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Real-Data Assimilation Experiments with Implicit and Explicit Integration Methods

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REAL-DATA ASSIMILATION EXPERIMENTS WITH IMPLICIT AND EXPLICIT INTEGRATION METHODS

I. Introduction

In an earlier Office Note (86), experimental results were presented comparing the relative efficiencies of implicit and explicit integration methods in four-dimensional assimilation. Those experiments were simulations, in the sense that the asynoptic "observations" that were assimilated were generated by the model itself. The main conclusion was that the adjustment of the model to the inserted data proceeds more slowly in the implicit than in the explicit model, but not enough to offset the implicit method's economic advantage.

We have repeated those experiments using real VTPR data. This Note presents the results of the later experiments in terms of a further comparison of explicit versus implicit methods. Beyond this comparison, the experiments yield insight into four-dimensional assimilation generally, and it is primarily for this reason that we document the results in this Note.

II. The Models

Both models are identical to those used in the earlier experiments, and are described in Office Note 86. The grid mesh for each is a polar stereographic projection of the Northern Hemisphere with a mesh length of 762 km at 60N. Integration of the explicit model is by the Euler-backward method with a time step of 10 minutes. The implicit-backward method (Kurihura, 1965; McPherson, 1973) is used in the implicit model with a time step of 60 minutes. A time filter (Asselin, 1971) is included in the implicit model to inhibit separation of solutions. The actual time advantage of the implicit method over the Euler-backward is approximately 8:1.

Initial data were NMC operational analyses of 500-mb heights and winds at 12Z, 8 April 1973. Initialization consisted of integrating forward and backward over the 12-hour period centered on 12Z for the equivalent of 5.25 days. This was done individually for each model. The resulting initialized fields at 12Z of both models exhibited good balance between mass and motion fields. Differences between initialized fields for the two models was small; the root-mean-square height difference was approximately 10 m. The last 12 hours of the initialization (i.e., 06Z-18Z) were stored at hourly intervals for later comparisons; this will be referred to as the "reference" state.

III. Asynoptic Data

Values of 500-mb height as calculated from VTPR temperature soundings during the period 06Z-18Z, 8 April 1973, were taken as the observations to be inserted into the models. The data were stratified by time into 2-hour blocks centered on odd hours; for example, observations between 12Z and 14Z were assumed synoptic at 13Z. This procedure effectively groups the observations on an orbit-by-orbit basis. No observations south of 20N were used. We did not perform any checking of the accuracy of the observations, beyond an examination for obviously erroneous values. The number of observations per insertion is given in the table.

Table 1

Insertion time		Number of Observations
07Z		16
09Z		14
11Z		32
13Z		18
15Z		3
17Z		5
	TOTAL	88

IV. Insertion Procedures

The present experiments, like the earlier simulation experiments, were based on the "repeated insertion" technique of Morel, Rabreau, and Lefevre (1971). Both models were integrated forward and backward between 06Z and 18Z for the equivalent 10.5 days. Each set of observations was inserted in the models repeatedly during this process, according to the procedures described below.

Local Objective Analysis. At each insertion time, the available data were interpolated to the grid points nearest the locations of the observations by means of a successive-approximation objective analysis method (Cressman, 1959). Four scans through the data were made, with scan radii of 2.375, 1.8, 1.1, and 0.9 grid increments respectively.

The first guess for the objective analysis is the current forecast height field. After analysis, the resulting correction to the first guess is smoothed and desmoothed twice, using a 9-point filter devised by Shuman (1957), with coefficients of 1/2 and -1/3 for smoothing and desmoothing respectively. This filtering completely removes two-grid increment waves, strongly damps waves up to four grid increments long, while longer waves remain nearly untouched. The combination of this process with the successive approximation method insures smoothness of the inserted fields along the boundaries of the local objective analysis area.

Geostrophic Wind Adjustment. In Office Note 86, we observed that assimilation using only observations of the mass field led to an exceedingly, even excruciatingly, slow adjustment. But when wind observations were included, the adjustment was greatly accelerated. Unfortunately, sounding radiometer data such as VTPR do not yield simultaneous measurements of both the mass field and the motion field. Hayden (1972) suggested the use of a geostrophic correction to the winds based on the gradients of the observed mass fields as a means of hastening the adjustment. Essentially, his proposal adjusts the winds according to the relationship

$$\vec{V}\,(\text{adjustment}) = \vec{V}_{\text{ageo}}\,(\text{fcst}) + \frac{g}{f}\,\vec{k} \times \nabla\,\{H(\text{anl}) - H(\text{fcst})\} \qquad (1)$$

where $\vec{V}_{\text{ageo}}$ (fcst) is the forecast ageostrophic wind component. This amounts to a one-for-one replacement of the forecast geostrophic wind by the observed geostrophic wind. It seems more reasonable to blend the two by a weighted average reflecting the relative accuracies of the observed and forecast height gradients, but this is reserved for later research. Unless otherwise indicated, the geostrophic wind correction has been employed in the experiments. Following each objective analysis, a revised geostrophic wind based on the gradients of the analyzed heights was calculated and directly replaced the forecast geostrophic wind component.

Insertion. In the case of the explicit model, the analyzed heights and geostrophic wind components were inserted into the model at only one time level (the current, or τ, time). At each grid point within the influence area of the orbit, the current forecast values were replaced by the objectively analyzed values, immediately prior to the calculation of tendencies for the next time step. Since the Euler-backward method does not permit a temporal computational mode, this proved sufficient.

