OPTIMAL INVESTMENT AND FINANCIAL STRATEGIES
IN SHRIMP FISHING

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## 1. Introduction

In April, 1970, Thompson, Callen and Wolken [1,2] published the first of two Texas A8M University Sea Grant Reports. The first bulletin contained a deterministic optimal control model of a shrimp fishing firm in addition to much background information on the industry and justification for the model specification. The second publication contained on extension of the model as first presented that took into account unknown, but random future catch and shrimp price and a constraint that required solvency to be maintained with a high probability based on the probability distributions of the random price and catch.

In this study, the original deterministic model is extended to require the purchase of integer (positive) numbers of vessels. Fractions could be purchased in the original application [1], but industry representatives suggested that a more realistic specification would require the purchase of integer numbers of vessels. This extension is significant in cases in which holding companies cannot be formed readily to overcome capital indivisibilities. Integer requirements clearly restrict the growth of the firm's physical capital and, in turn, net :orth over a finite planning horizon. If holding companies could be utilized without additional cost, vetsel owners could clearly experience a faster rate of net worth accumulation. However, because capital indivisibilities have not generally been overcome in the shrimp fishing industry the integer restriction is necessary for the model to be reflective of industry conditions.

The paper also serves to illustrate the importance of following an optimal strategy. Alternative strategies are compared with the optimal one in terms of net worth, net profits per year and accumulated net profits. The strategy that produces optimum net worth also performs best otherwise.

Using the same initial net worth and the same parameter values three alternative investment strategies were employed with respect to shrimp vessel purchases. Strategy I, a conservative one, was to purchase no additional fishing capacity and retain all cash flowe net of debt repayment as savings. In Strategies II and III, three additional boats were purchased. In Strategy II, a fairly confortable savings cushion $\$ 43,300$ was accumulated before the second boat was purchased. Additional purchases were made as soon as available cash was sufficient for a doun payment. Strategy III was the mixed-integer-linear programming solution to the investment problem. It reflects the optimal boat purchases for the given model and parameter values. In Strategy III the decision rule generated was to buy additional boats as soon as savings were sufficient to make a down payment. In each of the three strategies, borrowings were optinized using linear programing.
2. Dynamic Hodel for a Shrimp Fishing Firm

## Description of the model.

In the model, the objective of the fisherman is to maximize the amount of savings held in the last year of the decision-making period, $\mathrm{z}_{\mathrm{T}}$, less the anount of indebtedness outstanding at that time, $\mathrm{y}_{\mathrm{T}}$, plus the value of the boats ormed in the last year with an allowance being $\underset{T}{m a d e}$ for technological depreciation, $\psi_{t}$, and inflation in purchase prices, T $\sum_{t=0}{ }_{t}{ }^{T} t^{V} t$. There are three sets of difference equations and also three sets of inequality restrictions limiting the size of this objective. There
is one other set of constraints restricting the boat purchases in each period $v_{t}$ to have integer values. Indebtedness, $y_{t}$, savings, $z_{t}$, and boats owned, $x_{t}$, are the state (stock) variables in the model; boat purchases, $v_{t}$, and borrowings, $w_{t}$, are the control (flow) variables. Initial values of the state variables--number of boats, indebtedness, and savings--are taken as given; final values to the state variables are determined as a part of the solution to the problem.

In each year $t$, the shrimp fisherman in the model must repay a specified percentage of the indebtedness outstanding at the end of the previous year. In case the fisherman chooses to borrow in year $t$, he cannot borrow more than a fraction of the value of the boat investment in that year. That is, the fishing firm can only borrow money for the purchase of new boats; and in every case, the fisherman must have enough savings in the bank to cover the difference between the maximum loan value and the investment in boats. Letting $k$ denote the fraction (maximum) of the boat investment that can be borrowed, the upper-limit for borrowings in year $t$ is $k T_{t} v_{t}$, where $\tau_{t}$ is the purchase price (per boat) and $v_{t}$ is the number of boats bought. We may now state the inequality restrictions on $w_{t}$ as follows:

$$
\begin{equation*}
0 \leq w_{t} \leq K \tau_{t} v_{t}, \quad t=1,2, \ldots, T-1 . \tag{2.1}
\end{equation*}
$$

These restrictions mean that in any year $t$ borrowings, which must clearly be non-negative, may occur only if new boats are purchased, and then they cannot exceed the fraction $k$ of the investment $\tau_{t} v_{t}$ *

