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# NOAA Technical Memorandum NESS 21 

U.S. DEPARTMENT OF COMMERCE

National Oceanic and Atmospheric Administration
National Environmental Satellite Service

## Geostationary Satellite Position and Attitude Determination Using Picture Landmarks

WILLIAM J. DAMBECK

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## GEOSTATIONARY SATELLITE POSITION AND ATTITUDE

 DETERMINATION USING PICTURE LANDMARKSWilliam J. Dambeck

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# GEOSTATIONARY SATELLITE POSITION AND ATTITUDE DETERMINATION USING PICTURE LANDMARKS 

William J. Dambeck


#### Abstract

Given a truly geostationary satellite, it is theoretically possible to determine both its position and attitude from the relative orientation of landmarks in pictures from the satellite. This paper details several attempts to exploit this possibility.


## INTRODUCTION

This study, prompted by the initial planning for the Synchronous Meteorological Satellite - Geostationary Operational Environmental Satellite (SMS-GOES) series of geostationary satellites, had considerations of economy as a primary stimulus. It was generally recognized that NOAA would have the responsibility for doing its own ranging, rather than letting trilateration ranging equipment contracts. There was, of course, a monetary consideration involved: The equipment would have to be housed, personnel trained to operate and maintain it, provisions made for remote stations outside the United States, and software developed for analyzing the data. Concomitantly, it was felt that extant techniques for attitude determination, employed for Applications Technology Satellite (ATS-1 and ATS-3) operations, left much to be desired. This method relied on a combination of measured sun-pulse transit times between two slits on the satellite and polarization angle (POLANG) measurements. The procedure proved to be somewhat troublesome and admitted to an error as great as 0.5 to 1.0 degree in the spin-axis vector.

This error factor led to the development of a software program, which, while it accepts the Goddard Space Flight Center's (GSFC) satellite position measurements, made independent estimates of the attitude. Westinghouse Corporation, in particular, devoted extensive effort to a method based on earthhorizon profile measurements; NASA now employs this method. NESS evolved separate in-house approaches: One a horizon method, the other a geometrical method using picture landmark sample and line numbers as inputs. Although the horizon method was used initially as a diagnostic aid, operational use has been confined to the geometrical method which is somewhat more straightforward.

Finally, the transmission to NESS of orbital elements for ATS-1 and ATS-3, as determined by NASA, was normally delayed sufficiently to cast considerable doubt on the accuracy of the predicted positions based on these data. This was not acceptable to the NESS picture-pair wind velocity vector consortium, so it was thought desirable to have some other means for checking the accuracy of the calculated satellite positions.

For the above reasons and because it was anticipated that a landmark approach would in any event be used for finding the satellite attitude (the first SMS-GOES satellite is to include much of both landmark-rich Americas in its coverage), it was decided to investigate the feasibility of determining satellite attitude and position simultaneously. This investigation would assume as inputs the plate coordinates (sample and line numbers) of identified landmarks in the pictures to arrive at satellite position and orientation by variational procedures. Hopefully the traditional ranging approach could be abandoned.

THE SNAPSHOT APPROACH: THEORY
Initial attempts to obtain workable solutions relied on the central projection methods used in aerial photography. The sample and line number of a landmark are conceptually equivalent to the measured coordinates on a photographic plate. Consider figure 1. A satellite at $S$ with axes $x, y, z$ fixed in it, photographs a landmark $P$. The image of $P$ falls on the plate (represented here as a diapositive) at $I$ and has plate coordinates $x_{I}, y_{I}$, $d$. The plate is taken as perpendicular to the satellite's z-axis; the projection center is at $S$, the projection distance being $z_{I}=d$.


Figure 1.--Central projection.

Then the direction cosines of $P$ as seen from $S$, in the $x, y, z$ system, are:

$$
l_{I}=\frac{x_{I}}{\sqrt{x_{I}^{2}+y_{I}^{2}+d^{2}}}, m_{I}=\frac{y_{I}}{\sqrt{x_{I}^{2}+y_{I}^{2}}+d^{2}}, m_{I}=\sqrt{x_{i}^{2}+y_{x^{2}}^{2}+d^{2}},
$$

from which we obtain

$$
\begin{align*}
& \frac{l_{x}}{n_{x}}=\frac{x_{I}}{d}, \quad \frac{m_{x}}{n_{x}}=\frac{y_{x}}{d} \quad \text { or } \\
& x_{I}-d\left(\frac{l_{x}}{x_{I}}\right)=0 \quad \text { and } \quad y_{I}-d\left(\frac{m_{I}}{n_{I}}\right)=0 . \tag{1}
\end{align*}
$$

The direction cosines of $P$ as seen from $S$, in the earth-fixed or inertial coordinate system $X, Y, Z$, are:

$$
\begin{align*}
& \ell=\sqrt{\left(X_{p}-X_{s}\right)^{2}+\left(Y_{p}-Y_{s}\right)^{2}+\left(Z_{p}-Z_{s}\right)^{2}}, \\
& m=\sqrt{\left(X_{p}-X_{s}\right)^{2}+\left(Y_{p}-Y_{s}\right)^{2}+\left(Z_{p}-Z_{s}\right)^{2}}, \\
& m=\sqrt{\left(X_{p}-X_{s}\right)^{2}+\left(Y_{p}-Z_{s}\right)^{2}+\left(Z_{p}-Z_{s}\right)^{2}} \tag{2}
\end{align*}
$$

Now let $A=\left(a_{i j}\right)$ be the rotation matrix by which a free vector in the $x$, $Y, Z$ system is transformed into its $x, y, z$ system equivalent, that is, for any vector $\overrightarrow{\mathrm{T}}$, we have

$$
\left[\begin{array}{l}
x_{T} \\
y_{T} \\
Z_{T}
\end{array}\right]=\left[\begin{array}{lll}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
a_{31} & a_{12} & a_{23}
\end{array}\right] \cdot\left[\begin{array}{l}
X_{T} \\
Y_{T} \\
Z_{T}
\end{array}\right] .
$$

