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ADJUSTMENT OF THE GLOBAL SPECTRAL MODEL TO MORE ACCURATELY TREAT THE HYDROSTATIC AND THERMODYNAMIC INFLUENCE OF WATER VAPOR.

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1. Introduction

The fact that the atmosphere is a mixture of gases and water vapor water is well-known. The role of water within the atmosphere is of course an essential component of the production of what we loosely term weather.

One role of water vapor is to modify the density of the atmosphere gaseous mixture. The density is the fundamental variable relating geopotential and pressure increments for the hydrostatically balanced atmosphere.

We have recently considered whether or not the role of water vapor within the thermodynamics and hydrostatic dynamics formulated for the global spectral model was strictly correct.

In this note we review the current system of equations and recommend that certain adjustments be made to insure that the system of equations reflects the presence of water vapor to a high degree of accuracy.

2. The adiabatic equations

2.1 It is well-known that the density of moist air is a function of the relative concentration of water vapor. Since the ratio of the molecular weight of water vapor to that of a dry air mixture is 0.61, the gas constant for moist air is related to that of dry air by the expression

$$R_m = Rd\ (1. + 0.61\ w) \qquad (2.1)$$

were Rm is the gas constant for the moist air,

Rd is the gas constant for the dry air,

and w is the water vapor mixing ratio.

When the water vapor is not saturated, the equation of state for a moist air mixture is

$$\rho_m = p_m / (R_m T) \tag{2.2}$$

where ρ_m is the density of the mixture, p_m is the total pressure of the mixture and T is the temperature of the mixture.

The hydrostatic equation for moist air is then

$$d p_m = - \rho_m \, d\phi \tag{2.3}$$

or if the ideal gas equation (2.2) is used,

$$d \ln p_m = - d\phi / (R_m T) \tag{2.4}$$

Using equation (2.4), the system of quasi-static equations may be developed using the σ coordinate

$$\sigma = 1. - (p_m / p_m^*) \tag{2.5}$$

in which $\overset{*}{Pm}$ is the surface pressure.

One obtains

$$\frac{d\mathbb{V}}{dt} = - f \, \mathbb{K} \times \mathbb{V} - \nabla\phi - R_m T \nabla \ln p_m^* \tag{2.6}$$

$$\frac{\partial\phi}{\partial\sigma} = - R_m T \frac{\partial}{\partial\sigma} \ln p_m \tag{2.7}$$

$$\frac{d}{dt} \ln p_m^* = - \nabla \cdot \mathbb{V} - \frac{\partial\dot{\sigma}}{\partial\sigma} \tag{2.8}$$

for the horizontal momentum equation, the hydrostatic equation and the continuity equation.

Provided no condensation or evaporation takes place, the mixing ratio and specific humidity are constant for each parcel of moist air. Ignoring the small difference between these two parameters, i.e.

$$q \doteq w , \tag{2.9}$$

one has

$$\frac{dq}{dt} = 0 . \tag{2.10}$$

The first law of thermodynamics for adiabatic processes with no phase changes may be written

$$\frac{1}{T}\frac{dT}{dt} = \left(\frac{R}{C_p}\right)_m \frac{1}{p_m}\frac{d}{dt}p_m \qquad (2.11)$$

The ratio R/Cp is numerically very nearly the same for moist and dry air (Reference: Handbook of Meteorology pp. 356 and 341). We will write the Greek letter Kappa $\varkappa$ for this ratio.

The equations developed above differ from those employed in the global spectral model. The difference is related to the use of Rd in place of Rm in equations (2.6) and (2.7).

2.2 The "adiabatic" equations used in the spectral model may by "corrected" without introducing a variable gas constant Rm. (The use of a variable gas constant would upset the spatial invariance of the matrices involved in the discrete approximation (after Arakawa) of the hydrostatic equation.)

One may introduce the virtual temperature Tv which is defined so that

$$R_m T = R_d T_v \qquad (2.12)$$

or

$$T_v = (1. + 0.61 q) T \qquad (2.13)$$

Upon observing that $\frac{d}{dt} q = 0$, one may express the thermodynamic energy equation (2.11) as an equation for virtual temperature.

$$\frac{1}{T}\frac{dT}{dt} \equiv \frac{d}{dt}\ln T = \frac{d}{dt}\ln T_v \qquad (2.14)$$

So,

$$\frac{1}{T_v}\frac{dT_v}{dt} = \varkappa \frac{1}{p_m}\frac{dp_m}{dt} \qquad (2.15)$$

One may further introduce the virtual temperature into eqs 2.6 and 2.7:

$$\frac{d\mathbb{V}}{dt} = -f\,\mathbb{K}\times\mathbb{V} - \nabla\phi - R_d T_v \nabla \ln p_m^{*} \qquad (2.16)$$

$$\frac{\partial\phi}{\partial\sigma} = -R_d T_v \frac{\partial}{\partial\sigma}\ln p_m \qquad (2.17)$$

Thus, if everywhere in the global spectral model's adiabatic system of equations we regard the temperature to be really the virtual temperature, then no error will exist! More precisely, one may put the matter as follows.

At present, the global model's system of adiabatic equations does not <u>explicitly</u> recognize that water vapor is included in the model gas mixture. Taking this fact into account requires no formal change to the set of adiabatic equations; it is simply necessarly to recognize that temperature variable appearing in the equations is the virtual rather than the thermodynamic temperature of the model's moist air mixture.

