NOAA Technical Memorandum ERL BOMAP-5

U.S. DEPARTMENT OF COMMERCE
National Oceanic and Atmospheric Administration
Environmental Research Laboratories

Preliminary Velocity Divergence Computations for BOMEX Volume Based on Aircraft Winds

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The
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NOAA TECHNICAL MEMORANDA

Environmental Research Laboratories, BOMAP Series

The Barbados Oceanographic and Meteorological Analysis Project (BOMAP) was established in the Research Laboratories of the Environmental Science Services Administration (ESSA), now NOAA, to coordinate the reduction of data collected during the Barbados Oceanographic and Meteorological Experiment (BOMEX), May, June, July 1969; to analyze these data, particularly those pertaining to the Sea-Air Interaction Program (the BOMEX Core Experiment); and to provide a central contact point for BOMEX information exchange and publication of BOMEX results.

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ERL BOMAP-2 A Statistical Data Plan for BOMEX. Theodore W. Horner, December, 1970. (COM-71-00 188)

ERL BOMAP-3 Mass, Momentum, and Energy Budget Equations for BOMAP Computations. Eugene M. Rasmusson, January 1971. (COM-71-00 195)

ERL BOMAP-4 High-Level Cloud Photography Inventory, BOMEX Period II, May 24-June 10, 1969. Vance A. Myers, March 1971.

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PRELIMINARY VELOCITY DIVERGENCE COMPUTATIONS FOR BOMEX VOLUME BASED ON AIRCRAFT WINDS

Robert W. Reeves

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CONTENTS

NOTATION

p^* = vertical coordinate; $p^* = p_o - p$, where p is pressure at a point, and p_o is surface pressure vertically beneath that point (see Rasmusson, 1971)

ω = vertical velocity = dp^*/dt

g = acceleration of gravity

A = area of the BOMEX square

$\mathcal{C}$ = perimeter of the BOMEX square

∇_h = quasi-horizontal component of vector operator ∇, the component in a p^* surface

$\vec{V}$ = wind vector

V_n = component of the horizontal wind normal to the perimeter, positive outward from the BOMEX square

$\overrightarrow{TAS}$ = aircraft movement with respect to the air

$\overrightarrow{GS}$ = aircraft movement with respect to the ground

δ = aircraft drift angle, the angle between TAS and GS, positive when GS is to the right of TAS

$\vec{V}_m$ = measured wind vector obtained by subtraction of TAS_m from GS_m

V_M = component of $\vec{V}_m$ normal to aircraft track

VS = V_M measured at a sounding (see text)

$\vec{V}_a$ = computed wind

V_N = component on $\vec{V}_a$ normal to $\mathcal{C}$

V_{NBE} = average V_N for perimeter segment "BE"

hdg = aircraft heading

HDG = aircraft heading with respect to rotated axes

m = subscript indicating measured by aircraft instruments

V, TAS, GS, V_m, TAS_m, GS_m = magnitude of the respective vectors

T_E = magnitude of error in measured true airspeed, $TAS_m - TAS$

δ_E = magnitude of error in measured drift angle, $\delta_m - \delta$

V_x, V_y, V_{mx}, V_{my} = west and south components of $\vec{V}_a$ and $\vec{V}_m$, respectively.

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Abstract. These preliminary calculations of the horizontal velocity divergence within the Barbados Oceanographic and Meteorological Experiment (BOMEX) volume are based on a limited sample of wind measurements from line integral aircraft night flights around the perimeter of the BOMEX volume at a level of 1,000 ft and stepped soundings at seven different heights at the midpoint of each side of the volume. Results indicate that with careful systematic calibration wind data may be useful in determining divergences.

1. INTRODUCTION

One of the prime objectives of the BOMEX Sea-Air Interaction Program is the evaluation of the budgets of mass, water, momentum, and energy for the BOMEX volume based on volume integral techniques. These techniques require measurements on the vertical surfaces of the volume, with supplementary measurements within it. Therefore, observations of wind, temperature, and humidity were made as completely as operationally feasible along the vertical sides of the 500- by 500-km BOMEX volume.

Essential and difficult parts of the budget of mass are assessment of the horizontal mass divergence and the vertical velocity. The vertical velocity, intimately related to moisture and energy fluxes in the atmosphere, cannot be measured directly.

In the BOMEX design, as in many other experiments, the principal method of calculating the mean vertical velocity at a particular pressure level is to integrate the equation of continuity of mass,

$$\frac{\partial p}{\partial t} + \nabla \cdot (\rho \vec{V}) = 0 \tag{1}$$

from the surface to that level. One form of this integration is

$$\overline{\omega^*} = \int_0^{p^*} \nabla_h \cdot \vec{V} dp^* , \tag{2}$$

where p^* is the vertical coordinate in the pressure difference system described by Rasmusson (1971) and ω^* the vertical velocity in this system. See the Notation on page iv for further details.

The horizontal divergence, $\nabla_h \cdot \vec{V}$, is evaluated from the horizontal wind. By the divergence theorem,

$$\overline{\nabla_h \cdot \vec{V}} = \frac{1}{A} \oint V_n d\mathcal{C}, \tag{3}$$

where A is the area of the BOMEX square, $\mathcal{C}$ is the perimeter, and V_n is the component of the horizontal wind normal to the perimeter, directed outward. The bar over the horizontal divergence term indicates an average over the area A. The line integral is around $\mathcal{C}$ at a particular p^* level.

One objective in planning BOMEX was to measure wind on the vertical sides of an atmospheric volume with sufficient accuracy to yield meaningful derived values of horizontal velocity divergence (eq. 3) and thereby vertical velocity (eq. 2). This requirement was satisfied by

(a) selecting a region of relatively steady wind to make the wind observations as spatially and temporally representative as possible;

(b) limiting the size of the volume so that it was large enough to provide meaningful differences from side to side but small enough to be adequately sampled by the available observation system.

