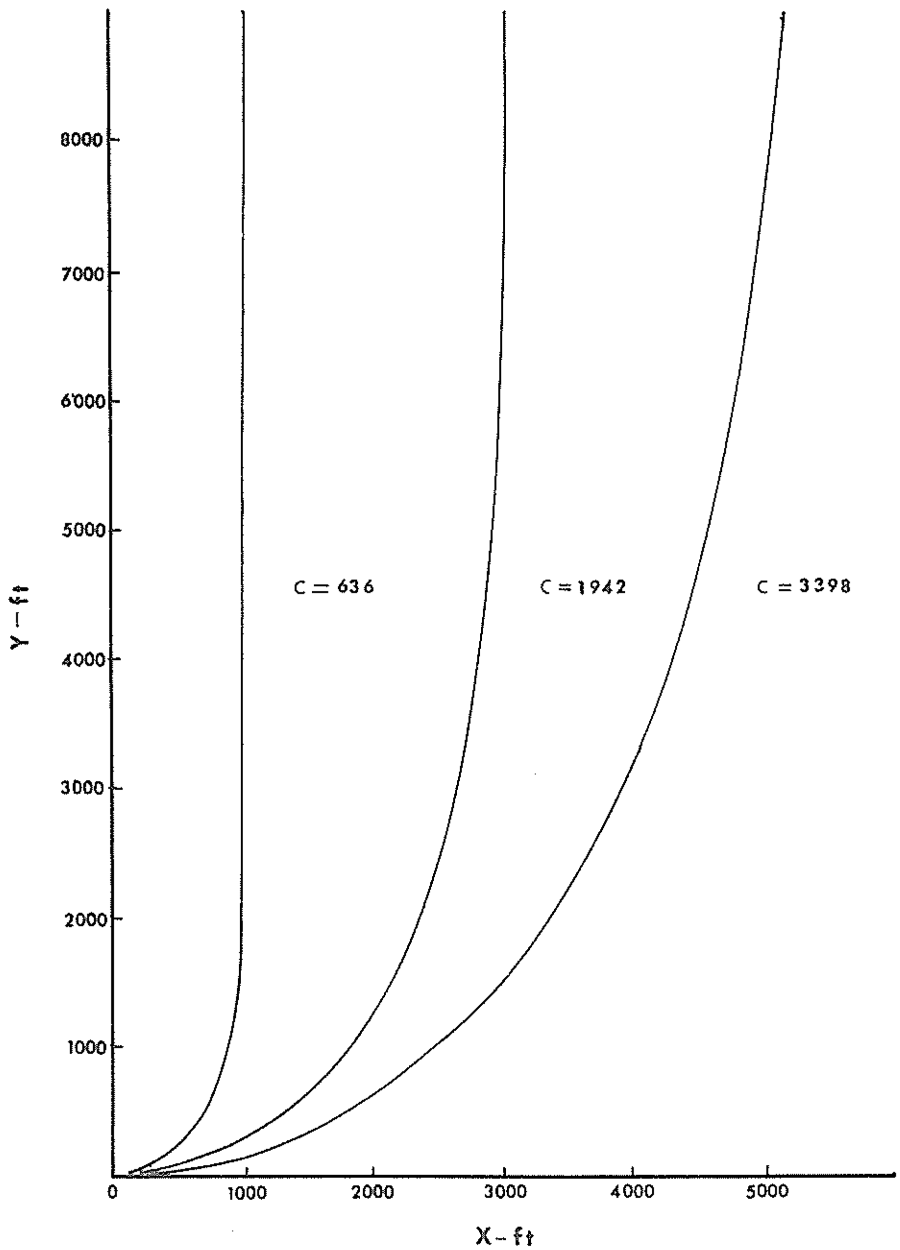

Fig. 16 Typical Ocean-Mooring Geometries, Constant-Stress Catenaries

Comparison

A comparison will now be made of the tapered cable to the common cable under two-dimensional conditions. As noted, Fig. 7 applies to both the E-tapered cable and the constant-stress catenary. Figs. 9 and 10 are then of general application and further discussion of load-carrying capacity and cable weight is not required.

All of the comparisons that follow are based on a mooring system geometry in which the mooring cable is tangent to the sea bottom. This has been specified for simplicity since, as Ref. 2 demonstrates, an infinite number of other possibilities exist.

The situations depicted are representative of mooring applications using tapered cables, and lend themselves to comparison with present mooring practice. These comparisons are intended to illustrate the trends to be expected and to provide a qualitative understanding of the relationships. Consideration is first given to the purely mathematical form of the catenary and constant-stress catenary geometrical expression. This is shown in Fig. 17 with both curves fitted between the points (0,0) and (1,000, 10,000). These curves are tangent to the x axis (0,0) as they might be in a slack moor. It should be noted that the axis scale is distorted. This has the effect of exaggerating the curve separation due to the closeness of the curves.

The distinction in geometry between the curves is largely due to the weight of the constant-stress catenary cable being concentrated

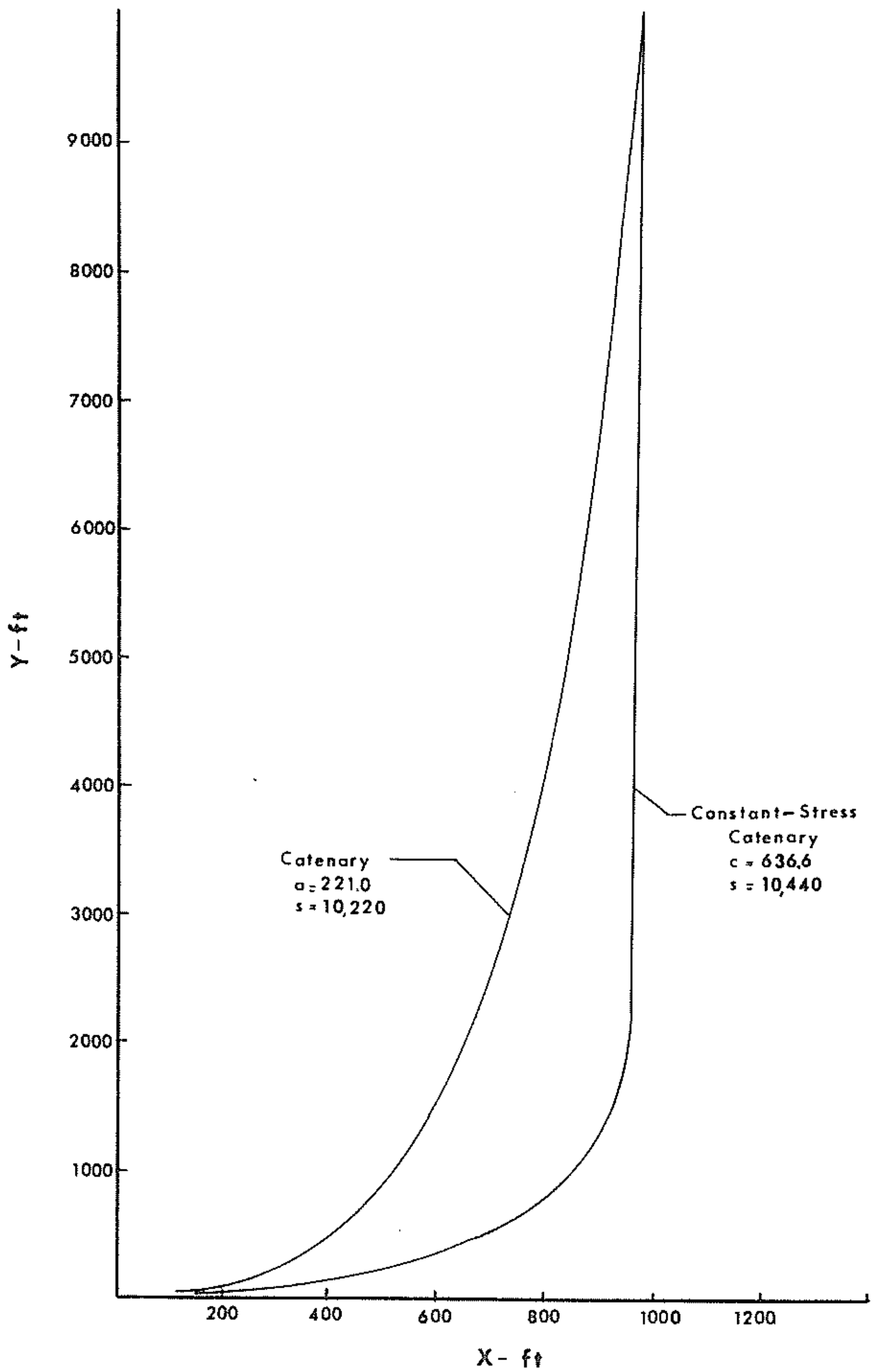

Fig. 17 Geometry of Curves Fitted Between Two Given Points

in the upper section of the cable. The curved portion will be much smaller in cross-sectional area than the upper, nearly straight section and contains very little of the total weight of the cable. This lack of bottom weight results in a smaller horizontal tension component than that of a catenary with a similar top diameter. However, these curves are not principally related to diameters or horizontal tensions. They can be considered as generic, as they fit many combinations of cable sizes, horizontal tensions, cable densities, etc. and should be viewed as mathematical shapes only. When considering factors other than the cable geometry, as defined by the curve parameter (a or c), account must also be taken of the material, the cable size, etc.

Fig. 18, also a generic curve, illustrates the effect of similar length cables on the geometry. In this case both curves or cables are 10,440 feet long and the horizontal tensions are again different. The catenary can be observed to extend much further in the horizontal direction.

Fig. 19 departs from the generic curve, as a comparison is made between two cables with the same horizontal tension component. This illustrates the major difference in the concentration of weight between the two types of cables. The weight near the bottom reduces the horizontal displacement of the catenary, whereas the constant stress cable is very sensitive to changes in horizontal force. This configuration could represent a moored buoy under the influence

