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# A New Mass-Field Variable for Normal Mode Procedures with a Sigma Coordinate Model

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## Abstract

A new mass-field variable is defined for use in normal mode initialization of a sigma coordinate model. The new variable is a nonlinear function of surface pressure and temperature whose Laplacian gives the full mass-field contribution to the divergence tendency. It is found that gravitational zeroing results in significantly smaller changes and a better balance with the new mass variable than with the old. For first order nonlinear initialization, use of the new mass variable yields only minor improvement in a global sense, but may be significant in the areas of high resolution steep orography.

## 1. Introduction

There are many forecast models with a vertical coordinate which is some function of P/Ps, where P is pressure, and Ps is surface pressure. A frequently used vertical coordinate, introduced by Phillips (1959), is $\sigma$ = P/Ps, which will be the only vertical coordinate addressed here. In addition, many such models use some form of normal mode initialization, the subject of which has been reviewed by Daley (1981). For normal mode initialization with such models, it has been conventional to employ a mass-field variable whose gradient gives the linearized pressure gradient force. For example, Temperton and Williamson (1981) use a mass-field variable $W = \phi + RT_0 \log Ps$, where $\phi$ is the geopotential height of the $\sigma$ surface, R the gas constant, and $T_0 = T_0(\sigma)$ only is a mean temperature. The model's linearized equations can then be written as functions of the winds and $W$, making possible a separation of horizontal and vertical dependence. Normal modes can then be calculated and used for initialization with the model.

Although the above conventional linearization works well in practice, it may not be optimal as $-\nabla W$ is not the full pressure force, $-\nabla\phi - RT\nabla \log Ps$. Thus $-\nabla W$ differs from the full pressure gradient force by $(T-T_0)\nabla \log Ps$, which can be significant, especially over mountains. The use of $W$ in linear initialization, where gravity mode amplitudes in the initial state are set to zero, can lead to error. For example, consider an initial model state that is geostrophic. Here we expect linear initialization to make small changes to the initial state. However, when a modal analysis is performed using $W$ as the mass field, the initial state will not be analyzed as being purely geostrophic as $-\nabla W$ is not the full pressure gradient force and does not fully balance the Coriolis force. Thus the linear initialization will produce unnecessary height and vorticity changes in areas that were geostrophic. These unnecessary changes tend to be largest over mountains and may be large enough to not be corrected by nonlinear initialization.

When one uses nonlinear normal mode initialization, the error in using $W$ as the mass-field variable is more subtle. We know that the nonlinear initialization adjusts the initial vertical velocity so that the mass field and wind field tendencies are approximately geostrophic, Leith (1980). However, when $W$ is used as the mass-field variable, it is only an approximation of the net mass field that is used, which could lead to error in the initial vertical velocity.

In this paper a new mass variable $W'$ is introduced, which is a nonlinear function of $\phi$, T, and the surface pressure. Here $W'$ is defined so that $\nabla^2 W'$ gives the full contribution of the mass field to the divergence tendency, which is relevant as the model and its modes are in divergence and vorticity form. To compare the use of $W'$ instead of $W$ in normal mode initialization, a 12-level version of the National Meteorological Center's global spectral model, Sela (1980), with rhomboidal trunction to 24 waves, is used. Earlier work on initialization of the model can be found in Ballish (1980), and more details on the model can be found in Sela (1982).

The normal mode initialization schemes used in tests here will be fairly basic. The first scheme to be examined is linear normal mode initialization, also referred to as gravitational zeroing, which was first tested by Williamson (1976) with a shallow water model. Some experiments will use Baer-Tribbia (1977) initialization, hereafter referred to as BTI, to first order accuracy. The first order BTI differs from the frequently used Machenhauer (1977) procedure in that BTI uses gravitational zeroing before any nonlinear balance is attempted. In addition first order BTI requires only one iteration of nonlinear correction. To illustrate the differences in these schemes, we write a gravity mode tendency as $\dot{Y} = i\nu Y + N(\underset{\sim}{X},\underset{\sim}{Y})$, where Y represents a single gravity mode, $\underset{\sim}{Y}$ represents them all, $\underset{\sim}{X}$ represents all the Rossby modes, $\nu$ is the single modes frequency, and N represents nonlinear contributions to $\dot{Y}$. First order BTI results in $Y = \frac{-1}{i\nu} N(\underset{\sim}{X},0)$. The Machenhauer procedure requires first calculating $\dot{Y}$ and then setting $\Delta Y = -\dot{Y}/i\nu$ in attempt to make $\dot{Y} = 0$. Since the system is nonlinear, one can iterate in attempt to make $\dot{Y}$ exactly 0. If the iterative procedure converges it results in $Y = \frac{-1}{i\nu} N(\underset{\sim}{X},\underset{\sim}{Y})$. The first order BTI is equivalent to gravitational zeroing followed by one iteration of Machenhauer's nonlinear correction. The conventional Machenhauer procedure uses no zeroing and usually involves a number of iterations. Here we are concerned with how the use of $W'$ instead of $W$ affects the performance of the initialization procedures.