We began the experiments with the implicit model also inserting at one time level only, assuming that the time filter mentioned earlier would effectively prevent separation of solutions. However, since the implicit model used a 1-hour time step, and insertion was done every odd hour, the result was a forced separation of solutions of large amplitude. The time filter proved inadequate to remedy this problem. It was treated by determining the correction to the forecast height and wind fields at the current (τ) time, and adding the same correction to both height and wind at the immediate past (τ-1) time. This procedure eliminated the temporal computational mode.

V. Experimental Results

Figure 1 summarizes the results of the experiments. In the top half of the diagram the "pooled" RMS fit of the first guess to the observations is plotted as a function of time. Each number represents the pooled RMS fit between first-guess fields (interpolated to observation locations) and observations during one 12-hour forward or backward half cycle. The values at the initial time actually represent the fit of first-guess to data during the initial 6-hour period from 12Z to 18Z. The difference between the implicit value and the explicit value for this initial period is due to the difference in the initial states used by the two models. The values plotted at $t = 0.5$ days represent the RMS fit during the first backward cycle from 18Z to 06Z, those at $t = 1$ the RMS fit during the following forward cycle (06Z to 08Z), and so on.

Our expectation was that, as the models accommodated themselves to the observations, the RMS fit of the first-guess to the observations should improve. The diagram indicates that this is the case. Curve 1 represents the behavior of the explicit model, with the geostrophic wind adjustment. Most of the adjustment has taken place by the end of the second day. Curve 2 shows the results of using the implicit model without the geostrophic wind correction, and inserting at only one time level. The adjustment is very slow, as was the case in the earlier simulation experiments. A dramatic improvement occurs when the geostrophic wind correction is used in the implicit model, as demonstrated by curve 3. Finally, the fourth curve shows the implicit experiment with the geostrophic wind correction and insertion at two time levels. The multiple insertion hastens the adjustment process noticeably. Qualitatively, comparison of curves 1 and 4 shows that the adjustment rates of the two models are very similar.

Figure 2 addresses the question of adjustment of the wind field to the observations. We have plotted the RMS vorticity difference between assimilation and reference runs at 13Z during the cycling process. Our expectation was that the difference in vorticity between the reference and assimilated states would increase during the adjustment process but level off as adjustment approaches completion. Curves 1, 3, and 4 exhibit this behavior, while curve 2 shows that without the geostrophic wind correction, the wind field responds very slowly to the updated height field.

The evidence therefore suggests that the implicit integration method with insertion at two time levels is as efficient as the explicit (Euler-backward) method, and is more economical by a factor of approximately eight. This conclusion is slightly at variance with the earlier conclusion in Office Note 86, based on simulated data; in that study we concluded that the implicit model adjusted more slowly than did the explicit model, but not enough to offset the economic advantage of the former. In retrospect, we believe this performance was due primarily to the one-time level insertion procedure which was used in the earlier study. A comparison of curves 1 and 3 of Figure 1 would yield the same conclusion as in Office Note 86.

In addition to the implicit-explicit comparison, there are three other points worth emphasizing in Figure 1. First, a comparison of curve 2 with any other curve demonstrates the absolute necessity of updating both mass and motion fields simultaneously if adjustment is to be achieved. Geostrophic balance is not a good approximation to the actual mass-motion balance in low latitudes, so this technique is limited in its applicability.

Second, a comparison of curves 3 and 4 suggests that the use of a centered-difference time integration method in an assimilation model requires insertion at both current and immediately-past time levels, or some analogous technique, to inhibit separation of solutions.

Finally, these results indicate the need for repeated use of the same data in order to force the model to fit the data. Figure 1 shows that if all the data are used only once, the RMS fit of forecast-to-observation is reduced only by a small amount; if each piece of data is used six times (3 days' cycling), practically all of the adjustment has taken place.

VI. Summary

The experiments reported here and in Office Note 86 suggest that the semi-implicit integration method would be as efficient as an explicit scheme in four-dimensional data assimilation, and substantially more economical. An extension of these experiments awaits the development of a suitable multi-level semi-implicit prediction model.

REFERENCES

Asselin, R., 1972: "A frequency filter for time integrations," Mon. Wea. Rev., vol. 100, no. 6.

Cressman, G., 1959: "An operational weather analysis system," Mon. Wea. Rev., vol. 87, no. 10.

Hayden, C. M., 1973: "Experiments in the four-dimensional assimilation of Nimbus 4 SIRS data," J. App. Meteor., vol. 12, no. 3.

Kurihara, Y., 1965: "On the use of implicit and iterative methods for the time integration of the wave equation," Mon. Wea. Rev., vol. 93, no. 1.

McPherson, R., 1973: "Damping properties of the implicit-backward integration method," NMC Office Note 81.

_____________, 1973: "A comparison of explicit and implicit integration schemes in four-dimensional data assimilation," NMC Office Note 86.

Morel, P., G. Rabreau, and G. Lefevre, 1971: "On initialization and non-synoptic data assimilation," Tellus, vol. 23, no. 3.

Shuman, F., 1957: "Numerical methods in weather prediction: II. smoothing and filtering," Mon. Wea. Rev., vol. 85, no. 11.

LIST OF FIGURES