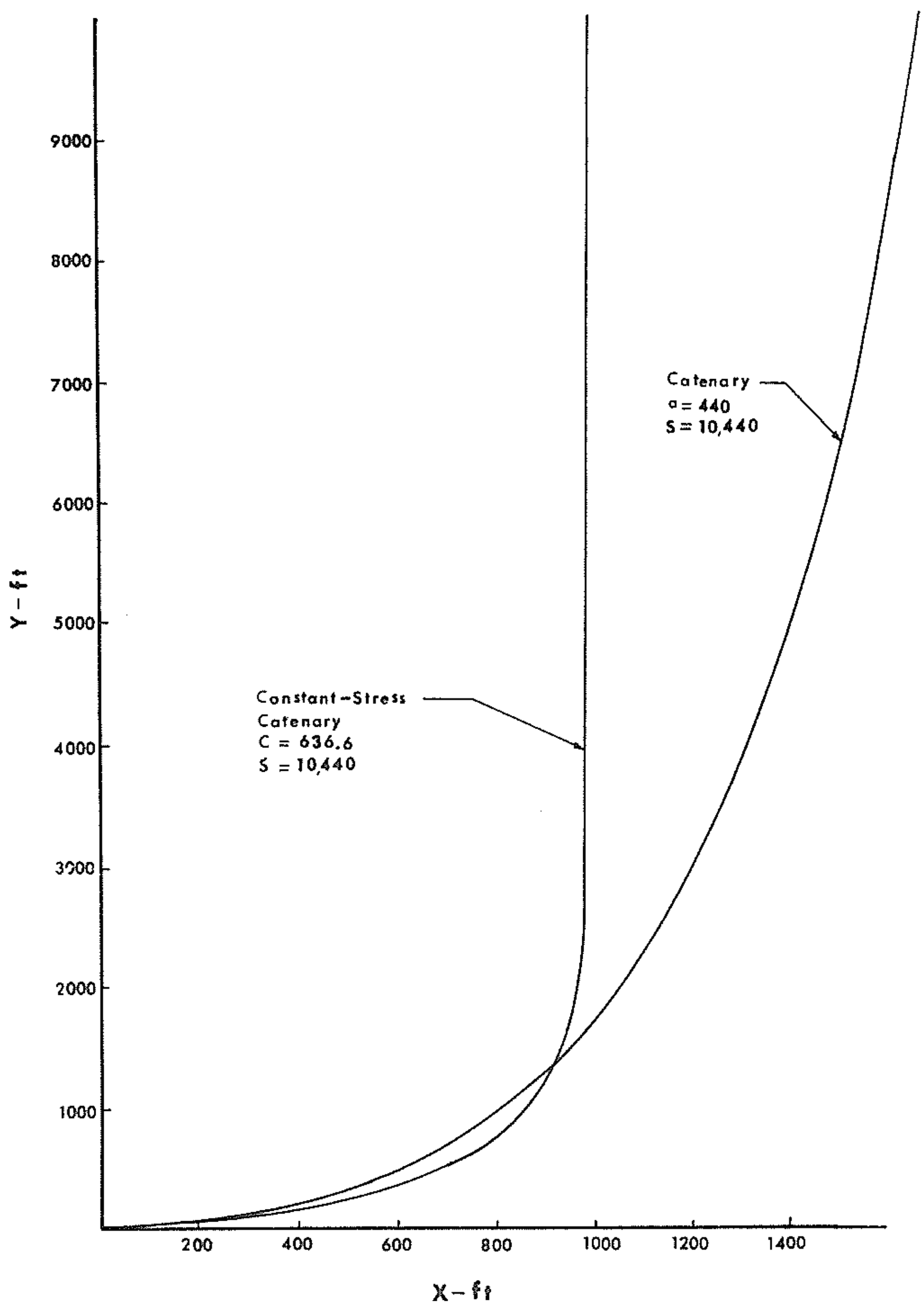

Fig. 18 Comparison of Curves of the Same Length

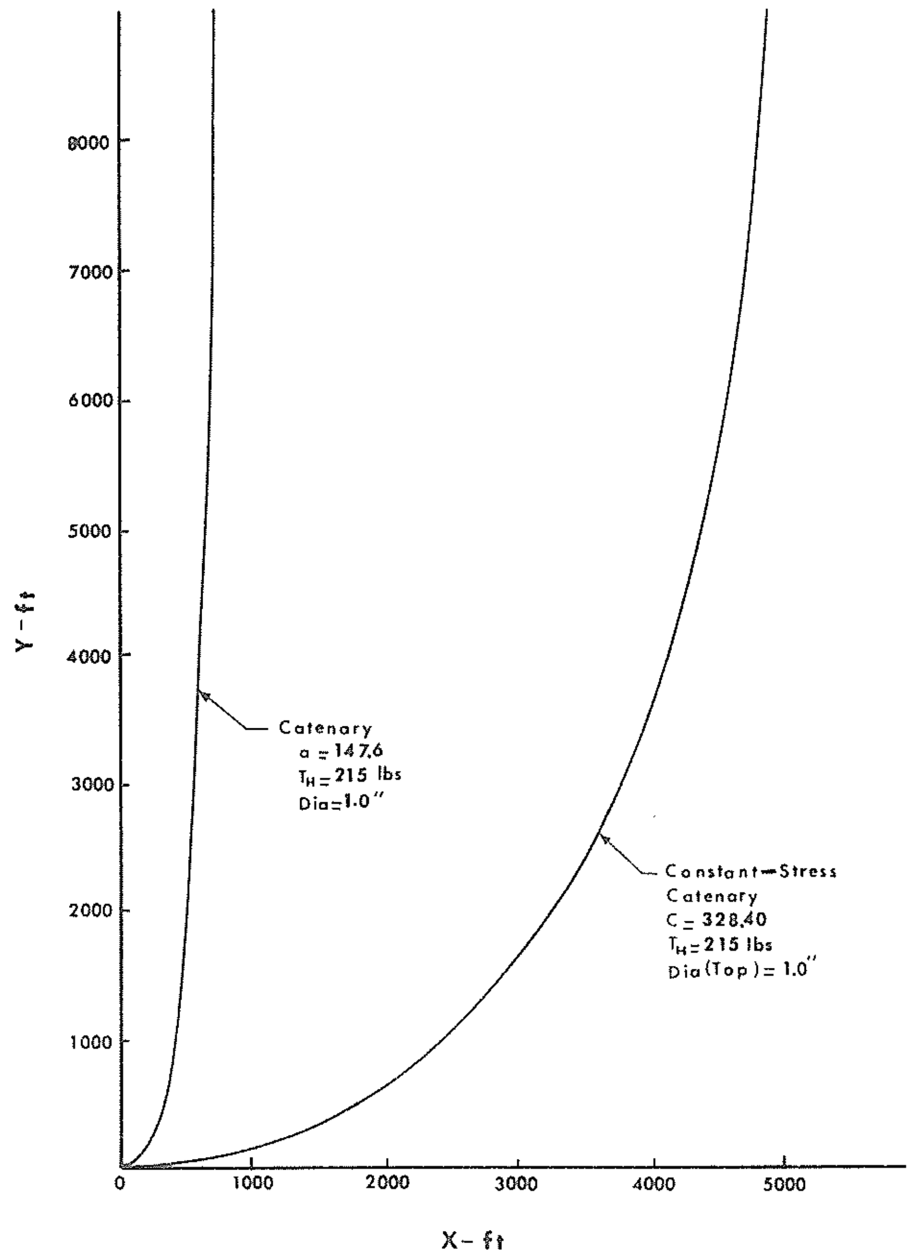

Fig. 19 Catenary vs Constant-Stress Catenary Comparison, with Same Horizontal Tension Component

of a steady wind load. If the cable is subjected to an underwater current load another configuration results.

The finite-element computer program described in Appendix VI was used to simulate the effects of current on the cable. Resulting configurations are shown in Fig. 20. Both .5-knot and 1-knot currents were imposed in the x direction. The large effect of the current on the constant stress cable is clearly indicated whereas a very small effect is observed on the catenary. This is as expected since, as Fig. 17 reveals, the constant-stress cable is very sensitive to T_H. Fig. 21 provides a plot of the corresponding cable stresses under the same loading conditions represented by Fig. 20. It should be noted that tapered cables subjected to current loads can no longer be considered to be under constant stress. This indicates that to obtain maximum benefit from a tapered cable, it is necessary to incorporate the combined influence of any anticipated currents as well as static loading into the design analysis.

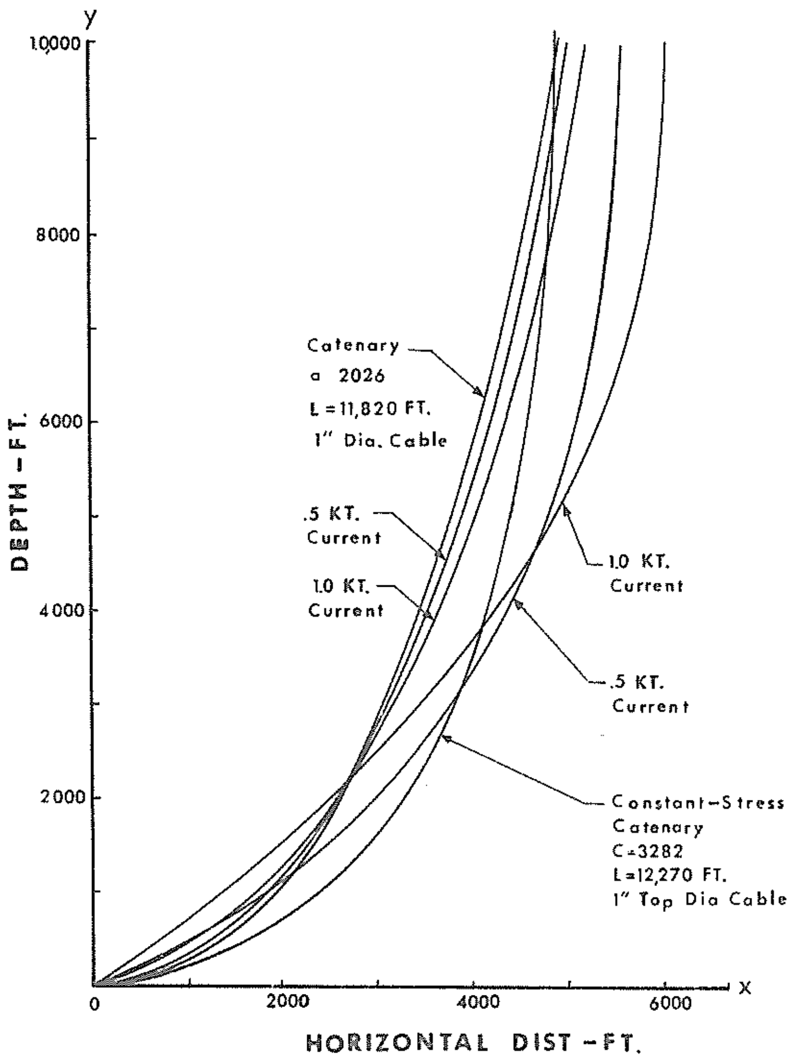

Fig. 20 Comparison - Cables With and Without Currents

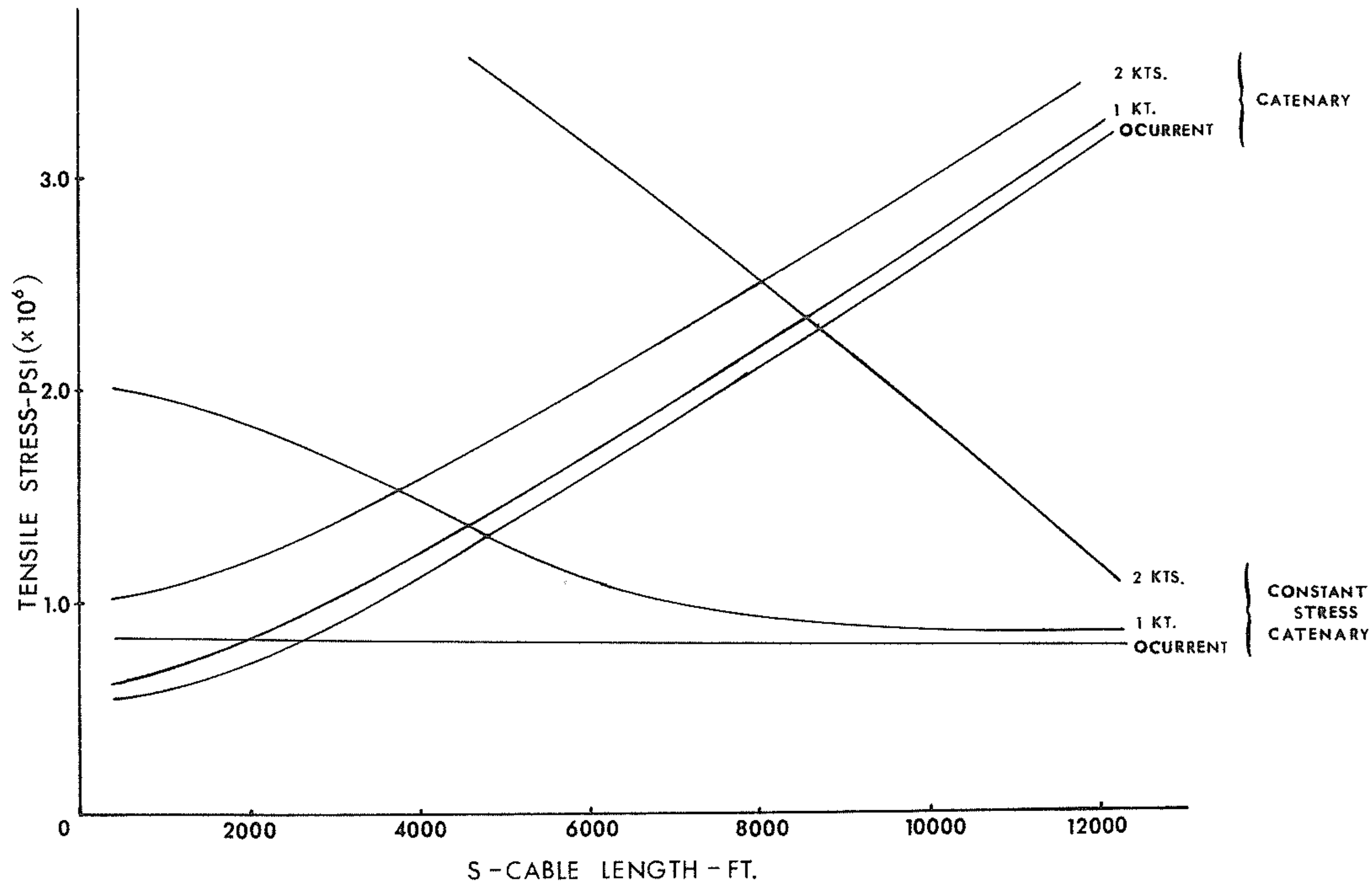

Fig. 21 Comparison - Cable Tensile Stress

CHAPTER V

CONCLUSIONS

This investigation has resulted in the development of analytical expressions describing the static behavior of tapered cables for two cases of practical importance. These equations have been substantiated by use of a finite-element computer program which has also enabled consideration of the effects of currents on such cables. Both common and tapered cables have been compared under several selected conditions leading to the following conclusions:

For any given length and top diameter, the tapered cable weighs less and has a greater load capacity than a similar common cable. This results in the tapered cable having a load capacity and resulting application at depths or cable lengths not possible with a common cable. In general, these advantages are all magnified for materials of increasing density and decreasing strength, or for increasing safety factors. There are no advantages in tapering for materials which are neutrally buoyant, since the cable weight does not have any effect on the loading. The tapered cable has the greatest value for applications requiring very long lengths or great depths.