## 2. Normal Mode Decomposition

### a. Model Equations

The model's basic dynamic variables are vorticity $\xi$, divergence D, temperature T, and the logarithm of surface pressure logPs, which will hereafter be referred to as Q. The model is hydrostatic with the geopotential given by $\tilde{\phi} = \tilde{\phi}_s + A\tilde{T}$, where the tilda indicates that we are dealing with a 12 level vector $\tilde{\phi} = (\phi_1, \phi_2, \cdots \phi_{12})^T$, $\tilde{\phi}_s$ is a vector with $\phi = \phi_s$ the surface geopotential at all 12 levels, and A is a constant 12x12 matrix. Model tendencies are written

$$\frac{\partial D}{\partial t} = \vec{K} \cdot \nabla \times f\vec{V} - \nabla^2 \phi - R T_0 \nabla^2 \varphi + NL_D \tag{1}$$

$$\frac{\partial \xi}{\partial t} = -\nabla \cdot f\vec{V} + NL_\xi \tag{2}$$

$$\frac{\partial \tilde{T}}{\partial t} = B\tilde{D} + \tilde{NL}_T \tag{3}$$

$$\frac{\partial Q}{\partial t} = -\tilde{\Delta}\cdot\tilde{D} + NL_Q \quad . \tag{4}$$

Here $\vec{V}$ is a 2-dimensional vector holding the 2 horizontal wind components, NL represents nonlinear terms which are calculated in the model, $\tilde{\Delta}$ is a vector designed so that $\tilde{\Delta}\cdot\tilde{D}$ is a finite difference version of $-\int_0^1 D\,d\sigma$, and B is a constant 12x12 matrix.

b. Standard Normal Mode Calculation

The first step in deriving normal modes is to note that the divergence tendency may be written

$$\frac{\partial D}{\partial t} = \vec{K}\cdot\nabla\times f\vec{V} - \nabla^2 W + NL_D \tag{5}$$

where $W = \phi + RT_0 Q$ . For the tendency of $W$, we see that $\partial\phi/\partial t = AB\tilde{D} + A\cdot\tilde{NL}_T$ and $\partial/\partial t\ R\tilde{T}_0 Q = RT_0(-\tilde{\Delta}\cdot\tilde{D} + NL_Q)$ . Now we can rewrite the model's linearized equations in terms of D, $\xi$, and $W$, see Sela (1982).

$$\frac{\partial D}{\partial t} = \vec{K}\cdot\nabla\times f\vec{V} - \nabla^2 W \tag{6}$$

$$\frac{\partial \xi}{\partial t} = -\nabla\cdot f\vec{V} \tag{7}$$

$$\frac{\partial \tilde{W}}{\partial t} = (AB - R\tilde{T}_0\tilde{\Delta}^T)\tilde{D} \equiv G\tilde{D} \tag{8}$$

Note that derivatives of $f\vec{V}$ will be evaluated spectrally in terms of $\xi$ and D.

Temperature and surface pressure are no longer directly involved in the linear equations, but we will see later that they can be calculated from $W$. The only vertical coupling that occurs in (6)-(8) is that $\partial W/\partial t$ depends on D at all levels. A separation of variables occurs if we expand D, $\xi$, and $W$ simultaneously in the eigenvectors of G. The eigenvalues of G are written as -gh, where h is the equivalent depth. The eigenvectors of G are the vertical modes. Then for a given vertical mode the horizontal linear equations are

$$\frac{\partial D}{\partial t} = \vec{K} \cdot \nabla \times f \vec{V} - \nabla^2 W \tag{9}$$

$$\frac{\partial \xi}{\partial t} = -\nabla \cdot f \vec{V} \tag{10}$$

$$\frac{\partial W}{\partial t} = -ghD \;. \tag{11}$$

Horizontal modes can now be solved on a computer.

After the horizontal modes have been computed, we still have the problem of how to decompose $W$ into its contribution to Q and T. We solve the problem by requiring all model fields in the linearized equations to oscillate with frequency $\nu$ for a given isolated mode of frequency $\nu$. Thus $\frac{\partial \varphi}{\partial t} = -\tilde{\Delta} \cdot \hat{D} = i\nu\varphi$ and $\frac{\partial W}{\partial t} = i\nu W = -ghD$ gives

$$\varphi = \tilde{\Delta} \cdot \tilde{W}/gh \;, \tag{12}$$

where Q is the logarithm of surface pressure associated with $\tilde{W}$, where $\tilde{W}$ is the vector of $W$ due to one vertical mode evaluated on model levels. Once Q has been evaluated, we can calculate T by using the definition of $W$.