In the model, we do not allow the fisherman to sell boats. He can only purchase boats during the decision-making period:

$$
\text { (2.2) } \quad v_{t} \geq 0, t=1,2, \ldots, T-1
$$

Furthermore the fisherman may only buy integer numbers of boats:
(2.3)

$$
v_{t} \text { a nember of } I, t=1, \ldots, T
$$

where $I$ is the set of integers.
Since some time is generally necessary between the time when the decision is made to buy a boat and the boat is operational, the number of boats operated in year $t$ was specified to be the number owned at the end of year t-1; and accordingly boat purchases in the last year of the planning period were specified to be zero. Thus, the change in the number of boats owned is described as follows:

$$
\begin{align*}
& x_{t}-x_{t-1}=v_{t}, x_{0} \text { given, } t=1,2, \ldots, T-1,  \tag{2.4}\\
& X_{T I}-X_{T-1}=0, \text { so that } v_{T} \equiv 0 .
\end{align*}
$$

In accordance with the final purchase assumption above, borrowings in the last year are also specified to be zero. Moreover, since the fisherman must always repay in year $t$ a fraction $\rho$ of the indebtedness owed at the end of the previous year, the change in indebtedness is as follows:

$$
\begin{align*}
& y_{t}-y_{t-1}=w_{t}-8 y_{t-1}, y_{0} \text { given, } t=1,2, \ldots, \mathrm{~T}-1,  \tag{2.5}\\
& y_{T}-y_{\mathrm{T}-1}=-8 y_{\mathrm{t}-1} .
\end{align*}
$$

To describe the fishing firm's cash flow, it is helpful to have the following symbols: $\gamma$ is the exvessel price received by the owner in year $t$ after the lay is paid; $\lambda$ is the expected catch per boat in pounds of shrimp; $\eta$ is the sundry expense assooiated with the fishing operation; $\zeta$ is the interest rate paid on debt; $\xi$ is the interest rate earned in savings; $\sigma$ is the income tax rate; $\theta_{t}$ is the cost of operating a fishing boat in year $t$; and $g_{t}\left(v_{i}\right)$ is the depreciation allowed in year $t$ on the boats purchased in year i. Then the difference equations describing the firm's cash flow are:

$$
\begin{aligned}
z_{t}-z_{t-1}= & w_{t}-\beta y_{t-1}-T-\tau_{t} v_{t}+\left(\gamma \lambda-\theta_{t}\right) x_{t-1}-5 y_{t-1} \\
& +\xi z_{t-1}-\sigma\left[\left(\gamma \lambda-\theta_{t}\right) x_{t-1}-\eta-5 y_{t-1}+\xi z_{t-1}\right. \\
& \left.-\sum_{i=0}^{t-1} g_{t}\left(v_{i}\right)\right], \\
& z_{0} \text { given, } t=1,2, \ldots, T-1, \\
z_{T T}-z_{T-1}= & -B y_{T-1}+\left(\gamma \lambda-\theta_{T}\right) x_{T-1}-\eta-6 y_{T-1}+\xi z_{T-1} \\
& -\sigma\left[\left(\gamma \lambda-\theta_{T}\right) x_{T-1}-\eta-\zeta y_{T-1}+\xi z_{T-1}-\sum_{i=0}^{T-1} E_{t}\left(v_{i}\right)\right] .
\end{aligned}
$$

In every year except the last one, the cash flow or change in savings is equal to the change in indebtedness less the boat investment plus the earnings retained after taxes. Before tax earnings equal net revenues to the boat owner and interest earnings on savings less interest payments on debt. In calculations in this paper discounted net profits after taxes will be regarded as the retained earnings after taxes. Such a definition implies that no personal allowances are used from the earnings in case the ownership is non-corporate and that no dividends are declared if ownership is corporate. If a boat is owner-operated, of course, the captain's share of the lay also goes to the owner and is an additional element of profit that our definition overlooks.

Initially, the fishing firm is regarded as having a given amount of fishing capacity, $x_{0}>0$, with possibly some indebtedness, $y_{0} \geq 0$. It may or may not have any savings at the beginning of the period, $z_{0} \geq 0$.

The parameters in the model, which are denoted by Greek letters, are all positive with $\sigma, 5,5, \rho$, and $k$ being less than unity. It is also assumed that $\zeta>5$.