3. Non-adiabatic processes

3.1 Since we have taken the view that the temperature history variable is the virtual temperature, we must adjust the value Tv to T whenever the thermodynamic temperature is needed in the computation of the physical process. Such computations will be needed within GFIDI and GWATER as well as in the calculation of radiation cooling rates.

Additionally, we must be careful to transform heating rates so that they apply to the virtual temperature, since it is the virtual temperature which is the history variable.

3.2 For non-adiabatic processes which do not involve phase changes of water vapor, the thermodynamic energy equation is

$$\frac{1}{T}\frac{dT}{dt} = \kappa \frac{1}{p_m}\frac{dp_m}{dt} + \frac{\dot{Q}}{c_{p_m} T} \tag{3.1}$$

where $\dot{Q}$ is the rate addition of energy per unit mass and C_{p_m} is the specific heat at constant pressure of the moist air mixture. Referring to Haurwitz, B., Dyn Meteorol p.37 the value of C_{p_m} is

$$C_{p_m} = (1 + w)^{-1} (C_p + C_{p_v} w) \tag{3.2}$$

In (3.2), w is the mixing ratio; C_p is the specific heat at constant pressure of dry air and C_{p_v} is the specific heat at constant pressure of water vapor.

Numerically

$$0 < w < 0.03$$

$$C_p = 0.239 \text{ cal gm}^{-1} \text{deg}^{-1}$$

$$C_{p_v} = 0.466 \quad ''$$

In current practice we compute the rate of change of temperature due to non-adiabatic processes by evaluating,

$$\left(\frac{\delta T}{\delta t}\right)_C = \frac{\dot{Q}}{C_p} \tag{3.3}$$

We may correct this by multiplying by C_p / C_{pm}

$$\left(\frac{\delta T}{\delta t}\right)_N = \frac{C_p}{C_{pm}}\left(\frac{\delta T}{\delta t}\right)_C \tag{3.4}$$

Since Tv = $(1. + 0.61 w)T$, the correct tendency of virtual temperature is

$$\left(\frac{\delta T_v}{\delta t}\right)_N = \left[\frac{(1. + 0.61 w)\, C_p}{C_{pm}}\right]\left(\frac{\delta T}{\delta t}\right)_C \tag{3.5}$$

When the factor in brackets on the right side of equation (3.5) is evaluated numerically with q = 0.01, one gets

$$\left(\frac{\delta T_v}{\delta t}\right)_N = 0.997\left(\frac{\delta T}{\delta t}\right)_C \qquad (3.6)$$

It seems sufficiently accutate therefore to leave in - place the current methods for augmenting the temperature history variable --- virtual temperature --- in response to those non-adiabatic processes which do not involve water vapor phase changes.

Specifically therefore one does not require any change in the application of the radiative cooling rates which are now used in the model. One must however insure that the radiative cooling rates are computed using the thermodynamic and not the virtual temperature. So when the virtual temperatures are passed to the radiation code they should be transformed to thermodynamic temperatures. The radiative cooling rates once computed may be used to good accuracy to affect the virtual temperature as indicated by equation (3.6) above.

3.3 The boundary layer exchanges of heat, vapor and momentum which are calculated in subroutine PROGTS and elsewhere in subroutine GFIDI are currently formulated on the basis that the temperatures are thermodynamic temperatures.

Rather than re-coding these grid-point (Gaussian grid) computations, the simplest course is to transform the virtual temperature to thermodynamic temperature prior to performing these computations.

A check of the GFIDI subroutine indicates that this transformation need be made only once and that it need not be reversed.

One must however modify within GFIDI one instruction to account for the transformation. This instruction (116) involves the coefficient DLAM and its dependence upon the depth of the model's first layer. Here the virtual temperature is required, but one may simply insert an extra factor in the denominator of the expression.

3.4 Within the subroutine GNATER a variety of physical process are computed. These are predominantly adjustment-computations to account for convection and condensation. All of these routines were designed on the basis that the temperature variable was the thermodynamic temperature.

The simplest way to correct these routines is to arrange to transform the virtual temperature to thermodynamic temperature upon entering the subroutine. All computations will then be correct, but before returning from 'GWATER', the temperature should be transformed back into virtual temperature.

4.0 Input and Output:

At present, the thermodynamic input to the spectral model is geopotential height and relative humidity. The temperatures are calculated on the basis of a hydrostatic approximation which presently yields virtual temperature. Thus the input method does not require any adjustment.

On output, the model's σ coordinate data set is postprocessed to provide isobaric data. The isobaric geopotential heights are computed as through the temperature data are virtual temperatures. This is, of course, precisely what they are.

Hence it is merely necessary to make it known that the temperature variable which is produced on output is in fact a virtual temperature. This distinction once noted is of minor consequence except for those users who need exact temperature data. It seems sufficient therefore to notify all users of the facts of the case and thereby caution them of the need to adjust the data for exact computation.

References:

1. Handbook of Meteorology (1945)
Berry, Bollay and Beers, eds., McGraw Hill, 1068 pp.

2. Dynamic Meteorology (1941)
B. Haurwitz, McGraw Hill, 365 pp.