The disadvantages of the tapered cable appear as follows: In a mooring situation the tapered cable exhibits greater motion on deflection due to currents and tends to be less desirable in any situation where weight is an advantage. The design of a tapered cable depends on the application with constant stress only achievable at the design conditions. Tapered cable manufacturing will probably prove difficult and/or expensive, although this was not examined in this study. Tapered cables appear to be applicable to both towing and mooring situations with no clear delineation of preferred usage between the two. However, this thesis should be considered as a beginning for tapered cable studies with more parametric evaluation being in order. Further work is also needed to investigate the practicality and possibility of manufacture of such cables, and to examine their elastic properties, particularly with regard to dynamic performance.

APPENDIX I

BIBLIOGRAPHY

1. Bohlen, W. F., "Data Center for Hydro-Wire Experience", *Geo-Marine Technology* October/November 1965, pp. 18-23.

2. Dominguez, R. F. and R. W. Filmer, "Steady-State Evaluation of a Multiple Cable Mooring by Discrete Parameter Techniques", Bulletin No. 46, August 1970, Engineering Experiment Station, Oregon State University.

3. Dominguez, R. F. and G. E. Owens, "Three-Dimensional Response of Deep Water Mooring Lines in Steady State Flows", Division of Coastal and Ocean Engineering, Texas A&M University, Technical Report No. COE-157, December, 1972.

4. Gilbert, D., "On the Mathematical Theory of Suspension Bridges, with Tables for Facilitating their Construction", *Philosophical Transactions of the Royal Society of London for the year 1826.* Part III, London, pp. 202-218.

5. Jakkula, A. A., "A History of Suspension Bridges in Bibliographical Form", Engineering Experiment Station Series No. 57, Texas Engineering Experiment Station, July 1, 1941.

6. Kullenberg, B., "On the Shape and the Length of the Cable during a Deep-Sea Trawling", *Reports of the Swedish Deep-Sea Expedition*; V. II(2), Zoology, Goteberg, 1951.

7. Landweber, L. and M. H. Protter, "The Shape and Tension of a Light Flexible Cable in a Uniform Current", Report 533, David Taylor Model Basin, U.S. Navy, Oct. 1944.

8. Markula, D. O., L. A. Rash and W. T. Hull, "Aerospace Targetry Techniques, Volume II. Tow Target Study", Beech Aircraft Corporation, Aerospace Division, Technical Report AFATL-TR-67-189, Vol. II, October 1967. (AD829291).

9. Montucla, J. F., "Histories Des Matematiques", Tome Deuxieme, Libraries Scientifique et Technique, Albert Blanchard, 1968.

10. Myers, J. J., C. H. Holm and R. F. McAllister, "Handbook of Ocean and Underwater Engineering", McGraw-Hill Book Co., New York, 1969.

11. Nowatzki, J. A., "Strength Member Design for Underwater Cables", Marine Technology Society 7th Annual Conference, August 16-18, 1971, Washington, D.C.

12. Phillips-Birt, D., "A History of Seamanship", Doubleday and Company, Inc., Garden City, New York, 1971.

13. Pode, L., "Tables for Computing the Equilibrium Configuration of a Uniform Stream", Report No. 687, David Taylor Model Basin, U.S. Navy, March 1951.

14. Routh, E. J., "A Treatise on Analytical Statics", Vol. I, Cambridge University Press, 1896.

15. Terry, R. D., "The Deep Submersible", Western Periodicals Co., North Hollywood, California, 1966.

16. Wilson, B. W., "Characteristics of Anchor Cables in Uniform Ocean Currents", Tech. Report No. 204-1, A&M Project 204, April 1960.

APPENDIX II

DERIVATION OF CATENARY EQUATIONS

The method used herein to derive the equations for the common catenary utilizes a vectorial format following the approach taken by Dominguez (2).

Consider Fig 22. Δs represents a small portion of a cable acted upon by a distributed load, w(s), whose resultant is $\bar{w}\,\Delta s$. It is assumed that the cable is completely flexible and inextensible. The cable tensions at either end are represented by $\bar{T}$ and $\bar{T} + \Delta\bar{T}$ respectively, with $\bar{r}$ being the position vector of any point a distance s along the cable.

For static equilibrium

$$\Sigma \bar{F} = 0$$

Where $\bar{F}$ represents the external forces acting on the element. Therefore,

$$-\bar{T} + (\bar{T} + \Delta\bar{T}) + \bar{w}\,\Delta s = 0$$

which reduces to

$$\frac{\Delta\bar{T}}{\Delta s} + \bar{w} = 0$$

and in the limit as $\Delta s \rightarrow 0$ becomes

$$\frac{d\bar{T}}{ds} + \bar{w} = 0 \quad \ldots\ldots\ldots\ldots \quad (23)$$

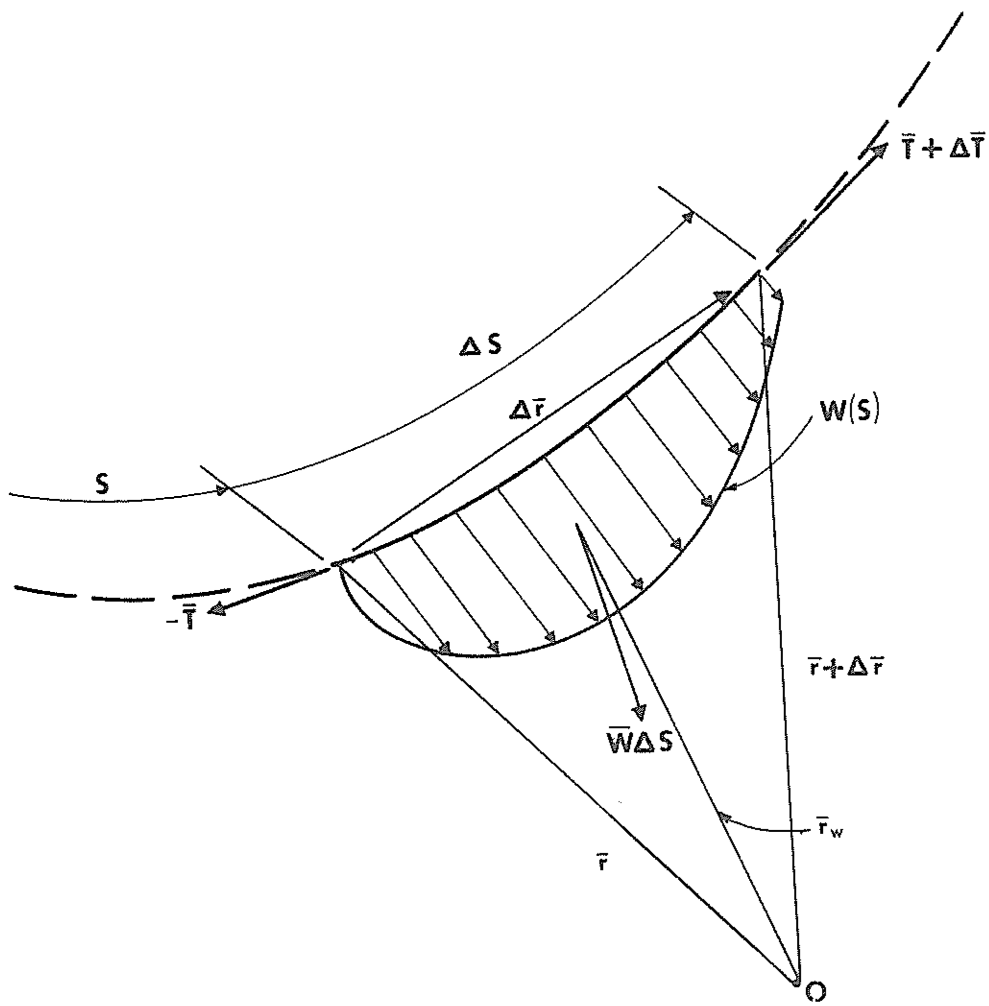

Fig. 22 Equilibrium of a Cable Element Influenced by an External Loading

Static equilibrium also requires

$$\Sigma \bar{M}_o = 0$$

where $\bar{M}_o$ is the moment produced by a force $\bar{F}$ about any arbitrary point 0. Therefore,

$$\bar{r} \times (-\bar{T}) + (\bar{r} + \Delta\bar{r}) \times (\bar{T} + \Delta\bar{T}) + \bar{r}_w \times \bar{w}\Delta s = 0$$

Expansion and division by Δs gives

$$\bar{r} \times \frac{\Delta\bar{T}}{\Delta s} + \frac{\Delta\bar{r}}{\Delta s} \times \bar{T} + \frac{\Delta\bar{r}}{\Delta s} \times \Delta\bar{T} + \bar{r}_w \times \bar{w} = 0$$

Taking the limit as $\Delta s \to 0$, $\Delta\bar{T} \to 0$ and $\bar{r}_w \to \bar{r}$ results in

$$\bar{r} \times \left(\frac{d\bar{T}}{ds} + \bar{w}\right) + \frac{d\bar{r}}{ds} \times \bar{T} = 0$$

Since $\frac{d\bar{T}}{ds} + \bar{w} = 0$ from Eq. 23 above,

Then

$$\frac{d\bar{r}}{ds} \times \bar{T} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (24)$$

Eqs (23) and (24) can be considered the basic differential equations governing the statics of a suspended cable subjected to a distributed loading.

If the additional assumption, that the cable is loaded solely by its own weight, is applied, then

$$\bar{w} = \bar{w}(s) = w(s)\bar{j}$$

where $\bar{i}$, $\bar{j}$ and $\bar{k}$ are unit vectors in the x, y and z directions respectively. The cable tension at any point can be represented as

$$\bar{T} = T_x\bar{i} + T_y\bar{j} + T_z\bar{k} \quad \ldots\ldots\ldots\ldots\ldots \quad (25)$$

where T_x, T_y and T_z are the components of tension $\bar{T}$, in the x, y and z directions (Fig 23).

