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Mass, Momentum, and Energy Budget Equations for BOMAP Computations

EUGENE M. RASMUSSON

The BOMAP Office

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551.5	Meteorology
.511.3	Atmospheric dynamics
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.465.7	Effect of sea on environment
.71	Energy exchange between sea and atmosphere

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NOTATION

Averaging of any quantity

$\overline{(\ \ )} = \frac{1}{A} \iint_A (\ \ )\, dA$ = area average, where A is the horizontal area of the BOMEX box

$(\ \ )'' = (\ \ ) - \overline{(\ \ )}$ = deviation from area average

$[(\ \ )] = \frac{1}{\epsilon} \oint_\epsilon (\ \ )\, d\epsilon$ = line integral average, where ϵ is the perimeter of the BOMEX box

$(\ \ )' = (\ \ ) - [(\ \ )]$ = deviation from line integral average

Basic quantities (cgs)

$(\ \ )_o$ = quantity at sea surface

$(\ \ )_T$ = quantity at top of box

X = general scalar quantity

A = horizontal area of the box

ϵ = perimeter of the box

g = acceleration of gravity

a = radius of the earth

λ = longitude

ϕ = latitude

Ω = angular velocity of the earth

f = $2\,\Omega \sin\phi$; the Coriolis parameter

z = height above sea surface

Φ = gz = geopotential

p = pressure

p^* = $p_o - p$ = pressure differential (atmospheric pressure at sea level minus atmospheric pressure at height of observation)

ρ = density of moist air

α = specific volume of moist air

T = temperature

H = specific enthalpy, c_pT

L = latent heat of change of phase

F_{RAD} = magnitude of vertical component of net radiation flux

F_H = magnitude of vertical component of microscale enthalpy flux

$\dot{Q}_F$ = frictional heating rate

c_p = specific heat at constant pressure

q = specific humidity

q_L = mixing ratio for water in liquid and solid form

D = vertical diffusion of water vapor

e = rate of evaporation (per unit mass) within the atmosphere

E = rate of evaporation (per unit area) from the sea surface

C = rate of condensation (per unit mass) within the atmosphere

P_o = rate of precipitation (per unit area) at the sea surface

P_T = rate of precipitation (per unit area) at the top of the box

$\vec{i}$ = unit vector directed eastward

$\vec{j}$ = unit vector directed northward

$\vec{n}$ = unit vector normal to the perimeter, directed outward

u = $a \cos\phi \, d\lambda/dt$ = eastward component

v = $a \, d\phi/dt$ = northward component along perimeter

V_n = outward normal wind component

$\vec{V}$ = $\vec{i}u + \vec{j}v$ = horizontal wind velocity

w = dz/dt = vertical velocity

ω^* = dp^*/dt

K = kinetic energy = $(u^2 + v^2)/2$

$\vec{\tau}$ = $\vec{i}\tau_\lambda + \vec{j}\tau_\phi$ = the microscale stress

MASS, MOMENTUM, AND ENERGY BUDGET EQUATIONS FOR BOMAP COMPUTATIONS

Eugene M. Rasmusson

Abstract: Atmospheric budget equations for mass, momentum, and energy are derived for an x, y, p* coordinate system, where p* is the pressure differential relative to sea level. These basic equations are then modified for use in computing the budgets of mass, momentum, and energy for the atmospheric part of the BOMEX volume, as related to the Barbados Oceanographic and Meteorological Experiment (BOMEX), May-July, 1969.

I. INTRODUCTION

Observations made during the Barbados Oceanographic and Meteorological Experiment (BOMEX) were planned to permit estimation of total transfer of water vapor, sensible heat, and momentum from sea to air over a 500- by 500-km square. It was contemplated (Davidson, 1968; Kuettner and Holland, 1969) that this would be done by measuring the quantities entering into the continuity or conservation equation for each property, with sufficient time and space resolution to allow the integrals over the 500-km square, through approximately the lower 6 km of the atmosphere, and over a time period of the order of 1 day, to be evaluated with acceptable accuracy. These integrated conservation equations are called the "budget equations."

A major objective of the Barbados Oceanographic and Meteorological Analysis Project (BOMAP) is the evaluation of these integrals. Observations used in the computations consist of:

1. Fixed-ship observations at the four corners and center of the 500-km square, including:

   (a) continuously recorded surface pressure, temperature, humidity, wind direction, wind speed, net radiation, and precipitation and

(b) rawinsonde temperature, humidity, wind direction, wind speed, and height as a function of pressure up to a pressure of 400 mb at 1½-hour intervals.