Using this relation, we obtain

$$
\begin{aligned}
& l_{I}=a_{11} l+a_{12} m+a_{13} n \\
& m_{I}=a_{21} l+a_{22} m+a_{23} n \\
& m_{I}=a_{31} l+a_{32} m+a_{33} n,
\end{aligned}
$$

so that

$$
\begin{align*}
& \frac{l_{I}}{m_{I}}=\frac{a_{11} l+a_{12} m+a_{13} m}{a_{31} l+a_{32} m+a_{33} m}, \\
& \frac{m_{I}}{m_{I}}=\frac{a_{31} l+a_{22} m+a_{23} m}{a_{31} l+a_{22} m+a_{33} m} . \tag{3}
\end{align*}
$$

Substituting the expressions in equations (2) for $1, m, n$ into equations (3) and substituting the resultant expressions into equations (1), we obtain

$$
\begin{align*}
& x_{I}-\alpha \cdot \frac{a_{11}\left(X_{p}-X_{s}\right)+a_{12}\left(Y_{p}-Y_{s}\right)+a_{13}\left(Z_{p}-Z_{s}\right)}{a_{31}\left(X_{p}-X_{s}\right)+a_{32}\left(Y_{p}-Y_{s}\right)+a_{33}\left(Z_{p}-Z_{s}\right)}=0, \\
& y_{I}-\alpha \cdot \frac{a_{21}\left(X_{p}-X_{s}\right)+a_{22}\left(Y_{p}-Y_{s}\right)+a_{23}\left(Z_{p}-Z_{s}\right)}{a_{31}\left(X_{p}-X_{s}\right)+a_{32}\left(Y_{p}-Y_{s}\right)+a_{33}\left(Z_{p}-Z_{s}\right)}=0 . \tag{4}
\end{align*}
$$

The equations (4) are our condition equations, two for each identifiable landmark. Now let us examine these equations and see what are the known and unknown quantities. Presumably we know d, the projection distance. We will also know $x_{I}$ and $y_{I}$ by direct measurement; $X_{p}, Y_{p}, Z_{p}$ are calculable from the latitude and longitude of the landmark (and the fime of the picture, if we are in an inertial system), obtainable from maps or an atlas. We do not know the satellite location $X_{S}, Y_{S}, Z_{s}$ nor do we know the rotation matrix. coefficients $a_{11}, a_{12}, \ldots, a_{33}$. These latter, however, are expressible in terms of a lesser number of rotation angles, such as the Eulerian angles $\theta, \phi, \psi$ illustrated in Figure 2.


Figure 2.--The Eulerian angles.

In terms of $\theta, \phi, \psi$, we would have

$$
\begin{aligned}
& a_{11}=\cos \theta \cos \phi \cos \psi-\sin \phi \sin \psi, \\
& a_{12}=\cos \theta \sin \phi \cos \psi+\cos \phi \sin \psi, \text { etc. }
\end{aligned}
$$

These rotation angles may be considered an expression of the satellite attitude.

Summing up, we have a total of six unknown quantities- $-X_{s}, Y_{s}, Z_{s}$ and $\theta$, $\phi, \psi$--which are the same for each landmark, and we have two independent equations for each landmark. Therefore, a picture with four or more landmarks will overdetermine the six unknowns. This overdetermination permits
us to employ least-squares iterative methods and to obtain successively better approximations to the six unknowns, starting from some initial guesses or estimates of their values. A detailed description of the procedure will be postponed to the discussion of the second approach; anyone interested in the special application of least-squares methods to this particular approach should consult Duane Brown's excellent monograph (1957) on the subject. Essentially one makes educated guesses as to the unknown parameters and utilizes the least-squares procedure to compute corrections to these estimates. The procedure is then iterated to the extent warranted by the desired degree of covergence.

TESTING AND RESULTS
The feasibility of the abovementioned approach was tested utilizing a "paper" (i.e., computer-simulated) satellite viewing 23 well-distributed hypothetical landmarks (fig. 3). The theory previously considered is, however, a "snapshot" theory --the image on the photographic plate represents the landmarks as seen simultaneously by the satellite at some given instant of time. The situation for a spin-scan camera is considerably different. The picture consists of a series of stepped scans (one per satellite-spin revolution); approximately 20 minutes separate the times of the first and the last scan.


Figure 3.--Landmark configuration used in testing.
A truly geostationary satellite has, by definition, a fixed position in geographic space. Its spin axis, however, is fixed in inertial space, which results in a complete rotation (precession) of this axis in geographic space every siderial day. Because of this precession it became expedient to introduce a time-dependent rotational matrix into the position-attitude determination software to change the original sample and line numbers into the values they would assume were the spin axis nonrotating. This was easily accomplished and initial testing for a zero-inclination angle "paper" satellite exhibited excellent convergence properties. A more extensive series of tests was then devised, with small random discrepancies introduced into the line and sample
numbers to simulate the inevitable imprecision characteristic of real data. These discrepancies tend to approximate the effects of moderate line and frame synchronization jitter, of errors in reading the landmark coordinates, etc., under fairly good operating conditions.

Because no real satellite is perfectly geostationary, and with the passage of time (and the diminishing amount of gas available for manuevering) becomes less geostationary, it was decided to run tests using a series of satellite inclination angles; the other orbital elements of the "paper" satellite were those then current for ATS-1. The first inclination angle was $i=0^{\circ}$ (the geostationary case), and the last was $i=19600$ (the current ATS-1 value). The results are tabulated in table 1.

Although the results reflect to some extent the randomness of the discrepancies introduced, several observations may be made. On the one hand, the computed subpoint location is relatively unaffected by the inclined orbits. On the other, the calculated earth-satellite distance and the spin-axis right ascension and declination solutions are very significantly influenced by these orbits, the discrepancies tending to increase with the inclination of the orbit. For this reason, and also bearing in mind that the landmark distribution of figure 3 is likely to be far better than anything actually encountered, it was decided that this approach could not be recommended as an operational procedure. Also, experience with ATS-1 suggested that operating conditions are usually considerably less favorable than those simulated above.

