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U.S. DEPARTMENT OF COMMERCE
NATIONAL OCEANIC AND ATMOSPHERIC ADMINISTRATION
NATIONAL WEATHER SERVICE

OFFICE NOTE 210

A Modified Threat Score and a Measure of Placement Error

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This is an unreviewed manuscript, primarily intended for informal exchange of information among NMC staff members.

Introduction

It is well known and can easily be shown that, with present forecast techniques and accuracy, higher threat scores can be achieved by increasing the bias above unity. Figure 1 indicates the effect of increased bias on threat score. For instance, it indicates that a forecast with unit bias and with threat score of 0.2, can be improved to a threat score of 0.27 by increasing the bias to 2.6. Skill, therefore, cannot be judged by threat scores alone, but only in consideration with other statistics, especially bias.

Records of bias as well as threat score, prefigurance, and post agreement are kept at NMC but in comparing techniques and forecasters, if biases differ substantially, relative technical skill is unclear. The same can be said for determining progress in technical skill from a time series of such statistics.

It is important to distinguish between measures of technical skill and measures of the usefulness, or service value, of forecasts. Measures of both should be available. The need for our keeping watch on the utility of our forecasts is self evident. As for our technical skill, it may be too low in some forecast products to be useful for certain purposes. In such cases, we want to know how we are progressing toward the goal of utility, especially in the present era of steady advances in the state of the art. The modified threat score and measure of placement error that I will develop here, are intended to measure technical skill, not necessarily service value.

I should also add that a verification statistic is only useful to the extent that it agrees with the judgement of skilled practicing meteorologists. Thus, verification statistics are not entirely objective, for they are designed and selected subjectively. Like any other statistic, the modified threat score and measure of placement error must be judged useful in order to be useful to us.

My aim in modifying the threat score is to remove the effect of bias in overforecasting, and likewise to exact a penalty for underforecasting. I will introduce the notion of complex numbers for threat scores, although in practice they will occur only in individual cases, or for very short records, or for very large amounts of precipitation. I will suggest that, in the case of a complex number, its magnitude be taken as a negative threat score.

The threat score is

$$TS = \frac{H}{F + Q - H} \tag{1}$$

where F and Q are, respectively, forecast and observed areas or numbers of events, and H is "hits," i.e., the area or number of events correctly forecast.

One of my aims is to be able easily to compute the modified threat score from the past record. The only parameters that have been measured and for which records have been kept over the years at NMC are F, Q, and H.