2. Dropsonde observations, centered around midday and midnight, of temperature and humidity as functions of pressure from a height of 6 km to sea level at eight points above the diagonals of the square.

3. Aircraft line-integral data on radar altitude, pressure, temperature, humidity, wind direction, and wind speed at selected nominal flight altitudes along the boundary of the square.

4. Precipitation estimates based on surface and airborne radar coverage of the square, supported by extensive satellite coverage and high-altitude aircraft photography, and calibrated by means of ship and island rain gages.

5. Radiative flux divergence estimates based on:

   (a) radiometersonde ascents once daily at the three fixed-ship stations and from Barbados, and

   (b) radiation models employing meteorological input data and validated by several aircraft radiation measurement experiments conducted as part of BOMEX.

6. Direct measurements of microscale vertical eddy fluxes of heat, water vapor, and momentum by means of fast-responding sensors mounted on buoys and aircraft, sampled over various time and space intervals.

The equations developed here are meant to apply to observations with time and space resolutions insufficient to account explicitly for cumulus- and smaller-scale variations (horizontal scale less than, very roughly, 10-30 km). Thus the "microscale" (including molecular) contributions to fluxes, flux divergences, and kinetic energy dissipation rates are represented by separate symbols in the equations. It is presumed that values for these variables will be developed by appropriate statistical analysis of observations of the type described in item 6. More details on the experiment are given by Kuettner and Holland (1969) and in BOMEX Bulletins Nos. 4, 5, and 6.

II. PRESSURE DIFFERENTIAL COORDINATE SYSTEM

Tha basic mass and energy budget relationships on which the BOMAP analyses are based are formulated in the pressure differential coordinate system x, y, p^*, t, where x and y are horizontal coordinates, p^* is the position on the vertical axis specified by pressure differential relative to sea level, and t is time. In this system, p^* is the difference in atmospheric pressure between the sea surface (p_o) and any height of observation (p) -- that is, p^* is equal to $p_o - p$ and is 0 at the sea surface (fig. 1). The hydrostatic equation becomes

$$dp^* = g\rho(z)dz,$$

where $\rho(z)$ is air density as a function of height z.

The analyses here will be concerned with defining the independently measurable terms of the conservation equations for mass, water vapor, latent heat, sensible heat, mechanical energy, and momentum, integrated over the "BOMEX atmospheric volume." The sides of this volume are vertical surfaces passing through the nominal positions of the corner ships of the fixed-ship array. The bottom surface is sea level and the top is a p^* surface designated p_T^*.

The total mass contained in the vertical atmospheric column of unit cross section extending from sea level up to a p_T^* surface, at which the height $z = z_T$ would be

$$\int_o^{z_T} \rho(z)dz = \int_o^{p_T^*} \frac{dp^*}{g} .$$

If variations of g are neglected, this mass is equal to p_T^*/g and is constant in space and time. Even if variations of g are considered, the mass contained in the column is constant with respect to time at any fixed latitude and longitude. Hence, the mass contained in the "BOMEX atmospheric volume" is constant for any given value of p_T^*; the volume, however, is not constant.