Table 1--Test results for 23 well-distributed points (latitudes positive North; longitudes positive East)

| Incl. $=0^{\circ}$ | Initial guess | Computed | True | Discrepancy |
| :---: | :---: | :---: | :---: | :---: |
| Spin-axis R.A. | 335.0000 | 339.5875 | 338:9310 | $0 \% 6565$ |
| Spin-axis Decl. | 8800000 | $86: 3626$ | 86.3280 | $0{ }^{\circ} 0346$ |
| Subpt. 1at. | $-1!0000$ | 0.0470 | 0.0000 | 0.0470 |
| Subpt. long. | -151.0000 | -149:9028 | -149.9252 | 0.0224 |
| Radius vector | 41,000.00 (km) | 42,184.09 | 42,159.20 | 24.89 (km) |
| Inc1. $=0.10$ | Initial guess | Computed | True | Discrepancy |
| Spin-axis R.A. | 335:0000 | 340.5932 | 338.9310 | $1: 6622$ |
| Spin-axis Decl. | 88.0000 | 86:3767 | $86^{\circ} 3280$ | $0_{0}^{0} 0487$ |
| Subpt. lat. | -100000 | -0.0124 | 0.0221 | -0.0345 |
| Subpt. long. | -151:0000 | -149?8675 | -140?9240 | 0.0565 |
| Radius vector | 41,000.00(km) | 41,628.96 | 42,159.20 | -530.24 (km) |


| Inc1. $=0.25$ | Initial guess | Computed | True | Discrepancy |
| :---: | :---: | :---: | :---: | :---: |
| Spin-axis R.A. | 335:0000 | 33796626 | $338: 9310$ | -192684 |
| Spin-axis Decl. | $88: 0000$ | $86: 2579$ | $86: 3280$ | -0:0701 |
| Subpt. 1at. | -190000 | $0: 0023$ | 090553 | -0:0530 |
| Subpt. 1ong. | -151:0000 | -149:9678 | -14999242 | -0:0436 |
| Radius vector | 41,000.00 (km) | 41,877.59 | 42,159.19 | -281.60(km) |
| Inc1. $=0950$ | Initial guess | Computed | True | Discrepancy |
| Spin-axis R.A. | 33590000 | 33696034 | 33899310 | -293276 |
| Spin-axis Decl. | $88: 0000$ | $86: 2373$ | 8693280 | -0:0907 |
| Subpt. lat. | -190000 | $0: 0698$ | $0 \% 1106$ | -0:0408 |
| Subpt. long. | -151:0000 | -14998947 | -149\%9247 | 090300 |
| Radius vector | 41,000.00(km) | 41,628.73 | 42,159.19 | -530.46 (km) |
| Inc1. $=1900$ | Initial guess | Computed | True | Discrepancy |
| Spin-axis R.A. | 33590000 | 33298383 | 33899310 | -6:0927 |
| Spin-axis Decl. | $88: 0000$ | $86: 0706$ | 8693280 | -0:2574 |
| Subpt. lat. | -190000 | 092109 | 092211 | -0:0102 |
| Subpt. long. | -15190000 | -14999189 | -149:9265 | 0:0076 |
| Radius vector | 41,000.00(km) | 41,158.77 | 42,159.17-1 | ,000.40 (km) |
| Incl. $=1.609 *$ | Initial guess | Computed | True | Discrepancy |
| Spin-axis R.A. | 33590000 | 32694830 | 338:9310* | -12:4480 |
| Spin-axis Dec1. | $88: 0000$ | $85: 7066$ | 86:3280* | -0:6214 |
| Subpt. 1at. | -1:0000 | 0.2942 | 00.3557* | -0:0615 |
| Subpt. long. | -151:0000 | -149?9626 | -14999301* | -0.0325 |
| Radius. vector | 41,000.00 (km) | 41,214.83 | 42,159.15* | -944.32 (km) |
| * Actual values for ATS-1 at one time. |  |  |  |  |

## A MORE GENERAL APPROACH

It is seen that the classical central projection theory is inadequate for determining simultaneously the attitude and position of current geostationary satellites because such satellites are really only approximately geostationary; their excursion in latitude during the 20 -minute scanning period required for the picture is sufficient to destroy the coherency of the data. Because of this, some other approach whose validity would be independent of the motion of the satellite was sought. The most reasonable approach seemed to be to try to determine the elements of the orbit for a given epoch, together with the attitude vector. The position at any subsequent time could then be computed with the available orbital prediction software, while the attitude vector presumably would remain unchanged.

We would try to solve then for eight variables:
a, the semimajor axis;
$\varepsilon$, the eccentricity of the orbit;
$i$, the inclination angle;
$\omega$, the argument of perigee;
$\Omega$, the right ascension of the ascending node;
$m$, the mean anomaly at the epoch $\mathrm{T}_{0}$;
$\alpha$, the right ascension of the positive spin vector; and
$\Delta$, the declination of the positive spin vector.
About this time it became known from the research of Doolittle et al. (1970) that for ATS-1 some nonlinear longitudinal shifting of the vectors represented by the central sample numbers was taking place. To avoid this source of error, it was suggested by Doolittle et al. that only the line numbers (rather than both line and sample numbers) of the observed landmarks be used in the determination. Also, because of the sensitivity of the orbital elements to the data from which they are computed, it was decided to use the data from a series of pictures covering a good-sized sector of an orbit. It was assumed that the cumulative effect of the perturbing forces during this period of time would be sufficiently small to permit the use of the classical two-body motion approximation.

