U.S. DEPARTMENT OF COMMERCE
NATIONAL OCEANIC AND ATMOSPHERIC ADMINISTRATION
NATIONAL WEATHER SERVICE
NATIONAL METEOROLOGICAL CENTER

OFFICE NOTE 138

On the Parameterization of Surface Frictional Drag and the Existence of an Inertial Oscillation

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1. The parameterization of surface frictional drag in NMC models is based on the formula

$$\vec{\tau}_0 = - \rho C |\vec{V}_1| \vec{V}_1 \qquad (1)$$

where C is a drag coefficient, ρ is the air density, and $\vec{V}_1$ is the wind vector at the first level of the numerical model.

The acceleration due to the surface frictional drag is applied to the equation for $\vec{V}_1$ as follows,

$$\frac{\partial V_1}{\partial t} = - \left(\frac{g}{\Delta p}\right) \rho C |\vec{V}_1| \vec{V}_1 \qquad (2)$$

in which g is the gravity acceleration and Δp is the difference (positive) of static pressure across the first layer of the model.

2. The formulation given above has been criticized because it uses $\vec{V}_1$ in the multiple role of estimating the drag, determining its direction, and reflecting the impact of the frictional acceleration. A second criticism involves the fact that C, the drag coefficient, has not been modified from the values originally introduced by Cressman for use with the barotropic model.

We have recently experimented with a modified version of this parameterization in which an estimate of the surface geostrophic wind was made by downward extrapolation of the winds in upper layers of the model. These winds ought to be close to the gradient winds in the relevant layers. The extrapolation downward ought therefore to approximate the baroclinic structure of the atmosphere. These were our hypotheses.

The results obtained to date have focused our attention on the existence of an inertial oscillation of appreciable magnitude in the wind field of the several numerical models, 6L PE, LFM, and 9L GLOPEP.

3. The inertial oscillation manifests itself by a characteristic rotation of the horizontal wind vector with a frequency given by the Coriolis parameter. The existence of such oscillations is to be expected in PE models. Simple analysis suggests that the wave motion will also involve a gravitational component. For internal gravity inertia waves* the frequency will approximate the Coriolis parameter and dispersion will be minimal especially when account is taken of horizontal truncation error. Furthermore, the theory of geostrophic adjustment given by Obukhov indicates that a damped inertial oscillation is to be expected in the wave train left by dispersing external gravity waves.

In the figures shown later, we provide limited empirical evidence of the presence of such oscillations in the NMC models.

4. We now turn to the problem of parameterization of surface frictional effects in the presence of an inertial oscillation.

First we note that, if friction is modeled by use of the geostrophic wind estimated by extrapolation of the upper layer winds, the presence of an inertial oscillation in those winds leads to the excitation of a large oscillation in the lowest level wind. This forced motion is not

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*of high wave number

dissipative and in limited experiments has shown some tendency to grow linearly with time through 2 days of numerical prediction.

Second, we note that the use of a formula such as eq. (2) will tend to damp the inertial oscillation in the boundary layer (i.e., layer one), since it always acts against that wind component.

5. The Evidence

Figure 1(a,b,c) shows the wind hodograph for two points--one in each hemisphere at about the same latitude. The v component is defined as positive toward the respective poles. Eleven hours are shown. At both points, the inertial oscillation is most pronounced in the lowest layer. In the third layer, the oscillation is masked by a pronounced drift at the Northern Hemisphere gridpoint.

Figure 2 shows the u and v components of the horizontal wind taken from an LFM rerun. The gridpoint is located at 72°N latitude. Three levels of wind are presented. Layer 1 is the boundary layer. We notice the existence of an inertial oscillation at all three levels. The oscillation is damped slowly over the 36 hrs shown.

Figure 3 shows the same data as Figure 2, but the formulation of friction was changed to use an estimate of the surface geostrophic wind. based on extrapolation of Layer 2 and Layer 3 winds to the surface. The behavior of Level 2 and Level 3 winds from the experimental run is very much the same as in the operational run. We note, however, the excitation

of a large oscillation in the boundary layer wind (Layer 1).

We also printed out winds from the coarse mesh (381 km) 6L PE, but the inertial oscillation was masked by a drift in the wind over the 11 hours printed. These results are shown in Figure 4.

6. Conclusions

At this point in the investigation, there are many more questions than substantiated conclusions. Some of the following are more nearly speculations:

a. Frictional drag is needed in the models to damp the amplitude of the inertial oscillation.

b. The 9-layer model doesn't seem to have sufficiently large surface friction to accomplish the needed damping.

c. The assimilation of wind data in middle and high latitudes will be adversely affected by the presence of the inertial oscillation. If the period of the oscillation is 12 hrs approximately, only multiples of 6 hrs will find the oscillation at a nodal point.