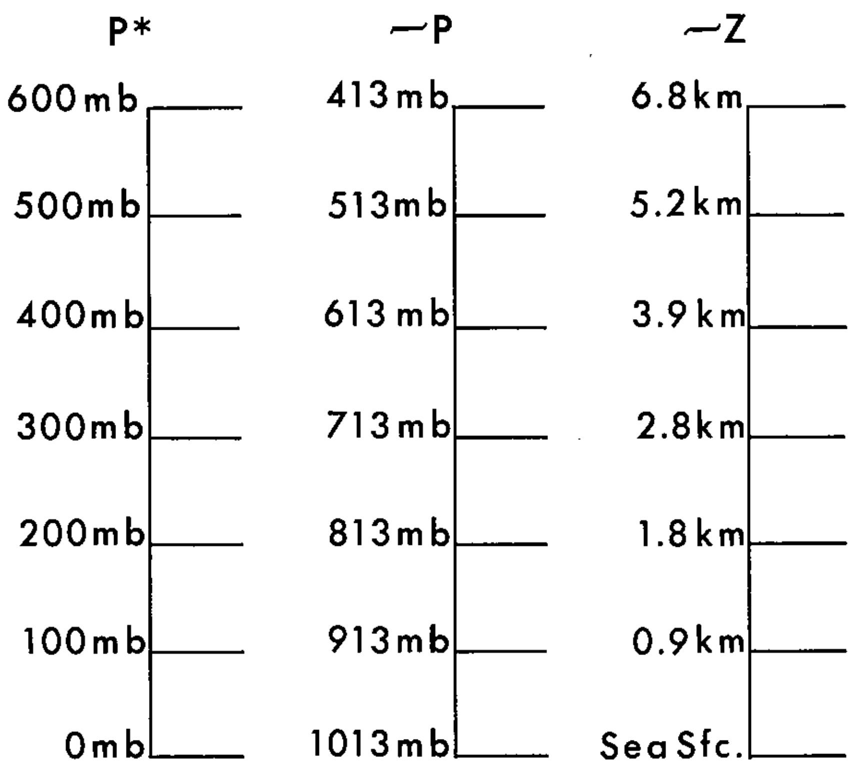

Figure 1.--Illustration of pressure differential (p*) concept where $p^* = p_o - p$ (the atmospheric pressure at sea level minus the atmospheric pressure at the height of observation).

Observation schedules for the BOMEX field observations of May 3 through July 2, 1969, in support of the Sea-Air Interaction Program (described by Kuettner and Holland, 1969) were predicated on a choice of p_T^* in the neighborhood of 500 mb. Rawinsonde observations were terminated normally at a pressure of 400 mb ($p^* \approx 613$ mb). Dropsondes were released from 20,000 ft (6 km, or $p^* \approx 550$ mb).

Conversion from the x, y, z, t system of Cartesian coordinates, where z is the vertical coordinate, to the x, y, p*, t system is accomplished through the following relationships:

$$\left[\frac{\partial}{\partial x}\right]_z = \left(\frac{\partial}{\partial x}\right)_{p^*} - \rho g \left(\frac{\partial z}{\partial x}\right)_{p^*} \frac{\partial}{\partial p^*}$$

$$\left(\frac{\partial}{\partial y}\right)_z \quad \left(\frac{\partial}{\partial y}\right)_{p^*} - \rho g \left(\frac{\partial}{\partial y}\right)_{p^*} \frac{\partial}{\partial p^*}$$

$$\frac{\partial}{\partial z} = \rho g \frac{\partial}{\partial p^*}$$

$$\left(\frac{\partial}{\partial t}\right)_z \quad \left(\frac{\partial}{\partial t}\right)_{p^*} - \rho g \left(\frac{\partial z}{\partial t}\right)_{p^*} \frac{\partial}{\partial p^*}$$

This type of coordinate transformation is discussed by Hess (1959, pp. 259-264). In the above relationships, x is measured positively eastward, y positively northward; thus,

$$dx = a \cos\phi \, d\lambda,$$

$$dy = a d\phi,$$

where a is the radius of the earth, λ the longitude measured eastward, and ϕ the latitude measured northward.

III. BASIC EQUATIONS FOR MASS, MOMENTUM, AND ENERGY

In this section, the basic equations for mass continuity, momentum, and energy are written in λ, ϕ, z, t, and λ, ϕ, p^*, t coordinates. The relationships are discussed by Lorenz (1967). Among the assumptions inherent in deriving these relationships are:

> The acceleration of gravity, g, is a constant.
>
> There is no variation in the heat of condensation, L.
>
> Lateral boundaries are fixed and vertical.

The effects of these assumptions are reviewed in appendices A, B, and C, respectively.

The "del" operator (∇) is used to denote the two-dimensional gradient:

$$\nabla X = \vec{i}\,\frac{1}{a\,\cos\phi}\,\frac{\partial X}{\partial \lambda} + \vec{i}\,\frac{1}{a}\,\frac{\partial X}{\partial \phi}.$$

III.1. Mass Continuity, Equation (1)

(a) x, y, z, t

$$-\frac{\partial\rho}{\partial t} = \nabla\cdot\rho\vec{V} + \frac{\partial\rho w}{\partial z}$$

(b) x, y, p*, t

$$\nabla\cdot\vec{V} + \frac{\partial\omega^*}{\partial p^*} = 0 \ . \tag{1}$$

III.2. Momentum, Equations (2) - (3)

(a) x, y, z, t

$$\frac{du}{dt} = fv + \frac{uv \tan\phi}{a} - \frac{\alpha}{a \cos\phi}\frac{\partial p}{\partial\lambda} + \alpha\frac{\partial\tau_\lambda}{\partial z}$$

$$\frac{dv}{dt} = -fu - \frac{u^2 \tan\phi}{a} - \frac{\alpha}{a}\frac{\partial p}{\partial\phi} + \alpha\frac{\partial\tau_\phi}{\partial z}$$