## LEAST-SQUARES ELEMENTS: THEORY

Each scan line corresponds to a certain angular displacement: Let $\nu$ be the number of radians per line. Then, if $j$ is the line number for a landmark and $j_{C L}$ the line number for the scan perpendicular to the spin axis, the cosine of the angle between the positive spin axis and a vector extending from the satellite to the landmark is given by

$$
\begin{equation*}
\cos \theta^{\prime}=\sin \{\nu(j c t-j)\} . \tag{5}
\end{equation*}
$$

If this angle is computed in terms of the unknown parameters, that is,

$$
\theta=\theta(a, \varepsilon, i, \omega, \Omega, m, \alpha, \Delta),
$$

then our condition equation may be witten

$$
\begin{equation*}
f=\cos \theta-\cos \theta^{\prime}=0 \tag{6}
\end{equation*}
$$

where the primed angle represents the angle as computed from the ouserved line number and the unprimed angle represents the angle as computed from the unknown parameters.

1. If $a_{0}, \varepsilon_{o}, i_{o}, \omega_{0}, \Omega_{0}, m_{o}, \alpha_{0}$, and $\Delta_{0}$ are good estimates of $a, \varepsilon, i, \omega, \Omega, m, \infty$, and $\Delta$, all at the epoch $T_{o}$, we may write to a sufficient annroximation

$$
\begin{aligned}
f(a, \varepsilon, i, \omega, \Omega, m, \alpha, \Delta, j)=f\left(a_{0}, \varepsilon_{0}, i_{0}, \omega_{0}, \Omega_{0}, m_{0}, \alpha_{0}, \Delta_{0}, j\right) & +\left.\frac{\partial f}{\partial a}\right|_{0} \delta a+\left.\frac{\partial f}{\partial \varepsilon}\right|_{0} \delta \varepsilon+\left.\frac{\partial f}{\partial i}\right|_{0} \delta i+\left.\frac{\partial f}{\partial \omega}\right|_{0} \delta \omega \\
& +\left.\frac{\partial f}{\partial \Omega}\right|_{0} \delta \Omega+\left.\frac{\partial f}{\partial m}\right|_{0} \delta_{m}+\left.\frac{\partial f}{\partial \alpha}\right|_{0} \delta \alpha+\left.\frac{\partial f}{\partial \Delta}\right|_{0} \delta_{\Delta}
\end{aligned}
$$

where $a=a_{0}+\delta a, \varepsilon=\varepsilon_{0}+\delta \varepsilon$, etc.
Let $f_{0}=f\left(a_{0}, \cdots, \Delta_{0}, j\right)$. Then, because $f(a, \cdots, \Delta, j)=0$, we may write

$$
f_{0}+\left.\frac{\partial f}{\partial a}\right|_{0} \delta a+\cdots-\cdots+\left.\frac{\partial f}{\partial a}\right|_{0} \Delta=0 .
$$

There will be $N$ such equations, one for eaci: landmar:. [f lie define

$$
\begin{equation*}
f_{0 i} \equiv f\left(a_{0}, \varepsilon_{0}, i_{0}, \omega_{0}, s_{0}, m_{0}, \alpha_{0}, \Delta_{0}, j_{i}\right), \tag{7}
\end{equation*}
$$

these equations may be written

$$
\begin{equation*}
f_{0 i}+\frac{\partial f}{\partial a_{0 i}} \|_{a}+\cdots+\cdots+\left.\frac{\partial f}{\partial \Delta}\right|_{0 i} \delta \Delta=r_{i} \tag{8}
\end{equation*}
$$

where $r_{i}$ is the ith residual. From equations (5) and (6), we see that

$$
\frac{\partial f}{\partial a}=\frac{\partial}{\partial a}(\cos \theta), \ldots, \ldots, \frac{\partial f}{\partial \Delta}=\frac{\partial}{\partial \Delta}(\cos \theta)
$$

Now

$$
\begin{equation*}
\cos \theta_{i}=l_{s} l_{i}+m_{s} m_{i}+m_{s} m_{i} \tag{9}
\end{equation*}
$$

where $1_{s}, m_{s}, n_{s}$ are the satellite spin-vector direction cosines and $1_{i}, m_{i}, n_{i}$ are the direction cosines of the satellites--ith landmark vector. We have

$$
\begin{equation*}
l_{s}=\cos \alpha \cos \Delta, \quad m_{5}=\sin \alpha \cos \Delta, \quad m_{3}=\sin \Delta, \tag{10}
\end{equation*}
$$

and

$$
\begin{align*}
& l_{i}=\sqrt{\left(x_{i}-X_{i}\right)^{2}+\left(y_{i}-Y_{i}\right)^{2}+\left(z_{i}-Z_{i}\right)^{2}}, \\
& m_{i}=\sqrt{\left(x_{i}-X_{i}\right)^{2}+\left(\frac{y_{i}-Y_{i}}{\left.y_{i}-Y_{i}\right)^{2}+\left(z_{i}-Z_{i}\right)^{2}},\right.} \\
& m_{i}=\sqrt{\left(x_{i}-X_{i}\right)^{2}+\left(z_{i}-Z_{i}\right.}, \tag{11}
\end{align*}
$$

where ( $x_{i}, y_{i}, z_{i}$ ) is the landmark position and ( $X_{i}, Y_{i}, Z_{i}$ ) is the satellite position, both in inertial space at the time the ith landmark is scanned.
Let us show how $f_{o i}$ and $\left.\frac{\partial f}{\partial a}\right|_{0 i}$ are actually computed--the expressions and computational procedure for $\left.\frac{\partial f}{\partial \varepsilon}\right|_{0 i},\left.\frac{\partial f}{\partial i}\right|_{0 i}$, etc. should then be easily derivable by the interested reader.
Suppose we have a landmark (the ith one) on an ATS-type picture. We find the line number $j_{i}$ passing through the landmark. Finding the picture start-time and the satellite spin-rate, we compute the time
(elapsed since our chosen epoch $T_{o}$ ) corresponding to $\mathfrak{j}_{i}$,
$\Delta t=\left(j_{i} \div\right.$ spin rate $)+$ picture start time $-T_{0}$.
Obtaining an ephemeris, we express $T_{o}$ as its sidereal equivalent; the time since the epoch is also expressed in sidereal units,

$$
\Delta t=1.0027379093 \times \Delta t_{\text {(mean solar) }}
$$

Using a good map, we determine the 1 atitude and longitude ( $\phi \mathrm{gd}, \lambda$ ) of the 1 andmark, changing the latitude (which is geodetic) into its geocentric equivalent by means of the equation

$$
\begin{align*}
\phi=\phi_{g \alpha}-0.00335851579910756 \sin \left(2 \phi_{g} \alpha\right) & +0.000005639814186428 \sin \left(4 \phi_{g \alpha}\right) \\
& -0.000000012627603366 \sin \left(6 \phi_{g \alpha}\right) \\
& +0.000000000031807504 \sin \left(8 \phi_{g \alpha}\right) . \tag{13}
\end{align*}
$$