Now that the normal mode calculation has been discussed, we can examine how to use the modes for initialization purposes. For linear modal initialization, we calculate $W$ from model fields and then use D, $\zeta$, and $W$ to modally analyze the gravitational projection of the model fields. The gravitational projection for $W$ is converted to the projection in T and Q. These gravitational projections are then subtracted from the given initial fields, with the result being the linearly initialized fields. To perform the first order correction of BTI or to do one iteration of Machenhauer's procedure, one calculates model tendencies $\dot{D}$, $\dot{\zeta}$, $\dot{T}$, and $\dot{Q}$. Then $\dot{W}$ is calculated and the gravitational projection of $\dot{D}$, $\dot{\zeta}$, and $\dot{W}$ is performed. For a given gravity mode tendency $\dot{Y}$, a change $\Delta Y = -\dot{Y}/i\nu$ is made. The gravitational changes are converted to changes in D, $\zeta$, and $W$, with the changes in $W$ converted to changes in T and Q.

c. Mass Field Modified Modal Calculations

The model divergence equation can be written

$$\frac{\partial D}{\partial t} = \vec{k}\cdot\nabla\times f\vec{V} - \nabla^2\phi - RT\nabla^2\varphi - R\nabla T\cdot\nabla\varphi + NL'_D \tag{13}$$

where $NL'_D$ is a nonlinear function of D, $\zeta$, and $\dot{\sigma}$ only. Equation (13) can be written more compactly as

$$\frac{\partial D}{\partial t} = \vec{k}\cdot\nabla\times f\vec{V} - \nabla^2 W' + NL'_D \tag{14}$$

where

$$W' = \phi + R(\nabla^2)^{-1}(T\nabla^2\varphi + \nabla T\cdot\nabla\varphi), \tag{15}$$

with $(\nabla^2)^{-1} = 0.0$ for mean spectral fields and is $-a^2/n(n+1)$ for spectral coefficients with $n \neq 0$, with a the radius of the Earth and n a spherical harmonic meridional integer index. The special case of setting $(\nabla^2)^{-1} = 0.0$ for $n = 0$ is to prevent division by 0.0 but does not harm the results of (15) as a mean mass field with $n = 0$ has no gravitational projection or contribution to $\partial D/\partial t$. Now the full contribution to the pressure gradient force is contained in $\nabla^2 W'$, and $NL_D'$ contains less nonlinear terms than does $NL_D$.

Although $W'$ contains more information on the pressure gradient force and gives a more efficient linearization of $\partial D/\partial t$ than does $W$; it adds some complexity to the mass field tendency. Before we had $\partial \tilde{W}/\partial t = G\tilde{D} + \widetilde{NL}_W$, which is replaced by

$$\frac{\partial \tilde{W}'}{\partial t} = G\,\tilde{D} + \widetilde{NL}_{W'} \qquad (16)$$

where $NL_{W'}$ has some cubic nonlinear terms that are not present in $NL_W$. It is possible that the added complexity of cubic nonlinearity may lessen the net beneficial effects of using $W'$ instead of $W$.

When $W'$ is used as the mass-field variable the linearized equations remain unchanged except that $W$ is replaced by $W'$ in (9)-(11). Thus one does not have to recompute normal modes. However, there is some difference in how the modes are used. The standard linear and nonlinear initialization procedures require respectively input values of D, $\xi$, and $W$ or $\dot{D}$, $\dot{\xi}$, and $\dot{W}$. When using $W'$ instead of $W$, $W$ and $\dot{W}$ must be replaced by $W'$ and $\dot{W}'$ respectively. On output of both the standard linear and nonlinear procedures will be desired changes in D, $\xi$, and $W$. When $W'$ is used, these procedures request changes in D, $\xi$, and $W'$. The changes in temperature and surface pressure due to the change in $W$ are derived from (12) and the definition of $W$. It is more complicated when changes in $W'$ are requested, as it is a nonlinear function of temperature and surface pressure. In this paper changes $\delta T$ and $\delta Q$ based on $\delta W'$ are calculated the same way as with $\delta W$. Thus $\delta T$ and $\delta Q$ only approximately give the desired change in $\delta W'$. One could try iterative procedures for finding exact values of $\delta T$ and $\delta Q$ that would give the desired $\delta W'$, but that is not considered here.