(c) obtaining wind observations as accurately as operationally and economically feasible, and as closely spaced vertically, horizontally, and in time as feasible;

(d) extracting the data to rigid specifications to retain the maximum accuracy possible; and

(e) judiciously averaging wind observations over space and time to reduce random variability.

Reported here are the results of a preliminary test of the BOMEX-designed calculation of divergence from perimeter wind observations. A sample of the first available aircraft wind data was analyzed to determine the apparent horizontal velocity divergence within the BOMEX volume on each of four successive nights. The data consist of wind observations at the 1000-ft level, which were hand-tabulated aboard WC-121 weather reconnaissance aircraft operated by the Navy VW-4 Squadron. All reasonable techniques were applied to correct and judiciously average this limited set of wind observations. Wind observations were automatically recorded on WC-121 and Environmental Science Services Administration Research Flight Facility (RFF) aircraft, and on the fixed ships, but require time-consuming data reduction procedures before they are available for analysis.

2. DESCRIPTION OF DATA

Data for the preliminary line-integral calculations were obtained on four consecutive line integral night (LIN) flights by Navy WC-121 aircraft on the nights of May 31 to June 3. The LIN MOD 2 flight pattern shown in figure 1 was used. Observations were made along 100-n-mi segments on each side of the BOMEX square, beginning on the southern side. Stepwise soundings were taken at midpoints HOTEL, KILO, JULIETT, and INDIA, and again at HOTEL. These consisted of observations at 1,000, 2,000, 3,000, 4,000, 6,000, 8,000, and 10,000 ft, with aircraft flying opposite headings at each level parallel to the sides of the BOMEX square. Each flight required about 7 hours and provided the following data:

(a) at 5-min intervals during the flight the meteorologist recorded temperature, dew point, pressure, altitude, clouds, and sea-surface temperature;

(b) at 5-min intervals on the 100-n-mi legs the navigator used the ASN-41 on-board computer to tabulate a series of 10 consecutive readings of heading, ground speed, drift angle, calibrated airspeed, pressure-altitude, temperature, and wind speed, using as input continuously displayed heading, drift angle, ground speed, and true airspeed -- 4 to 7 series of 10 consecutive readings being tabulated along each leg; and

(c) during each of the two headings flown at each sounding level the navigator used the computer to tabulate a series of 5 consecutive readings in the same manner as the 100-n-mi readings.

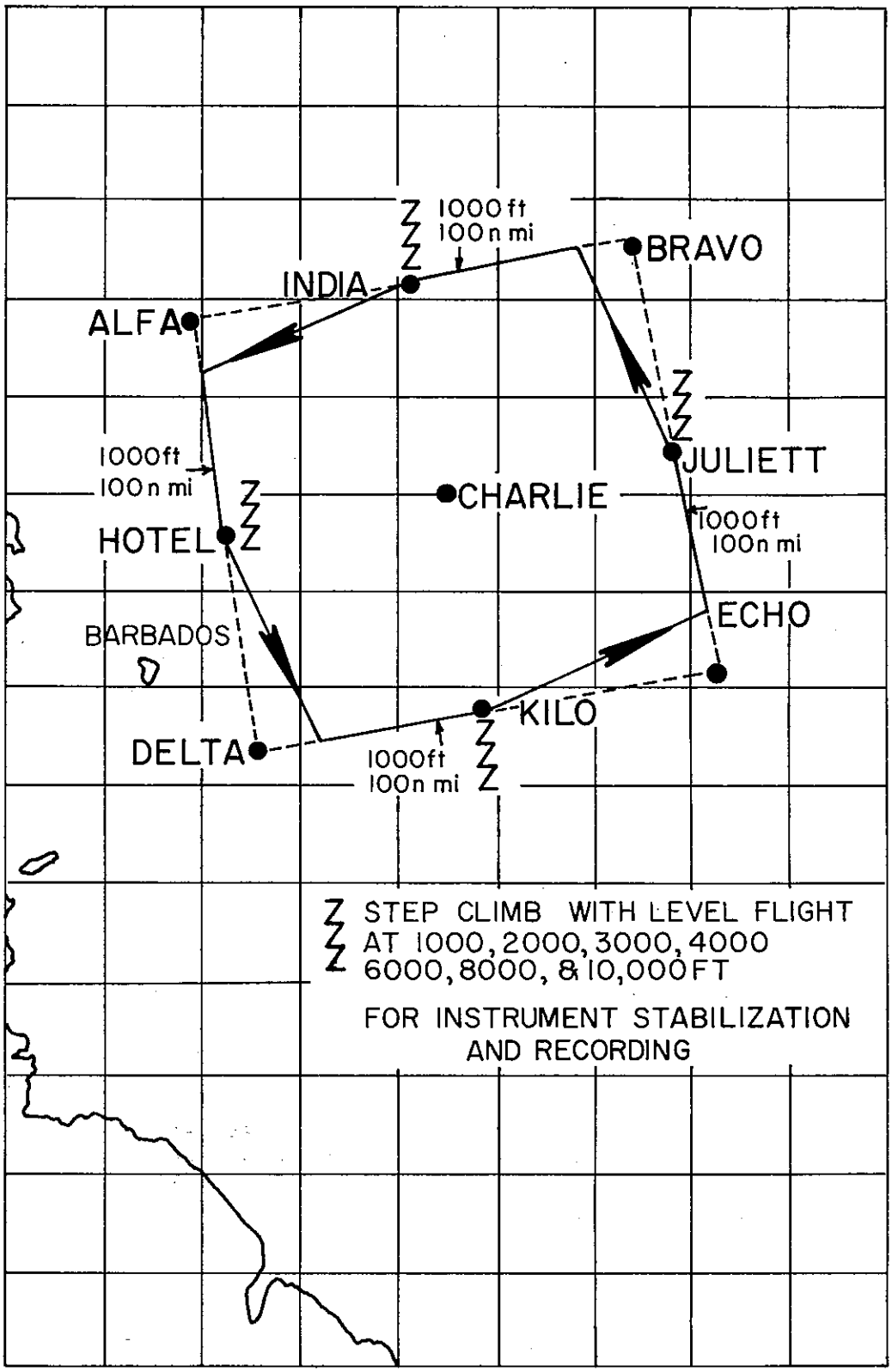

1000 ft
100 n mi
BRAVO
INDIA
ALFA
JULIETT
1000ft
100 n mi
CHARLIE
HOTEL
1000ft
100n mi
ECHO
BARBADOS
KILO
DELTA
1000ft
100n mi
STEP CLIMB WITH LEVEL FLIGHT
AT 1000, 2000, 3000, 4000
6000, 8000, & 10,000 FT
FOR INSTRUMENT STABILIZATION
AND RECORDING

Figure 1.