$\underset{\sim}{\text { ㄱ․ }}$ Defining the constant $K_{T}=2 \pi /$ sidereal day, we compute $x_{i}, y_{i}, z_{i}$,

$$
\begin{align*}
& x_{i}=\frac{a_{0} b_{0} \cos \phi_{i} \cos \left(\lambda_{i}+k_{T}\left(t_{0}+\Delta t\right)\right)}{\sqrt{a_{i}^{2} \sin ^{2} \phi_{i}+b_{0}^{2} \cos ^{2} \phi_{i}}}, \\
& y_{i}=\frac{a_{0} b_{0} \cos \phi_{i} \sin \left(\lambda_{i}+K_{\tau}\left(T_{0}+\Delta t\right)\right)}{\sqrt{a_{0}^{2} \sin ^{2} \phi_{i}+b_{i}^{2} \cos ^{2} \phi_{i}}}, \\
& z_{i}=\sqrt{a_{0}^{2} \sin ^{2} \phi_{i}+b_{0} b_{i}^{2} \phi_{i} \cos ^{2} \phi_{i}} \tag{14}
\end{align*},
$$

where $a_{0}$ and $b_{\infty}$ are the semimajor and semiminor axes of the earth, respectively, the $\lambda_{i}$ is considered positive eastward. We note that $K_{T}\left(T_{o}+\Delta t\right)$ is the Greenwich Hour Angle (G.H.A.) at the time the landmark is scanned.

To determine the position of the satellite in inertial space, we must first ascertain its eccentric anomaly $E_{i}$ when the ith landmark is scanned. We define the satellite's mean motion as

$$
\begin{equation*}
n_{0} \equiv 0.07436574 \times(6378.270)^{3 / 2} \times a_{0}^{-3 / 2} \tag{15}
\end{equation*}
$$

and solve Kepler's Equation

$$
E_{i}-\varepsilon_{0} \sin E_{i}=m_{0}+n_{0} \Delta t \text { (mean solar) }
$$

for $E_{i}$; various iterative schemes for accomplishing this are extant in the literature. The $a_{0}, \varepsilon_{0} \geqslant v$ and $m_{0}$ are initial orbital element estimates. Using this computed value of $E_{i}$, we may compute

Also, from here on, the semimajor acis should be considered asjbeing expressed in earth radius
$X_{i}=a_{0}\left(\cos \omega_{0} \cos \Omega_{0}-\sin \omega_{0} \sin \Omega_{0} \cos i_{0}\right)\left(\cos E_{i}-\varepsilon_{0}\right)$ units of 6378.270 km .
$-a_{0} \sqrt{1-E_{0}^{2}}\left(\sin \omega_{0} \cos \Omega_{0}+\cos \omega_{0} \sin \Omega_{0} \cos i_{0}\right) \sin E_{i}$,
$Y_{i}=a_{0}\left(\cos \omega_{0} \sin \Omega_{0}+\sin \omega_{0} \cos \Omega_{0} \cos i_{0}\right)\left(\cos E_{i}-\varepsilon_{0}\right)$
$+a_{0} \sqrt{1-\varepsilon_{0}{ }^{2}}\left(-\sin \omega_{0} \sin \Omega_{0}+\cos \omega_{0} \cos \Omega_{0} \cos i_{0}\right) \sin E_{i}$,
$Z_{i}=a_{0}\left(\sin \omega_{0} \sin i_{0}\right)\left(\cos E_{i}-\varepsilon_{0}\right)+a_{0} \sqrt{1-\varepsilon_{0}^{2}} \cos \omega_{0} \sin i_{0} \sin E_{i}$

So finally, combining equations (5), (6), (7), (10), and (11), we obtain for $f_{o i}$,

$$
\begin{equation*}
f_{0 i}=\frac{\left(x_{i}-X_{i}\right) \cos \alpha_{0} \cos \Delta_{0}+\left(y_{i}-Y_{i}\right) \sin \alpha_{e} \cos \Delta_{0}+\left(z_{i}-Z_{i}\right) \sin \Delta_{i}}{\sqrt{\left(x_{i}-X_{i}\right)^{2}+\left(y_{i}-Y_{i}\right)^{2}+\left(z_{i}-Z_{i}\right)^{2}}}-\sin \left\{\nu\left(j_{c L}-j_{i}\right)\right\} \tag{18}
\end{equation*}
$$

where $x_{i}, y_{i}, z_{i}$ have the values computed from equations (14), and $X_{i}, Y_{i}, Z_{i}$ are computed from
equations (17).
Concerning the computation of we obtain now
$\left.\frac{\partial f}{\partial a}\right|_{0 i}$, we have seen that $\frac{\partial f}{\partial a}=\frac{\partial(\cos \theta)}{\partial a}$ so by using equation (9),