3. Results of Gravitational Zeroing

In this section we examine gravitational zeroing applied to various cases. Standard zeroing is compared to zeroing using $W'$ instead of $W$. Attention will be given to the magnitude of the changes produced by the zeroing as well as the state of balance after zeroing. The assumption is made that it is beneficial and desirable for any change in the initialization to result in both smaller changes to the model fields and a better balance. Before testing zeroing with actual analysis cases, we examine the dynamics of linear initialization when using standard atmospheric data.

a. Standard Atmosphere Case

Before investigating model normal mode initialization with standard atmosphere data, we examine how much $-\nabla W$ differs from the pressure gradient force in a constant lapse rate standard atmosphere. Here we assume that there are zero winds and a uniformly constant lapse rate $\partial T/\partial z = -\gamma$. Using the hydrostatic assumption, one finds

$$\log(P_1/P_2) = \frac{g}{R\gamma} \log\left(\frac{T_{oo} - \gamma Z_1}{T_{oo} - \gamma Z_2}\right) \tag{17}$$

where $T_{oo}$ is T at Z = 0 and is a constant, with $P_1$ and $P_2$ being respectively pressures at heights $Z_1$ and $Z_2$. In addition, at a constant $\sigma$ level we have

$$Z = Z_s\, \sigma^{R\gamma/g} \quad \text{and} \quad T = \sigma^{R\gamma/g}\,(T_{oo} - \gamma Z_s)$$

where $Z_s$ is the surface height. Furthermore, $P_s = P_{oo}\left(\frac{T_{oo} - \gamma Z_s}{T_{oo}}\right)^{g/R\gamma}$ where Ps is the surface pressure and Poo is the pressure at Z = 0. Now $-\nabla W$ differs from the full pressure gradient force by $\delta\vec{F} = R(T - T_o)\,\nabla \log P_s$ , where $T_o$ is the mean temperature used in the linearization. Here we take $T_o = T_o(\sigma) = T_{oo}\,\sigma^{R\gamma/g}$ , then

$$\delta\vec{F} = \gamma Z_s\, \sigma^{R\gamma/g}\, \frac{g\,\nabla Z_s}{(T_{oo} - \gamma Z_s)} \quad . \tag{18}$$

We now estimate the magnitude of $\delta\vec{F}$ in the spectral model over the Himalayas with the above lapse rate condition. Based on maps of the model's orography, we take a point with Z $\approx$ 2 km, and a gradient of roughly 2 km in height over 500 km horizontal distance. If we take $\gamma$ = -6.5°/km and assume $T_{oo}$ = 285°, we find $\delta F \approx$ 2.88 x $10^{-3}$ m/sec$^2$. This is large and corresponds to a geostrophic wind of roughly 39 m/sec. Thus $-\nabla W$ differs significantly from the full pressure gradient force. This difference can be significantly higher in models with high resolution orography.

Now we examine performing gravitational zeroing with the model when its initial data is derived from a standard atmosphere, which has zero winds and constant heights. These fields are used to extrapolate the fields of temperature and surface pressure required by the model. This extrapolation is not perfect, as the model has small nonzero pressure gradient forces in the $\sigma$ levels. The amplitude of imbalance due to the above error is small; for example, if Machenhauer's initialization is applied to this case for all 12 vertical modes, the vertically averaged temperature change is .097°, and the maximum surface pressure change is only .122 mb.

Although Machenhauer's nonlinear initialization results in small changes with the standard atmosphere case, we will see that simple linear initialization results in some significant changes. When standard gravitational zeroing is applied in this case for all 12 vertical modes, the following maximum changes in model fields result: in surface pressure, 12.3 mb; in temperature, 4.66° at level 9; in wind speed, 14.0 m/sec at level 8, all occurring over the Himalayas. When only 4 vertical modes are zeroed, the maximum changes are: in surface pressure, 12.6 mb; in temperature 4.87°; and in wind speed, 6.65 m/sec. These changes are unnecessary and are large enough to have impact on possible further initialization steps.

When gravitational zeroing is performed for all 12 vertical modes with the standard atmosphere case, but with $W'$ used instead of $W$, the maximum changes are: in surface pressure, .029 mb; in temperature, .094° at level 8; in wind speed, .429 m/sec at level 7. These changes are due to small imbalances in the model rather than due to error in the modified zeroing. Thus the mass field modified linear initialization is making small changes to the standard atmosphere case.

b. Application to a 24 Hour Forecast

Here we examine gravitational zeroing applied to a 24 hour forecast from an analysis valid 0000 GMT 11 March 1981. The forecast should be fairly balanced as first order BTI for all 12 vertical modes was used at the initial time plus the model tends to disperse transient gravity waves. By applying initialization to this forecast, we can judge the initialization scheme according to the size of the changes produced. Here smaller changes are preferred. If we were initializing an analysis, such judgment is less valid as the initialization may be correctly making large changes.

For gravitational zeroing for N vertical modes, we use the notation ZN. Thus Z12 represents gravitational zeroing for 12 vertical modes. For gravitational zeroing using $W'$ instead of $W$, we use the notation Z'6 to represent zeroing for 6 vertical modes, etc.