In the computations that follow, each series of 5 and 10 consecutive wind direction and speed readings were averaged separately to arrive at a wind direction to the nearest degree and a wind speed to the nearest half knot. This would differ from actual vector averaging only if the speed and direction had varied widely within each series. For example, consider a series of 5 consecutive readings:

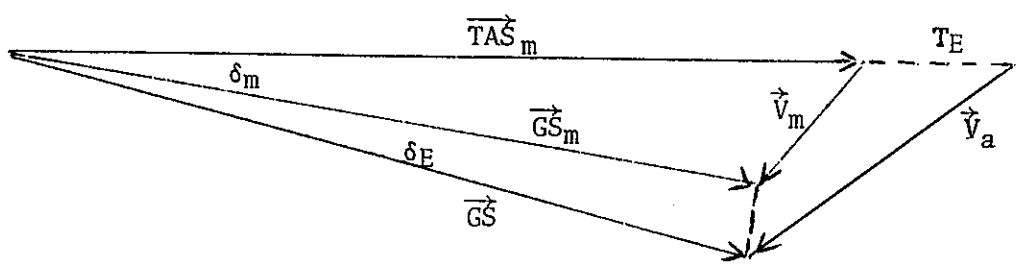
Direction	Speed
087°	12 kt
088°	10 kt
090°	10 kt
093°	8 kt
094°	8 kt

The vector average of this series is a 9.6-kt wind from 090.1°. Averaging the directions and speeds separately yields a 9.6-kt wind from 090.4°. Wind variability as great as the one exhibited by this example rarely occurred in the data studied.

3. METHOD OF CORRECTING WINDS FOR INSTRUMENT ERRORS

A change in aircraft heading produces an apparent change in the wind vector. This inconsistency is the result of systematic errors in the instruments from which the wind is computed.

Consider the following figure:



Here the vector $\overrightarrow{TAS}_m$ represents the aircraft's measured movement with respect to the air, the vector $\overrightarrow{GS}_m$ represents the measured movement with respect to the ground, and δ_m, the measured drift angle, is the angle between the heading and the apparent track of the aircraft. Thus, the vector $\vec{V}_m$ is a measured wind vector obtained by vector subtraction of $\overrightarrow{TAS}_m$ from $\overrightarrow{GS}_m$. The usual procedure to refine the wind measurement is to assume errors in both the measured drift angle and the measured true airspeed, indicated in the above figure by T_E and δ_E. As used here, T_E and δ_E are corrections applied to the measured quantities to compute the wind, $\vec{V}_a$.

By taking readings of $\overrightarrow{TAS}_m$, $\overrightarrow{GS}_m$, and δ_m on two headings, one can solve for T_E and δ_E with certain assumptions. These values of T_E and δ_E are then used to correct the winds on the various aircraft flight legs. This procedure is used by the RFF and National Hurricane Research Laboratory to correct true airspeed and drift angle measurements (see Friedman et al., 1969).

The following assumptions are applied in the calibration box procedure to measure T_E and δ_E:

(a) the horizontal wind field remains constant for the period of the calibration run;

(b) T_E, δ_E, and $\vec{V}_a$ are the same on both headings, but are unknown; and

(c) the direction of $\overrightarrow{TAS}$ (the aircraft heading) and the magnitude of $\overrightarrow{GS}$ are accurately measured on the two headings.

Once values of T_E and δ_E are solved from the calibration boxes, the following equations for the west and south components of the computed wind, V_x and V_y, can be solved at desired points on the flight legs (fig. 2):

$$V_x = GS_m \sin (hdg_m + \delta_m) - TAS_m \sin hdg_m + T_E \sin hdg_m - \delta_E GS_m \cos (hdg_m + \delta_m), \tag{4}$$

and

$$V_y = GS_m \cos (hdg_m + \delta_m) - TAS_m \cos hdg_m + T_E \cos (hdg_m) - \delta_E GS_m \sin (hdg_m + \delta_m). \tag{5}$$

The drift angle error, δ_E, is in radians.