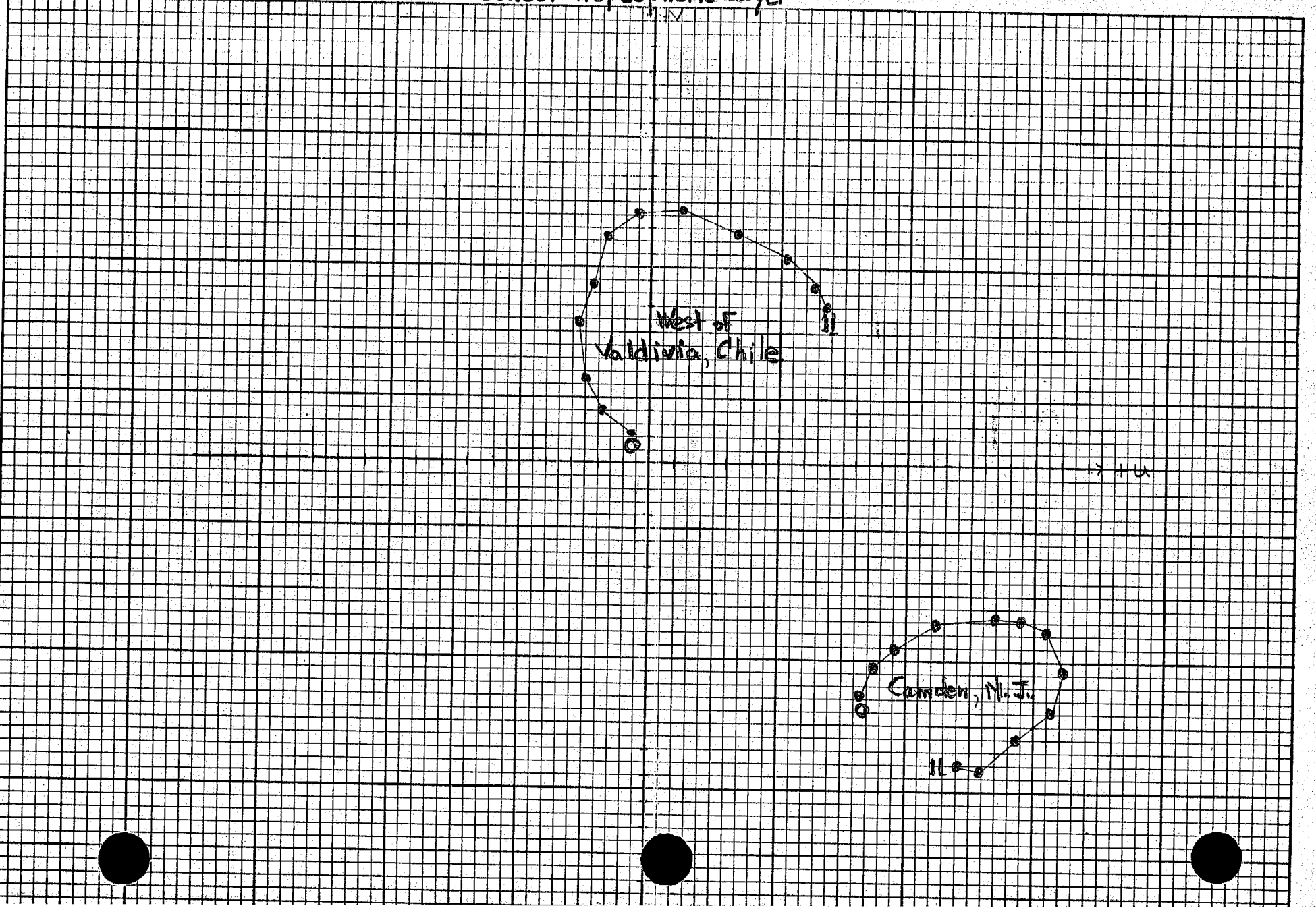
d. It seems impossible to obtain good geostrophic wind estimates from either the wind or mass field produced by current NMC primitive equation models. The only possibility still open is to calculate the nondivergent component of the wind for this purpose, but this would be an expensive calculation. In a

primitive equation model based on the divergence and vorticity equations (e.g., Sela's spectral model) the calculation would be inexpensive.

2.5° Global Model

Fig 1a

Lowest Tropospheric Layer



2.5° Global Model

DRAWING PAPER NO. 1280-10-5
TRACING PAPER NO. 1227-10-6
CROSS SECTION-10X10 TO 1 INCH
5TH LINE ACCT'D, 10TH HEAVY

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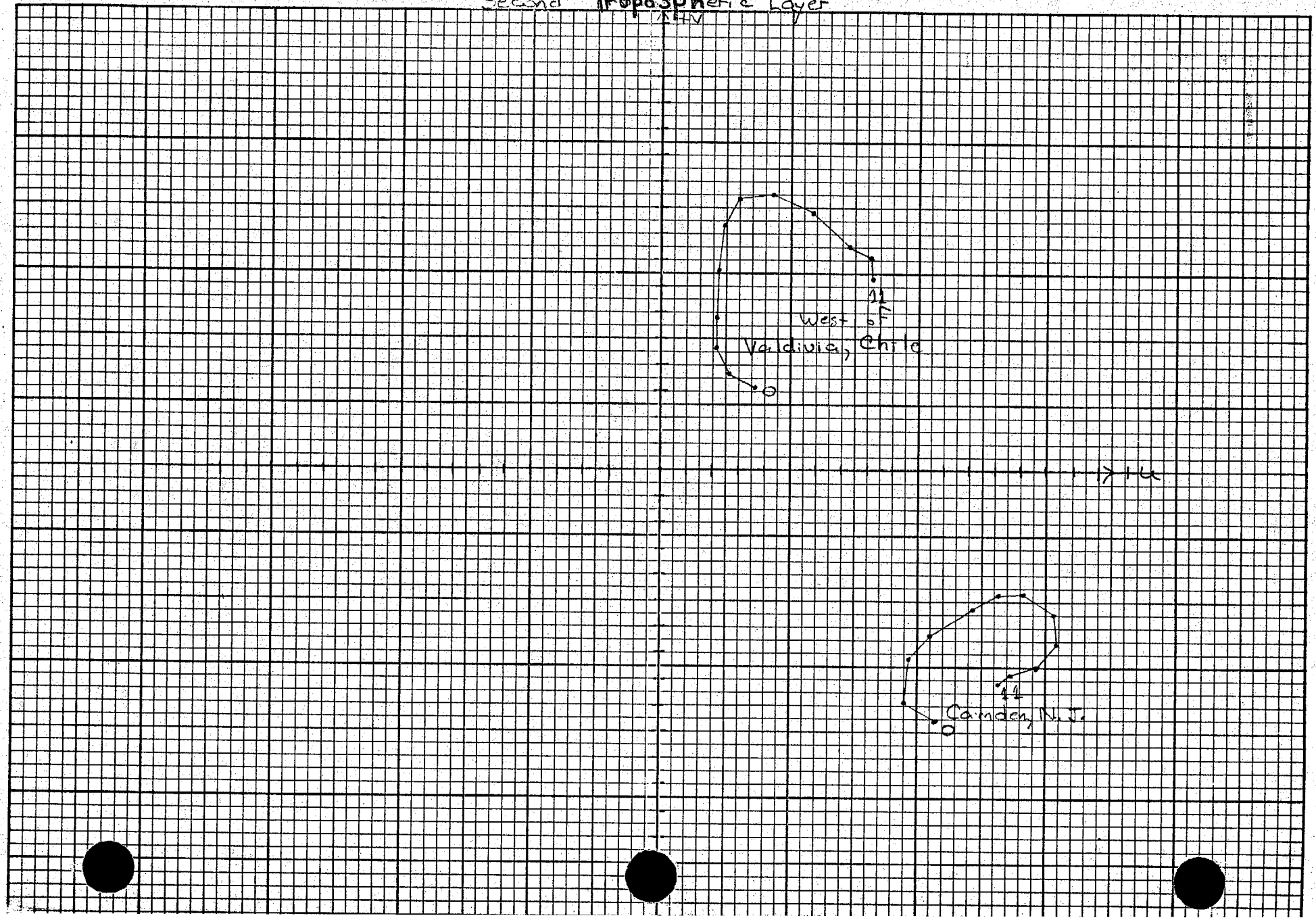
Fig 1b