In Table 1 we compare RMS changes to model fields due to Z12 and Z'12 gravitational zeroing of the 24 hour forecast. Vertically averaged RMS changes in temperature, and vorticity are denoted as $\delta T$ and $\delta \xi$ respectively, while $\delta Q$ denotes the RMS change in Q. In addition, the table lists $(\delta Ps)_{max}$ the global maximum change in surface pressure due to the zeroing and $\dot{D}$ the vertically averaged RMS divergence tendency after initialization. The Z*12 initialization procedure will be discussed in Section 3d. The RMS changes in divergence and the RMS vorticity, temperature, and surface pressure tendencies are similar after the two zeroings, and are not shown in the table. From the numbers in Table 1, it is clear the Z' zeroing results in both smaller changes and a better balance then the standard zeroing. The Z' zeroing results in distinctly smaller mass field and vorticity changes and a smaller divergence tendency. If less vertical modes are zeroed, the RMS changes are reduced.

Table 1
Some Diagnostics on Results of Gravitational Zeroing on a 24-Hour Forecast from 0000 GMT 11 March 1981

| Diagnostic | | Initialization | |
|---|---|---|---|
| | Z12 | Z'12 | Z*12 |
| $\delta Q$ | $.243\times10^{-2}$ | $.126\times10^{-2}$ | $.199\times10^{-2}$ |
| $\delta T$ | .748° | .5991° | .7270° |
| $\delta \xi$ | $.6298\times10^{-5}$ sec$^{-1}$ | $.5786\times10^{-5}$ sec$^{-1}$ | $.6180\times10^{-5}$ sec$^{-1}$ |
| $(\delta Ps)_{MAX}$ | 15.43 mb | 5.60 mb | 11.03 mb |
| $\dot{D}$ | $.1562\times10^{-8}$ sec$^{-2}$ | $.7694\times10^{-9}$ sec$^{-2}$ | $.1161\times10^{-8}$ sec$^{-2}$ |

c. Application to Analyses

In this section, gravitational zeroing is applied to analyses resulting from the NMC assimilation system described in McPherson et al. (1979) and Kistler and Parrish (1982). Again we find that the mass field modified zeroing results in significantly smaller changes to the initial state and a noticeably better balance.

For the analysis valid 0000 GMT 11 March 1981, we apply Z6, Z'6, Z12, and Z'12 initializations. Table 2 gives vertically averaged RMS changes in temperature, divergence, and vorticity, denoted as $\delta T$, $\delta D$, and $\delta \xi$, while $\delta Q$ denotes the RMS change in Q. Also, $\dot{D}$ is the vertically averaged RMS divergence tendency after the zeroing. We see that the Z and Z' zeroings result in similar RMS changes in divergence, but the Z' zeroing results in smaller changes to vorticity, temperature, and surface pressure. Not shown in the table is the result that the Z and Z' zeroings result in similar vorticity and mass field tendencies. However the Z' zeroing results in a significantly smaller RMS divergence tendency. We see that the 12 vertical mode zeroing produces bigger RMS changes than does 6 vertical mode zeroing, except for the RMS change in Q. The Z' zeroing results in a smaller RMS divergence tendency, with Z'12 zeroing giving the smallest value of $\dot{D}$.

For the analysis at 0000 GMT 9 August 1982, Z6 and Z'6 zeroings were performed. In this case, the analysis is less noisy and energetic than in the 11 March case. Table 3 gives the same diagnostics as in Table 2, but for this experiment. Here there appears to be a larger difference between Z'6 and Z6 zeroing than in the previous comparison. For example the Z'6 zeroing has less than one half the RMS change in Q of the Z6 zeroing. The larger relative difference is probably due to this analysis having a fairly good balance.

Table 2
Some Diagnostics on Results of Gravitational Zeroing of the 0000 GMT 11 March 1981 Analysis

| Diagnostic | Initialization Z6 | Z'6 | Z12 | Z'12 |
|---|---|---|---|---|
| $\delta D$* | .7927 | .7915 | 1.199 | 1.199 |
| $\delta\zeta$* | .5177 | .4597 | .9528 | .9124 |
| $\delta T$ | 1.014° | .8543° | 1.032° | .9304° |
| $\delta Q$ | $.355\times10^{-2}$ | $.249\times10^{-2}$ | $.353\times10^{-2}$ | $.249\times10^{-2}$ |
| $\dot{D}$** | .1477 | .1208 | .1387 | .0846 |