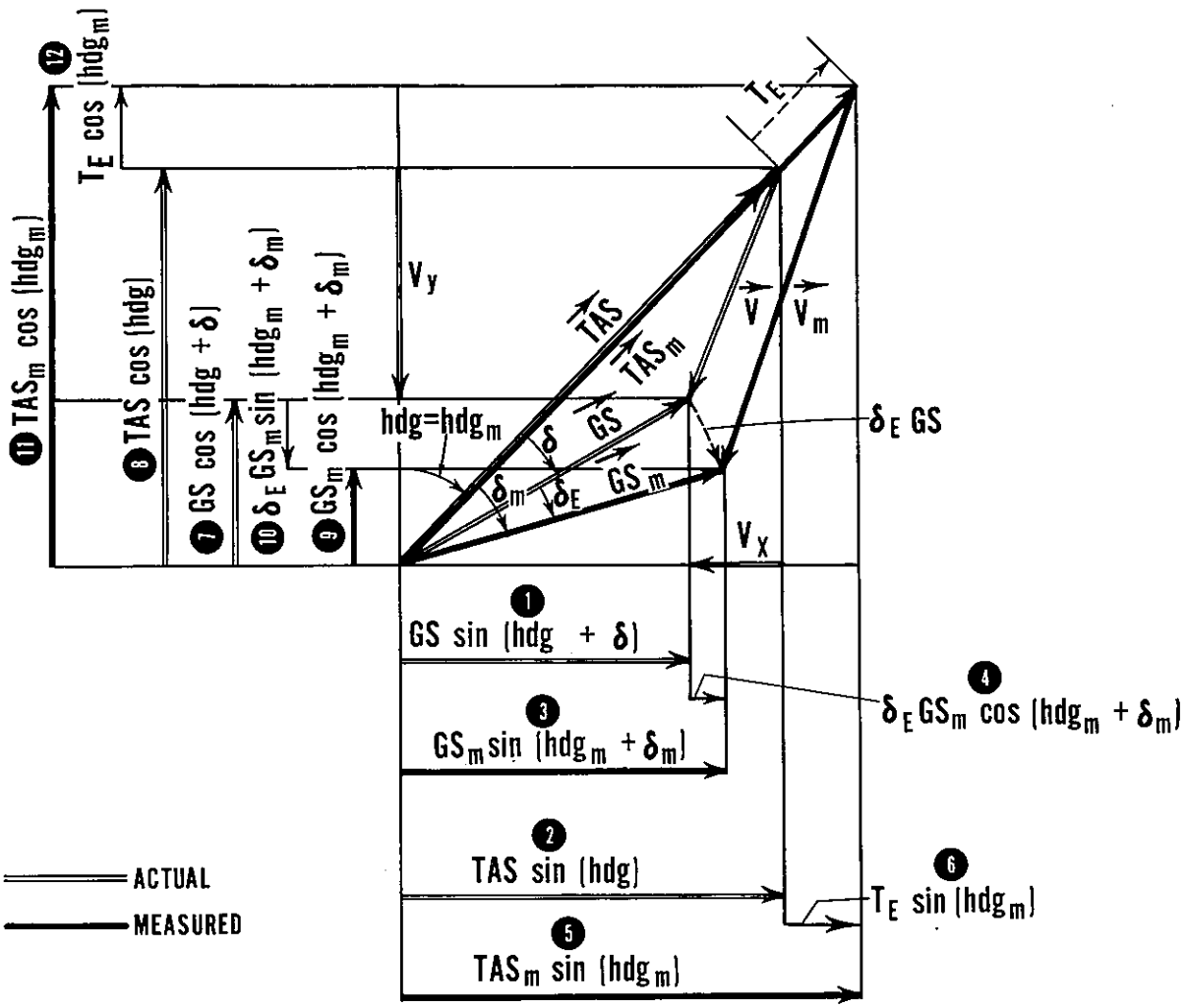
Components of the measured wind, $\vec{V}_m$, are

$$V_{mx} = GS_m \sin (hdg_m + \delta_m) - TAS_m \sin hdg_m \tag{6}$$

and

$$V_{my} = GS_m \cos (hdg_m + \delta_m) - TAS_m \cos hdg_m. \tag{7}$$

This method may be described as an "internal calibration" of the wind-measuring system since it makes use of inputs from the compass, airspeed meter, and Doppler radar and does not make use of navigation fixes. In the divergence computations performed here, the pair of readings made on opposite headings at each level during the aircraft soundings was used as an internal calibration of the winds.



$$V_x = (1) - (2) = (3) - (4) - (5) + (6)$$

$$V_y = (7) - (8) = (9) - (10) - (11) + (12)$$

Figure 2.

The BOMEX array was oriented along the axes defined by 350°-170° and 260°-80°. By rotating the reference axes 10°, we can write the components of the wind normal to the array and outward as follows:

```
A. . . . . . . . . . .B
 .                   .
 .                   .
 .                   .
D. . . . . . . . . . .E
```

Along BE, for example,

$$V_{NBE} = GS_m \sin (HDG_m + \delta_m) - TAS_m \sin (HDG_m) \tag{8}$$

$$+ T_E \sin (HDG_m) + \delta_E GS \sin (HDG_m + \delta_m),$$

and along AD the signs would be reversed. HDG_m indicates the measured heading with respect to the rotated axes:

$$HDG_m = hdg_m + 10°$$

We have defined V_{NBE} as the computed normal wind component along BE. Consider readings on opposite headings, identified by subscripts 1 and 2. Calling the measured normal wind component on heading 1 V_{M1}, by anology to (6), we obtain

$$V_{M1} = GS_m \sin (HDG_1 + \delta_m) - TAS_m \sin (HDG_1). \tag{9}$$

Substituting in (8), we have

$$V_{NBE1} = V_{M1} + T_E \sin HDG_1 - \delta_E GS_1 \cos (HDG_1 + \delta_1). \tag{10}$$

Similarly,

$$V_{NBE2} = V_{M2} + T_E \sin HDG_2 - \delta_E GS_2 \cos (HDG_2 + \delta_2) . \tag{11}$$

Averaging these two equations gives

$$\frac{1}{2} (V_{NBE1} + V_{NBE2}) = \frac{1}{2} (V_{M1} + V_{M2}) + T_E \frac{(\sin HDG_1 + \sin HDG_2)}{2} \tag{12}$$

$$- \delta_E \left[\frac{GS_1 \cos (HDG_1 + \delta_1) + GS_2 \cos (HDG_2 + \delta_2)}{2} \right] .$$

Suppose we now approximate V_{NBE} by the average of the measured normal wind components on opposite headings. The last terms of (12) become the error error, E, in applying this assumption:

$$V_{NBE} - \frac{1}{2}(V_{NBE1} + V_{NBE2}) = \frac{1}{2}(V_{M1} + V_{M2}) + E \tag{13}$$

and

$$E = T_E \frac{[\sin HDG_1 + \sin HDG_2]}{2}$$

$$- \delta_E \left[\frac{GS_1 \cos (HDG_1 + \delta_1) + GS_2 \cos (HDG_2 + \delta_2)}{2}\right]. \tag{14}$$