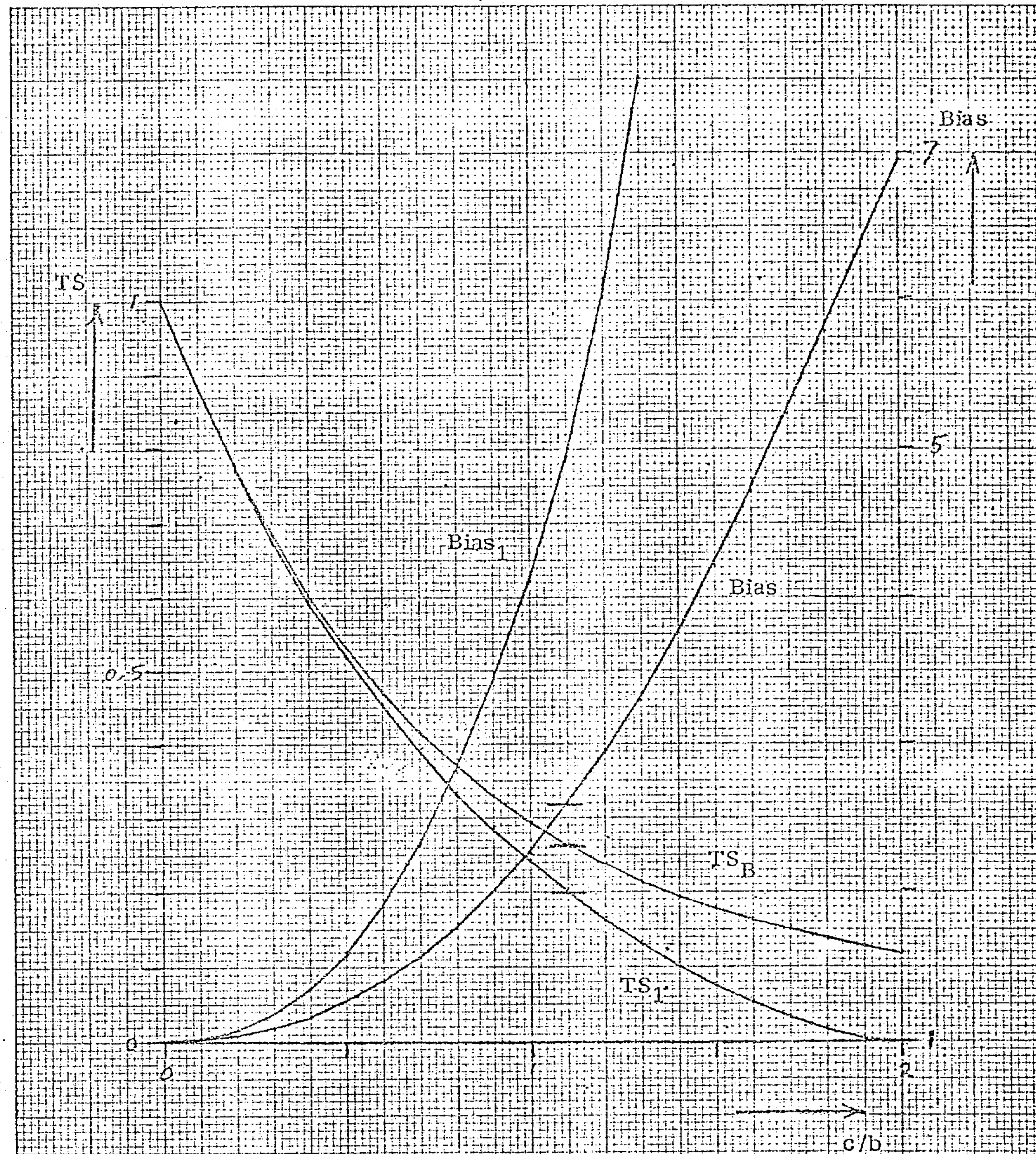


Figure 1. The effect of bias on threat score. The curves were calculated from my model (Figure 2). The curve labeled TS_1 shows threat score with unit bias and that labeled TS_B, the threat score with bias as shown by the Bias curve. TS_B is the maximum threat score that can be achieved by increasing bias, given the placement error, c, divided by the radius, b, of Q. For biases above the Bias curve shown, threat scores are lower than TS_B. For biases above the $Bias_1$ curve, the threat score is less than TS_1.

The modified threat score is designed to require only these three parameters, and only these are needed for placement error for an individual case. For a set of cases the number, N, of cases for which both F and Q are not zero in the set will also be needed to calculate placement error, but N can easily be counted from available records.

The approach I take is to model F and Q as two circles with areas equal to F and Q, respectively; H is thus modeled as the area of overlap, as shown in Figure 2.