$$
\begin{align*}
& \frac{\partial l_{i}}{\partial a_{0}}=\frac{\partial l_{i}}{\partial X_{i}} \frac{\partial X_{i}}{\partial a_{0}}+\frac{\partial l_{i}}{\partial Y_{i}} \frac{\partial Y_{i}}{\partial a_{0}}+\frac{\partial l_{i}}{\partial Z_{i}} \frac{\partial Z_{i}}{\partial a_{0}} \\
& \frac{\partial m_{i}}{\partial a_{0}}=\frac{\partial m_{i}}{\partial X_{i}} \frac{\partial X_{i}}{\partial a_{0}}+\frac{\partial m_{i}}{\partial Y_{i}} \frac{\partial Y_{i}}{\partial a_{0}}+\frac{\partial m_{i}}{\partial Z_{i}} \frac{\partial Z_{i}}{\partial a_{0}} \\
& \frac{\partial m_{i}}{\partial a_{0}}=\frac{\partial m_{i}}{\partial X_{i}} \frac{\partial X_{i}}{\partial a_{0}}+\frac{\partial m_{i}}{\partial Y_{i}} \frac{\partial Y_{i}}{\partial a_{0}}+\frac{\partial m_{i}}{\partial Z_{i}} \frac{\partial Z_{i}}{\partial a_{0}} \tag{19}
\end{align*}
$$

Defining

$$
\begin{equation*}
R_{i} \equiv \sqrt{\left(x_{i}-X_{i}\right)^{2}+\left(y_{i}-Y_{i}\right)^{2}+\left(z_{i}-Z_{i}\right)^{2}} \tag{20}
\end{equation*}
$$

and referring to equations (11), we obtain

$$
\begin{align*}
& \frac{\partial l_{i}}{\partial X_{i}}=-\left[\left(y_{i}-Y_{i}\right)^{2}+\left(z_{i}-Z_{i}\right)^{2}\right] / R_{i}^{3}, \\
& \frac{\partial l_{i}}{\partial Y_{i}}=\left(x_{i}-X_{i}\right)\left(y_{i}-Y_{i}\right) / R_{i}^{3}, \\
& \frac{\partial l_{i}}{\partial Z_{i}}=\left(x_{i}-X_{i}\right)\left(z_{i}-Z_{i}\right) / R_{i}^{3}, \\
& \frac{\partial m_{i}}{\partial X_{i}}=\frac{\partial \ell_{i}}{\partial Y_{i}}, \\
& \frac{\partial m_{i}}{\partial Y_{i}}=-\left[\left(x_{i}-X_{i}\right)^{2}+\left(z_{i}-Z_{i}\right)^{2}\right] / R_{i}^{3}, \\
& \frac{\partial m_{i}}{\partial Z_{i}}=\left(y_{i}-Y_{i}\right)\left(z_{i}-Z_{i}\right) / R_{i}^{3}, \\
& \frac{\partial m_{i}}{\partial X_{i}}=\frac{\partial l_{i}}{\partial Z_{i}}, \\
& \frac{\partial m_{i}}{\partial Y_{i}}=\frac{\partial m_{i}}{\partial Z_{i}}, \\
& \frac{\partial m_{i}}{\partial Z_{i}}=-\left[\left(x_{i}-X_{i}\right)^{2}+\left(y_{i}-Y_{i}\right)^{2}\right] / R_{i}^{3}, \tag{21}
\end{align*}
$$

Using equations (17) and the relation

$$
\begin{equation*}
\frac{\partial E_{i}}{\partial a_{0}}=-\frac{3}{2} \frac{E_{i}-\varepsilon_{0} \sin E_{i}-m_{0}}{a_{0}\left(1-\varepsilon_{0} \cos E_{i}\right)} \tag{22}
\end{equation*}
$$

obtainable from Kepler's Equation (16), we obtain

$$
\frac{\partial X_{i}}{\partial a_{0}}=\left[\left(\cos \omega_{0} \cos \Omega_{0}-\sin \omega_{0} \sin \Omega_{0} \cos i_{0}\right) \cdot\left(\cos E_{i}-\varepsilon_{0}+\frac{3}{2} \sin E_{i} \cdot \frac{E_{i}-\varepsilon_{0} \sin E_{i}-m_{0}}{1-E_{0} \cos E_{i}}\right)\right]
$$ $-\left[\sqrt{1-\varepsilon_{0}^{2}}\left(\sin \omega_{0} \cos \Omega_{0}+\cos \omega_{0} \sin \Omega_{0} \cos i_{0}\right) \cdot\left(\sin E_{i}-\frac{3}{2} \cos E_{i} \cdot \frac{\left.E_{i}-\varepsilon_{0} \sin E_{i}-m m_{0}\right)}{1-E_{0} \cos E_{i}}\right)\right]$,

$\frac{\partial Y_{i}}{\partial a_{0}}=\left[\left(\cos \omega_{0} \sin \Omega_{0}+\sin \omega_{0} \cos \Omega_{0} \cos i_{0}\right) \cdot\left(\cos E_{i}-\varepsilon_{0}+\frac{3}{2} \sin E_{i} \cdot \frac{E_{i}-\varepsilon_{0} \sin E_{i}-m_{0}}{1-\varepsilon_{0} \cos E_{i}}\right)\right]$ $+\left[\sqrt{1-\varepsilon_{0}{ }^{2}}\left(-\sin \omega_{0} \sin \Omega_{0}+\cos \omega_{0} \cos \Omega_{0} \cos i_{0}\right) \cdot\left(\sin E_{i}-\frac{3}{2} \cos E_{i} \cdot \frac{E_{i}-\varepsilon_{0} \sin E_{i}-m m_{0}}{1-\varepsilon_{0} \cos E_{i}}\right)\right]$,

$$
\begin{align*}
\frac{\partial Z_{i}}{\partial a_{0}}= & {\left[\left(\sin \omega_{0} \sin i_{0}\right) \cdot\left(\cos E_{i}-\varepsilon_{0}+\frac{3}{2} \sin E_{i} \cdot \frac{E_{i}-\varepsilon_{0} \sin E_{i}-m_{0}}{1-\varepsilon_{0} \cos E_{i}}\right)\right] } \\
& +\left[\sqrt{1-\varepsilon_{0}^{2}}\left(\cos \omega_{0} \sin i_{0}\right) \cdot\left(\sin E_{i}-\frac{3}{2} \cos E_{i} \cdot \frac{E_{i}-\varepsilon_{0} \sin E_{i}-m_{0}}{1-E_{0} \cos E_{i}}\right)\right] . \tag{23}
\end{align*}
$$