(b) x, y, p*, t

$$\frac{du}{dt} = fv + \frac{uv \tan\phi}{a} - \frac{g}{a \cos\phi}\left(\frac{\partial z}{\partial\lambda} + \frac{\alpha}{g}\frac{\partial p_o}{\partial\lambda}\right) + g\frac{\partial\tau_\lambda}{\partial p^*} \tag{2}$$

$$\frac{dv}{dt} = -fu - \frac{u^2 \tan\phi}{a} - \frac{g}{a}\left(\frac{\partial z}{\partial\phi} + \frac{\alpha}{g}\frac{\partial p_o}{\partial\phi}\right) + g\frac{\partial\tau_\phi}{\partial p^*} \tag{3}$$

III.3. Energy, Equations (4) - (8)

(a) x, y, z, t

$$\frac{dK}{dt} = - \alpha\vec{V}\cdot\nabla p + \alpha \frac{\partial(\vec{V}\cdot\vec{\tau})}{\partial z} - \dot{Q}_F$$

$$\frac{d\Phi}{dt} = gw$$

$$\frac{dH}{dt} = \alpha \frac{\partial}{\partial z}(F_{RAD}) + \frac{\alpha\partial F_H}{\partial z} + \dot{Q}_F + L(C-e) + \alpha \frac{dp}{dt}$$

$$\frac{d(Lq)}{dt} = - L(C-e) + \alpha L \frac{\partial D}{\partial z}$$

(b) x, y, p*, t

$$\frac{dK}{dt} = - \vec{V}\cdot\nabla\Phi - \alpha\vec{V}\cdot\nabla\, p_o + g \frac{\partial(\vec{V}\cdot\vec{\tau})}{\partial p^*} - \dot{Q}_F \tag{4}$$

$$\frac{d\Phi}{dt} = \frac{\partial\Phi}{\partial t} + \vec{V}\cdot\nabla\Phi + \alpha\omega^* \tag{5}$$

$$\frac{dH}{dt} = g \frac{\partial}{\partial p^*}(F_{RAD} + F_H) + L(C-e) + \alpha \left[\frac{\partial p_o}{\partial t} + \vec{V}\cdot\nabla p_o - \omega^*\right] + \dot{Q}_F \tag{6}$$

$$\frac{d(Lq)}{dt} = -L(C-e) + Lg \frac{\partial D}{\partial p^*} \tag{7}$$

$$\frac{d(K + \Phi + H + Lq)}{dt} = g\frac{\partial}{\partial p^*}(F_{RAD} + F_H) + Lg \frac{\partial D}{\partial p^*} + \alpha \frac{\partial p_o}{\partial t} + \frac{\partial\Phi}{\partial t} + g\frac{\partial(\vec{V}\cdot\vec{\tau})}{\partial p^*} \tag{8}$$

IV. BASIC FORMULATION OF BUDGET EQUATIONS

IV.1. Operations Performed on Equations (1)-(7)

(a) Expand substantial derivatives and use mass continuity equation to obtain

$$\frac{dX}{dt} = \frac{\partial X}{\partial t} + \nabla \cdot X\vec{V} + \frac{\partial X\omega^*}{\partial p^*} \tag{9}$$

(b) Integrate over the volume of the box, expressing the results as averages per unit horizontal area by:

using the Gauss Theorem to transform the second term on the right side of (9) to

$$\frac{1}{A}\iint_A (\nabla \cdot X\vec{V})\, dA = \frac{1}{A}\oint_{\epsilon} XV_n\, d\epsilon; \tag{10}$$

using the operators

$$[X] = \frac{1}{\epsilon}\oint_{\epsilon} X d\epsilon,$$

$$X' = X - [X],$$

to transform (10) to

$$\frac{1}{A}\oint_{\epsilon} XV_n d\epsilon = \frac{\epsilon}{A}\left\{ [X]\,[V_n] + [X'V_n'] \right\}; \tag{11}$$

transforming the third term on the right side of (9) to a boundary term (in which $\omega^* = 0$ at $p^* = 0$ and micro-scale fluxes are not included)

$$\frac{1}{A}\iint_A \left\{ \int^{p_T^*} \frac{\partial X\omega^*}{\partial p^*}\, dp^* = \right\} dA = \frac{1}{A}\iint_A \left\{ (X\omega^*)_T dA \right\} ; \tag{12}$$

using the operators

$$\overline{X} = \frac{1}{A}\iint_Z XdA$$

$$X'' = x - \overline{X}$$

to transform (12) to

$$\frac{1}{A}\iint_A (X\omega^*)_T dA = \overline{X}_T\overline{\omega}^*_T + \overline{(X''\omega^{*''})}_T ; \tag{13}$$

and using the first and third steps under (b) above to transform those additional terms that can be transformed to boundary terms and for which such a transformation appears to be convenient.