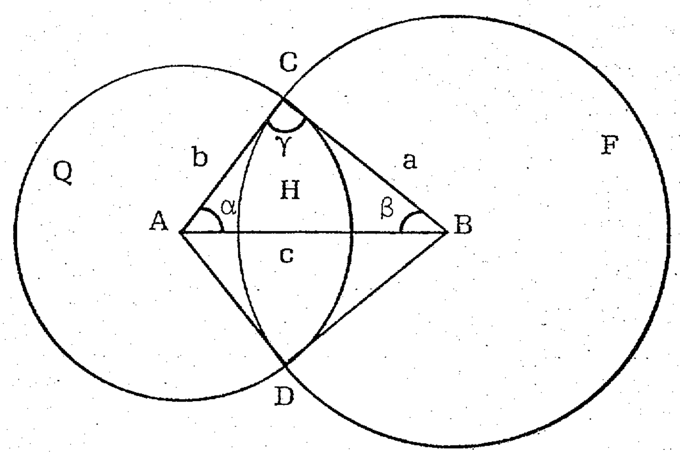


Figure 2. Model for calculating the modified threat score and placement error. Forecast events, F, and observed events, Q, are represented as areas of circles with radii a and b, respectively. The placement error, c, is the distance between the centers of the circles. "Hits," i.e., events correctly forecast is represented as H, the overlapping area.

It is evident from Figure 2 that, given a and b, H will vary monotonically with c, so long as circles A and B intersect. Thus, given H, F, and Q, it is possible to determine c.

To modify the threat score, the placement error, c, is first determined. Then holding c constant, the larger of F(a) and Q(b) is reduced to equal the other, thus changing the bias to unity. The modified threat score is then calculated from the new configuration.

Determination of placement error

In Figure 2, H is the area, $b^2\alpha$, of circular sector CAD; plus the area, $a^2\beta$, of circular sector CBD; less the area of quadrilateral ACBD. Because of the symmetry of ACBD about line segment AB, its area is twice the area, $\frac{1}{2}ab \sin\gamma$, of triangle ABC. But $\gamma = \pi - (\alpha + \beta)$, thus $\sin\gamma = \sin(\alpha+\beta)$. Therefore

$$H = b^2\alpha + a^2\beta - ab\sin(\alpha+\beta) \tag{2}$$

The cosine law gives

$$\cos\alpha = \frac{c^2 - (a^2 - b^2)}{2\,b\,c} \tag{3}$$

$$\cos\beta = \frac{c^2 + (a^2 - b^2)}{2\,a\,c}$$

and therefore α and β are functions of a, b, and c. The parameters a and b are given by

$$F = \pi a^2$$
$$Q = \pi b^2$$

H is thus a transcendental function of c, and given F, Q, and H, (2) can be solved for c. The method I use, based on Newton's iteration, is

$$c^{\nu+1} - c^{\nu} = \frac{1}{2}\left[\frac{b^2\alpha^{\nu} + a^2\beta^{\nu} - H}{b\sin\alpha^{\nu}} - c^{\nu}\right] \tag{4}$$

where
$$\alpha^{\nu} = \cos^{-1}\frac{(c^{\nu})^2 - (a^2 - b^2)^{\nu}}{2\,b\,c^{\nu}}$$
$$\beta^{\nu} = \cos^{-1}\frac{c^{\nu} - b\cos\alpha^{\nu}}{a}$$

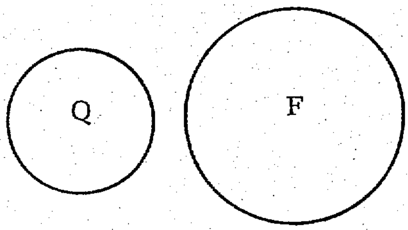
and where ν is the iteration count. As a flag to readers, the law of sines was used in deriving (4):

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

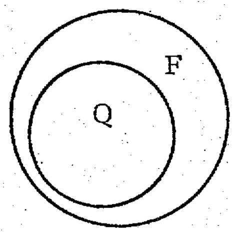
As a first guess, I use

$$c^{\nu=0} = \sqrt{a^2 - b^2} \quad (\alpha^{\nu=0} = \tfrac{1}{2}\pi) \text{ if } F > Q$$
$$c^{\nu=0} = \sqrt{b^2 - a^2} \quad (\beta^{\nu=0} = \tfrac{1}{2}\pi) \text{ if } F \leq Q$$

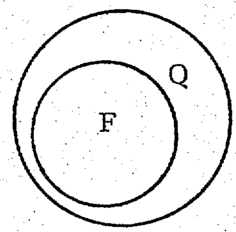
There are three cases for which c cannot be determined with my model of circles, without further assumptions. They are when the circles do not intersect:



Case I. H=0.