Then

$$T = \sqrt{T_x^2 + T_y^2 + T_z^2} \quad \ldots\ldots\ldots\ldots \quad (26)$$

or

$$T = \sqrt{T_H^2 + T_y^2} \quad \ldots\ldots\ldots\ldots \quad (27)$$

Where

$$T_H = \sqrt{T_x^2 + T_z^2} \quad \ldots\ldots\ldots\ldots \quad (28)$$

Note that T_H is the total horizontal tension component. The substitution of Eq. 25 into Eq. 23 results in

$$\frac{dT_x}{ds}\,\bar{i} + \frac{dT_y}{ds}\,\bar{j} + \frac{dT_z}{ds}\,\bar{k} + w(s)\bar{j} = 0$$

If coefficients of like unit vectors are equated, then

$$\frac{dT_x}{ds} = 0 \quad \ldots\ldots\ldots\ldots \quad (29)$$

$$\frac{dT_z}{ds} = 0 \quad \ldots\ldots\ldots\ldots \quad (30)$$

$$\frac{dT_y}{ds} + w(s) = 0 \quad \ldots\ldots\ldots\ldots \quad (31)$$

Eqs 29 and 30 indicate that the horizontal components of tension are constant. Rearranging terms and integrating Eq. 31 yields

$$T_y = -\int w(s)ds + C_1 \quad \ldots\ldots\ldots\ldots \quad (32)$$

Where C_1 represents a constant of integration. The position vector $\bar{r}$ can be expressed as

$$\bar{r} = x\bar{i} + y\bar{j} + z\bar{k} \quad \ldots\ldots\ldots\ldots \quad (33)$$

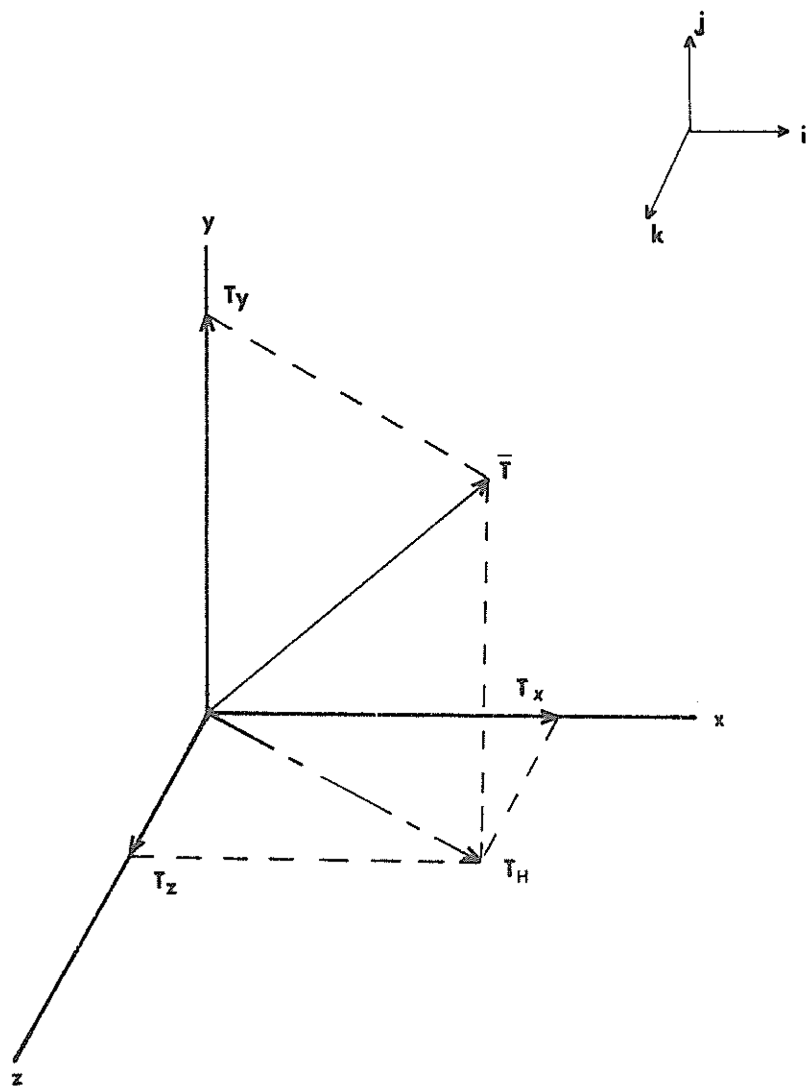

Fig. 23 Tension Components and Coordinate System

Substituting Eq. 33 into 24 yields

$$\left(\frac{dx}{ds}\,\bar{i} + \frac{dy}{ds}\,\bar{j} + \frac{dz}{ds}\,\bar{k}\right) \times (T_x\bar{i} + T_y\bar{j} + T_z\bar{k}) = 0$$

Expanding and separating terms results in

$$\frac{T_x}{dx} = \frac{T_y}{dy} = \frac{T_z}{dz} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (34)$$

Since T_x = constant and

T_z = constant from Eq. 29 and 30, then

$$\frac{dx}{dz} = \frac{T_x}{T_z} = \text{constant} \quad \ldots\ldots\ldots\ldots\ldots \quad (35)$$

Integrating yields:

$$x = \frac{T_x}{T_z}\,z + C_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (36)$$

This reveals that a cable hanging under its own weight projects as a straight line in the x-z plane.

Selecting axis location such that z = 0 when x = 0, then C_2 = 0 and

$$x = \frac{T_x}{T_z}\,z \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (37)$$

Substituting Eq. 34 into Eq. 32 yields

$$\frac{dy}{dz} = -\frac{1}{T_z}\int w(s)ds + C_1 \quad \ldots\ldots\ldots\ldots \quad (38)$$

Considering the case of a homogeneous cable of constant diameter, results in uniform loading

$$w(s) = -w = \text{constant weight/unit length of cable}$$

and

$$\frac{dy}{dz} = -\frac{1}{T_z}\int -w ds + C_1$$

or

$$\frac{dy}{dz} = \frac{ws}{T_z} + C_1$$

If the origin of the coordinate system is selected at the point where

$$\frac{dy}{dz} = 0$$

Then $s = 0$, $C_1 = 0$ and

$$\frac{dy}{dz} = \frac{ws}{T_z} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (39)$$

Refering then to Eq. 34

$$T_y = ws$$

A similar substitution of Eq. 34 into Eq. 32, integrating and evaluating boundary conditions yields

$$\frac{dy}{dx} = \frac{ws}{T_x} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (40)$$

Next consider the geometrical relationship

$$ds^2 = dx^2 + dy^2 + dz^2$$

Dividing by dz^2 gives

$$\frac{ds^2}{dz^2} = \frac{dx^2}{dz^2} + \frac{dy^2}{dz^2} + 1 \quad \ldots\ldots\ldots\ldots \quad (41)$$

Substituting Eqs. 35 and 39 into Eq. 41 and combining terms yields

$$\frac{ds}{dz} = \frac{\sqrt{T_x^{\,2} + T_z^{\,2} + (ws)^2}}{T_z}$$

which when inverted, and the variables separated gives

$$dz = \frac{T_z \, ds}{\sqrt{T_x^{\,2} + T_z^{\,2} + (ws)^2}}$$

Inserting Eq. 28

$$dz = \frac{T_z \, ds}{\sqrt{T_H^{\,2} + (ws)^2}}$$

Rearranging and integrating gives

$$z = \frac{T_z}{w} \sinh^{-1} \frac{ws}{T_H} + C_3 \quad \text{. . . . . (42)}$$

If z = 0 at s = 0, then $C_3 = 0$.

$$z = \frac{T_z}{w} \sinh^{-1} \frac{ws}{T_H} \quad \text{. . . . . (43)}$$

and inverting

$$s = \frac{T_H}{w} \sinh \frac{wz}{T_z} \quad \text{. . . . . (44)}$$

By a similar approach

$$x = \frac{T_x}{w} \sinh^{-1} \frac{w}{T_H} s \quad \text{. . . . . (45)}$$

and inverting

$$s = \frac{T_H}{w} \sinh \frac{wx}{T_x} \quad \text{. . . . . (46)}$$

Substituting Eq. 44 into Eq. 39

$$\frac{dy}{dz} = \frac{T_H}{T_z} \sinh \frac{wz}{T_z}$$

Integrating gives

$$y = \frac{T_H}{w} \cosh \frac{wz}{T_z} + C_4 \quad \ldots\ldots\ldots\ldots\ldots \quad (47)$$

Letting $y = \frac{T_H}{w}$ at $z = 0$, then $C_4 = 0$

and

$$y = \frac{T_H}{w} \cosh \frac{wz}{T_z} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (48)$$

Similarly, inserting Eq. 46 into 40

$$y = \frac{T_H}{w} \cosh \frac{wx}{T_x} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (49)$$

Finally, it can be seen that

$$y = \frac{T_H}{w} \cosh \frac{wz}{T_z} = \frac{T_H}{w} \cosh \frac{wx}{T_x} \quad \ldots\ldots\ldots \quad (50)$$

$$s = \frac{T_H}{w} \sinh \frac{wz}{T_z} = \frac{T_H}{w} \sinh \frac{wx}{T_x} \quad \ldots\ldots\ldots \quad (51)$$

which constitute the symmetric equations of the catenary in three-dimensional space.

APPENDIX III

DERIVATION OF CONSTANT-STRESS CATENARY

The method of solution used below is basically similar to that used in Appendix II. This constant-stress catenary derivation starts with the basic cable equations

$$\frac{d\bar{T}}{ds} + \bar{w} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (23)$$

and

$$\frac{d\bar{r}}{ds} \times \bar{T} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (24)$$

discussed in Appendix I. Assuming that the cable is loaded solely by its own weight, it was shown that

$$\frac{dT_x}{ds} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (29)$$

and

$$\frac{dT_z}{ds} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (30)$$

$$\frac{dT_y}{ds} + w(s) = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (31)$$

In the case of the tapered cable, it must be recognized that w(s) varies throughout the total length of the cable. If it is assumed that the material is of <u>constant density</u>, then

$$w(s) = -\gamma A(s) \quad \ldots\ldots\ldots\ldots\ldots\ldots (52)$$

where γ = weight per unit volume of the cable material and A(s) is the cross-sectional area of the cable (Fig. 24).

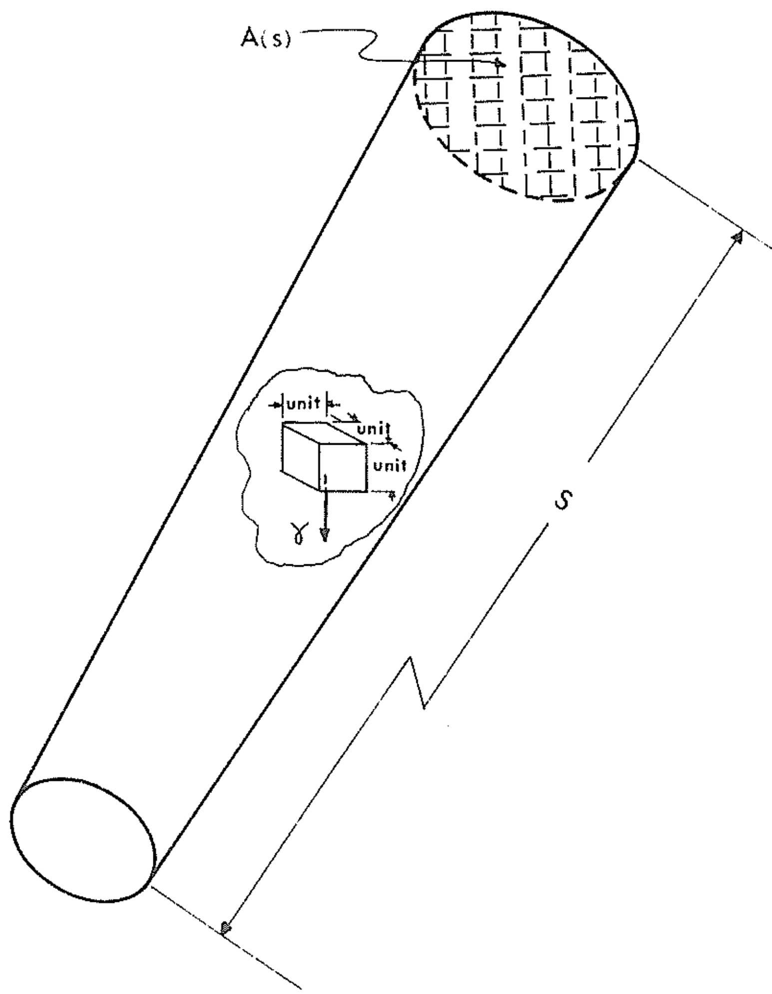

Fig. 24 Typical Section of Cable

Inserting Eq. 52 into Eq. 31 results in

$$dT_y = \gamma \; A(s) \; ds \quad \ldots\ldots\ldots \quad (53)$$

If constant stress is required, then the cross-sectional area and cable tension at any point are related:

$$A = \frac{T}{\sigma} \quad \ldots\ldots\ldots \quad (54)$$

Substituting Eqs. 54 and 27 into Eq. 52,

$$dT_y = \gamma \cdot \frac{\sqrt{T_H^{\;2} + T_y^{\;2}}}{\sigma} \; ds$$

rearranging and integrating yields

$$\text{Log}_e \left(T_y + \sqrt{T_H^{\;2} + T_y^{\;2}}\right) = \frac{\gamma}{\sigma} s + C_5 \quad \ldots\ldots \quad (55)$$