Second Tropospheric Layer



2.5° Global Model

Fig 1c

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5TH LINE ACCT'D, 10TH HEAVY

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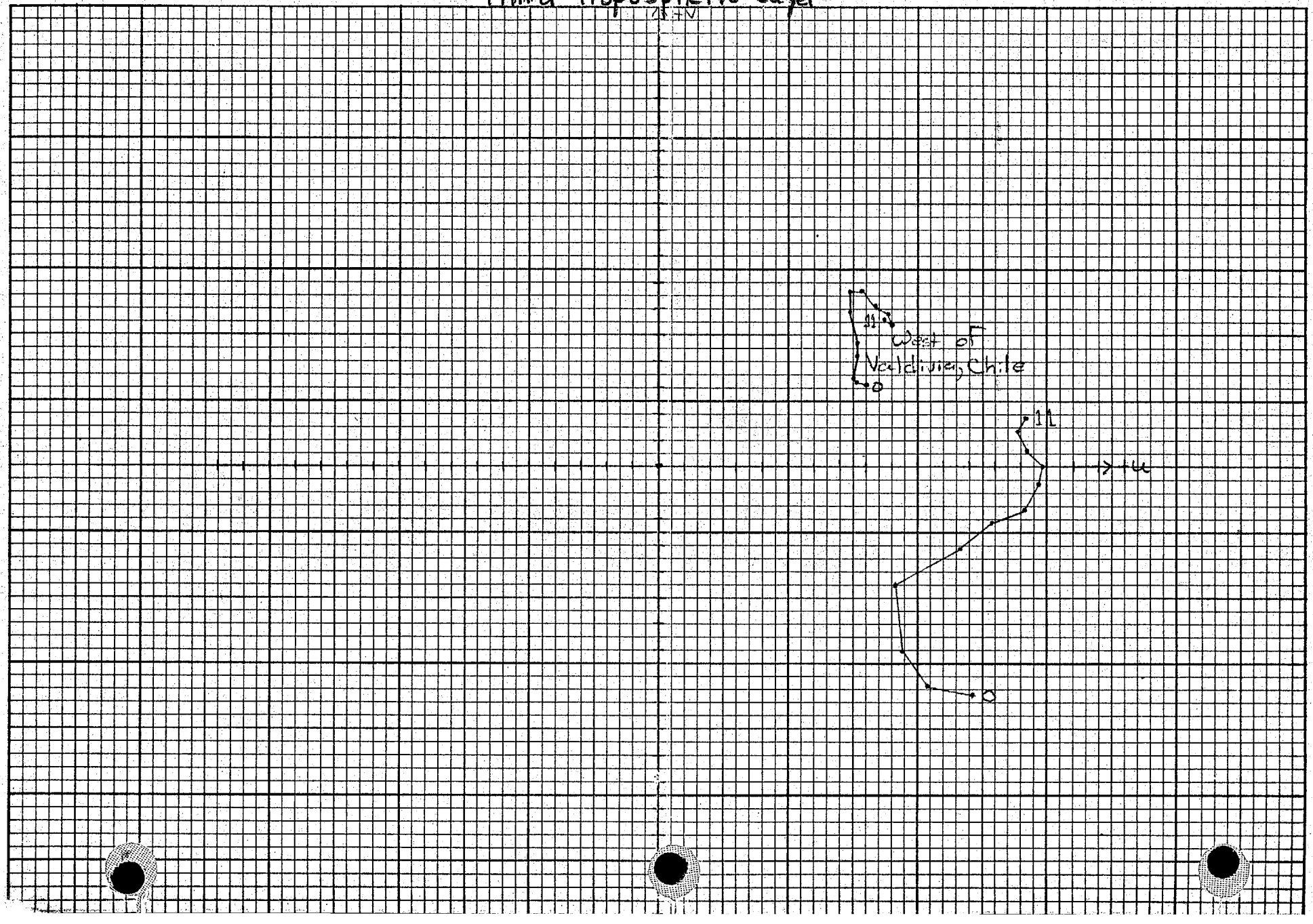
Third Tropospheric Layer