*units of $10^{-5}$ $sec^{-1}$

**units of $10^{-8}$ $sec^{-2}$

Table 3
Some Diagnostics on Results of Gravitational Zeroing of the 0000 GMT 9 August 1982 Analysis

| Diagnostic | Initialization Z6 | Z'6 |
|---|---|---|
| $\delta D$* | .5725 | .5723 |
| $\delta\zeta$ * | .3245 | .2884 |
| $\delta T$ | .7299° | .5876° |
| $\delta Q$ | $.433\times10^{-2}$ | $.200\times10^{-2}$ |
| $\dot{D}$** | .1190 | .0679 |

*units of $10^{-5}$ $sec^{-1}$

**units of $10^{-8}$ $sec^{-2}$

The results of these gravitational zeroing experiments indicate that the Z' zeroing is superior to the standard zeroing. The conventional zeroing uses a simplified mass-field variable which makes the vorticity and mass field relationship seem less geostrophic than it really is. The modal analysis then makes unnecessary vorticity and mass field changes. As a result, the conventional zeroing causes larger RMS vorticity and mass field changes as well as a larger divergence tendency. However, the divergence changes are not appreciably altered by the mass field modification.

d. Rest-state Modified Zeroing

As pointed out in Section 3a, a standard atmosphere is analyzed to have significant gravitational projection when $W$ , not $W'$ , is used in modal analysis. These gravitational fields have their largest amplitudes over mountains. Thus standard gravitational zeroing can result in unnecessary sizable changes over mountain areas. In attempt to correct this defect in zeroing, Ballish (1980) introduced a modification of the zeroing step. After setting to zero the amplitude of gravity modes being adjusted, he added on the gravitational fields of the model with standard atmosphere data. This modification was an improvement over pure zeroing, but we will see that it is considerably less effective than the mass field modified initialization introduced here.

When the above rest-state modified zeroing is applied to the 24-hour forecast used in Section 3b, we find that the RMS changes are smaller and the balance is better than when pure zeroing is performed. Table 1 gives some diagnostics on the results of rest-state modified zeroing for 12 vertical modes, denoted Z*12, applied to the forecast. We see that the Z*12 zeroing gives smaller changes and a better balance than regular Z12 zeroing, but the Z'12 zeroing is better yet. Evidentally the balanced gravity wave fields associated with mountains are only roughly approximated by the gravitational fields of a standard atmosphere with mountains.

## 4. Results After Nonlinear Balancing

In this section, the results of experiments with first order BTI using mass-field variables $W$ and $W'$ are discussed. We will use the notation 1BT6 to denote first order BTI for 6 vertical modes where $W$ is the mass-field variable employed. The notation 1BT'12 denotes first order BTI for 12 vertical modes with $W'$ the mass-field variable, etc. With the new mass-field variable $W'$ we found that gravitational zeroing led to smaller changes and a better balance than standard zeroing. Here we find that 1BT' initialization results in smaller changes and a better balance than 1BT initialization, but the differences are smaller than with gravitational zeroing.

For the analysis at 0000 GMT 9 August 1982, 1BT6 and 1BT'6 initializations are applied, with diagnostics on these experiments given in Table 4. These diagnostics include vertically averaged RMS divergence and temperature tendencies along with RMS tendencies in Q for the analysis, the initialized analysis, and the results of six hour forecasts after initialization. Based on the tendencies shown in Table 4, it is evident that both the 1BT6 and 1BT'6 initializations result in a significantly better balance than the analysis. The 1BT'6 experiment appears to have a better state of balance at both t=0 and t=6 hours than that of the 1BT6 experiment. In addition, Table 5 shows that the 1BT'6 initialization resulted in smaller RMS changes to model fields.

Table 4
RMS Model Tendencies for the 0000 GMT 9 August 1982 Case

| | $\dot{D}$ | $\dot{T}$ | $\dot{Q}$ |
|---|---|---|---|
| Analysis | 2.142 | 1.894 | 2.953 |
| 1BT6 (t=0) | .4938 | .4555 | .0926 |
| 1BT'6 (t=0) | .4749 | .4546 | .0867 |
| 1BT6 (t=6) | .4951 | .4704 | .1527 |
| 1BT'6 (t=6) | .4876 | .4630 | .1273 |
| | $\times10^{-8}$ sec$^{-2}$ | $\times10^{-4}$ °/sec | $\times10^{-6}$ sec$^{-1}$ |

Table 5
RMS Model Changes Due to Initialization of 0000 GMT 9 August 1982 Analysis

| Initialization | $\delta D$* | $\delta\xi$ * | $\delta T$ | $\delta Q$ |
|---|---|---|---|---|
| 1BT6 | .5302 | .2708 | .5124° | $.1886\times10^{-2}$ |
| 1BT'6 | .5288 | .2696 | .5088° | $.1799\times10^{-2}$ |

*units of $10^{-5}$ sec$^{-1}$

The above 1BT6 and 1BT'6 initializations were also applied to the analysis valid 0000 GMT 11 March 1981. This case is unusually noisy and may present problems for the initialization schemes. In Table 6 we list some RMS model tendencies for 1BT6 and 1BT'6 initialization experiments both at t=0 and t=6 hours after initialization. The tendencies indicate a better balance in the 1BT'6 experiment, but the differences are smaller in this noisy case. In addition, as indicated in Table 7, the 1BT'6 initialization again results in smaller RMS changes.