Applying the boundary conditions (Fig. 25)

$T_y = 0$ at $s = 0$, gives

$C_5 = \text{Log}_e \; T_H$

thus

$$\text{Log}_e \left(T_y + \sqrt{T_H^{\;2} + T_y^{\;2}}\right) = \frac{\gamma}{\sigma} s + \text{Log}_e \; T_H$$

Combining terms results in

$$\text{Log}_e \left(\frac{T_y + \sqrt{T_H^{\;2} + T_y^{\;2}}}{T_H} \right) = \frac{\gamma}{\sigma} s$$

which by identity is

$$\text{Sinh} \; \frac{\gamma s}{\sigma} = \frac{T_y}{T_H} \quad \ldots\ldots\ldots \quad (56)$$

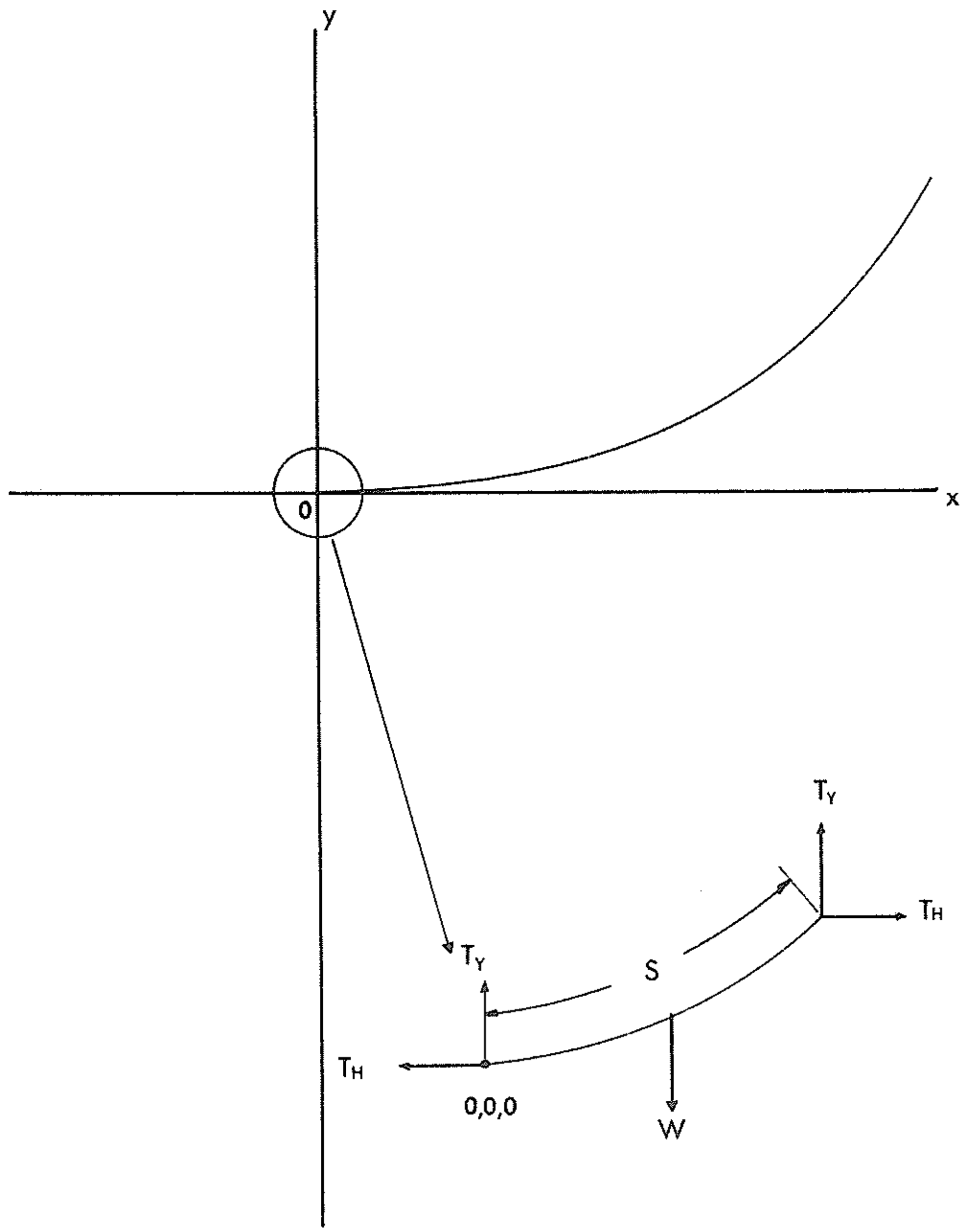

Fig. 25 Boundary Conditions at Origin

Eq. 34 can be inserted into Eq. 41 and with some manipulation produce

$$\frac{T_y}{T_H} = \frac{1}{\sqrt{\left(\frac{ds}{dy}\right)^2 - 1}} \quad \ldots\ldots\ldots\ldots\ldots \quad (57)$$

Substituting Eq. 57 back into Eq. 56 gives

$$\text{Sinh } \frac{\gamma s}{\sigma} = \frac{1}{\sqrt{\left(\frac{ds}{dy}\right)^2 - 1}}$$

which by rearrangement and use of identity becomes

$$\frac{dy}{ds} = \text{Tanh } \frac{\gamma s}{\sigma} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (58)$$

The square root of $\left(\frac{dy}{ds}\right)^2$ produces both positive and negative roots. Since the negative root represents negative values of Tanh $\frac{\gamma s}{\sigma}$, it will represent, when integrated, the negative (or mirror image) of the curve of interest, Cosh $\frac{\gamma s}{\sigma}$. Therefore, the negative root is discarded since it has no physical meaning for mooring applications. These functions are illustrated in Fig. 26. Separating variables in Eq. 58 and integrating gives

$$y = \frac{\sigma}{\gamma} \text{Log}_e \text{ Cosh } \frac{\gamma s}{\sigma} + C_6 \quad \ldots\ldots\ldots\ldots \quad (59)$$

Applying the boundary conditions of $y = 0$ when $s = 0$, $\cosh = 1$, $\text{Log } 1 = 0$, $C_6 = 0$ and

$$y = \frac{\sigma}{\gamma} \text{Log}_e \text{ Cosh } \frac{\gamma s}{\sigma} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (60)$$

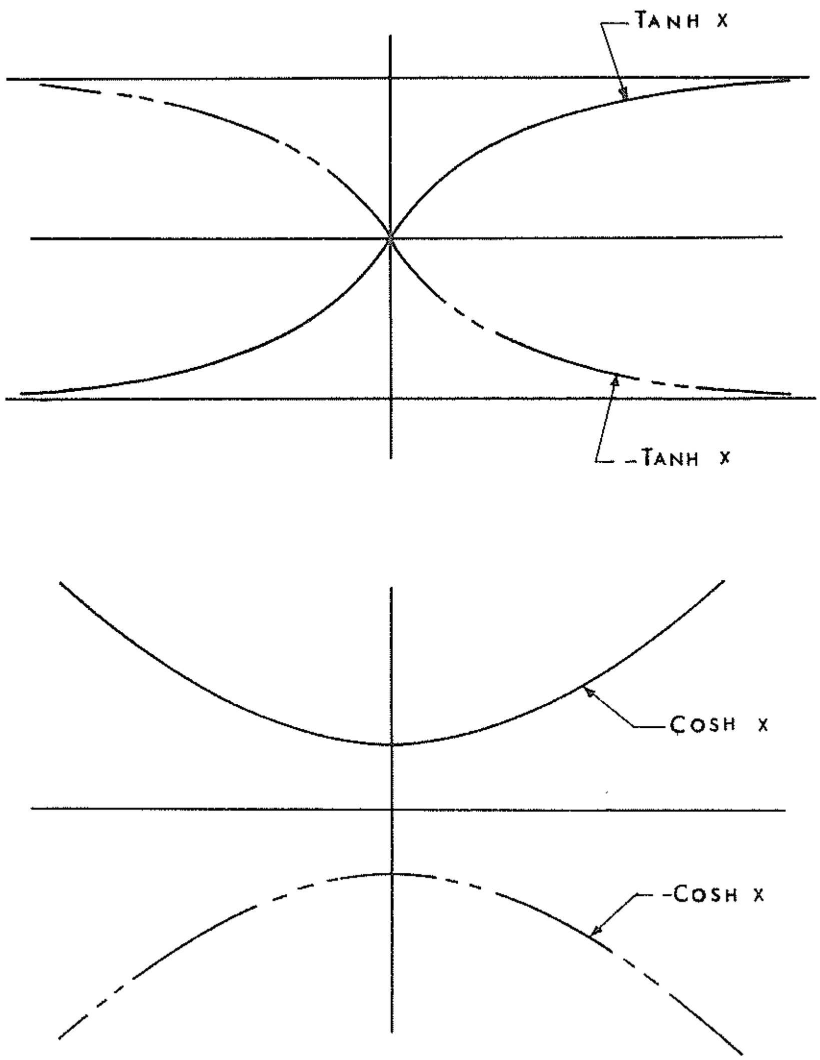

Fig. 26 Hyperbolic Functions

Inverting Eq. 60 produces

$$e^{\frac{\gamma y}{\sigma}} = \text{Cosh}\,\frac{\gamma s}{\sigma}$$

and

$$\text{Cosh}^{-1}\, e^{\frac{\gamma y}{\sigma}} = \frac{\gamma s}{\sigma}$$

Differentiation then yields

$$\frac{ds}{dy} = \frac{e^{\frac{\gamma}{\sigma} y}}{\sqrt{e^{\frac{2\gamma y}{\sigma}} - 1}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (61)$$

Substituting Eq. 41 for $\frac{ds}{dy}$ gives

$$\sqrt{1 + \left(\frac{dx}{dy}\right)^2 + \left(\frac{dz}{dy}\right)^2} = \frac{e^{\frac{\gamma y}{\sigma}}}{\sqrt{e^{\frac{2\gamma y}{\sigma}} - 1}}$$

which reduces to

$$dz\sqrt{\left(\frac{dx}{dz}\right)^2 + 1} = \frac{dy}{\sqrt{e^{\frac{2\gamma y}{\sigma}} - 1}}$$

Inserting Eq. 34 for $\frac{dx}{dz}$ yields

$$dz\sqrt{\left(\frac{T_x}{T_z}\right)^2 + 1} = \frac{e^{\frac{\gamma}{\sigma} y}\,dy}{e^{\frac{\gamma}{\sigma} y}\sqrt{e^{\frac{2\gamma}{\sigma} y} - 1}}$$

Note: Since T_x and T_z are constants, then

$$\sqrt{\left(\frac{T_x}{T_z}\right)^2 + 1} \quad \text{is also a constant}$$

Integrating yields

$$z\sqrt{\left(\frac{T_x}{T_z}\right)^2 + 1} = \frac{\sigma}{\gamma} \sec^{-1} e^{\frac{\gamma}{\sigma} y} + C_7 \quad \ldots\ldots (62)$$

Considering the boundary conditions

$z = 0$ and $C_7 = 0$ when $y = 0$

i.e. curve goes through the origin, and

$$z\sqrt{\left(\frac{T_x}{T_z}\right)^2 + 1} = \frac{\sigma}{\gamma} \sec^{-1} e^{\frac{\gamma}{\sigma} y} \quad \ldots\ldots\ldots (63)$$

inverting

$$\sec \frac{\sigma}{\gamma} z\sqrt{\left(\frac{T_x}{T_z}\right)^2 + 1} = e^{\frac{\gamma}{\sigma} y}$$

Taking the log of each side

$$y = \frac{\sigma}{\gamma} \operatorname{Log}_e \sec \frac{\gamma}{\sigma} z\sqrt{\left(\frac{T_x}{T_z}\right)^2 + 1} \quad \ldots\ldots\ldots (64)$$

Using Eq. 37, Eq. 64 becomes

$$y = \frac{\sigma}{\gamma} \operatorname{Log}_e \sec \frac{\gamma}{\sigma} \sqrt{x^2 + z^2} \quad \ldots\ldots\ldots\ldots (65)$$

This constitutes the principal equation of a constant-stress catenary.