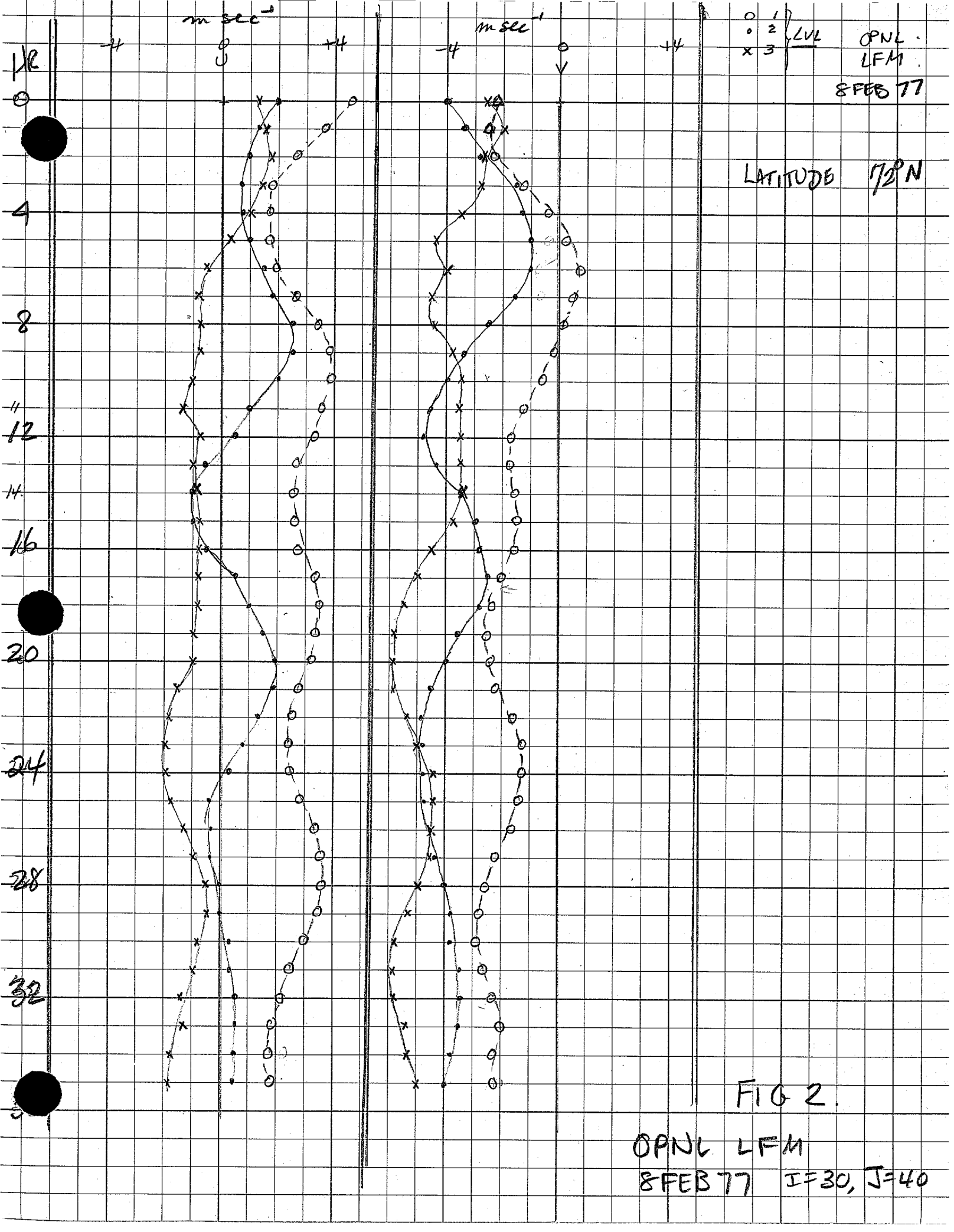


FIG 2.

OPNL LFM
8FEB 77 I=30, J=40

FIG (3)