IV.2. Relationships Derived From Operations on Equations (1) - (7)

By performing the above operations on equations (1) - (7), the following relationships are derived.

Mass Continuity

$$\frac{\epsilon}{A} \int_{o}^{p_T^*} [V_n] \, dp^* = - \overline{\omega_T^*} \tag{14}$$

Momentum

(a) Zonal:

$$\frac{\partial}{\partial t} \int_{o}^{p_T^*} \overline{u} \frac{dp^*}{g} = - \frac{\epsilon}{A} \int_{o}^{p_T^*} \left\{ [u] \, [V_n] + [u'V_n'] \right\} \frac{dp^*}{g} - \frac{\overline{u_T}\overline{\omega_T^*}}{g} \tag{15}$$

$$+ \int_{o}^{p_T^*} \overline{\left(\frac{uv \tan\phi}{a}\right)} \frac{dp^*}{g} + \int_{o}^{p_T^*} \overline{fv} \frac{dp^*}{g} - \int_{o}^{p_T^*} \overline{\left(\frac{1}{a \cos\phi} \frac{\partial z}{\partial \lambda}\right)} \, dp^*$$

$$- \int_{o}^{p_T^*} \overline{\left(\frac{\alpha}{a \cos\phi} \frac{\partial p_o}{\partial \lambda}\right)} \frac{dp^*}{g} - \frac{\overline{(u''\omega^{*''})}_T}{g} + (\overline{\tau}_\lambda)_T - (\overline{\tau}_\lambda)_o \ .$$

(b) Meridional:

$$\frac{\partial}{\partial t} \int_{o}^{p_T^*} \overline{v} \frac{dp^*}{g} = \frac{\epsilon}{A} \int_{o}^{p_T^*} \left\{ [v] \, [V_n] + [v'V_n'] \right\} \frac{dp^*}{g} - \frac{\overline{v_T}\overline{\omega_T^*}}{g}$$

$$- \int_{o}^{p_T^*} \overline{\left(\frac{u^2 \tan\phi}{a}\right)} \frac{dp^*}{g} - \int_{o}^{p_T^*} \overline{fu} \frac{dp^*}{g} - \int_{o}^{p_T^*} \overline{\frac{1}{a} \frac{\partial z}{\partial \phi}} \, dp^*$$

$$- \int_{o}^{p_T^*} \overline{\left(\frac{\alpha}{a} \frac{\partial p_o}{\partial \phi}\right)} \frac{dp^*}{g} - \frac{\overline{(v''\omega^{*''})}_T}{g} + (\overline{\tau}_\phi)_T - (\overline{\tau}_\phi)_o \ . \tag{16}$$

Energy

(a) Mechanical Energy:

$$\frac{\partial}{\partial t}\int_o^{p_T^*} \overline{K}\,\frac{dp^*}{g} = -\frac{\epsilon}{A}\int_o^{p_T^*}\left\{ [K]\,[V_n] + [K'V_n'] \right\}\frac{dp^*}{g} - \frac{\overline{K}_T\overline{\omega}_T^*}{g}$$

$$-\frac{\overline{(K''\omega^{*''})}_T}{g} - \int_o^{p_T^*} \overline{\alpha\vec{V}\cdot\nabla p_o}\,\frac{dp^*}{g} - \int_o^{p_T^*} \overline{\vec{V}\cdot\nabla\Phi}\,\frac{dp^*}{g}$$

$$-\int_o^{p_T^*} \overline{\dot{Q}}_F\frac{dp^*}{g} + \overline{\vec{\tau}_T\cdot\vec{V}_T} - \overline{\vec{\tau}_o\cdot\vec{V}_o} \qquad (17)$$