To evaluate $\left.\frac{\partial f}{\partial a}\right|_{o i}$, one then computes (20) and (21), and $\frac{\partial X_{i}}{\partial a_{0}}, \frac{\partial Y_{i}}{\partial a_{0}}>\frac{\partial Z_{i}}{\partial a_{0}}$
calculated values of $x_{i}, y_{i}, z_{i}, X_{i}, Y_{i}, z_{i}$, and $E_{i}$ in this computation. Using these values and equations (19) enables us to compute $\frac{\partial l_{i}}{\partial a_{0}}, \frac{\partial m_{i}}{\partial a_{0}}$, and $\frac{\partial m_{i}}{\partial a_{0}}$. Using equations (10), we may then finally calculate

$$
\begin{equation*}
\left.\frac{\partial f}{\partial a}\right|_{0 i}=\left(\cos \alpha_{0} \cos \Delta_{0}\right) \frac{\partial l_{i}}{\partial a_{0}}+\left(\sin \alpha_{0} \cos \Delta_{0}\right) \frac{\partial m_{i}}{\partial a_{0}}+\sin \Delta_{0} \frac{\partial m_{i}}{\partial a_{0}} \tag{24}
\end{equation*}
$$

In a similar manner, one may derive expressions for and calculate $\left.\frac{\partial f}{\partial \varepsilon}\right|_{0 i},\left.\frac{\partial f}{\partial i}\right|_{0 i},\left.\frac{\partial f}{\partial \omega}\right|_{0 i},\left.\frac{\partial f}{\partial \Omega}\right|_{0 i},\left.\frac{\partial f}{\partial m}\right|_{0 i},\left.\frac{\partial f}{\partial \alpha}\right|_{0 i}$, and $\left.\frac{\partial f}{\partial \Delta}\right|_{0 i}$.
Having computed all these quantities, let us look again at equation (8),

$$
f_{0 i}+\left.\frac{\partial f}{\partial a}\right|_{0 i} \delta a+----+\left.\frac{\partial f}{\partial \Delta}\right|_{\alpha i} \delta \Delta=r_{i} .
$$

The theory of least squares postulates that

$$
\begin{align*}
& \frac{\partial}{\partial(\delta a)}\left(\sum_{i}^{\pi} w_{i} r_{i}^{2}\right)=0 \\
& \frac{\partial}{\partial(\delta \varepsilon)}\left(\sum_{i}^{k} w_{i} r_{i}^{2}\right)=0 \\
& ----\cdots  \tag{25}\\
& \frac{\partial}{\partial(\delta \Delta)}\left(\sum_{i}^{r} w_{i} r_{i}^{2}\right)=0
\end{align*}
$$

where $w_{i}$ is the weight of the $i$ th condition equation and $N$ is the total number of landmarks. From this, we obtain

$$
\sum_{i}^{N} w_{i} r_{i} \frac{\partial r}{\partial(\delta a)}=\sum_{i}^{N} w_{i} r_{i} \frac{\partial r}{\partial(\delta \varepsilon)}=\ldots . . .=\sum_{i}^{N} w_{i} r_{i} \frac{\partial r_{i}}{\partial(\delta \Delta \Delta)}=0
$$

But from equation (8), we have $\frac{\partial r_{i}}{\partial(\delta a)}=\left.\frac{\partial f}{\partial a}\right|_{0 i}, \frac{\partial r_{i}}{\partial(\delta \varepsilon)}=\left.\frac{\partial f}{\partial \varepsilon}\right|_{0 i}$, etc.

So equations (25) finally reduce to $\left.\sum_{i}^{N} w_{i} r_{i} \frac{\partial f}{\partial a}\right|_{0 i}=0$,

$$
\begin{align*}
& \left.\sum_{i}^{N} w_{i} r_{i} \frac{\partial f}{\partial \varepsilon}\right|_{0 i}=0, \\
& \left.\sum_{i}^{N} w_{i} r_{i} \frac{\partial f}{\partial i}\right|_{0 i}=0, \text { etc. } \tag{26}
\end{align*}
$$

Substituting for $r_{i}$ on the right side of equation (8), we obtain

We have then eight linear equations with eight unknowns, that is

$$
\begin{align*}
& a_{11} \delta a+a_{12} \delta \varepsilon+a_{13} \delta i+a_{14} \delta \omega+a_{18} \delta \Omega_{6}+a_{16} \delta m+a_{17} \delta \delta+a_{18} \delta \Delta=c_{1}, \\
& a_{21} \delta a+a_{22} \delta \varepsilon+a_{23} \delta i+a_{24} \delta \omega+a_{25} \delta \Omega+a_{26} \delta m+a_{27} \delta \alpha+a_{28} \delta \Delta=c_{2}, \\
& \cdots \cdots+\cdots \\
& a_{11} \delta a+a_{12} \delta \varepsilon+a_{83} \delta i+a_{84} \delta \omega+a_{85} \delta \Omega+a_{16} \delta m+a_{88} \delta \alpha+a_{88} \delta \Delta=c_{8} \tag{28}
\end{align*}
$$

where from equations (27)

$$
\begin{align*}
& c_{1} \equiv-\sum_{i}^{N} w_{i} \frac{\partial f_{0 i}}{\partial a_{0 i}} f_{0 i}, c_{2} \equiv-\left.\sum_{i}^{N} w_{i} \frac{\partial f}{\partial \varepsilon}\right|_{0 i} f_{0 i}, \ldots, \ldots, c_{0} \equiv-\left.\sum_{i}^{N} w_{i} \frac{\partial f}{\partial \Delta}\right|_{0 i} f_{i i} . \tag{29}
\end{align*}
$$

Having already computed $f_{o i},\left.\quad \frac{\partial f}{\partial a}\right|_{o i},---,\left.\frac{\partial f}{\partial \Delta}\right|_{o i} \quad$ for every value of $i$, we may use their values and calculate all the $a_{i j}$ 's and $c_{i}{ }^{\prime} s ; \delta a, \delta \varepsilon, \delta i$, $\delta \omega, \delta \Omega, \delta m, \delta \alpha$, and $\delta \Delta$ may then be computed at once through the use of determinants, and the values obtained used to compute improved estimates of $a, \varepsilon, \ldots, \alpha, \Delta$ by means of $a=a_{0}+\delta a, \varepsilon=\varepsilon_{0}+\Delta \varepsilon$, etc. The entire process may then be iterated until a satisfactory degree of convergence is obtained.