## Mathematical statement of the decision-making model.

In this section, the model described above is formally stated as a discrete-time control problem.
Maximize $I=z_{T}-y_{T}+\sum_{i=0}^{T} \psi_{i} \tau_{i} v_{i}$
satisfying the difference equations
(I.1) $x_{t}-x_{t-1}=v_{t}, x_{o}$ given and positive,
$x_{T}-x_{T-1}=0$,
(I.2) $y_{t}-y_{t-1}=w_{t}-R y_{t-1}, y_{o}$ given and non-negative, $y_{T}-y_{T-1}=-{ }^{8} y_{T-1}$,
(I.3) $z_{t}-z_{t-1}=w_{t}-8 y_{t-1}-\tau_{t} v_{t}+\left(\gamma \lambda-\theta_{t}\right) x_{t-1}-\eta$

$$
-\zeta y_{t-1}+\xi z_{t-1}-\sigma\left[\left(y \lambda-\theta_{t}\right) x_{t-1}-\eta-\zeta y_{t-1}+\xi z_{t-1}\right.
$$

$$
t-1
$$

$$
\left.-\sum_{i=0} g_{t}\left(v_{i}\right)\right]
$$

$$
z_{0} \text { given and non-negative, }
$$

$$
z_{T}-z_{T-1}=-B y_{T-1}+\left(\gamma \lambda-\theta_{T}\right) x_{T-1}-y_{T-1}+\xi y_{T-1}-\eta
$$

$$
T-1
$$

$$
-\sigma\left[\left(\gamma \lambda-\theta_{\mathrm{T}}\right) \mathrm{x}_{\mathrm{T}-1}-\eta-\sum_{i=0}^{T-1} \mathrm{~g}_{\mathrm{T}-1}\left(\mathrm{v}_{\mathrm{i}}\right)-\zeta \mathrm{y}_{\mathrm{T}-1}+\xi \mathrm{z}_{\mathrm{T}-1}\right]
$$

and satisfying the inequalities
(I.4) $\quad w_{t} \geq 0$
$, t=1,2, \ldots, T-1$,
(I.5) $w_{t} \leq K_{T v_{t}}$
, $t=1,2, \ldots, T-1$,
(I.6) $z_{t} \geq 0$
$, t=1,2, \ldots, T$,
(I.7) $\quad v_{t} \geq 0$
, $t=1,2, \ldots, T-1$,
(I.8) $v_{t}$ is a member of $I$, the set of integers , $\mathbf{t}=1,2, \ldots$, T-1.

Solving the difference equations in I.1, I.2, and I. 3 for their respective "closed-form" solutions, the state variables can be stated in terms of their initial values and the unknown control variables:

$$
\begin{align*}
& x_{t}=x_{o}+\sum_{i=1}^{t} v_{i},  \tag{2.6}\\
& y_{t}=y_{0}(1-\beta)^{t}+\sum_{i=1}^{t} w_{i}(1-\beta)^{t-i}, \\
& z_{t}=z_{1} Q_{t 1}+\sum_{i=2}^{t}\left[w_{i}-\tau_{i} v_{i}+\Delta_{i} x_{i-1}+\pi y_{i-1}+.091 \sigma \sum_{j=0}^{i-\uparrow} \tau_{j} v_{j}\right.  \tag{2.8}\\
& +(\sigma-1) \pi] Q_{t i} \text {, }
\end{align*}
$$

where

$$
\begin{aligned}
v_{T} & =0=w_{T}, g_{t}\left(v_{i}\right)=.091 \tau_{i} v_{i}, \\
\Delta_{i} & =(\gamma \lambda)(1-\sigma)-(1-\sigma) \theta_{i}, \\
\pi & =\zeta(\sigma-1)-B, \\
Q_{t i} & =(1+\Gamma)^{t-i}, \\
\Gamma & =\xi(1-\sigma), i=1,2, \ldots, t \text { and } t=1,2, \ldots, T .
\end{aligned}
$$

Substituting the closed-form solution for $x_{t}$ and also $y_{t}$ from (2.6) and (2.7) into (2.8), we obtain the following solution for $z_{t}$ in terms of the initial values for the states, the unknown controls, and the parameters: (2.9)

$$
z_{t}=C_{t}+\sum_{i=1}^{t} w_{i} P_{t i}+\sum_{i=1}^{t} v_{i} D_{t i}
$$

where

$$
\begin{aligned}
C_{t}= & \sum_{i=1}^{t} Q_{t i}\left[\left(\Lambda_{i}+.0910 T_{o}\right) x_{0}+(\sigma-1) \pi\right]+\eta_{o} \sum_{i=1}^{t} Q_{t i} x^{i-1} \\
& +(1+\Gamma) z_{o} Q_{t 1}, t=1,2, \ldots, T-1, \\
x= & 1-\rho, \\
P_{t t}= & Q_{t t}, t=1,2, \ldots, T-1,
\end{aligned}
$$