Case II. H=Q.



Case III. H=F.

In each of these cases, I assume that the two circles are tangent to each other. Thus I assume the configuration most favorable to the forecast in Case I, and least favorable in the other two cases. The result is

For Case I.	$H = 0$	$c = a + b$
For Case II.	$H = Q$	$c = a - b$
For Case III.	$H = F$	$c = b - a$

For these cases, iteration (4) is not required to find c.

<u>Determination of the modified threat score</u>

If $F > Q$, then $F' = Q$ and $a' = b$, where the primes indicate modified parameters. Then, from (1)

$$TS' = \frac{H'}{2Q - H'}$$

From (2),

$$H' = b^2 \left(2\alpha' - \sin 2\alpha'\right)$$

Thus,

$$TS' = \frac{2\alpha' - \sin 2\alpha'}{2\pi - (2\alpha' - \sin 2\alpha')} \tag{5}$$

where from (3)

$$\alpha' = \cos^{-1} \frac{c}{2b} \tag{6}$$

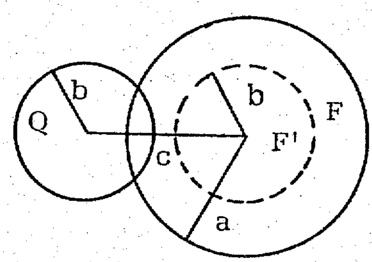
Likewise, if Q > F, then $Q' = F$ and $b' = a$, and

$$TS' = \frac{2\beta' - \sin 2\beta'}{2\pi - (2\beta' - \sin 2\beta')}$$

where

$$\beta' = \cos^{-1} \frac{c}{2a} \qquad (7)$$

Cases can and do occur in which the two circles in the modified configuration do not overlap nor are tangent. The figure below illustrates an example.



In such cases,

if F > Q	$c > 2b$	and	$\cos \alpha' > 1$
if Q > F	$c > 2a$	and	$\cos \beta' > 1$

and $\alpha' = \beta'$ is therefore an imaginary number, and TS' is a complex number. I note that

$$\cos iz = \cosh z$$
$$\sin iz = i \sinh z$$

for any z. I let

$$\alpha' = \beta' = iz$$

where z is real. The parameter, z, is found, in the case of F > Q, by substituting iz for α' into (6),

$$z = \cosh^{-1} \frac{c}{2b}$$

and in the case of Q > F, by substituting iz for β' in (7),

$$z = \cosh^{-1} \frac{c}{2a}$$

The positive root is always taken for z. Equation (5) then gives

$$TS' = \frac{-iq}{2\pi + iq}$$

where $q = \sinh 2z - 2z$

Thus,

$$TS' = -\frac{q^2 + 2i\pi q}{4\pi^2 + q^2}$$

The function, q, by the way, is always positive for positive z. Instead of dealing with complex numbers, I take the magnitude of TS' as a negative modified threat score:

$$TS'' = \frac{-q}{\sqrt{4\pi^2 + q^2}}$$

TS" is always negative, and it can easily be shown that $\partial TS''/\partial c$ is always negative, so that a forecast is penalized in the modified threat score by placement error, c. Also, $\partial TS''/\partial a$ or $\partial TS''/\partial b$ is always positive for $F > Q$ or $Q > F$, respectively, so that the smaller the precipitation areas, the lower the algebraic threat score. In the limit when Q or F, but not both, are zero the modified threat score is $TS'' = -1$, the minimum algebraic value for TS". When both Q and F are zero, such cases are simply not counted. The modified threat score is thus nicely limited:

$$-1 \leq TS'', TS' \leq +1$$

Figure 3 shows the variation of TS" with c/b $(F > Q)$. $TS'' = -0.996$ for $c/b = 12$, and approaches -1 asymptotically as c/b approaches infinity.