Cable Tension

Combining Eq. 27 with Eq. 56 yields

$$T = T_H \sqrt{\operatorname{Sinh}^2 \frac{\gamma s}{\sigma} + 1} = T_H \operatorname{Cosh} \frac{\gamma s}{\sigma}$$

Substituting Eq. 28 for T_H

$$T = \sqrt{T_x^{\,2} + T_z^{\,2}} \;\; \text{Cosh}\, \frac{\gamma s}{\sigma} \quad \ldots\ldots\ldots\ldots \quad (66)$$

Eq. 66 depicts the tension at any point a distance s along the cable.

Cable Area

As noted previously, Eq. 54

$$A = \frac{T}{\sigma}$$

Substituting Eq. 66 for T gives

$$A = \frac{\sqrt{T_x^{\,2} + T_z^{\,2}}}{\sigma} \;\; \text{Cosh}\, \frac{\gamma s}{\sigma} \quad \ldots\ldots\ldots\ldots \quad (67)$$

"A" represents the cross sectional area of the cable at a distance s from the bottom end, (Fig. 25).

Cable Weight

The total cable weight W can be expressed as

$$W = \int \gamma \, A \, ds = \int \gamma \, \frac{T_H}{\sigma} \, \text{Cosh}\, \frac{\gamma s}{\sigma} \, ds$$

which upon integration becomes

$$W = T_H \, \text{Sinh}\, \frac{\gamma s}{\sigma} + C_8$$

Considering the boundary conditions at

$$s = 0, \; W = 0 \text{ and } C_8 = 0$$

Therefore

$$W = T_H \, \text{Sinh}\, \frac{\gamma s}{\sigma}$$

Substituting Eq. 28 for T_H

$$W = \sqrt{T_x^{\,2} + T_z^{\,2}} \ \text{Sinh} \frac{\gamma s}{\sigma} \qquad (68)$$

which represents the weight of a length s of cable, measured from the origin.

APPENDIX IV

Notation

A	area
A_L	cable area at distance L
A_o	cable area at bottom end of tapered cable
A_s	cable area distance s along a tapered cable
D_L	diameter at distance L along a tapered cable
D_o	bottom diameter of tapered cable
D_T	top diameter of tapered cable
F_s	factor of safety
L	total cable length
P	load supported by the cable
P_a	allowable load
T	cable tension
T_H	horizontal cable tension component
T_V	vertical component of cable tension, $= T_y$
T_x	component of cable tension in X direction
T_y	component of cable tension in Y direction
T_z	component of cable tension in Z direction
W	total cable weight
a	catenary parameter
c	constant-stress catenary parameter
s	length along cable

Notation (Continued)

w	weight per unit length of cable
w(s)	weight as a function of distance along cable
x, y, z	base coordinates
$(\ )_o$	subscript o indicates end condition
$\bar{i}, \bar{j}, \bar{k}$	unit vectors x, y, z directions
$\bar{r}$	position vector for a point in space
$\bar{T}$	vectorial representation of tension
σ	cable tensile stress = T/A
γ	cable unit weight

APPENDIX V

Tables of

Log (sec x)

Functions

X = IN DEGREES	X = IN RAD.	SEC(XRAD) =	LOGSEC =
0.0000000000000000D 00	0.0000000000000000D 00	0.1000000000000000D 01	0.0000000000000000D 00
0.1000000000000000D 02	0.1745329251994333D 00	0.1015426611885745D 01	0.1530883146598568D-01
0.2000000000000000D 02	0.3490658503988666D 00	0.1064177772475912D 01	0.6220245635789949D-01
0.3000000000000000D 02	0.5235987755982998D 00	0.1154700538379252D 01	0.1438410362258910D 00
0.4000000000000000D 02	0.6981317007977331D 00	0.1305407289332280D 01	0.2665150911870572D 00
0.5000000000000000D 02	0.8726646259971662D 00	0.1555723826860415D 01	0.4419409208388763D 00

0.5000000000000000D 02	0.8726646259971662D 00	0.1555723826860415D 01	0.4419409208388763D 00
0.5500000000000000D 02	0.9599310885968827D 00	0.1743446795621102D 01	0.5558640708211494D 00
0.6000000000000000D 02	0.1047197551196599D 01	0.2000000000000005D 01	0.6931471805599476D 00
0.6500000000000000D 02	0.1134464013796316D 01	0.2366201583152505D 01	0.8612859619068086D 00
0.7000000000000000D 02	0.1221730476396032D 01	0.2923804400163099D 01	0.1072885645040924D 01

0.7000000000000000D 02	0.1221730476396032D 01	0.2923804400163099D 01	0.1072885645040924D 01
0.7100000000000000D 02	0.1239183768915976D 01	0.3071553486757255D 01	0.1122183455358310D 01
0.7200000000000000D 02	0.1256637061435919D 01	0.3236067977499804D 01	0.1174359005619553D 01
0.7300000000000000D 02	0.1274090353955862D 01	0.3420303619833284D 01	0.1229729324863035D 01
0.7400000000000000D 02	0.1291543646475805D 01	0.3627955278543315D 01	0.1288669205424797D 01
0.7500000000000000D 02	0.1308996938995749D 01	0.3863703305156292D 01	0.1351626129022358D 01
0.7600000000000000D 02	0.1326450231515692D 01	0.4133565494438770D 01	0.1419140350374897D 01
0.7700000000000000D 02	0.1343903524035635D 01	0.4445411482585827D 01	0.1491872436691653D 01
0.7800000000000000D 02	0.1361356816555579D 01	0.4809734344744159D 01	0.1570641852809530D 01
0.7900000000000000D 02	0.1378810109075522D 01	0.5240843064167886D 01	0.1656482375499901D 01
0.8000000000000000D 02	0.1396263401595465D 01	0.5758770483143677D 01	0.1750723994134888D 01

0.8000000000000000D 02	0.1396263401595465D 01	0.5758770483143677D 01	0.1750723994134888D 01
0.8050000000000000D 02	0.1404990647855437D 01	0.6053885795668034D 01	0.1801521326327818D 01
0.8100000000000000D 02	0.1413716694115408D 01	0.6392453221499719D 01	0.1855118110440313D 01
0.8150000000000000D 02	0.1422443340375380D 01	0.6765469075058643D 01	0.1911831597734637D 01
0.8200000000000000D 02	0.1431169986635352D 01	0.7185296534327781D 01	0.1972036789984540D 01
0.8250000000000000D 02	0.1439896632895323D 01	0.7661297575540456D 01	0.2036181365691224D 01
0.8300000000000000D 02	0.1448623279155295D 01	0.8205509048125166D 01	0.2104805763827226D 01
0.8350000000000000D 02	0.1457349925415266D 01	0.8833671471997653D 01	0.2178570723440203D 01
0.8400000000000000D 02	0.1466076571675238D 01	0.9566772233505723D 01	0.2258295863897807D 01
0.8450000000000000D 02	0.1474803217935210D 01	0.1043343305246234D 02	0.2345015124293729D 01
0.8500000000000000D 02	0.1483529864195182D 01	0.1147371324567003D 02	0.2440059614205367D 01
0.8550000000000000D 02	0.1492256510455153D 01	0.1274549484318262D 02	0.2545177963530422D 01
0.8600000000000000D 02	0.1500983156715125D 01	0.1433558702620394D 02	0.2662745049051744D 01
0.8650000000000000D 02	0.1509709802975096D 01	0.1639040823939860D 02	0.2796086001154557D 01
0.8700000000000000D 02	0.1518436449235068D 01	0.1910732260929791D 02	0.2950071644265975D 01
0.8750000000000000D 02	0.1527163095495040D 01	0.2292558567605398D 02	0.3132253563118498D 01
0.8800000000000000D 02	0.1535889741755011D 01	0.2865370834789447D 02	0.3355292871009552D 01
0.8850000000000000D 02	0.1544616388014983D 01	0.3820155001411185D 02	0.3642876091074985D 01
0.8900000000000000D 02	0.1553343034274955D 01	0.5729868849855382D 02	0.4048277735126358D 01

X = IN DEGREES	X = IN RAD.	SEC(XRAD) =	LOGSEC =
0.8900000000000000D 02	0.1553343034274953D 01	0.5729868849854867D 02	0.4048277735126268D 01
0.8910000000000002D 02	0.1555088363526947D 01	0.6366459530599980D 02	0.4153628604388527D 01
0.8920000000000003D 02	0.1556833692778942D 01	0.7162205154995412D 02	0.4271403009090971D 01
0.8930000000000005D 02	0.1558579022030937D 01	0.8185314984304099D 02	0.4404926786192644D 01
0.8940000000000006D 02	0.1560324351282931D 01	0.9549471120672642D 02	0.4559070865918876D 01
0.8950000000000010D 02	0.1562069680534926D 01	0.1145930134801467D 03	0.4741386838025610D 01
0.8960000000000011D 02	0.1563815009786921D 01	0.1432406123421915D 03	0.4964525820059600D 01
0.8970000000000013D 02	0.1565560339038916D 01	0.1909868043777635D 03	0.5252204338632595D 01
0.8980000000000014D 02	0.1567305668290910D 01	0.2864794793428259D 03	0.5657666908259127D 01
0.8990000000000016D 02	0.1569050997542905D 01	0.5729580860200189D 03	0.6350812565732150D 01
0.8990000000000016D 02	0.1569050997542905D 01	0.5729580860200189D 03	0.6350812565732150D 01
0.8991000000000013D 02	0.1569225530468104D 01	0.6366200341681041D 03	0.6456172984927897D 01
0.8992000000000013D 02	0.1569400063393304D 01	0.7161974766254503D 03	0.6573955934276152D 01
0.8993000000000013D 02	0.1569574596318504D 01	0.8185113395248390D 03	0.6707487250746792D 01
0.8994000000000013D 02	0.1569749129243703D 01	0.9549298330868101D 03	0.6861637864573913D 01
0.8995000000000016D 02	0.1569923662168902D 01	0.1145915735709243D 04	0.7043959365521771D 01
0.8996000000000015D 02	0.1570098195094102D 01	0.1432394604188350D 04	0.7267102871144514D 01
0.8997000000000018D 02	0.1570272728019301D 01	0.1909859404379597D 04	0.7554784908058843D 01
0.8998000000000018D 02	0.1570447260944501D 01	0.2864789033854497D 04	0.7960249990784722D 01
0.8999000000000015D 02	0.1570621793869700D 01	0.5729577980485468D 04	0.8653397156121290D 01
0.8999000000000013D 02	0.1570621793869700D 01	0.5729577980485468D 04	0.8653397156121290D 01
0.8999100000000018D 02	0.1570639247162222D 01	0.6366197749962472D 04	0.8758757670815826D 01
0.8999200000000018D 02	0.1570656700454774D 01	0.7161972462556225D 04	0.8876540705613290D 01
0.8999300000000013D 02	0.1570674153747260D 01	0.8185111379553986D 04	0.9010072097477339D 01
0.8999400000000018D 02	0.1570691607039780D 01	0.9549296603222509D 04	0.9164222776649352D 01
0.8999500000000018D 02	0.1570709060332300D 01	0.1145915591749800D 05	0.9346544332887506D 01
0.8999600000000015D 02	0.1570726513624820D 01	0.1432394489045633D 05	0.9569687883753782D 01
0.8999700000000016D 02	0.1570743966917340D 01	0.1909859318071036D 05	0.9857369955861834D 01
0.8999800000000015D 02	0.1570761420209859D 01	0.2864788976431812D 05	0.1026283506373447D 02
0.8999900000000016D 02	0.1570778873502379D 01	0.5729577952420336D 05	0.1095598224421704D 02
0.8999900000000016D 02	0.1570778873502379D 01	0.5729577952420336D 05	0.1095598224421704D 02
0.8999910000000016D 02	0.1570780618831631D 01	0.6366197724956092D 05	0.1106134275988188D 02
0.8999920000000016D 02	0.1570782364160883D 01	0.7161972440573423D 05	0.1117912579553796D 02
0.8999930000000016D 02	0.1570784109490135D 01	0.8185111360797472D 05	0.1131265718817984D 02
0.8999940000000016D 02	0.1570785854819387D 01	0.9549296587852081D 05	0.1146680786803381D 02
0.8999950000000016D 02	0.1570787600148639D 01	0.1145915590635084D 06	0.1164912942490878D 02
0.8999960000000016D 02	0.1570789345477891D 01	0.1432394488330247D 06	0.1187227297624839D 02
0.8999970000000016D 02	0.1570791090807143D 01	0.1909859317989514D 06	0.1215995504881320D 02
0.8999980000000016D 02	0.1570792836136395D 01	0.2864788977644916D 06	0.1256542015715197D 02
0.8999990000000016D 02	0.1570794581465647D 01	0.5729577959238604D 06	0.1325856733840983D 02