$$
\begin{aligned}
& P_{t i}=Q_{t i}+\prod_{j=i+1}^{t} Q_{t j} R_{j-1, i}, \\
& \\
& \quad i=1,2, \ldots, t-1 \text { and } t=2, \ldots, T-1, \\
& D_{t t}=-\tau_{t} Q_{t t}, t=1,2, \ldots, T-1, \\
& D_{t i}=\sum_{j=i+1}^{t} \Delta_{j} Q_{t j}+.091 \sigma \tau_{i} \sum_{j=i+1}^{t} Q_{t j}-\tau_{i} Q_{t i}, \\
& \quad i=1,2, \ldots, t-1 \text { and } t=2,3, \ldots, T-1 ;
\end{aligned}
$$

$$
\text { (2.9), } \quad z_{T}=\sum_{i=1}^{T-1} w_{i} P_{T i}+\sum_{i=1}^{T-1} v_{i} D_{T i}+C_{T},
$$

where

$$
\begin{aligned}
& C_{\mathrm{T}}=(1+\Gamma) \mathrm{C}_{\mathrm{T}-1}+.091 \tau_{0} \mathrm{x}_{0} \sigma+(\sigma-1) \eta+\Delta_{\mathrm{T}} \mathrm{x}_{0}+\pi_{y_{o}} \mathrm{x}^{\mathrm{T}-1}, \\
& \mathrm{P}_{\mathrm{Ti}}=\pi_{\mathrm{T}-1, i}+(1+\Gamma) P_{\mathrm{T}-1, i}, i=1,2, \ldots, \mathrm{~T}-1, \\
& \mathrm{D}_{\mathrm{Ti}}=\Delta_{\mathrm{T}}+(1+\Gamma) \mathrm{D}_{\mathrm{T}-1, i}+.091 \sigma_{\mathrm{i}}{ }_{i}, i=1,2, \ldots, \mathrm{~T}-1, \\
& R_{\mathrm{ti}}=(1-\beta)^{t-i}, i=1,2, \ldots, t \text { and } t=1,2, \ldots, \mathrm{~T} .
\end{aligned}
$$

## The Linear Programming Model.

Substituting the solutions above for the state variables-- $x_{t}, y_{t}$, $z_{t}$--into the objective function and the inequality restrictions of the control model, the state variables (and the difference equations describing them) are removed from the problem. The resulting problem is the following linear programming model:

$$
\text { Maximize } I=2+\sum_{t=1}^{T-1} B_{t} v_{t}+\sum_{t=1}^{T-1} A_{t} W_{t}
$$

subject to the inequality restrictions
(II.1) $w_{t} \geq 0$,
$t=1,2, \ldots, T-1$,
(II.2) $K T_{t}{ }^{v} t-w_{t} \geq 0, \quad t=1,2, \ldots, T-1$,
(II.3) $\sum_{i=1}^{t} P_{t i} w_{i}+\sum_{i=1}^{t} D_{t i} v_{i} \geq-C_{t}, \quad t=1,2, \ldots, T$,
(II.4) $v_{t} \geq 0, \quad t=1,2, \ldots, T-1$,
(II.5) $v_{t}$ is a nember of $I, \quad t=1,2, \ldots, T-1$
vhere

$$
\begin{aligned}
A_{t} & =P_{T t}-R_{T-1, t}(1-\beta), t=1,2, \ldots, T \\
B_{t} & =P_{T t}+\psi_{t} T_{t}, t=1,2, \ldots, T, \\
a & =C_{T}+\psi_{0} T_{0} x_{0}-Y_{0} X^{T}, \text { and } \\
I & \text { is the set of all integers. }
\end{aligned}
$$

## Letting

$$
\begin{gathered}
h_{t} \equiv h_{t}\left(w_{1}^{0}, \ldots, w_{t-1}^{0}, v_{1}^{0}, \ldots, v_{t-1}^{0}\right) \equiv c_{t}+\sum_{i=1}^{t-1} P_{t i} w_{i} \\
\\
+\sum_{i=1}^{t-1} D_{t i} v_{i}, t=1,2, \ldots, T-1,
\end{gathered}
$$

inequality IT. 3 may be expressed as follows in terms of the non-negative function $h_{t}$ :
(II.3)'

$$
r_{t}-r_{t} v_{t}+h_{t} \geq 0, t=1,2, \ldots, T-1
$$

## 3. An Investment Strategy From the Model

As done in the first report [1], the model developed above is applied to a relatively small shrimp fishing firm operating 73 foot steel hull trawlers. Our aim is to illustrate how a shrimp fisherman having a given amount of physical and money capital might use the model to obtain guidelines for investment and financial decision-making.

