U. S. DEPARTMENT OF COMMERCE
NATIONAL OCEANIC AND ATMOSPHERIC ADMINISTRATION
NATIONAL WEATHER SERVICE
NATIONAL METEOROLOGICAL CENTER

OFFICE NOTE 57

DIGITAL FILTERS

FOR USE IN

POST-PROCESSING FORECAST FIELDS

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June 1971

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1. Introduction

The growth of noise, that is high wave number variability, in the course of the numerical integration of limited-area forecasts has been discussed in previous papers [1,2]. As long as the integration is computationally stable, the noise is not overly significant except when it detracts from the appearance of the post-processed charts. To avoid confusion in the interpretation of such charts, it is desirable to suppress the noise components while leaving the meteorological components relatively unaltered.

In principle, one may utilize Fourier analysis to accomplish the desired smoothing. More efficient methods exist for accomplishing the desired end to a reasonable degree of accuracy. Shuman [3] and, more recently, Shapiro [4] have discussed the construction of cascaded linear digital filters to meet prescribed specifications. The approach used in our filter design differs somewhat from theirs and documentation seems warranted.

2. A broad-band, low-pass filter

Suppose that in one-dimension, a set of data is sampled at equal intervals Δx. Consider that the data are available at L+1 points. This situation is typical of that encountered in post-processing. The data will generally be such that the end-point values are unequal and the mean value is non-zero. A unique Fourier representation of such data is non-existent. To perform a Fourier analysis, the data must be extended by some (arbitrary) rule to be periodic or of infinite extent.

We may proceed by a somewhat different course and simply define a linear operation by which the given data are to be transformed. Some idea of the effect of the linear operator may be gleaned by analysis of its effect upon periodic functions. We reserve the right to judge the performance of the operator by consideration of its effect on realistic data.

Let's consider the datum at gridpoint $j\Delta x$ $(0 \leq j \leq L)$ to be denoted by d_j and define the linear transformation

$$y_j = w_o d_j + \sum_{m=1}^{M} w_m (d_{j+m} + d_{j-m}) \tag{1}$$

in which the weights, w_m, are real numbers. The operator can only be applied for

$$M < j < (L-M) \tag{2}$$

Now, suppose that

$$d_j = D_k e^{ikj\Delta x} \tag{3}$$

where D_k is a complex constant and k is a real number. From sampling theory, one knows that the value of k is bounded between $\pm\pi/\Delta x$. The term given in (3) is one element in a complex Fourier representation of the data.

Application of (1) gives

$$y_j = d_j \ [w_o + \sum_{m=1}^{M} 2w_m \cos(km\Delta x)] \tag{4}$$

Since the quantity in brackets is a real number, we may note that the phase of d_j is unaffected. The "response" of the linear operator is defined as the value of the bracketed term; it is a function of $k\Delta x$, M and the w_m. If we define

$$\alpha = k\Delta x \tag{5}$$

we may note the trigonometric identity,

$$\cos m\alpha = 2 \cos (m-1)\alpha \quad \cos \alpha - \cos (m-2)\alpha \tag{6}$$

If we further define

$$\zeta = \cos \alpha \tag{7}$$

one may show that (4) is a polynomial of degree M in ζ. By specification of the w_m's, one may cause that polynomial to pass through any M+1 points at prescribed values of ζ. Thus we see that a trade-off is possible between the length of the data set which may be transformed and the constraints to which the response may be subjected. It should be noted that the "cascaded, elemental filters" discussed by Shuman and Shapiro are subject to the same trade-off.

Let's now consider the design criteria used for our broad-band, low-pass filter [cf. ref. 1]. We require the response, R, to be unity for $\zeta=1$ and vanish for $\zeta=-1$. We further require that $\frac{\partial R}{\partial \zeta}=0$, at $\zeta=\pm1$. These four conditions require that M=3. Thus, the filtered field will be available only from j=3 to j=L-3. It so happens that the boundary conditions used in the integration of the LFM [5] are artificial and consequently, that close to the boundaries, the forecast fields have little validity.

From eq. (4) with M=3, one gets with the use of (6) and (7),

$$R = (w_0-2w_2) + (2w_1-6w_3)\zeta + 4w_2\zeta^2 + 8w_3\zeta^3 \tag{8}$$

If we define

$$\begin{aligned} a &= w_0 - 2w_2 \\ b &= 2w_1 - 6w_3 \\ c &= 4w_2 \\ d &= 8w_3 \end{aligned} \tag{9}$$

then the conditions imposed on R may be expressed:

$$\begin{aligned} R = 1,\ \zeta = 1 &: \quad a + b + c + d = 1 \\ R = 0,\ \zeta = -1 &: \quad a - b + c - d = 0 \\ \frac{\partial R}{\partial \zeta} = 0,\ \zeta = 1 &: \quad b + 2c + 3d \quad = 0 \\ \frac{\partial R}{\partial \zeta} = 0,\ \zeta = -1 &: \quad b - 2c + 3d \quad = 0 \end{aligned} \tag{10}$$

The solution of (10) is

$$\begin{aligned} a &= .5 \\ b &= .75 \\ c &= 0. \\ d &= -.25 \end{aligned} \tag{11}$$

$$R = .5 + .75\zeta - .25\zeta^3 \tag{12}$$

The weights, w_m, may then be determined from (9) and (11):

$$\begin{aligned} w_2 &= 0 \\ w_3 &= -1/32 \\ w_1 &= 9/32 \\ w_0 &= 16/32 \end{aligned} \tag{13}$$

We will discuss details of the response later on.