Some examples

To get a feeling for how the threat score is modified in practice using my model, I calculated the modified threat score from the annual records of QPF for the 1" area of accumulated precipitation in the first 24 hr for the 10 years, 1970 to 1979. The records are in the form of annual sums of F, Q, and H. I had the Quantitative Precipitation Branch count the number of days, N, in each year for which both F and Q were not zero, and then divided F, Q, and H by N before calculating. This was done so that the placement error, c, would be in the same unit of length that was used in the daily measurements of the area, F, Q, and H. That unit of length is a latitude degree.

Figure 4 shows the result. Although the forecasts are substantially penalized in varying amounts in the modified threat score for biases greater than one, especially so for the years 1974-6, there remains a high correlation between the two threat scores. The table below shows the correlation coefficient for that relationship, as well as for others.

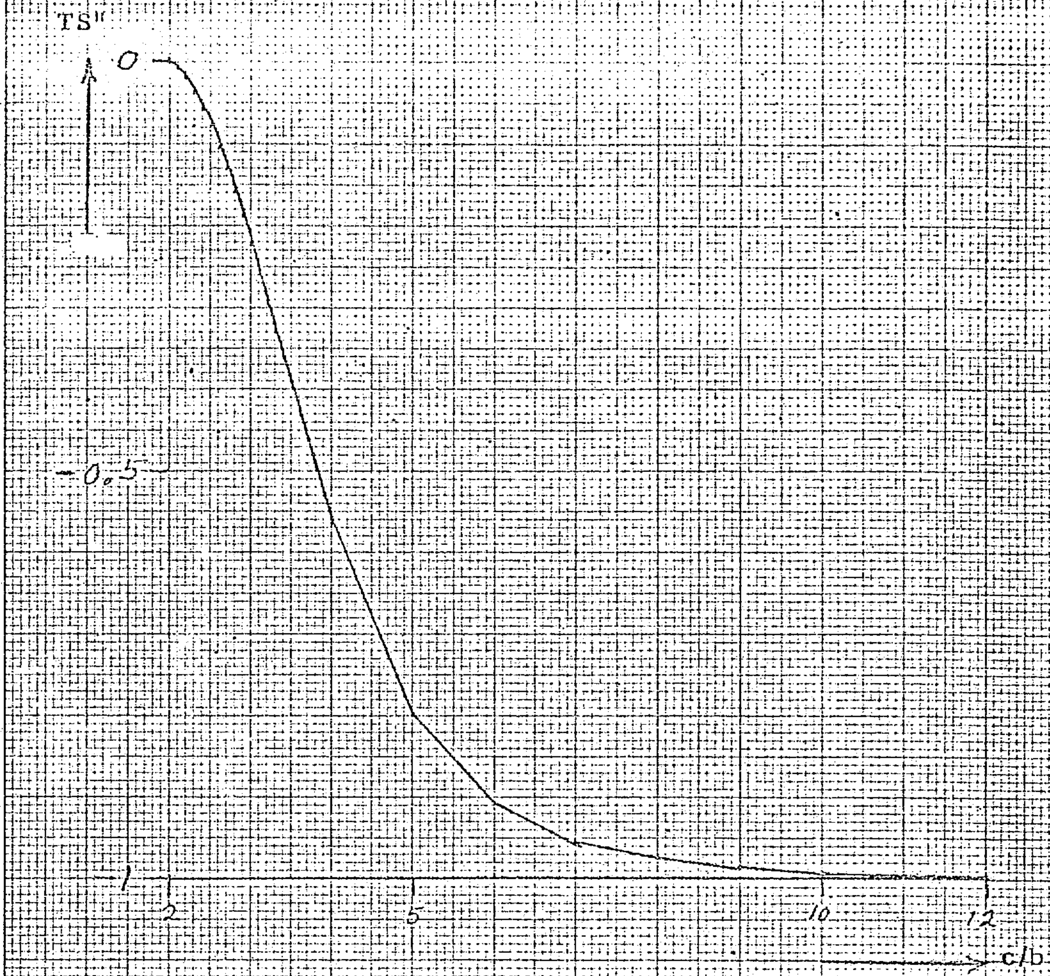


Figure 3. The variation of TS" with c/b.