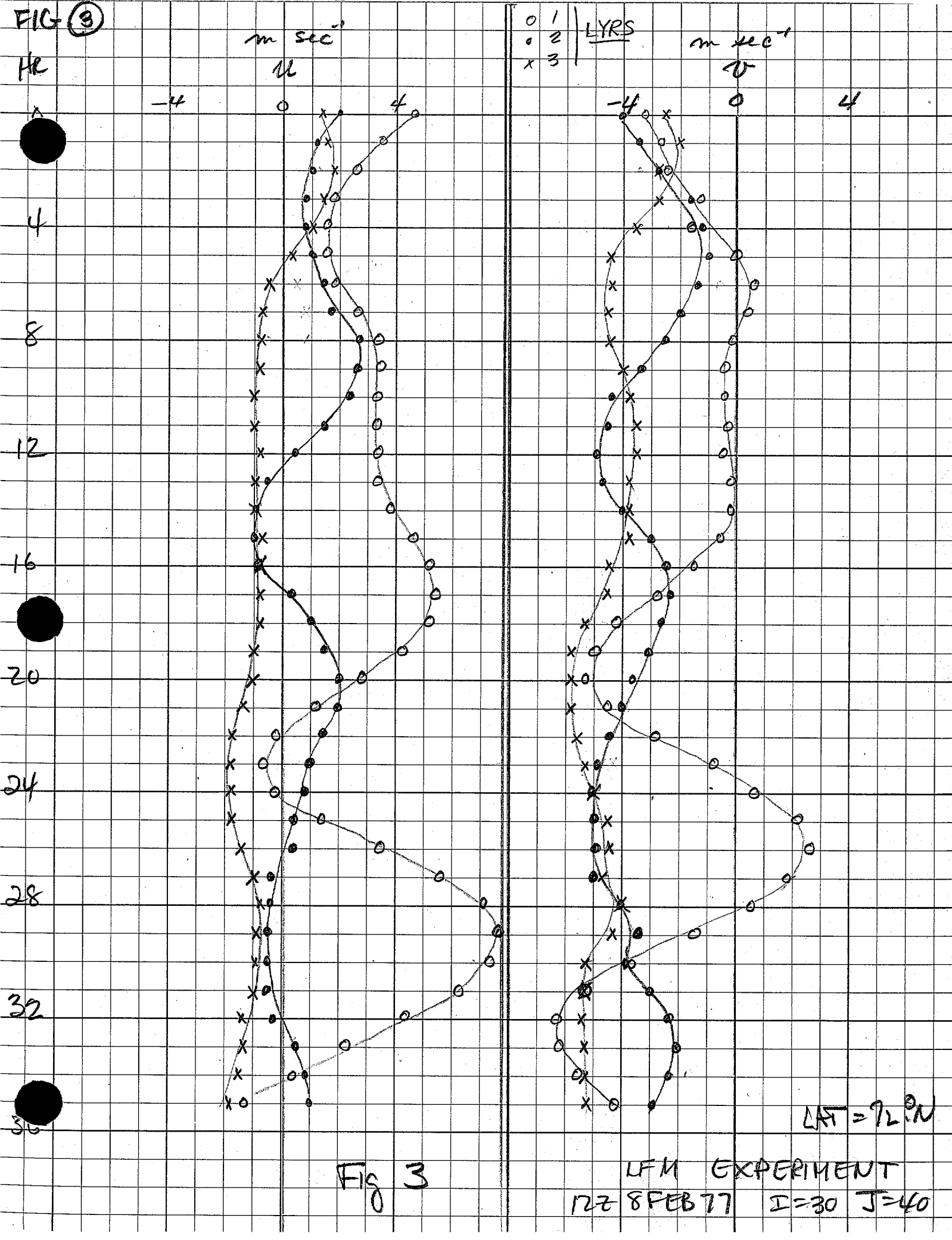


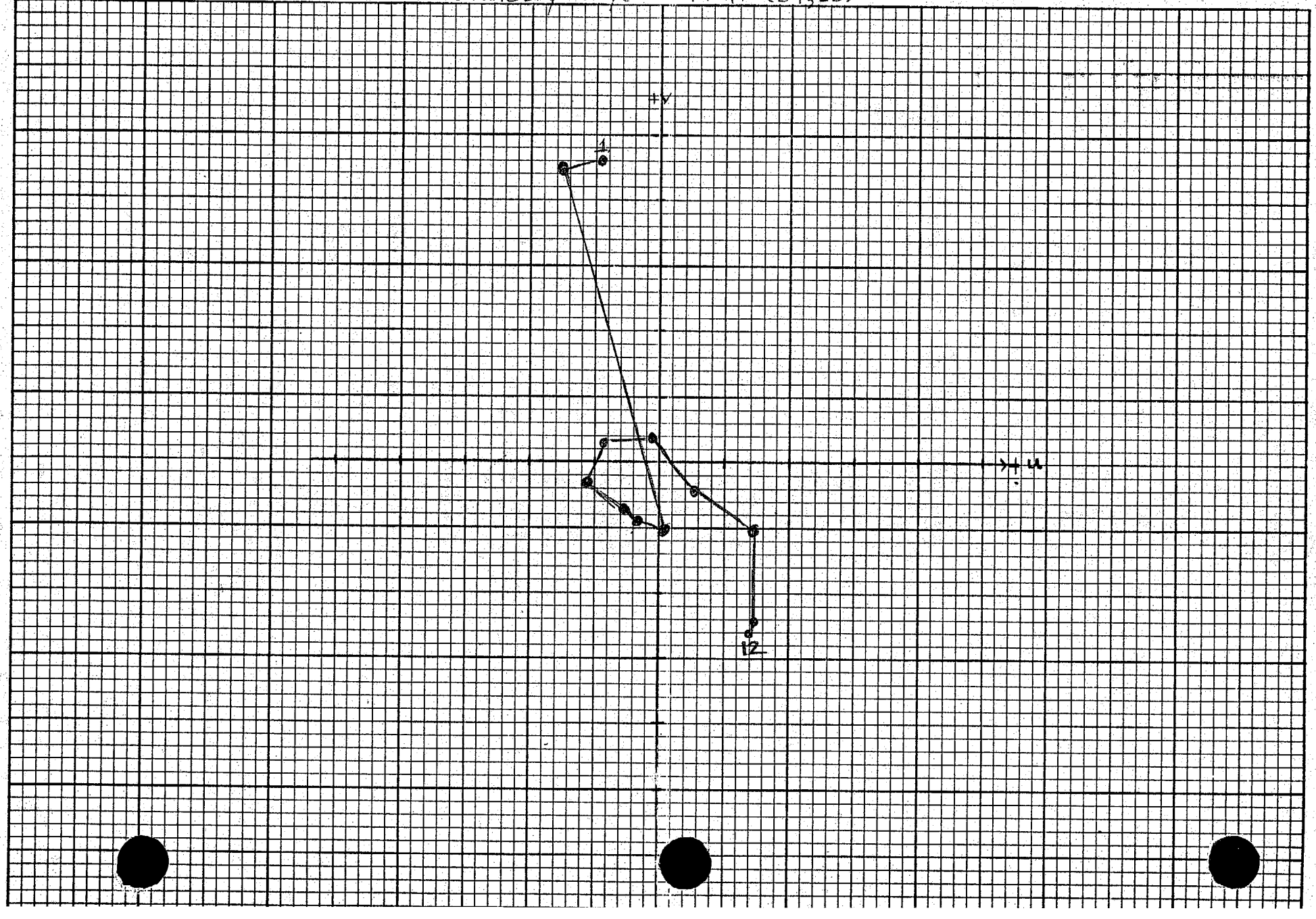
Fig 3

LFM EXPERIMENT
12Z 8FEB77 I=30 J=40

Fig 4a

6-Layer Coarse Mesh Model

Boundary Layer - Point (34,18)