Initial state values and values of the parameters considered.
In this application, the value of $\mathbf{x}_{0}$, is specified to be one boat. That is, the model firm is initially operating one 73 foot steel hull trawler. It is further visualized that this boat was purchased at the end of 1969 for a price of $\$ 100,000$ and was completely outfitted for shrimp fishing. The model fisherman had $\$ 30,000$ in cash with a minimum downpayment of $\$ 25,000$ being made on the boat: $k=.75, y_{0}=\$ 75,000$, and $z_{0}=5,500$. The loan contract requires the indebtedness to be repaid at a rate of $10 \%$ yearly starting at the end of the first year with interest (including mortgage incurance) at $9 \%$ percent annually: $\theta=.10$ and $\zeta=.095$. This borrowing rate, which reflects 1969 conditions may be somewhat high at the end of 1970 and may continue to decline. The interest rate on savings is specified to be $5 \%$ percent annually, the present maximum rate on savings deposits: $5=.055$.

Since it is quite common for owners of vessels like this one to obtain 65 percent of the gross revenues with the captain and first mate (who pay for all of the groceries) receiving the other 35 percent, the net price per pound of shrimp landed is specified to be 65 percent of the exvessel price in year $t$, $\epsilon_{t}$. That is, $\gamma_{t}=.65 \varepsilon_{t}$. The exvessel price for shrimp in year $t \epsilon_{t}$ was determined by the equation developed in Thompson et al. $[2, p .10]:$

$$
\ln \varepsilon_{t}=4.4725+0.0176 t
$$

The above equation gives estimates of the exvessel average price of shrimp with landings at the mean value of the period 1958 through 1967 and a projected 1.5 percent rate of growth in real per capita income. The 1.5 percent rate of growth in real per capita income reflects the slow rate of growth of the late 1950 's. This rate of growth appears reasonable as opposed to a faster rate of growth observed in the middle 1960's.

To convert to money terms, the projected prices from this equation are multiplied by the value of the consumer price index (with base 1957/59 $=$ 100) for $1969,1.277$, and by a price inflating factor of 1.5 percent in onch year thereafter. Taking the product of the projected price and the expected annual landing per vessel with an adjustment for the lay fraction, the owner's expected annual revenue per vessel was obtained. The expected annual landing per vessel $\lambda_{t}$ used in this study was the average of the landings per vessel obtained by the cooperating firms in the period 1958 through 1969 (57,560 pounds of heads off shrimp). There was, of course, a steady rate of technological improvement in that period so that this average is likely to be an underestimate of a 73 foot vessel's annual catch potential. Thus, the value of the expected annual owner's revenue per vessel for the stipulated 1.5 nercont econonic roth rote is a onscrvotive estimate. It might have been further increased for expected technological improvements.

From the cost records of cooperating firms, the annual cost of operating a 73 foot trawler was found to be $\$ 30,000$ in 1969. This cost figare includes an allowance for overhead and insurance costs. Representatives of firms interviewed indicated these costs have increased by 3 percent per year in recent years. Thus, the annual production cost per vessel, $\tau_{t}$, was specified to be $30,000(1.03)^{t}$.

To reflect inflation, the purchase price of new vessels was specified to increase at 3 percent per year: (1.03) $\tau_{0}=\tau_{t}$.

Straight line depreciation methods were used for tax purposes with an 11 year depreciation period being used for a fully outfitted vessel. This average was estimated on a value weighted basis from the records of a number of firms. The reciproical of this figure, .091, was the depreciation fraction used for $g\left(v_{:}\right)$.

The initial value of the technical depreciation rate $\psi_{0}$ is .65 and is based on the argument in Thompson et al. [1, p. 29], where ${ }_{t}=1 /(1.044)^{t}$.

Income for tax purposes is the sum of the revenue received by the owner after the "lay" less operating costs, interest costs, and depreciation. The income tax rate, which is denoted by $\sigma$, was taken to be 25 percent of this figure. This rate was paid in the late $1960^{\circ} \mathrm{s}$ by a number of the small fishing firms studied.

In shrimp fishing, as in every business, there are sundry expenses for a number of factors related to the firm. Some of these costs, it might be argued, are not absolutely necessary for the operation of the business; but for the sake of convenience (or acceptance), they are commonly incurred. Such costs are difficult to estimate. Thus, in this study, a base allowance of $\$ 1200$ per year was specified for sundry expenses: $\eta=\$ 1200$.

In shrimp fishing, the captain and first mate of the vessel are commonly paid on a "lay" basis wherein they receive an agreed upon percentage of the revenue earned by the vessel. The third crew member, who is called a header, is typically paid on a per box basis. An allowance for his wages was included in the value of the production cost per vessel.

> 4. Application of the Model to the Selection of Optimal Investment Strategies

The gequential linear programming form of the model is evaluated for the alternative investment strategies. In Strategy I the fisherman enters year 1 with one boat, purchases no other boats and borrows no additional capital. Capital accumulation is based strictly on the accumulation of net retained earnings as savings and the amortization of the initial debt in Strategy I. Solutions for Strategy I for the period are presented in Table 2.