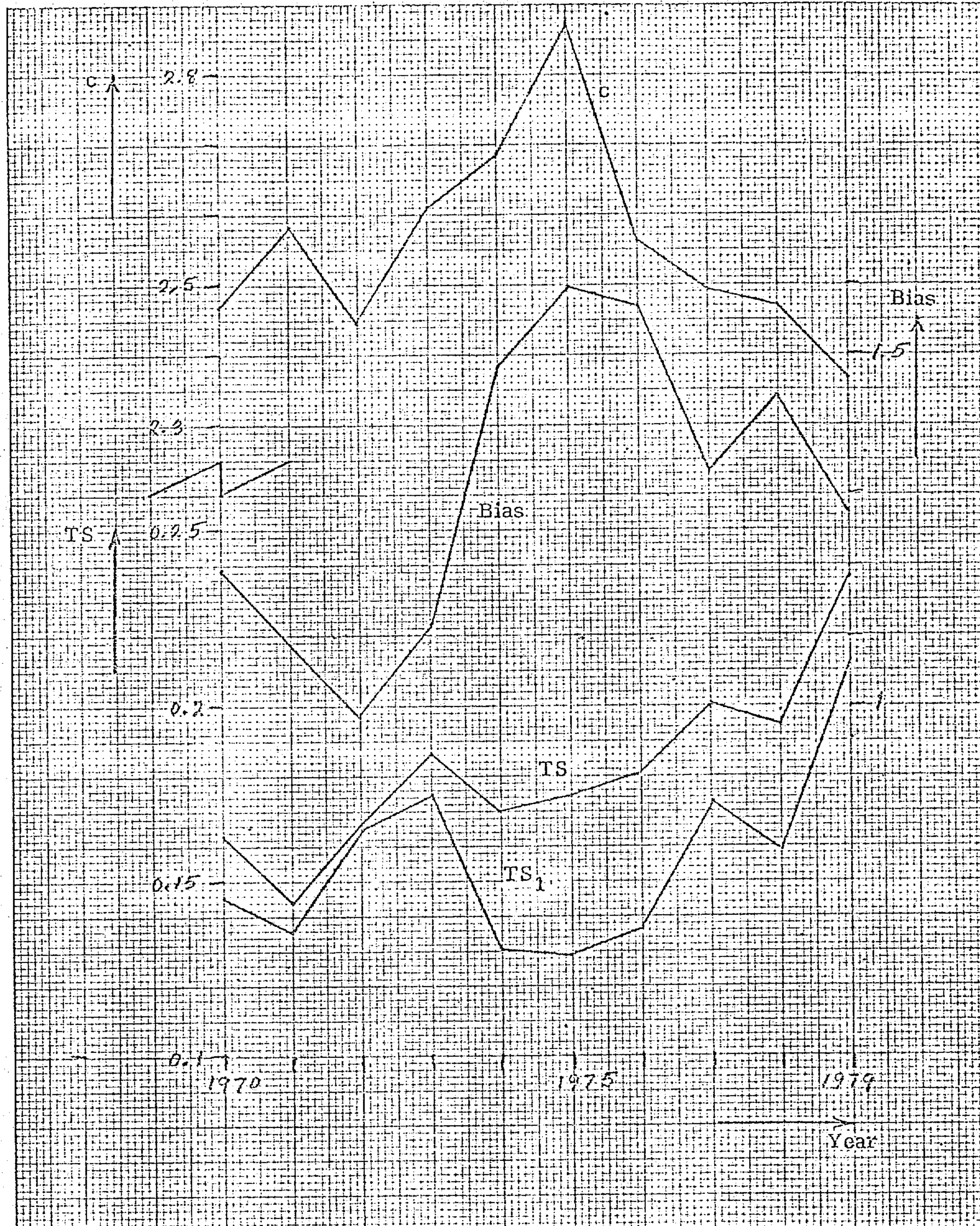


Figure 4. Verification statistics for accumulation of precipitation in the first 24 hr greater than one inch.