6-LAYER COARSE MESH MODEL

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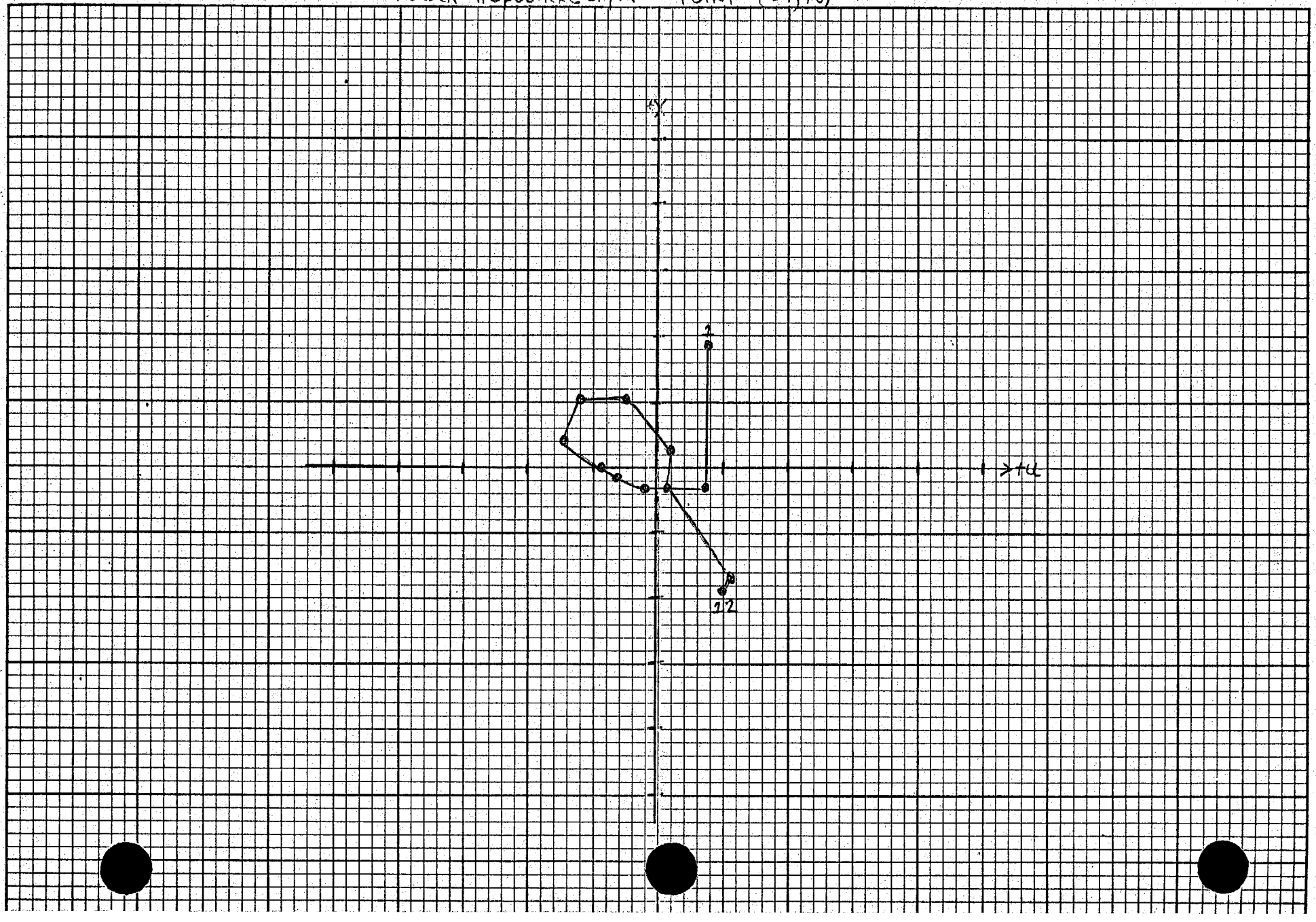
Fig 4b

LOWER TROPOSPHERE LAYER - POINT (34,18)