Table 6
RMS Model Tendencies for the 0000 GMT 11 March 1981 Case

| | $\dot{D}$ | $\dot{T}$ | $\dot{Q}$ |
|---|---|---|---|
| Analysis | 3.059 | 2.368 | 3.22 |
| 1BT6 (t=0) | .9465 | .8043 | .114 |
| 1BT'6 (t=0) | .9358 | .7954 | .106 |
| 1BT6 (t=6) | .9490 | .8248 | .142 |
| 1BT'6 (t=6) | .9403 | .8105 | .138 |
| | $\times 10^{-8}$ sec$^{-2}$ | $\times 10^{-4}$ °/sec | $\times 10^{-6}$ sec$^{-1}$ |

Table 7
RMS Changes Due to Initialization of 0000 GMT 11 March 1982 Analysis

| Initialization | $\delta D$* | $\delta \xi$ * | $\delta T$ | $\delta Q$ |
|---|---|---|---|---|
| 1BT6 | .7324 | .4229 | .7800° | $.217 \times 10^{-2}$ |
| 1BT'6 | .7321 | .4213 | .7791° | $.214 \times 10^{-2}$ |

*units of $10^{-5}$ sec$^{-1}$

Nonlinear initialization expeirments were also performed with the 24-hour forecast discussed in Section 3 as input to the initialization. This time a modification of 1BT12 and 1BT'12 initializations was used. The modification was an additional iteration of the Machenhauer procedure for the external made only after performing first order initialization. These initializations with the extra external mode step are denoted 1BT12$^+$ and 1BT'12$^+$. Here we do not explain all the reasons for the modification, but note that it results in a reduction of surface pressure oscillations during forecasts. Table 8 lists some RMS model tendencies for the forecast, the 1BT12$^+$ and 1BT'12$^+$ initializations of the forecast, plus the tendencies after 6 hours of forecast after initialization. The tendencies indicate that the initialization is improving the balance of the forecast. But we should note these diagnostic tendencies do not include the effects of latent heating, convection, horizontal diffusion, or the semi-implicit time intergration, all of which are significant in the forecast. The $1BT'12^{+}$ initialization indicates a better balance initially and at t=6 hours of forecasting. In addition, the $1BT'12^{+}$ initialization resulted in smaller RMS changes as indicated in Table 9.

Table 8
Some RMS Model Tendencies Associated with the 24-Hour
Forecast from 0000 GMT 11 March 1981

| | $\dot{D}$ | $\dot{T}$ | $\dot{Q}$ |
|---|---|---|---|
| Forecast | .6103 | .7619 | .992 |
| $1BT12^{+}$ (t=0) | .4727 | .7615 | .984 |
| $1BT'12^{+}$ (t=0) | .4551 | .7496 | .968 |
| $1BT12^{+}$ (t=6) | .4963 | .7647 | 1.01 |
| $1BT'12^{+}$ (t=6) | .4787 | .7526 | 1.01 |
| | $\times 10^{-8}$ sec$^{-2}$ | $\times 10^{-4}$ °/sec | $\times 10^{-7}$ sec$^{-1}$ |

Table 9
Some RMS Changes Due to Initialization of the 24-Hour
Forecast from 0000 GMT 11 March 1981

| Initialization | $\delta D$* | $\delta \zeta$ * | $\delta T$ | $\delta Q$ |
|---|---|---|---|---|
| $1BT12^{+}$ | .8063 | .5313 | .4526° | $.314 \times 10^{-3}$ |
| $1BT'12^{+}$ | .8026 | .5285 | .4482° | $.308 \times 10^{-3}$ |

*units of $10^{-5}$ sec$^{-1}$

## 5. Discussion

We have found that conventional gravitational zeroing employing $W$ as the mass field has problems in mountain areas. Such zeroing leads to unnecessary vorticity and mass field changes due to $W$ not containing sufficient pressure gradient information. Over the Himalayas, $-\nabla W$ may differ considerably from the actual pressure gradient. The unnecessary vorticity and mass field charges are undesirable in terms of mass and wind analyses but also lead to imbalances and a large divergence terdency. These problems with the conventional zeroing are no doubt related to the fact that the normal modes are calculated from a linearization that does not include orography.