Table of correlation coefficients

	TS	TS'	Bias	c	c/b
TS	1				
TS'	+0.821	1			
Bias	+0.222	-0.374	1		
c	-0.417	-0.688	+0.502	1	
c/b	-0.799	-0.9981	+0.408	+0.702	1

The low correlation, 0.222, between Bias and TS, and relatively high correlations between c and TS and between c and Bias, support the notion that the forecaster tends to include more misses than hits when he increases the bias. Also, to the extent that this notion is correct, these correlations lend some credibility to the placement error as a valid concept. The remarkably high correlation between c/b and TS' is explained by the fact that TS' is a function of c/b only, if $F \geq Q$, which was true for all years except 1972, when $F = 0.988\ Q$. The departure of the correlation coefficient from unity mostly reflects the departure of the function from linearity, which is small because of the relatively small range of the numbers given (see the curve, TS_1, Figure 1). Note the high negative correlation coefficient between c/b and TS. This points up the fact that there is far more to be gained, in terms of threat score, by the forecaster reducing placement error relative to the dimensions of Q, than by increasing the bias.

I also went to the daily NMC records for 24 hr accumulated precipitation amounts verifying at 1200 GMT for the first week of January 1979. The table below shows a few of my more interesting calculations.

Sample of the daily record (1979)

Date		Amount (>)	F	Q	H	Bias	TS	TS',TS"	c	c/b
Jan 1	Day 1	½"	148.2	114.4	100.8	1.295	0.623	0.623	2.214	0.367
		3"	3.3	1.5	0	2.200	0	-0.074	1.716	2.483
Jan 2	Day 1	1"	59.5	51.7	39.6	1.151	0.553	0.545	1.895	0.467
		3"	3.2	0	0	∞	0	-1	1.009	∞
Jan 3	Day 1	½"	61.5	51.2	50.5	1.201	0.812	0.841	0.548	0.136
Jan 3	Day 1	2"	8.4	1.9	1.7	4.421	0.198	0.110	1.071	1.377
Jan 3	Day 1	3"	0	0.1	0	0	0	-1	0.178	0.998
Jan 5	Day 1	½"	10.5	2.6	0.2	4.038	0.016	-0.134	2.469	2.714
Jan 5	Day 2	½"	17.1	2.6	0	6.577	0	-0.419	3.243	3.565
Jan 6	Day 1	1"	1.0	7.2	0.4	0.139	0.051	-0.153	1.569	1.036

Note that TS' may be greater than TS, for example, in the case of the excellent forecast on Jan 3 for Day 1, for the $> \frac{1}{2}$" area. In that case, Figure 1 shows that the forecaster actually hurt his threat score, even with a bias as close to unity as 1.2. On the other hand, with a forecast not quite as good, on Jan 1, Day 1, $\geq \frac{1}{2}$", TS' is the same as TS with the accuracy shown, although the bias is higher, 1.3.

In conclusion, use of the modified threat score developed here appears to be a good way of removing the effect of bias on threat score, and the placement error appears to be related to skill in locating precipitation areas.

U.S. DEPARTMENT OF COMMERCE
National Oceanic and Atmospheric Administration
NATIONAL WEATHER SERVICE

National Meteorological Center
Washington, D. C. 20233

March 5, 1980

TO: Distribution

FROM: OA/W3 - Frederick G. Shuman F G Shuman

SUBJECT: Correction to NMC Office Note 210

The iteration given on page 4 will not converge if the bias exactly equals one (a = b). The problem is the first guess given near the bottom of page 4. A better first guess is given on the corrected page 4, attached.

Attachment

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As a first guess, I use

$$c^{\nu=0} = \sqrt{a^2 + b^2} \quad (\gamma^{\nu=0} = \alpha^{\nu=0} + \beta^{\nu=0} = \tfrac{1}{2}\pi)$$

[Corrected March 5, 1980]

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