3. A recursive, narrow band filter

The filter presented above does not appreciably suppress that portion of the spectrum about $\zeta=0$ (the $4\Delta x$ wave). In some of the LFM fields, post-processed with this filter, an excessive residue of noise seems to be present with a wave length near $4\Delta x$. To suppress that wave, we have formulated a "recursive filter." The description of the theoretical basis for its design would take us off on a tangential course and will therefore not be attempted here*.

Using the same notation as earlier, the "recursive filter" may be written,

$$y_j = \left(d_j + d_{j-2} - y_{j-2}\right)\left(\frac{1 + (1+\varepsilon)^2}{2(1+\varepsilon)^2}\right) \tag{14}$$

In practice, we apply this filter from right to left (j increasing) and then a second time in the opposite direction (making the obvious modifications in indexing). The mean of the two passes is taken as the final value of y_j. Whenever a value of d or y is required outside the domain ($o \le j \le L$), it is taken to be equal to the mean value of d or zero respectively. By varying the value of ε, the filter's response can be made sharper (reducing ε), but with less damping of the $4\Delta x$ wave, or broader (increasing ε), with greater damping of the $4\Delta x$ wave. The filter's response is theoretically symmetric about $\zeta=0$. It has been normalized to yield unity at $\zeta=\pm1$. There is a wavy character, "ripple", to the response.

In Table 1, we present the response calculated for various values of ε as a function of harmonic index, k, for trigonometric fields defined on a grid of 52 intervals. The harmonic index, 13, corresponds to that of a wave of length four grid intervals.

The calculation was made by construction of a data set with a particular wave length (trigonometric functions). The recursive filter was applied. The reduction in the amplitude, i.e. the response, is not uniform over the entire field. The tabulated values were read near the midpoint of the field - at gridpoint 29 - and are relative minima. The symmetry of the response about k, the harmonic index, equal to 13 was not checked.

*The theory was presented in an ESSA sponsored short-course in time-series analysis taught by Dr. Flinn.

k ↓ \ ε →	0.01	.02	.03	.04	.05	.06	.07	.08	.09	0.1
1	.991	.983	.976	.970	.964	.959	.954	.950	.946	.942
2	.999	.998	.995	.992	.988	.984	.980	.976	.972	.967
3	.991	.983	.976	.970	.964	.959	.954	.950	.946	.942
4	.999	.998	.995	.992	.988	.984	.980	.976	.971	.967
5	.991	.983	.976	.969	.964	.959	.954	.950	.945	.941
6	.999	.998	.995	.992	.988	.984	.979	.975	.970	.965
7	.991	.983	.976	.969	.964	.958	.953	.949	.944	.940
8	.999	.998	.994	.991	.987	.982	.977	.972	.967	.962
9	.991	.983	.976	.969	.963	.957	.952	.947	.942	.937
10	.999	.996	.993	.988	.983	.977	.970	.964	.957	.950
11	.991	.982	.975	.967	.960	.952	.945	.937	.929	.920
12	.996	.986	.972	.953	.933	.911	.887	.863	.839	.815
13	.871	.760	.665	.584	.514	.453	.400	.355	.315	.281

Table 1:

Response of recursive filter estimated at grid point 29 for various values of ε and the harmonic index k; $k=13$ corresponds to the four grid interval wave. The response is theoretically symmetric about $k=13$.

4. Two-dimensional application

The use of the filters in two dimensions poses no essential problem (cf. [3]). The broad-band, low-pass filter may be written as a 49-point stencil, or passed in both coordinate directions as programming ease suggests. The recursive filter does not lend itself to a stencil application. Other schemes for suppressing the $4\Delta x$-wave might be developed along the line used in Section 2, but they will generally require a rather large stencil.

In combined application of the two filters presented here, it seems best to first apply the recursive scheme, since no data points are lost.

5. Response of combined application

It is possible that these filters will find application in the LFM post-processor package. At that time, some specific examples may be worked out. For the moment, we must be content with a somewhat theoretical estimate of the response. For this purpose, Table 2 has been constructed. Using L=52, the value of $\cos(2\pi m/L)$ --- ζ_m -- was evaluated for m=1,2,...,26. The response, R_1, was calculated from eq. 12. The estimate of the recursive filter's response given in Table 1 for ε=.1 was tabulated as R_2. The response R_3 of the combination of the two linear operators was calculated by multiplication of R_1 and R_2.

		ε=.1				ε=.1	
Wavelength/Δx	R_1	R_2	R_3	W.L./Δx	R_1	R_2	R_3
52	.999	.942	.999	3.7	.422	.815	.345
26	.998	.967	.965	3.5	.335	.920	.308
17.3	.998	.942	.940	3.3	.252	.950	.239
13	.991	.967	.958	3.1	.184	.937	.172
10.4	.977	.941	.919	2.9	.124	.962	.119
8.7	.956	.965	.922	2.7	.079	.940	.074
7.4	.921	.940	.866	2.6	.045	.965	.043
6.5	.876	.962	.843	2.5	.023	.941	.022
5.8	.816	.937	.764	2.4	.009	.967	.009
5.2	.748	.950	.711	2.3	.002	.942	.002
4.7	.665	.920	.613	2.2	.002	.967	.002
4.3	.578	.815	.471	2.1	.001	.942	.001
4.0	.5	.281	.140	2.0	.0	1.	0.

Table 2;

The theoretical response of the combination of the two filters presented in the text.

References:

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