(b) Enthalpy:

$$\frac{\partial}{\partial t}\int_o^{p_T^*} \overline{H}\,\frac{dp^*}{g} = -\frac{\epsilon}{A}\int_o^{p_T^*}\left\{ [H]\,[V]_n + [H'V_n'] \right\}\frac{dp^*}{g}$$

$$-\frac{\overline{H}_T\overline{\omega}_T^*}{g} - \frac{\overline{(H''\omega^{*''})}_T}{g} + (\overline{F}_{RAD})_o - (\overline{F}_{TAD})_T$$

$$+ (\overline{F}_H)_o - (\overline{F}_H)_T + \int_o^{p_T^*} \overline{\dot{Q}}_F\,\frac{dp^*}{g} + \int_o^{p^*} \overline{L(C-e)}\,\frac{dp^*}{g}$$

$$+\int_o^{p_T^*} \overline{\alpha\,\frac{\partial p_o}{\partial t}}\,\frac{dp^*}{g} + \int_o^{p_T^*} \overline{\alpha\vec{V}\cdot\nabla p_o}\,\frac{dp^*}{g} - \int_o^{p_T^*} \overline{\alpha}\,\overline{\omega} + \overline{\alpha''\omega''}\quad\frac{dp^*}{g}\,. \qquad (18)$$

(c) Latent energy:

$$\frac{\partial}{\partial t}\int_0^{p_T^*} L\bar{q}\,\frac{dp^*}{g} = -\frac{\mathcal{C}}{A}\int_0^{p_T^*} L\left\{[q]\;[V_n] + [q'V_n']\right\}\frac{dp^*}{g}$$

$$-\frac{\overline{Lq_T}\,\overline{\omega_T^*}}{g} - \frac{L\overline{(q''\omega^{*''})}_T}{g} - \int_0^{p_T^*} L\overline{(C-e)}\,\frac{dp^*}{g} + L\bar{E} - L\bar{D}_T \tag{19}$$

where $\bar{D}_o \equiv \bar{E}$.

(d) Total energy:

$$\frac{\partial}{\partial t}\int_0^{p_T^*}(\bar{K} + \bar{\Phi} + \bar{H} + \overline{Lq})\,\frac{dp^*}{g} = -\frac{\mathcal{C}}{A}\int_0^{p_T^*}\left\{[K] + [\Phi] + [H] + L[q]\right\}\;[V_n]\,\frac{dp^*}{g}$$

$$-\frac{\mathcal{C}}{A}\int_0^{p_T^*}[(K' + \Phi' + H' + Lq')V_n']\,\frac{dp^*}{g} - \left\{\bar{K}_T + \bar{\Phi}_T + \bar{H}_T + L\bar{q}_T\right\}\frac{\overline{\omega_T^*}}{g}$$

$$-\overline{\left\{K'' + \Phi'' + H'' + Lq''\right\}\frac{\omega_T^{*''}}{g}} + \left\{(\bar{F}_{RAD})_o - (\bar{F}_{RAD})_T + (\bar{F}_H)_o - (\bar{F}_H)_T\right\}$$

$$+\int_0^{p_T^*}\alpha\,\overline{\frac{\partial p_o}{\partial t}}\,\frac{dp^*}{g} + \int_0^{p_T^*}\overline{\frac{\partial\Phi}{\partial t}}\,\frac{dp^*}{g} + L\bar{E} - L\bar{D}_T + \overline{\vec{\tau}_T\cdot\vec{V}_T} - \overline{\vec{\tau}_o\cdot\vec{V}_o}\;. \tag{20}$$

V. MODIFICATION OF BASIC BUDGET EQUATIONS

To take further advantage of the concentration of observations on the boundary, it appears desirable to modify the basic form of the budget equations. Furthermore, the pressure gradient on a p* surface is merely the surface pressure gradient, ∇p_o, and therefore not a function of p*. This simplifies the computation of certain terms and should be reflected in the formulation of the equations.

V.1. Modification of the Momentum Equations

Pressure gradient terms

(a) Vertical integration:

Consider the term

$$\int_o^{p_T^*} \left(\frac{\alpha}{a}\frac{\partial p_o}{\partial \phi}\right) \frac{dp^*}{g}$$

from the meridional momentum equation. Since $\alpha = g\,\dfrac{\partial z}{\partial p^*}$, and $\dfrac{\partial p_o}{\partial \phi}$ is not a function of p*, we can write

$$\int_o^{p_T^*} \frac{\alpha}{a}\frac{\partial p_o}{\partial \phi}\frac{dp^*}{g} = \frac{\partial p_o}{\partial \phi}\int_o^{p_T^*}\frac{\partial z}{\partial p^*}\,dp^* = \frac{z_T}{a}\frac{\partial p_o}{\partial \phi} \quad . \tag{21}$$