X = IN DEGREES	X = IN RAD.	SEC(XRAD) =	LOGSEC =
0.8999990000000016D 02	0.1570794581465647D 01	0.5729577959288604D 06	0.1325856733840983D 02
0.8999991000000016D 02	0.1570794755998572D 01	0.6366197734097216D 06	0.1336392785431181D 02
0.8999992000000016D 02	0.1570794930531497D 01	0.7161972451256320D 06	0.1348171089002362D 02
0.8999993000000016D 02	0.1570795105064423D 01	0.8185111375784659D 06	0.1361524228300492D 02
0.8999994000000016D 02	0.1570795279597348D 01	0.9549296606444568D 06	0.1376939296297486D 02
0.8999995000000016D 02	0.1570795454130273D 01	0.1145915593231284D 07	0.1395171452016844D 02
0.8999996000000016D 02	0.1570795628663198D 01	0.1432394492755602D 07	0.1417485807233192D 02
0.8999997000000016D 02	0.1570795803196124D 01	0.1909859325741428D 07	0.1446254014586613D 02
0.8999998000000016D 02	0.1570795977729049D 01	0.2864788994814151D 07	0.1486800525613921D 02
0.8999999000000016D 02	0.1570796152261974D 01	0.5729578032565705D 07	0.1556115244419314D 02
0.8999999000000016D 02	0.1570796152261974D 01	0.5729578032565705D 07	0.1556115244419314D 02
0.8999999100000014D 02	0.1570796169715266D 01	0.6366197818909417D 07	0.1566651296062812D 02
0.8999999200000014D 02	0.1570796187168559D 01	0.7161972553230552D 07	0.1578429599725595D 02
0.8999999300000014D 02	0.1570796204621851D 01	0.8185111511313221D 07	0.1591782739255690D 02
0.8999999400000014D 02	0.1570796222075144D 01	0.9549296781373350D 07	0.1607197807428740D 02
0.8999999500000014D 02	0.1570796239528436D 01	0.1145915616818134D 08	0.1625429963374589D 02
0.8999999600000014D 02	0.1570796256981729D 01	0.1432394533903914D 08	0.1647744319405290D 02
0.8999999700000014D 02	0.1570796274435021D 01	0.1909859394441326D 08	0.1676512527483136D 02
0.8999999800000013D 02	0.1570796291888314D 01	0.2864789139370313D 08	0.1717059039959287D 02
0.8999999900000013D 02	0.1570796309341606D 01	0.5729578679490589D 08	0.1786373765009687D 02
0.8999999900000013D 02	0.1570796309341606D 01	0.5729578679490589D 08	0.1786373765009687D 02
0.8999999910000013D 02	0.1570796311086935D 01	0.6366198539835223D 08	0.1796909816686492D 02
0.8999999920000013D 02	0.1570796312832265D 01	0.7161973456706986D 08	0.1808688121639909D 02
0.8999999930000013D 02	0.1570796314577594D 01	0.8185112667996763D 08	0.1822041262686649D 02
0.8999999940000013D 02	0.1570796316322923D 01	0.9549298339845446D 08	0.1837456333048424D 02
0.8999999950000011D 02	0.1570796318068252D 01	0.1145915836948125D 09	0.1855688492058490D 02
0.8999999960000011D 02	0.1570796319813582D 01	0.1432394870247671D 09	0.1878002852185915D 02
0.8999999970000011D 02	0.1570796321558911D 01	0.1909859858802459D 09	0.1906771061096428D 02
0.8999999980000011D 02	0.1570796323304240D 01	0.2864790169970519D 09	0.1947317585229926D 02
0.8999999990000011D 02	0.1570796325049569D 01	0.5729583202244066D 09	0.2016632353245990D 02
0.8999999990000011D 02	0.1570796325049569D 01	0.5729583202244066D 09	0.2016632353245990D 02
0.8999999991000011D 02	0.1570796325224102D 01	0.6366204264840266D 09	0.2027168415914013D 02
0.8999999992000011D 02	0.1570796325398635D 01	0.7161979897337838D 09	0.2038946720867432D 02
0.8999999993000010D 02	0.1570796325573168D 01	0.8185122365457669D 09	0.2052299890462800D 02
0.8999999994000010D 02	0.1570796325747700D 01	0.9549308517635014D 09	0.2067714948929316D 02
0.8999999995000010D 02	0.1570796325922234D 01	0.1145917556450973D 10	0.2085947151238148D 02
0.8999999996000010D 02	0.1570796326096767D 01	0.1432397599628539D 10	0.2108261551333688D 02
0.8999999997000010D 02	0.1570796326271299D 01	0.1909863611866488D 10	0.2137029766905557D 02
0.8999999998000010D 02	0.1570796326445832D 01	0.2864798757401060D 10	0.2177576394290121D 02
0.8999999999000003D 02	0.1570796326620365D 01	0.5729617552503523D 10	0.2246891462068174D 02

X = IN DEGREES	X = IN RAD.	SEC(XRAD) =	LOGSEC =
0.89999999999000003D 02	0.1570796326670365D 01	0.5729617552503523D 10	0.2246891462068174D 02
0.89999999999100003D 02	0.15707963266537818D 01	0.6366246672605656D 10	0.2257427591350136D 02
0.89999999999200003D 02	0.1570796326655271D 01	0.7162025518804671D 10	0.2269205867159989D 02
0.89999999999300003D 02	0.1570796326672725D 01	0.8185169100531234D 10	0.2282558970736443D 02
0.89999999999400006D 02	0.1570796326690178D 01	0.9549375309917413D 10	0.2297974157677546D 02
0.89999999999500006D 02	0.1570796326707631D 01	0.1145920945557541D 11	0.2316206479886906D 02
0.89999999999600006D 02	0.1570796326725084D 01	0.1432408681946927D 11	0.2338520835018328D 02
0.89999999999700006D 02	0.1570796326742538D 01	0.1909890965151518D 11	0.2367289708406140D 02
0.89999999999800006D 02	0.1570796326759991D 01	0.2864860302876600D 11	0.2407837051901490D 02
0.89999999999900005D 02	0.1570796326777444D 01	0.5729306486293077D 11	0.2477153268807118D 02
0.89999999999900005D 02	0.1570796326777444D 01	0.5729806486293077D 11	0.2477153268807118D 02
0.89999999999910005D 02	0.1570796326779189D 01	0.6366409240940457D 11	0.2487688654214734D 02
0.89999999999920005D 02	0.1570796326780935D 01	0.7162329676770745D 11	0.2499468623184108D 02
0.89999999999930005D 02	0.1570796326782680D 01	0.8185461206832910D 11	0.2512821048698706D 02
0.89999999999940005D 02	0.1570796326784425D 01	0.9549319486087860D 11	0.2528238495860758D 02
0.89999999999950005D 02	0.1570796326786170D 01	0.1145961297258615D 12	0.2546467986863112D 02
0.89999999999960005D 02	0.1570796326787916D 01	0.1432465935354149D 12	0.2568783341240102D 02
0.89999999999970005D 02	0.1570796326789661D 01	0.1910050011399573D 12	0.2597556544862908D 02
0.89999999999980005D 02	0.1570796326791406D 01	0.2864931870708298D 12	0.2638098059296097D 02
0.89999999999990003D 02	0.1570796326793152D 01	0.5730722705599300D 12	0.2707427767233949D 02
0.89999999999990003D 02	0.1570796326793152D 01	0.5730722705599300D 12	0.2707427767233949D 02
0.89999999999991003D 02	0.1570796326793326D 01	0.6367752514406063D 12	0.2717968260677759D 02
0.89999999999992003D 02	0.1570796326793501D 01	0.7162329676770746D 12	0.2729727132483512D 02
0.89999999999993003D 02	0.1570796326793675D 01	0.8186980495749085D 12	0.2743098117098190D 02
0.89999999999994003D 02	0.1570796326793850D 01	0.9550568186580046D 12	0.2758503667163429D 02
0.89999999999995003D 02	0.1570796326794024D 01	0.1146373678373182D 13	0.2776762475295008D 02
0.89999999999996002D 02	0.1570796326794199D 01	0.1432465935354149D 13	0.2799041850539507D 02
0.89999999999997002D 02	0.1570796326794373D 01	0.1910113637316009D 13	0.2827818385219424D 02
0.89999999999998002D 02	0.1570796326794548D 01	0.2864216353258072D 13	0.2868331590451248D 02
0.89999999999999002D 02	0.1570796326794722D 01	0.5734161139222659D 13	0.2937746258540551D 02
0.89999999999999002D 02	0.1570796326794722D 01	0.5734161139222659D 13	0.2937746258540551D 02
0.89999999999999101D 02	0.1570796326794740D 01	0.6364218800469100D 13	0.2948171260677930D 02
0.89999999999999202D 02	0.1570796326794757D 01	0.7149826856886110D 13	0.2959810925652113D 02
0.89999999999999301D 02	0.1570796326794774D 01	0.8168320711143389D 13	0.2973128446036183D 02
0.89999999999999400D 02	0.1570796326794792D 01	0.9541033509521896D 13	0.2988662292985244D 02
0.89999999999999501D 02	0.1570796326794809D 01	0.1144543141561409D 14	0.3006861176330278D 02
0.89999999999999600D 02	0.1570796326794827D 01	0.1426408243587726D 14	0.3028876577576862D 02
0.89999999999999701D 02	0.1570796326794844D 01	0.1905036923329787D 14	0.3057810759963877D 02
0.89999999999999800D 02	0.1570796326794862D 01	0.2838693633278544D 14	0.3097695016698644D 02
0.89999999999999899D 02	0.1570796326794879D 01	0.5567146737109378D 14	0.3165048887615801D 02