In Strategy II, three boats are purchased in periods 7,8, and 9 in addition to the boat that was owned in the initial period. Through the purchase of additional boats net worth at the terminal period was increased by some $\$ 40,845$ and accumulated net profits by some $\$ 36,699$. Solutions for Strategy II are given in Table 3.

In Strategy III the second boat was purchased as soon as sufficient savings had accumulated to meet a down payment. Similarly the third and fourth trawlers were acquired at the first opportunity. By acquiring additional capacity as rapidly as possible, the terminal net worth was increased over that of Strategy II by $\$ 20,961$ and accumulated net profits by $\$ 21,187$. Strategy III is the optimal mixed integer programing solution to the problem. The purchase of fewer boats or boats in other periods will either be infeasible or less profitable than Strategy III. Solutions for the optimal strategy are given in Table 4.

It should be noted that the solutions to Strategies I and II given in Thable 4 and 3 respectively are optimal in a sense also. The numbers of boats to be purchased was first chosen in each case. Then the optimal level for borrowings was obtained by linear programing techniques. In Strategy III mixed integer programming was used to obtain both the optimal boat purchases and the optimal borrowings per period.

The progress of the firm after the planning period with respect to net worth, net profits and accumulated net profits aregiven in Figures 1, 2, and 3 respectively. The grouth of the firm from Strategy I arises in the growth of savings rather then additions to the number of boats. In Strategy II, the firm grows faster as a result of the increased revenue earning power of the added boats. In Strategy III, the firm adds boats at the optimum time and in the optimum number with correspondingly improved results.

TABLE 1. Values of parameters and initial state values specified.

| Parameter | Value |
| :---: | :---: |
| B -- Debt repayment rate | . 10 |
| $\tau_{t}$-- purchase price per boat | \$100,000 (1.03) ${ }^{\text {t }}$ |
| ${ }_{\text {t }}{ }^{\text {-- technological }}$ depreciation | $1 /(1.044)^{t}$ |
| 5 -- interest rate on debt | . 095 |
| \% -- interest rate on savings | . 055 |
| K -- financeable fraction of investment | . 75 |
| $\theta_{t}$-- operating costs per boat | \$30,000 (1.03) ${ }^{t}$ |
| o -- rate of withdrawel from income for taxes | . 25 |
| t-1 | t-1 |
| $\sum_{i=0} g\left(v_{i}\right) \text {-- the depreciation function for taxes }$ | $.091 \sum_{i=0} \tau_{i} v_{i}$ |
| $x_{0}--$ Initial fishing capacity | 1 boat |
| $y_{0}$ - - initial indebtedness | \$75,000 |
| $z_{0}--$ initial savings | \$5,000 |

From the above discussion, it may appear that a shrimp fisherman should buy additional boats, as many as he can, as rapidly as he can save up a down payment. If such were actually the case, this model would be of limited use for the answer to the problem would be very well known. The reason for this simplicity is that average price and catch were assumed to be known and to be so high that the rate of return on additional shrimp boats was greater than the rate of return on savings. Thompson et al. [1] was able to demonstrate periods in which the rate of return on savings was higher than the rate of return on additional boats by utilizing altered price and catch assumptions with a somewhat different model specification. Thompson's model delineated years in which the best decision was to deplete all savings in order to buy boats, years in which it was best not to buy boats regardless of cash on hand, and years in which it was best to invest in a limited way and also to maintain cash balances for future obligations.

The prices used by Thompson et al [1] varied from $\$ .52$ to $\$ .65$ per 1 lb . net of lay. Prices used in the present study varied from $\$ .76$ to $\$ 1.16$ in periods 1 and 10, respectively. In the present study, the catch was assumed at the industry mean 57,560 pounds but in Thompson's first study catch was specified at $60,000,70,000$, and 80,000 pounds. It appears that the investment climate in our model is only slightly more attractive than in Thompson's because of the higher catches he assumed. In both studies, catch and price are specified to be known in advance; a specification they did not need in the second study [2]. In the earlier models the boat purchases may be any fraction of a boat and thus, do not appear to be as reflective of the industry as they might.