In our case, $(\sin HDG_1 + \sin HDG_2) = 0$ because the headings are opposite. Hence ,

$$E = - \delta_E \left[\frac{GS_1 \cos (HDG_1 + \delta_1) + GS_2 \cos (HDG_2 + \delta_2)}{2}\right] \tag{15}$$

We can estimate the magnitude of E from typical values:

$$\delta_E \sim 2° = .035;$$

$$GS_1 \text{ and } GS_2 \sim 100 \text{ m sec}^{-1} .$$

Since a crosswind of 10 m sec^{-1} would produce a drift of approximately 6°, we use that along with hdg = 0° and hdg = 180°, as follows:

$$\delta_1 = - 4°; \qquad \delta_2 = + 8° .$$

Then the magnitude of the error term is

$$.035 \; [100 \cos (-4°) + 100 \cos (188°)] \; (.5)$$

$$= .035 \; [100 \; (.99756 - .99027)] \; (.5)$$

$$= 3.5 \; [00729] \; (.5) = .0125 \text{ m sec}^{-1} = (.5) \; .0125 \; .$$

Even a difference of 2 m sec^{-1} in the ground speeds on opposite headings would produce an error of < 0.1 m sec^{-1}. The error term E can therefore be neglected and V_{NBE} can be assumed to be satisfactorily approximated by the average of V_{M1} and V_{M2}.

Finally we rearrange (13):

$$V_{NBE} \approx V_{M1} - \frac{1}{2}\left(V_{M1} - V_{M2}\right) \tag{16}$$

and interpret $\frac{1}{2}\left(V_{M1} - V_{M2}\right)$ as a correction to apply to V_{m1} to obtain our best estimate of the crosswind, V_{NBE}. The above method allows us to use winds measured on opposite headings to determine the crosstrack component. It was used during BOMEX field operations in the absence of computing facilities to obtain estimates of the divergence quickly.

The 5-min average normal wind components along the four sides of the BOMEX volume for the four nights at 1,000 ft are shown in table 1. The sounding data used as an internal calibration on the winds are presented in table 2. If we form the differences of the normal components on opposite headings in the latter table, we note the variability of those differences within a given sounding, and at any level for successive soundings. Recall that only 10 readings of the wind (5 each on opposite headings) were made at each level. We are aware that the airspeed error is height-dependent, but since the drift angle error should not be dependent on height, we use all levels of the sounding for calibrating the 1,000-ft winds in order to reduce the random error. The divergence is thus based on winds adjusted as follows:

Write (16) in notation applicable to any side of the BOMEX square:

$$V_N = V_M(HDG) - \frac{1}{2}\left[V_M(HDG) - V_M(HDG \pm 180°)\right], \tag{17}$$

We now improve our correction term in brackets in (17) by replacing it with the equivalent correction term averaged over available levels at the sounding at midpoint of the same side of the BOMEX square:

$$V_N = V_M(hdg) - \frac{1}{2j}\sum_{k=1}^{k=j}\left[VS_k\ (hdg) - VS_k\ (hdg \pm 180°)\right] \tag{18}$$

where the k's are individual levels and j is the total number of levels in the aircraft sounding.

Table 1. Measured components of the wind in knots, normal to the sides of the BOMEX volume (5-min averages) along 100-n-mi segments at 1,000 ft

Leg	May 31	June 1	June 2	June 3
South:	- 0.5	+ 1.7	+ 1.7	- 2.6
K1	- 0.8	+ 3.4	+ 2.5	- 1.1
to	- 0.3	+ 1.3	+ 1.5	- 0.4
K	- 0.6	+ 2.0	+ 1.2	+ 0.7
	- 1.5	+ 3.4	- 0.3	+ 0.3
	- 1.1	+ 1.6	+ 1.1	- 1.7
	.0	+ 2.2	+ 3.9	
North:	+ 3.4	+ 2.9	+ 4.7	+ 7.7
I1	+ 3.4	+ 2.7	+ 5.2	+ 9.5
to	+ 2.6	+ 2.7	+ 4.4	+ 9.8
I	+ 3.0	+ 2.9	+ 4.8	+ 9.4
	+ 3.1	+ 5.2	+ 5.7	+ 8.0
			+ 5.9	
East:	-13.8	-14.0	-10.6	-19.0
J1	-16.0	-14.0	- 9.8	-18.2
to	-11.5	-12.0	-10.4	-19.0
J	-13.0	-12.9	-12.0	-18.0
	-14.0	-16.8	- 9.5	-17.4
	-14.7		-10.0	-18.9
West:	+13.8	+19.0	+14.8	+17.5
H1	+14.8	+20.8	+15.0	+16.2
to	+14.6	+18.7	+15.4	+17.8
H	+14.6	+17.6	+15.9	+19.9
	+15.5		+17.4	+20.4
	+12.7		+17.3	+19.3
				+17.4

Table 2. Measured components of the wind normal to the BOMEX array during soundings (2-min averages in knots)