X = IN DEGREES	X = IN RAD.	SEC(XRAD) =	LOGSEC =
0.8999999999999895D 02	0.1570796326794879D 01	0.5567146737109374D 14	0.3165048887615801D 02
0.8999999999999910D 02	0.1570796326794881D 01	0.6301275977167758D 14	0.3177435835787080D 02
0.8999999999999920D 02	0.1570796326794882D 01	0.6992879438076412D 14	0.3187849861712339D 02
0.8999999999999930D 02	0.1570796326794884D 01	0.7964112693364804D 14	0.3200855174537159D 02
0.8999999999999940D 02	0.1570796326794886D 01	0.9101343078131205D 14	0.3214208313799611D 02
0.8999999999999950D 02	0.1570796326794888D 01	0.1102723296004358D 15	0.3233397414580622D 02
0.8999999999999960D 02	0.1570796326794889D 01	0.1333525846330851D 15	0.3252401774869408D 02
0.8999999999999970D 02	0.1570796326794891D 01	0.1849729399749245D 15	0.3285123065990250D 02
0.8999999999999979D 02	0.1570796326794893D 01	0.2606436881464846D 15	0.3319417541102933D 02
0.8999999999999989D 02	0.1570796326794895D 01	0.4778467616018882D 15	0.3380031121459964D 02
0.8999999999999989D 02	0.1570796326794895D 01	0.4778467616018882D 15	0.3380031121459964D 02
0.8999999999999990D 02	0.1570796326794895D 01	0.4778467616018882D 15	0.3380031121459964D 02
0.8999999999999990D 02	0.1570796326794895D 01	0.5734161139222659D 15	0.3398263277139360D 02
0.8999999999999992D 02	0.1570796326794895D 01	0.5734161139222659D 15	0.3398263277139360D 02
0.8999999999999992D 02	0.1570796326794895D 01	0.5734161139222659D 15	0.3398263277139360D 02
0.8999999999999994D 02	0.1570796326794895D 01	0.7167701424028324D 15	0.3420577632270781D 02
0.8999999999999994D 02	0.1570796326794895D 01	0.7167701424028324D 15	0.3420577632270781D 02
0.8999999999999995D 02	0.1570796326794895D 01	0.8191658770318085D 15	0.3433930771533233D 02
0.8999999999999995D 02	0.1570796326794895D 01	0.8191658770318085D 15	0.3433930771533233D 02
0.8999999999999995D 02	0.1570796326794895D 01	0.8191658770318085D 15	0.3433930771533233D 02
0.8999999999999995D 02	0.1570796326794895D 01	0.8191658770318085D 15	0.3433930771533233D 02
0.8999999999999995D 02	0.1570796326794895D 01	0.8191658770318085D 15	0.3433930771533233D 02
0.8999999999999995D 02	0.1570796326794895D 01	0.8191658770318085D 15	0.3433930771533233D 02
0.8999999999999995D 02	0.1570796326794895D 01	0.8191658770318085D 15	0.3433930771533233D 02
0.8999999999999995D 02	0.1570796326794895D 01	0.8191658770318085D 15	0.3433930771533233D 02
0.8999999999999995D 02	0.1570796326794895D 01	0.8191658770318085D 15	0.3433930771533233D 02
0.8999999999999995D 02	0.1570796326794895D 01	0.8191658770318085D 15	0.3433930771533233D 02
0.8999999999999995D 02	0.1570796326794895D 01	0.8191658770318085D 15	0.3433930771533233D 02
0.8999999999999995D 02	0.1570796326794895D 01	0.8191658770318085D 15	0.3433930771533233D 02
0.8999999999999995D 02	0.1570796326794895D 01	0.8191658770318085D 15	0.3433930771533233D 02
0.8999999999999995D 02	0.1570796326794895D 01	0.8191658770318085D 15	0.3433930771533233D 02
0.8999999999999995D 02	0.1570796326794895D 01	0.8191658770318085D 15	0.3433930771533233D 02
0.8999999999999995D 02	0.1570796326794895D 01	0.8191658770318085D 15	0.3433930771533233D 02
0.8999999999999995D 02	0.1570796326794895D 01	0.8191658770318085D 15	0.3433930771533233D 02
0.8999999999999995D 02	0.1570796326794895D 01	0.8191658770318085D 15	0.3433930771533233D 02
0.8999999999999995D 02	0.1570796326794895D 01	0.8191658770318085D 15	0.3433930771533233D 02
0.8999999999999995D 02	0.1570796326794895D 01	0.8191658770318085D 15	0.3433930771533233D 02
0.8999999999999995D 02	0.1570796326794895D 01	0.8191658770318085D 15	0.3433930771533233D 02
0.8999999999999995D 02	0.1570796326794895D 01	0.8191658770318085D 15	0.3433930771533233D 02
0.8999999999999995D 02	0.1570796326794895D 01	0.8191658770318085D 15	0.3433930771533233D 02

APPENDIX VI

COMPUTER PROGRAM FOR A TAPERED CABLE

Current loading on cable systems presents one of the more difficult problems to the analyst. This is due to both the non-linearity of hydrodynamic loads and to the configuration dependence of such loads. These difficulties have lead to the frequent use of finite-element models with a digital computer to handle cable/current problems. Although these methods are approximate, the solutions tend to be very good and in general can be made increasingly accurate by decreasing the size and increasing the number of elements within cost constraints.

To enable a simulation of the tapered cable concept, a finite-element computer program has been prepared. This is largely based on the program discussed by Dominguez (3), using the same hydrodynamic sub-program for calculating drag forces but having an additional sub-program to generate cable diameter for various types of tapered cables. The method, accuracy to be obtained, and the hydrodynamic loading criteria have been fully described in reference (3). The program has provision for accepting any type of taper and presently contains four types -- common cable, constant-stress catenary, E-tapered and straight-taper "D".

Results

The program was first used (Fig. 27) to compare its results to

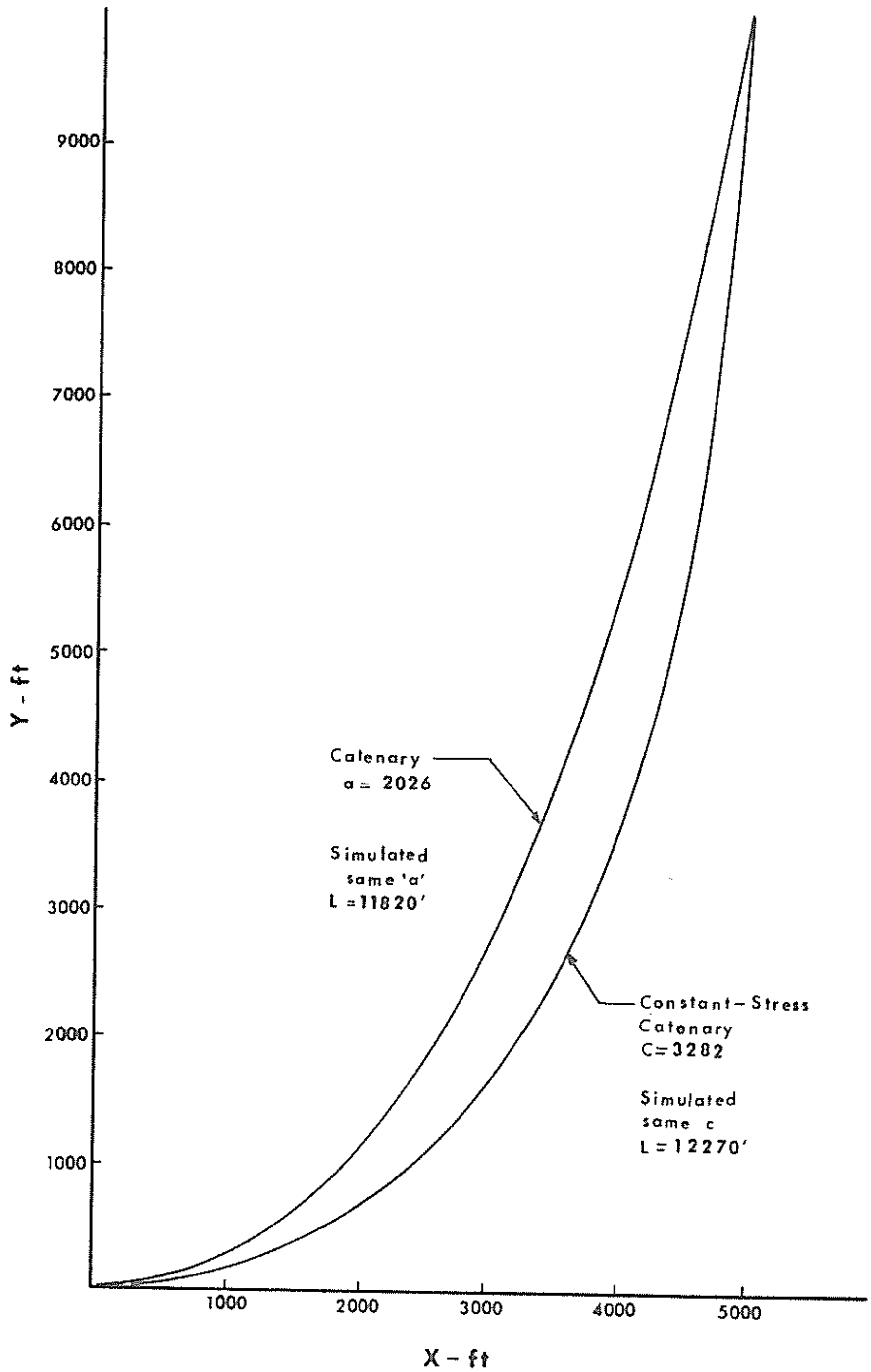

Fig. 27 Analytical and Computer Simulation

the analytical expression for static cable configurations previously derived. It was then used to simulate the effects of current on the same cable configuration.

Fig. 27 illustrates two curves fitted through (0.0) and (5,000, 10,000). In this case, the cables were selected and were also simulated, using a finite element model, to confirm the equations and trends previously defined. Table II lists the coordinates along the constant-stress catenary and shows the very good fit obtained. Fig. 28 depicts the stress obtained from the same finite-element simulations. The common cable has less stress at the bottom, as excess material is present as illustrated in Fig. 6.

The tapered cable, on the other hand, has a nearly constant stress throughout its length as anticipated. The computer simulation therefore, presents a very satisfactory cable model, confirming the analytical results presented elsewhere in this report.

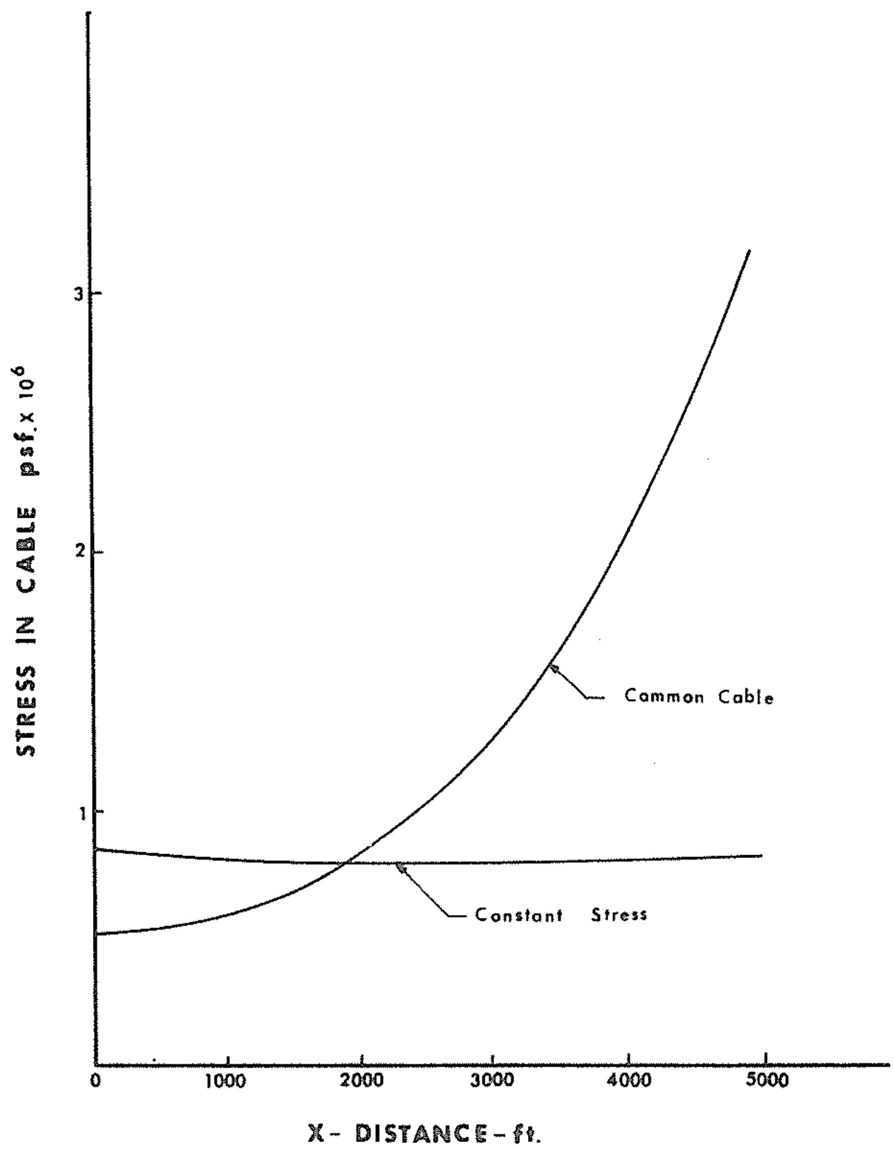

Fig. 28 Cable Tensile Stress

Table 2. - Comparison - Analytical to Finite-Element Simulation

Calculated		Finite Element	
X =	Y =	X =	Y =
0.00	0.00	0.00	0.00
500.00	38.21	488.90	34.96
1000.00	154.67	967.15	143.76
1500.00	355.19	1535.33	373.22
2000.00	650.86	1957.40	623.08
2500.00	1060.78	2526.05	1089.24
3000.00	1618.09	3012.88	1640.62
3500.00	2384.27	3481.24	2359.73
4000.00	3490.12	3988.75	3475.53
4500.00	5298.95	4507.02	5366.95
5000.00	9955.38	4994.43	9999.77