Our objective in this paper has been to illustrate a method obtaining optimal investment strategies for shrimp fishermen. The objectives of our
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| Year | States |  |  | Strategy Farameter | Contr | Objective | Companion Values |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Bosts Owned | Indebtedness | Savings | Boats Purchased | Borrowings | Net Worth | Profit | $\begin{aligned} & \text { Accumulated } \\ & \text { Profits } \end{aligned}$ |
|  | $\begin{gathered} \mathrm{x}_{\mathrm{t}}^{0} \\ \text { (number) } \end{gathered}$ | $\begin{aligned} & y_{t}^{0} \\ & \text { (dollars) } \end{aligned}$ | $\begin{aligned} & z_{t}^{\circ} \\ & \text { (dollars) } \end{aligned}$ | $\begin{gathered} \mathrm{v}_{\mathrm{t}} \\ \text { (number) } \end{gathered}$ | $\begin{gathered} \mathrm{w}_{\mathrm{t}}^{\circ} \\ \text { (dollars) } \end{gathered}$ | (dollars) | (dollars) | (dollars) |
| 1 | 1.00 | 67499.94 | 17080.70 | 0.00 | 0.00 | 49580.76 | 12080.68 | 12080.68 |
| 2 | 1.00 | 60749.99 | 18033.70 | 0.00 | 0.00 | 57283.71 | 952.95 | 13033.67 |
| 3 | 1.00 | 54674.99 | 21097.20 | 0.00 | 0.00 | 66422.19 | 3063.51 | 16097.18 |
| 4 | 1.00 | 49207.49 | 26300.50 | 0.00 | 0.00 | 77093.00 | 5203.31 | 21300.49 |
| 5 | 2.00 | 44286.73 | 33688.80 | 0.00 | 0.00 | 89402.06 | 7388.33 | 28688.82 |
| 6 | 1.00 | 39858.06 | 43323.20 | 0.00 | 0.00 | 103465.10 | 9634.42 | 38323.24 |
| 7 | 1.00 | 35872.25 | 55279.90 | 0.00 | 0.00 | 119407.60 | 11956.67 | 50279.91 |
| $\varepsilon$ | 1.00 | 32285.02 | 69650.63 | 0.00 | 0.00 | 137365.50 | 14370.72 | 64650.63 |
| 9 | 1.00 | 29056.52 | 86542.63 | 0.00 | 0.00 | 157486.00 | 16891.93 | 8.542 .56 |
| 10 | 1.00 | 26150.87 | 106077.00 | 0.00 | 0.00 | 179926.10 | 19535.07 | 101077.63 |

Teble 3. Optimal solution to investment Strategy II

| Year | States |  |  | Strategy <br> Parameter | Control | Objective | Companion Values |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Boats Owned $\begin{gathered} \mathrm{x}_{\mathrm{t}}^{0} \\ \text { (number) } \end{gathered}$ | Indebtedness $\begin{gathered} y_{t}^{o} \\ (\text { dollers }) \end{gathered}$ | $\begin{gathered} \text { Savings } \\ \mathrm{z}_{\mathrm{t}}^{\mathrm{o}} \\ \text { (dollars) } \end{gathered}$ | Boats Furchased $\begin{gathered} v_{t} \\ \text { (number) } \end{gathered}$ | $\begin{aligned} & \text { Borxowings } \\ & w_{t}^{\circ} \\ & \text { (dollars) } \end{aligned}$ | Net Worth (dollars) | Profit (dollars) | Accumviated Profits <br> (dollars) |
| 1 | 1.00 | 67499.94 | 17080.70 | 0.00 | 0.00 | 49580.76 | 12080.68 | 12080.88 |
| 2 | 1.00 | 60749.99 | 18033.70 | 0.00 | 0.00 | 57283.71 | 952.99 | 13033.67 |
| 3 | 1.00 | 54674.99 | 21097.20 | 0.00 | 0.00 | 66422.19 | 3063.51 | 16097.18 |
| 4 | 1.00 | 49207.49 | 26300.50 | 0.00 | 0.00 | 77093.00 | 5203.31 | 21300.49 |
| 5 | 1.00 | 44286.73 | 33688.80 | 0.00 | 0.00 | 89402.06 | 7388.33 | 28688.82 |
| 6 | 1.00 | 39858.06 | 43323.20 | 0.00 | 0.00 | 103465.10 | 9834.42 | 38323.24 |
| 7 | 2.00 | 122803.90 | 19224.90 | 1.00 | 86931.69 | 99941.00 | 12956.69 | 50279.93 |
| 8 | 3.00 | 205530.60 | 5449.20 | 1.00 | 95007.13 | 114757.80 | 17893.38 | 68173.31 |
| 9 | 4.00 | 282834.80 | 0.00 | 1.00 | 97857.31 | 151710.60 | 27169.92 | 93343.23 |
| 10 | 4.00 | 254551.20 | 40777.40 | 0.00 | 0.00 | 220771.50 | 40777.42 | 1361.20 .65 |