6-Layer Coarse Mesh Model

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CROSS SECTION-10X10 TO 1 INCH
5TH LINE ACCT'D, 10TH HEAVY

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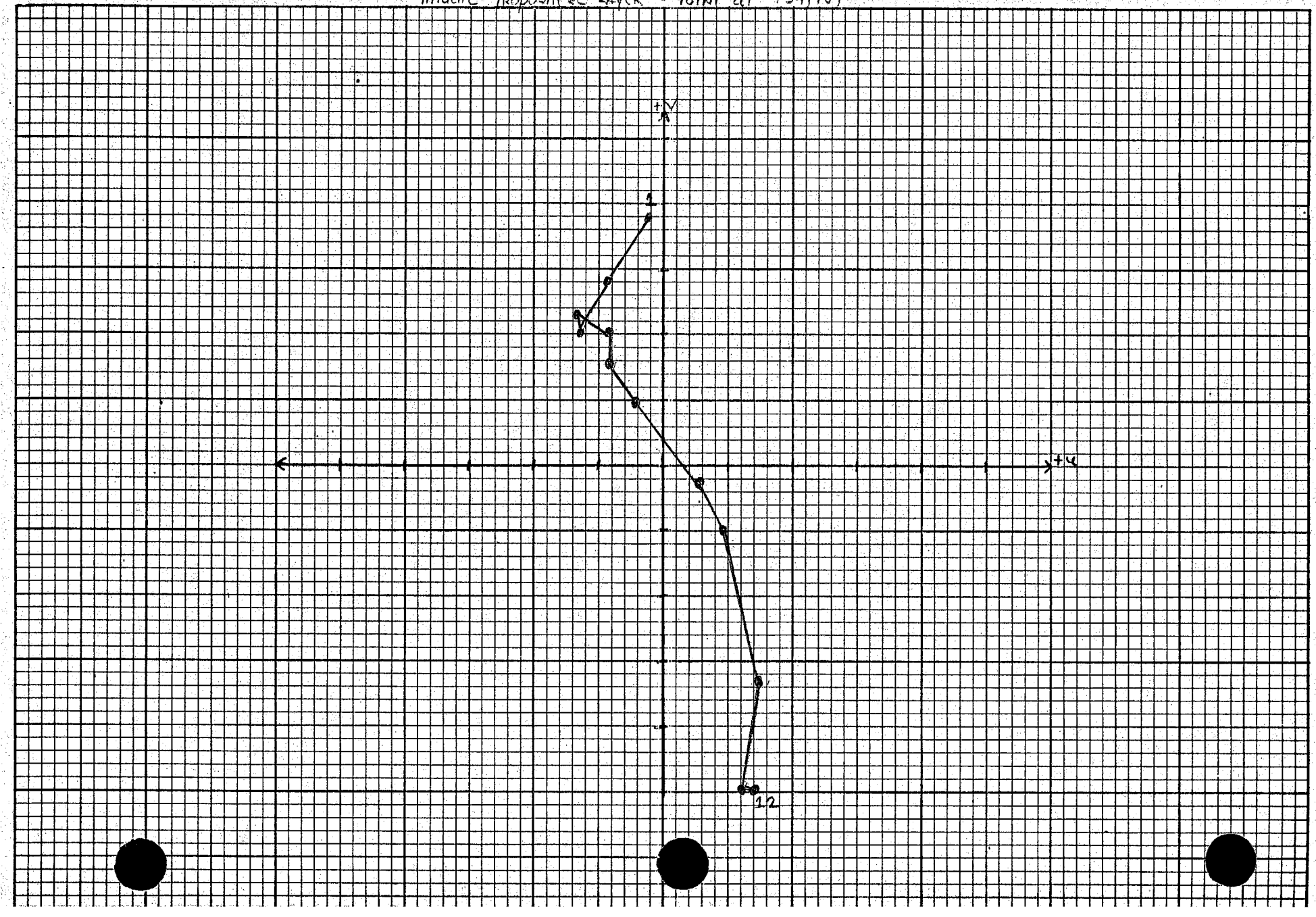
Fig 4C

Middle Troposhere Layer - Point at (34, 18)



6-Layer Coarse Mesh Model

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TRACING PAPER NO. 1227-10-5
CROSS SECTION-10X10 TO 1 INCH
5TH LINE ACCT'D, 10TH HEAVY

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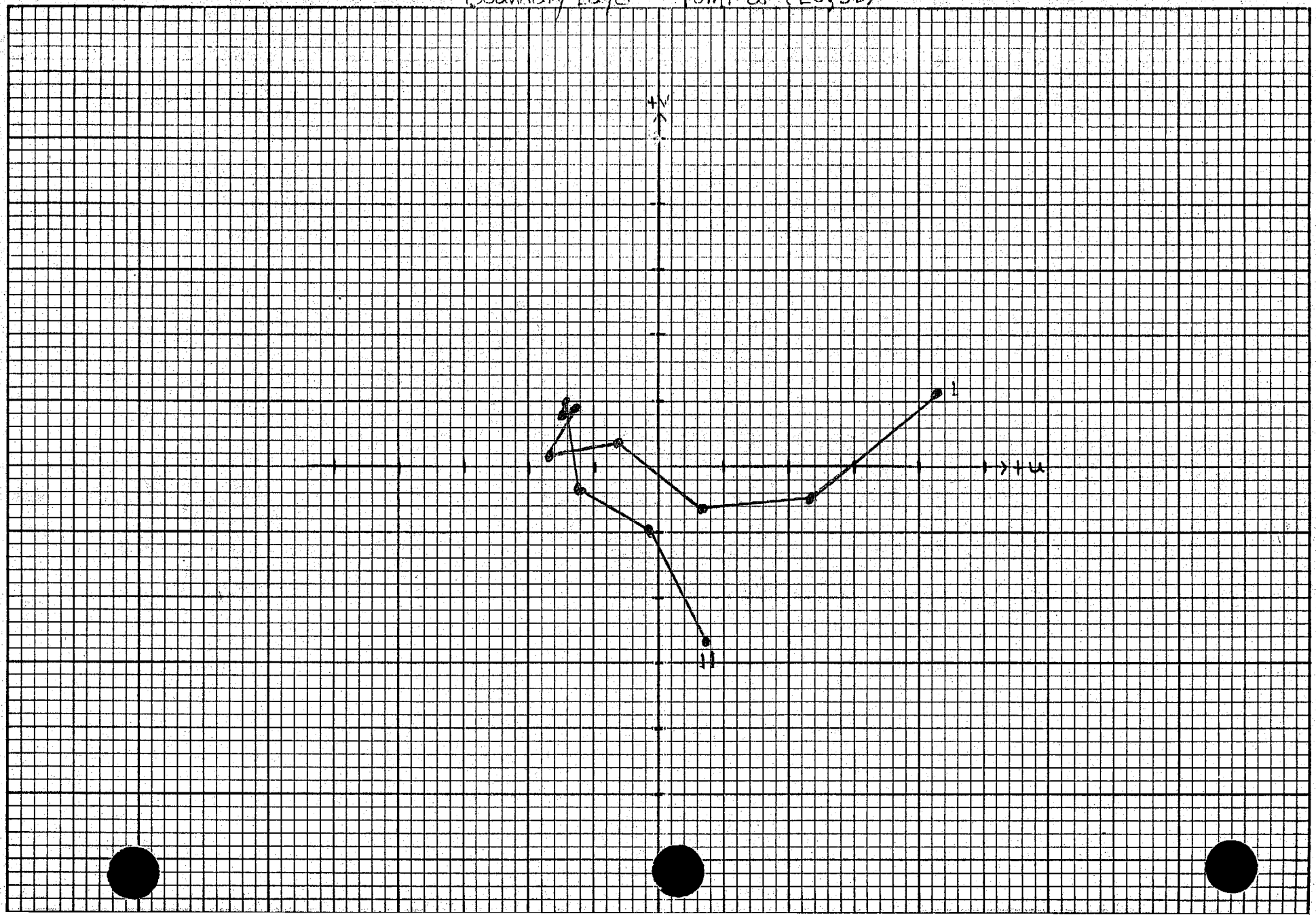
Fig 4d