	May 31		June 1		June 2		June 3	
	Aircraft Heading							
	0°	180°	0°	180°	0°	180°	0°	180°
HOTEL								
10,000 ft								
8,000 ft	8.8	8.0			5.7	9.7		
6,000 ft	15.6	16.1	6.0	12.5	6.1	13.2	10.8	17.4
4,000 ft	20.4	16.5	10.5	12.7	8.9	17.2	15.9	18.7
3,000 ft	19.3	19.8	10.6	16.9	11.0	19.0	18.6	21.6
2,000 ft		18.8	13.0	10.9	9.0	14.9	19.0	21.0
1,000 ft	18.3	10.7	12.0	15.9	9.5	16.9	16.9	19.9
JULIETT								
10,000 ft	4.9	8.3	0.5	4.1	4.7	8.3	13.6	18.1
8,000 ft	8.0	7.9	0.2	3.3	4.3	9.2	15.4	17.5
6,000 ft	11.6	13.8	2.5	8.4	-1.3	5.8	15.3	17.8
4,000 ft	16.4	18.8	10.0	14.6	6.3	11.6	21.0	21.0
3,000 ft	16.0	19.0	11.3	15.7	5.3	11.3	23.5	26.0
2,000 ft	17.4	7.5	12.0	17.0	9.9	15.0	22.4	22.5
1,000 ft	15.6	16.3	14.0	22.6	9.4	15.5	20.5	23.2
HOTEL								
10,000 ft	6.4	6.9	4.4	9.2	5.5	11.5	9.2	11.2
8,000 ft	3.0	5.9	9.0	14.3	8.0	11.6	12.7	12.8
6,000 ft	7.7	14.6	10.5	17.0	11.2	16.1	13.2	16.8
4,000 ft	9.6	12.0	11.7	15.6	9.4	15.2	20.2	18.9
3,000 ft	12.2	16.7	9.1	14.3	12.8	18.4	20.4	22.8
2,000 ft	13.0	13.5	9.5	15.9	14.3	19.3	21.2	24.2
1,000 ft	15.5	13.4	10.7	15.9	13.9	18.0	24.8	22.0
	Aircraft Heading							
	90°	270°	90°	270°	90°	270°	90°	270°
KILO								
10,000 ft	-1.0	1.5	3.3	9.5	-1.2	8.8	7.4	5.9
8,000 ft	-0.6	1.5	-1.7	0.5	4.5	9.0	7.2	8.5
6,000 ft			-9.2	-2.6	1.9	10.0	8.5	12.0
4,000 ft	1.9	-0.3	-7.0	-1.9	2.4	7.9	1.1	4.5
3,000 ft	1.7	0.3	-2.3	-1.2	1.7	6.3	1.2	1.8
2,000 ft	1.2	0.7	-0.9	-2.6	1.4	4.5	2.0	1.2
1,000 ft			-1.9	-4.4	-3.1	4.6	-0.7	4.1
INDIA								
10,000 ft	0.7	1.0	-8.2	0.5	4.2	11.1	10.5	14.6
8,000 ft	0.4	4.5	-7.4	5.5	6.0	12.2	12.8	18.2
6,000 ft	2.1	5.0	-1.7	0.6	0.1	6.7	1.5	10.0
4,000 ft	1.8	3.9	-2.7	2.1	-1.2	5.0	-1.4	5.2
3,000 ft	2.9	6.3	-3.4	3.9	2.8	7.4	4.8	11.5
2,000 ft	5.1	5.7	-2.2	3.7	2.2	8.8	5.5	4.6
1,000 ft	3.0	5.1	-4.5	2.7	0.9	6.8	2.9	7.4

4. COMPUTED DIVERGENCE AT 1,000 FT

The computed divergences at 1,000 ft for the four nights are given in table 3. The "uncalibrated data" are divergences computed from the normal components of the wind vectors obtained by averaging each group of 10 readings of wind speed and direction from the ASN-41 computer (see section 2). The V_M's for this are listed in table 1. The "calibrated data" are divergences via equation (3) from 1000-ft normal wind components adjusted to multi-level sounding data by equation (18). The VS's for this are listed in table 2. These latter divergences are all of the order of 10^{-6} sec^{-1}, expected a priori. Note that using uncalibrated winds gave divergences an order of magnitude greater.

To gain some idea of the range of validity of the calculations, 95 percent confidence limits for the divergences were computed by Theodore W. Horner (see appendix). These confidence limits indicate that there is still some uncertainty even concerning the sign of the divergence. It should be noted, however, that the standard deviation of the calibrated divergences for the four nights is only $.4 \times 10^{-6}$ sec^{-1}. Such an improbably low value may indicate that the 95 percent confidence limits for individual runs are too broad in the aggregate. This would be true if part of the variance treated as random in the computation of confidence limits was actually systematic and persistent from night to night (e.g. the meridional gradients of wind velocity).

5. CONCLUSION

Horizontal divergences that appear realistic were obtained from a sample of aircraft wind measurements during BOMEX by careful systematic calibration and averaging. Other writers (e.g. Riehl) have estimated the horizontal divergence in the Trades to be of the order of 10^{-6} sec^{-1}. If these measurements at 300 m altitude are taken as an approximation of the average over the lowest 600 m, they imply a sinking rate at cloud base level of the order of 50 m/day.

The data used for this study were manual tabulations of the on-board computer readout and therefore are probably the least reliable of the BOMEX aircraft wind measurements, which were automatically recorded with greater precision and time resolution in other line integral flights. This preliminary result gives promise that the BOMEX wind observations will fulfill their purpose of giving meaningful average divergence values for the BOMEX volume. This result also demonstrates, however, that very careful calibration incorporated into both the flight plan and the subsequent evaluations is mandatory.

Table 3. Divergences computed from data hand-tabulated from the ASN-41 on-board computer 1,000-ft wind measurements

Date	Divergences From Uncalibrated Data	Divergences From Calibrated Data	95% Confidence Limits For Calibrated Divergences
May 31	$+ 3.0 \times 10^{-6} sec^{-1}$	$+ 0.8 \times 10^{-6} sec^{-1}$	$- 2.8 \times 10^{-6} sec^{-1}$ to $+ 4.4 \times 10^{-6} sec^{-1}$
June 1	$+10.8 \times 10^{-6} sec^{-1}$	$+ 1.1 \times 10^{-6} sec^{-1}$	$- 2.5 \times 10^{-6} sec^{-1}$ to $+ 4.7 \times 10^{-6} sec^{-1}$
June 2	$+12.7 \times 10^{-6} sec^{-1}$	$+ 0.4 \times 10^{-6} sec^{-1}$	$- 1.7 \times 10^{-6} sec^{-1}$ to $+ 2.6 \times 10^{-6} sec^{-1}$
June 3	$+ 7.4 \times 10^{-6} sec^{-1}$	$+ 1.6 \times 10^{-6} sec^{-1}$	$- 2.0 \times 10^{-6} sec^{-1}$ to $+ 5.3 \times 10^{-6} sec^{-1}$
$\overline{Div}$	$+ 8.5 \times 10^{-6} sec^{-1}$	$+ 1.0 \times 10^{-6} sec^{-1}$	
$\sigma_{\overline{Div}}$	$3.7 \times 10^{6} sec^{1}$	$0.4 \times 10^{6} sec^{1}$	

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7. REFERENCES

Friedman, Howard A., Ahrens, Merlin R., and Davis, Harlan W., "The ESSA Research Flight Facility: Data Processing Procedures," ESSA Technical Report ERL 132-RFF 2, Research Laboratories, Environmental Science Services Administration, U.S. Department of Commerce, November 1969.