## TESTING AND RESULTS

Using the preceding considerations, a computer program for determining orbital elements was originated and debugged for use on a CDC 6600 computer. Initial testing with multiple-pass data from a "paper" satellite yielded satisfactory results at first, i.e., perfect data generated by a hypothetical satellite with known attitude and orbital characteristics together with estimates of $a, \varepsilon$, $i$, $\omega, \Omega, m, \alpha$, and $\Delta$, when used as input for the program in question, resulted in perfect convergence to these same a priori values of $a, \boldsymbol{\varepsilon}, \mathrm{i}, \boldsymbol{\omega}, \boldsymbol{\Omega}, \mathrm{m}$, $\alpha$, and $\Delta$. As with the previous approach, i.t was decided to simulate reality more faithfully by introducing small random discrepancies into the data. This situation resulted in considerable aberration in the elements computed from these data. In particular, because the orbit under test was nearly circular, the argument of perigee and the mean anomaly at the epoch were mutually poorly defined, and the discrepancies for these elements, arising from the randomness, were correspondingly gross.

It was determined that because the orbit was very nearly circular, very little error in mapping a picture would result if the argument of perigee was arbitrarily defined as some fixed quantity (this would effectively determine the mean anomaly at the epoch very closely) and the eccentricity was set $=0$. With these provisions, the errors in the other elements were considerably diminished, but not so much as to allay fears about their magnitude in an actual nonhypothetical situation. The situation indicated the need for an experimental test with a real satellite to see if the elements other than the argument of perigee and the eccentricity, and the spin vector, could be satisfactorily determined.

Accordingly, on June 14, 1970, a series of seven ATS-3 spin-scan pictures were taken between 1349 and 1957 Universal Time (U.T.). A typical picture is shown in figure 4. Seventy-nine landmarks were selected from this series of pictures. Software existed for enhancing a small sector of such a picture; figure 5 shows an enhanced landmark sector of the Baja California area. Such sectors enable one to determine easily the line and sample numbers corresponding to a given landmark, and so were used throughout for this purpose.


Figure 4.--ATS-3 spin-scan picture.


Figure 5.--Enhanced landmark sector of Baja California.

At about the time of these tests, certain information on anomalies of the ATS-3 spin axis was published by H. Ausfresser (1970). Briefly, it had been determined that the actual spin axis of ATS-3 no longer coincided with the satellite's geometrical axis. As a result, it became necessary to interpose an additional matrix multiplication in the previous chain of logic, the pertinent rotational matrix being that required to ensure coherent data.

Because we envision in actual operations that the landmark selection would be made by subprofessional personnel, hand-massaging of the data was scrupulously avoided; all landmarks chosen were assigned the same weight, with no arbitrary subsequent rejections directed toward a "good" answer. Using as initial estimates the known orbital elements and spin-axis right ascension and declination, various attempts to secure convergence to the proper values of a, i, $\omega, \Omega, \infty, \Delta$ were undertaken. Unfortunately, every such attempt resulted in divergence.

## THOUGHTS

An account of the reasons for the failure of the second approach for position and attitude determination and the corresponding suggestions for improved future results follow.
a. The spin-scan picture start-times were known only to the nearest minute. An uncertainty of 1 minute of time translates into an uncertainty of 15 minutes of arc, so improvement in start-time information is indicated.
b. With ATS-3, the line number of a given line can be determined only indirectly and with a certain degree of uncertainty corresponding to the location of the vertical-frame synchronization point. In the forthcoming SMS-GOES satellites, however, every line is to be directly attributable.
c. When specialized maps for a given area were not readily available, a standard atlas was used. Although positions were obtained as carefully as possible, some inaccuracy was inevitable. A more extensive set of cartographic references is indicated.
d. As previously indicated, the landmarks utilized were not identical in quality. An investigation of some reliable weighting scheme should ameliorate this situation.
e. The skewing of the ATS-3 spin axis was to some degree indeterminate; in any event, it is felt strongly that the influence of this canting on the final results was distinctly negative.
f. The orbit computed was a simple two-body motion ellipse. A more complex implementation of this computation, taking into account the action of perturbing forces during the series of passes, should yield improved results.
g. The orbital elements computed were the so-called classical orbital elements which are not always well defined. Solving for a less ambiguous set of elements might well make a significant difference in the final outcome.

None of these comments are intended to condemn completely the concept of using landmarks as input. It is felt that the use of landmarks for satellite attitude determination will continue to be a valuable tool for some time to come. Position determination, however, is another matter entirely. Even granting the aforementioned improvements, it is felt that the results obtained will be far inferior to those attainable with existing trilateration equipment. Also, the process of obtaining landmark sectors and from them line and sample numbers, together with the required geographic coordinates, etc., would require the on-the-spot presence of a large number of personnel with immediate and exclusive access to a digital computer and display device. If positions (elements) and attitudes of any current usefulness are to be obtained, the procedure would be both costly and tedious.

Some effort is currently being devoted to the automating of the landmark detection and input operations, using the concepts of pattern recognition theory. Until this automation takes place, the procedure will remain tedious and time-consuming. For this reason, it is felt that anyone contemplating further effort in this direction should consider the perfected software as a diagnostic tool rather than an operational procedure.

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