Similarly,

$$\int_o^{p_T^*} \frac{\alpha}{a\cos\phi}\frac{\partial p_o}{\partial \lambda}\frac{dp^*}{g} = \frac{z_T}{a\cos\phi}\frac{\partial p_o}{\partial \lambda} \quad . \tag{22}$$

(b) Transformation to boundary terms:

Since

$$\nabla p_o = \vec{j}\,\frac{1}{a}\frac{\partial p_o}{\partial \phi} + \vec{i}\,\frac{1}{a\,\cos\phi}\frac{\partial p_o}{\partial \lambda}\ ,$$

it follows that

$$\frac{1}{a}\frac{\partial p_o}{\partial \phi} = \vec{j}\cdot\nabla p_o = \nabla\cdot p_o\vec{j} - p_o\nabla\cdot\vec{j}\ .$$

Evaluation of $\nabla\cdot\vec{j}$ gives

$$\frac{1}{a}\frac{\partial p_o}{\partial \phi} = \nabla\cdot p_o\vec{j} + \frac{p_o\ \tan\phi}{a}\ . \qquad (23)$$

Similarly,

$$\frac{1}{a\ \cos\phi}\frac{\partial p_o}{\partial \lambda} = \nabla\cdot p_o\vec{j}\ . \qquad (24)$$

Referring to (21) we can write

$$\overline{\frac{z_T}{a}\frac{\partial p_o}{\partial \phi}} = \bar{z}_T\,\overline{\frac{\partial p_o}{a\partial\phi}} + \overline{z_T''\ \frac{\partial p_o}{a\partial\phi}''}$$

Substitution from (23) gives

$$\overline{z_T\,\frac{1}{a}\frac{\partial p_o}{\partial \phi}} = \bar{z}_T\left\{\overline{\nabla\cdot p_o\vec{j}} + \overline{\frac{p_o\ \tan\phi}{a}}\right\} + \overline{z_T''\left\{\frac{\partial p_o}{a\partial\phi}\right\}''} \qquad (25)$$

(25)

Similarly, (22) and (24) give

$$\overline{z_T \frac{1}{a\cos\phi}\frac{\partial p_o}{\partial\lambda}} = \overline{z_T}\;\overline{\nabla\cdot p_o\vec{i}} + \overline{z_T''\left\{\frac{\partial p_o}{a\cos\phi\,\partial\lambda}\right\}''} \tag{26}$$

Finally, application of the Gauss theorem to the divergence terms of (25) and (26) gives

$$\frac{1}{A}\iint_A \left\{\int_o^{p_T^*} \frac{\alpha}{a}\frac{\partial p_o}{\partial\phi}\frac{dp^*}{g}\right\} dA = \frac{\epsilon}{A}\overline{z_T}\;[p_o\vec{j}\cdot\vec{n}] + \overline{z_T}\;\frac{\overline{p_o}\tan\phi}{a} + \overline{z_T''\left(\frac{\partial p_o}{a\partial\phi}\right)''} \tag{27}$$

$$\frac{1}{A}\iint_A \left\{\int_o^{p_T^*} \frac{\alpha}{a\cos\phi}\frac{\partial p_o}{\partial\lambda}\frac{dp^*}{g}\right\} dA = \frac{\epsilon}{A}\overline{z_T}\;[p_o\vec{i}\cdot\vec{n}] + \overline{z_T''\left\{\frac{\partial p_o}{a\cos\phi\,\partial\lambda}\right\}''}. \tag{28}$$

Geopotential gradient terms

Using arguments similar to those expressed by equations (23), (24), (25), (26), and application of the Gauss theorem to divergence terms, leads to the following formulation for the geopotential gradient terms in the momentum equations:

$$\int_o^{p_T^*}\left\{\frac{1}{A}\iint_A \frac{1}{a}\frac{\partial z}{\partial\phi}\,dA\right\} dp^* = \frac{\epsilon}{A}\int_o^{p_T^*} [z\vec{j}\cdot\vec{n}]\;dp^* + \int_o^{p_T^*} \frac{\overline{z\tan\phi}}{a}\,dp^* \tag{29}$$

$$\int_o^{p_T^*}\left\{\frac{1}{A}\iint_A \frac{1}{a\cos\phi}\frac{\partial z}{\partial\lambda}\,dA\right\} dp^* = \frac{\epsilon}{A}\int_o^{p_T^*} [z\vec{i}\cdot\vec{n}]\;dp^* \tag{30}$$