Rasmusson, Eugene M., "Mass, Momentum, and Energy Budget Equations for BOMAP Computations," NOAA Technical Memorandum ERL BOMAP-3, Environmental Research Laboratories, National Oceanic and Atmospheric Administration, U.S. Department of Commerce, January 1971.

Riehl, Herbert, Tropical Meteorology, McGraw-Hill, New York, 1954.

APPENDIX

CONFIDENCE LIMITS FOR HORIZONTAL DIVERGENCE ESTIMATED FROM WINDS MEASURED BY AIRCRAFT

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INTRODUCTION

Postulate a component of the wind, normal or perpendicular (directed outward) to the side of the BOMEX volume. The integral of this component around the perimeter of the BOMEX volume is the horizontal divergence.

Aircraft wind data permit an estimation of the horizontal divergence. The average normal component for a side multiplied by the length of that side gives that side's contribution to the estimate. Summation of the contributions from each of the four sides gives the complete estimate of the horizontal divergence.

When the aircraft flies at one altitude, the estimate of the horizontal divergence is representative of the layer through which the flight is made. These estimates are imperfect. The aircraft samples only a small portion of the perimeter of a layer. It does not provide an simultaneous estimate at all points. In an extreme case during the time of aircraft flight the wind field might reverse itself, providing zero divergence through time, yet the estimate deduced from aircraft data might indicate maximum convergence or maximum divergence. Whether a good or bad estimate of the divergence is obtained depends on the extent to which the wind field (divergence) changes or does not change in time and the extent to which the aircraft adequately or inadequately samples the perimeter space.

Divergences estimated from aircraft winds may be compared to divergences estimated from rawinsonde winds and to those estimated from other data. However, this discussion pertains only to summations of the normal wind components around a layer perimeter of the BOMEX volume and to the errors inherent in these summations. These are labeled horizontal divergence estimates and confidence limits are attached.

Flight time around a layer's perimeter was usually about 7 hours. The actual divergence either changed or did not change, as did the wind field. Addition by sampling in time or space is not continuous. Even if the exact component for every sampled period or interval were known, as each point is associated with a different time, integration (summation) of the component around the perimeter provides a very complicated estimate, $\overline{D}$, of the divergence as it changes in time. $\overline{D}$ is a random variable whose value is a function of the time at which the aircraft arrives at each point on the perimeter.

However complicated it may be, it is the only estimate that can be obtained from aircraft winds. The same difficulties will be inherent in any system of wind measurements that does not provide simultaneous winds at each and every point of the BOMEX or any other volume under consideration.

This appendix describes the computation of $\overline{D}$ and the attachment of confidence limits to the estimate. To the extent that $\overline{D}$ is invariant with time during the flight, the confidence limits have a broader interpretation.

VARIANCE OF WIND COMPONENT NORMAL TO BOMEX VOLUME PERIMETER

The term $v_i(x,t)$ represents the observed velocity normal (perpendicular) to the i-th side of the BOMEX volume at a time t, as measured from the start of the flight, and at a distance x from the corner of the BOMEX volume. Dropping the subscript i, a statistical model for this velocity on side i is:

$$v(x,t) = u + d(x) + w(t) + e(x,t),$$

where

u = the mean normal velocity along the side,
$d(x)$ = an adjustment for the effect of distance x,
$w(t)$ = an adjustment for the effect of time, and
$e(x,t)$ = an error deviation that includes all other sources of error.

The variance among the observations on a side includes variation due to $d(x)$, $w(t)$, and $e(x,t)$.

Draw a large number of pairs of values of x and t at random and assign the value of $v(x,t)$ to each. The variance among the v's as the number of pairs increases toward infinity would tend toward

$$\sigma_v^2 = \sigma_d^2 + \sigma_w^2 + \sigma_e^2 ,$$

where the subscript identifies the source to which the variance applies. In the construction of confidence limits for divergence, one would like to use σ_v^2 . However, σ_v^2 is unknown. The sample variance tends to underestimate σ_v^2. The underestimation is not serious if there is no systematic variation of $d(x)$ with x or of $w(t)$ with t, but it tends to produce confidence limits for the divergence that are too narrow.

The nature of the underestimation is as follows. The sample variance computed from the observations on a side estimates

$$f_d\ \sigma_d^2 + f_w\ \sigma_w^2 + f_e\ \sigma_e^2 ,$$

where the f's are fractions. The fraction f_e should be essentially 1 as variation of e is due to largely independent error deviations from one 5-min sample to another. The fractions f_d and f_w are expected to be less than 1, because

(a) only a portion of the side of the volume was sampled, and
(b) only a portion of the variation of the entire flight time was sampled.

The fact that f_d and f_w are less than 1 becomes unimportant if the variances and σ_d^2 and σ_w^2 are small, in which case the sample variance can be regarded as a good estimator of σ_v^2. When the variances σ_d^2 and σ_w^2 are not small, but f_d and f_w are, the sample variance underestimates σ_v^2, and the confidence limits based on the sample variance will be too small.