With the new mass-field variable $W'$, whose Laplacian gives the entire mass field contribution to the divergence tendency, gravitational zeroing results in considerably smaller vorticity and mass field changes as well as a better state of balance. This modification was successful even though changes in surface pressure and temperature were only approximate and do not give the exact change in $W'$ requested by the modal calculations. Further work on iterative procedures for calculating changes in surface pressure and temperature as a function of requested changes in $W'$ needs further investigation for both gravitational zeroing and nonlinear initialization.

Although the use of $W'$ in gravitational zeroing resulted in a significant improvement in performance over conventional zeroing, we found that use of $W'$ in first order nonlinear initialization only results in a minor improvement in performance in terms of global diagnostics. There are at least three reasons for the use of $W'$ having less positive impact on nonlinear initialization than it does on gravitational zeroing. First, in all of the nonlinear initialization experiments, model tendencies are calculated exactly employing all aspects of the pressure gradient force. Thus if there is an imbalance between the mass field and vorticity after gravitational zeroing, the nonlinear initialization attempts to correct the problem. Second, in the approximate sense, we know that the nonlinear initialization step adjusts the vorticity and mass field in attempt to make the divergence tendency zero. Given a nonzero divergence tendency, the nonlinear initialization requests a change in the mass field which is the same regardless of whether we are using $W$ or $W'$ as the mass field. Since the $W'$ initialization experiments adjust $\delta T$ and $\delta \varphi$ as the same function of $\delta W'$ as is done when using $W$, there is no difference in this particular aspect of the nonlinear initialization. The third reason involves the divergence changes produced by the nonlinear initialization. We know that in an approximate sense such initialization adjusts the divergence in attempt to make the vorticity and mass field tendencies geostrophically related. When using $W$ in nonlinear initialization, the change in divergence is proportional to $f\dot{\xi} - \nabla^2 \dot{W}$, while when using $W'$, the change is proportional to $f\dot{\xi} - \nabla^2 \dot{W}'$. With the linear initialization, $\nabla^2 W$ differs substantially from $\nabla^2 W'$, whereas in the nonlinear step, $\nabla^2 \dot{W}$ and $\nabla^2 \dot{W}'$ are not significantly different.

Apparently the biggest reason for the net improvement in first order BTI with $W'$ instead of $W$ is the improvement in the gravitational zeroing step. Zeroing using $W$ introduces unnecessary error that is not completely corrected by the nonlinear step. In a global sense, the nonlinear initialization succeeds in correcting most of the error introduced in the zeroing step. However, it is possible that gravitational errors introduced by simple zeroing will not be adequately corrected by the nonlinear adjustment. This problem could be most serious when we have a strong flow pattern such as a jet stream over a mountain area.

Conventional Machenhauer initialization without zeroing was not considered in this paper. This is in part due to the focus or the paper being $W'$ versus $W$ instead of zeroing versus no zeroing. In addition, use of $W'$ in the nonlinear initialization without zeroing can lead to problems due to increased cross vertical mode interaction. For example, suppose we are initializing 6 vertical modes and there is large amplitude divergence in vertical modes 7-12. This divergence can have significant linear effect on $\dot{Q}$ and $\dot{T}$. The linear contribution of this divergence to vertical modes 1-6 of $\dot{W}$ is zero, but it is not zero for $\dot{W}'$. Thus divergence in modes 7-12 could have some significant effect on the adjustment of modes 1-6 when $W'$ is used in the nonlinear step. This vertical mode interaction could easily lead to error but may be beneficial in some cases. An alternate example is the case where the analysis results in large erroneous divergence in the external mode. This divergence has a very large linear contribution to $\dot{Q}$ and $\dot{T}$ which affects only vertical mode 1 of $\dot{W}$ but affects all vertical modes of $\dot{W}'$. In this situation, convention Machenhauer initialization using $W'$ could make unnecessary changes to all 12 vertical modes, where as if $W$ was used instead of $W$ , the initialization would have essentially removed the erroneous divergence in the external mode. Further work needs to be done to investigate how to minimize any error due to increased cross vertical mode interaction associated with using $W'$.

The mass-field variable $W'$ may have other applications than normal mode initialization. For example, Phillips (1982) has suggested that an analysis should analyze only slow modes. Such an analysis may have problems in mountain areas if $W$ is used as the mass-field variable. In addition, $W'$ may be useful for modal analyses or modal time integrations.

In conclusion, we have found that use of $W'$ instead $W$ of is beneficial for first order BTI, even though we do not know how to fully optimize its use. Further work needs to be done on how to specify changes in surface pressure and temperature as a function of mass field changes. Further knowledge is needed concerning the problems and possible benefits of the increased cross vertical mode interaction associated with $W'$. The experiments discussed here were performed with a model with relatively smooth orography; thus we may see more substantial benefits with using $W'$ in some of the high resolution models in existence today.

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