Boundary Layer - Point at (20,33)



6-layer Coarse Mesh Model

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CROSS SECTION-10X10 TO 1 INCH
8TH LINE ACCENTED, 10TH HEAVY

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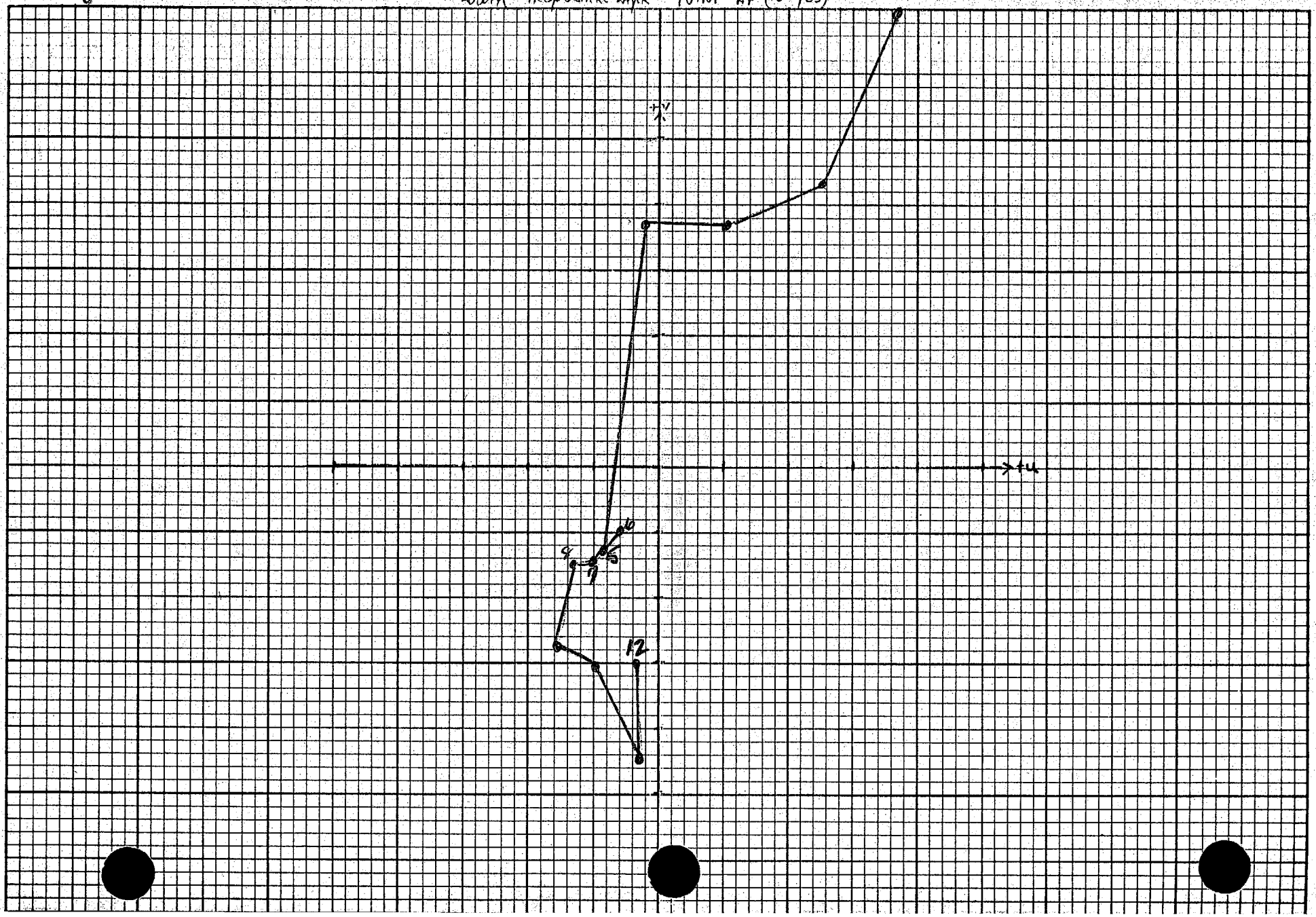
Fig 4e

Lower Troposhere Layer - Point At (80,33)



6-Layer Coarse Mesh Model

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CROSS SECTION-10X10 TO 1 INCH
5TH LINE ACCT'D, 10TH HEAVY

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Fig 4f

Middle Troposphere Layer - Point at (20, 33)