| Table 4. Optimal solution to investment Strategy III |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Year | States |  |  | Controls |  | Objective $\qquad$ New Worth | Companion Values |  |
|  | $\begin{gathered} \text { Boats } \\ \text { Owned } \\ \mathrm{x}_{\mathrm{t}}^{\circ} \\ \text { (number) } \end{gathered}$ | Indebtedness $\begin{gathered} y_{t}^{\circ} \\ (\text { dollars }) \end{gathered}$ | $\begin{gathered} \text { Savings } \\ z_{t}^{0} \\ \text { (dollars) } \end{gathered}$ | Boats Purchased $\mathrm{v}_{\mathrm{t}}^{\mathrm{o}}$ (number) | Borrovings $w_{t}^{o}$ <br> (dollars) | New Worth (dollars) | Profit (dollars) | Accumulated Profits (dollars) |
| 1 | 1.00 | 67499.94 | 17080.70 | 0.00 | 0.00 | 49580.76 | 12080.68 | 12080.86 |
| 2 | 1.00 | 60749.99 | 18033.70 | 0.00 | 0.00 | 57283.71 | 952.85 | 13033.67 |
| 3 | 1.00 | 54674.99 | 21097.20 | 0.00 | 0.00 | 66422.19 | 3063.51 | 16097.18 |
| 4 | 1.00 | 49207.49 | 26300.50 | 0.00 | 0.00 | 77093.00 | 5203.31 | 21300.49 |
| 5 | 2.00 | 126524.80 | 0.00 | 1.00 | 82238.19 | 62997.81 | 7388.51 | 28689.00 |
| 6 | 2.00 | 113872.30 | 11427.60 | 0.00 | 0.00 | 87077.94 | 12427.60 | 401.16 .60 |
| 7 | 2.00 | 102485.10 | 27877.80 | 0.00 | 0.00 | 112915.30 | 16450.20 | 56566.80 |
| 8 | 3.00 | 169491.00 | 0.00 | 1.00 | 77254.38 | 131350.90 | 21544.42 | 78111.52 |
| 9. | 4.00 | 250059.90 | 0.00 | 1.00 | 97518.13 | 170488.10 | 32958.77 | 111069.99 |
| 10 | 4.00 | 225053.80 | 46238.20 | 0.00 | 0.00 | 241732.30 | 46238.20 | 159308.19 |




NET PROFITS AFTER TAXES

Figure 3


NET PROFITS ACCUMULATED
three-year Sea Grant Research Project have included (1) the development of models of optimal investment decisions in shrimp fishing; (2) the refining of those models to be reflective of industry conditions and practices and be practicable as a menagement tool; and (3) to disseminate the information for use by fishermen. The first objective has been previously accomplished. This paper was concerned with objectives 2 and 3.

To develop a practical management tool several refinements may be relevant. Farameter values should be reevaluated with additional data, to insure their reflectiveness. A study of alternative sizes of boats would be oí interest but will require much additional data on parameters.

The possibility of trading old boats in on new ones should also be investigated. The present models do not allow such reversability in investments.

The integer restriction suggests that it would be meaningful to study the opportunities for increasing the net worth of fishermen through holding companies to reduce capital individibilities. If additional management costs were minimal, such an arrangement could be significant in increasing net worth.

A previous study [2] discribed a dynamic stochastic model that differed from the one presented here in that prices and catches did not have to be assumed known in advance. The dynamic model learns the prices and catches in each harvesting period, just as does the shrimp fisherman. Thus, random or actual sequences of prices and catches may be utilized to obtain optimal decision rules that closely simulate industry conditions. The integer refinement along with the refinements mentioned up to this point should be implemented with the dynamic model to more closely reflect industry conditions and to make definitive recommendations.

Finally the models should be very carefully monitored using parameter and initial state data from a variety of fishing firms and making comparisons of optimal prescriptions with actual decisions. Guidelines may be obtained for the industry in general using hypothetical initial conditions and parameter values. However exact prescriptions for any given firm should be obtained using that firm's particular initial asset position and it's own parameter values. Computer costs for individual application of such models, given that the firm has the expertice to obtain and apply the information, should generally be less than $\$ 100$ per year.

Footnotes

1. Partially supported by the National Science Foundation Sea Grant Program Institutional Grant GH-101 to Texas A8M University.
2. Robert R. Wilson is Assistant Professor, Institute of Statistios, Texas ApM University. Russell G. Thompson is Chief of the Forecast Division of the National Hater Commission, Arlington, Virginia. Richard U. Callen is Research Assistant. Institute of Statistics, Texas A\&M University.
3. Prepared for publication in the Proceedings of the $23 r d$ Annual Gulf and Caribbean Fisheries Institute, November 1970.

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