The data available from side i are not a random sample of values of $v_i(s,t)$, as values in the x,t plane were not sampled at random. What is available is a set of values of v_i, which will be identified as v_{ij}, where $j(i = 1,w,. . . p_i)$ is the j-th value. Both distance and time increase with the subscript j. The average component on a side is estimated as

$$\overline{v}_i = (1/p_i) \sum_j v_{ij} .$$

The sample variance of v_{ij} is calculated as

$$s^2_{v_{ij}} = [1/(p_i - 1)] \sum_j (v_{ij} - \overline{v}_i)^2 ,$$

which can be regarded as an estimator of σ_v^2, subject to the limitations discussed earlier.

The variance of the estimated average velocity on a side is estimated as

$$s^2_{\overline{v}_i} = (1/p_i) \; s^2_{v_{ij}} .$$

RATIONALE FOR USE OF CALIBRATION DATA OBTAINED DURING SOUNDINGS

Primes differentiate calibration data from other data. During the stepped soundings at the midpoint of each side of the BOMEX volume, the aircraft flew at seven different heights. On one of the sides, calibration data were obtained twice, once at the beginning of the sounding and once at its completion. At each height the aircraft flew 2 min in one direction and 2 min in the opposite direction along the same path. An average velocity normal to the face of the side was computed for each 2-min interval. Subtracting the average normal velocity in the clockwise direction from that in the counterclockwise direction provides the difference in average normal velocity. An average difference is then calculated over the seven heights for three of the sides and as an average of 14 differences for the side with the two sets of calibration data.

Models for the calibration data are

$$\overline{v}'_{io} = \overline{v}'_{i,cor} + b_i ,$$

and

$$\overline{v}'_{i1} = \overline{v}'_{i,cor} - b_i ,$$

where

v'_{io} = observed average normal velocity in the counter-clockwise direction for the i-th side,

v'_{i1} = observed average normal velocity in the clockwise direction for the i-th side,

$v'_{i,cor}$ = corrected average normal velocity, and

b_i = a bias correction for the i-th side.

It follows that

$$b_i = (1/2)(\overline{v}'_{io} - \overline{v}'_{i1}) = 1/2\ \overline{d}_i ,$$

where

$$\overline{d}_i = (1/q_i) \sum_j d_{ij} .$$

When the data were obtained along the sides of the BOMEX volume, the aircraft was flying counterclockwise. Thus the corrected average normal velocity was estimated as

$$\overline{v}_{i,cor} = \overline{v}_i - (1/2)\ \overline{d}_i .$$

At each height for the calibration data, a difference in normal velocity was computed by subtracting the normal velocity in the clockwise direction from the normal velocity in the counterclockwise direction. This difference for the j-th ($j = 1,2, \ldots q_i$) height on the i-th side was represented as d_{ij}. The $d_{ij}/2$ for a side was regarded as a random sample of independent estimates of the bias characteristic of side i. The implicit assumption was made that the bias was the same at all heights and independent of the distance x or the time t. To the extent that these assumptions are invalid, the sample variance of the d_{ij}'s underestimates the variance that should be used in the construction of confidence limits. The average of the d_{ij}'s is

$$d_i = (1/q_i) \sum_{j=1}^{q_i} d_{ij} = \overline{v}'_{io} - \overline{v}'_{i1} .$$

The sample variance of the d_{ij}'s is

$$s^2_{d_{ij}} = [1/(q_i-1)] \sum_j (\overline{d}_{ij} - d_i)^2 ,$$

and the estimated variance of $\overline{d}_i$ is

$$s^2_{d_{ij}} = (1/q_i)\ s^2_{d_{ij}}$$

ESTIMATION OF DIVERGENCE

In terms of the foregoing notation, the estimate of the divergence of the horizontal surface layer of the BOMEX volume was

$$\hat{D} = L \sum_{i=1}^{4} (v_i - (1/2)\overline{d}_i) ,$$

where L is the length of a side. The estimated variance of $\hat{D}$ is

$$s^2_{\hat{D}} = L^2 \left\{ \sum_{i=1}^{4} \left[s^2_{\overline{v}_i} + 1/4\ s^2_{\overline{d}_i} \right] \right\}$$

Ninety-five percent confidence limits for the true divergence are computed as

$$\hat{D} \pm t_f s_{\hat{D}} ,$$

where t_f is a value taken from a two-tailed Student t table with f degrees of freedom. The exact degrees of freedom that should be attached to it is unknown.

The number of degrees of freedom is approximated by a formula developed by Satterthwaite (see Satterthwaite, 1946) for estimating the degrees of freedom to be attached to a linear function F of independent estimated variances of the form

$$F = \sum_i a_i \hat{v}_i ,$$

where a_i are arbitrary coefficients, and $\hat{v}_i$ are independently estimated variances with f_i degrees of freedom. The approximation formula is

$$f = \frac{(\Sigma a_i v_i)^2}{\Sigma (a_i v_i / f_i)^2} ,$$

where f is the approximate number of degrees of freedom to be attached to F.

In the application here, the following identifications were made:

$a_1 = L^2$	$v_1 = s^2_{\bar{v}_1}$	$f_1 = p_1$
$a_2 = L^2$	$v_2 = s^2_{\bar{v}_2}$	$f_2 = p_2$
$a_3 = L^2$	$v_3 = s^2_{\bar{v}_3}$	$f_3 = p_3$
$a_4 = L^2$	$v_4 = s^2_{\bar{v}_4}$	$f_4 = p_4$
$a_5 = L^2/4$	$v_5 = s^2_{\bar{v}_1}$	$f_5 = q_1$
$a_6 = L^2/4$	$v_6 = s^2_{\bar{v}_2}$	$f_6 = q_2$
$a_7 = L^2/4$	$v_7 = s^2_{\bar{v}_3}$	$f_7 = q_3$
$a_8 = L^2/4$	$v_8 = s^2_{\bar{v}_4}$	$f_8 = q_4$

REFERENCE

Satterthwaite, F.E., "An Approximate Distribution of Estimates of Variance Components," Biomedical Bulletin, Vol. 2, 1946, pp. 110-114.

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