V.2. Modification of the Energy Equations

(a) $\alpha\vec{V} \cdot \nabla p_o$ term:

This term can be written

$$\alpha\vec{V} \cdot \nabla p_o = \nabla \cdot \alpha p_o \vec{V} - p_o \nabla \cdot \alpha\vec{V} \ . \tag{31}$$

Horizontal averaging and application of the Gauss theorem yields

$$\overline{\alpha\vec{V} \cdot \nabla p_o} = \frac{\epsilon}{A}\left\{ [\alpha p_o V_n] - \overline{p_o}\,[\alpha V_n] \right\} - \overline{p_o''(\nabla \cdot \alpha\vec{V})''}$$

$$= \frac{\epsilon}{A}\left\{ [p_o]\,[\alpha V_n] + [p_o'(\alpha V_n)'] - \overline{p_o}\,[\alpha V_n] \right\} - \overline{p_o''(\nabla \cdot \alpha\vec{V})''}$$

$$= \frac{\epsilon}{A}\left\{ [p_o'']\,[\alpha V_n] + [p_o'(\alpha V_n)'] \right\} - \overline{p_o''(\nabla \cdot \alpha\vec{V})''} \ .$$

Integrating vertically, and noting that p_o is not a function of p^*, we obtain

$$\int_o^{p_T^*} \overline{\alpha\vec{V} \cdot \nabla p_o}\,\frac{dp^*}{g} = \frac{\epsilon}{A}\left[p_o''\right] \int_o^{p_T^*} \left[\alpha V_n\right] \frac{dp^*}{g}$$

$$+ \frac{\epsilon}{A}\left[p_o' \int_o^{p_T^*} (\alpha V_n)' \,\frac{dp^*}{g}\right] - \overline{\left\{p_o'' \int_o^{p_T^*} (\nabla \cdot \alpha\vec{V})''\,\frac{dp^*}{g}\right\}} \ . \tag{32}$$

(b) $\vec{V} \cdot \nabla\Phi$ term:

This term can be written

$$\vec{V} \cdot \nabla\Phi = \nabla \cdot \vec{V}\Phi - \Phi\nabla \cdot \vec{V} .$$

Horizontal averaging and application of the Gauss theorem yields

$$\overline{\vec{V} \cdot \nabla\Phi} = \frac{\epsilon}{A} \left\{ \left[\Phi\right] \left[V_n\right] + \left[\Phi' V_n'\right] - \overline{\Phi} \left[V_n\right] \right\} - \overline{\Phi''(\nabla \cdot \vec{V})''} .$$

$$\overline{\vec{V} \cdot \nabla\Phi} = \frac{\epsilon}{A} \left\{ \left[\Phi''\right] \left[V_n\right] + \left[\Phi' V_n'\right] \right\} - \overline{\Phi''(\nabla \cdot \vec{V})''} . \tag{33}$$

(c) $\alpha(\partial p_o/\partial t)$ term:

This term can be written

$$\alpha \frac{\partial p_o}{\partial t} = \frac{g\partial z}{\partial p^*} \frac{\partial p_o}{\partial t} .$$

Integrating vertically, and noting that p_o is not a function of p^*, we have

$$\int_o^{p_T^*} \alpha \frac{\partial p_o}{\partial t} \frac{dp^*}{g} = \frac{\partial p_o}{\partial t} \int_o^{p_T^*} \frac{\partial z}{\partial p^*} dp^* = z_T \frac{\partial p_o}{\partial t} . \tag{34}$$

Averaging horizontally yields

$$\frac{1}{A} \int \int_A \left\{ \int_o^{p_T^*} \alpha \frac{\partial p_o}{\partial t} \frac{dp^*}{g} \right\} dA = \overline{z}_T \frac{\partial \overline{p}_o}{\partial t} + \overline{z_T'' \frac{\partial p_o''}{\partial t}} . \tag{35}$$

VI. PROPOSED BUDGET EQUATIONS FOR BOMAP COMPUTATIONS

In this section we shall write a set of budget equations that are believed to be a satisfactory framework for the basic BOMAP budget computations. These equations are based on the equations in section III but are modified as follows:

1. The modifications in section IV are incorporated into the set of equations.
2. The top of the box can be varied, but is assumed to be above the level at which the microscale stress, the microscale enthalpy transfer, and the diffusion of water vapor can be neglected. That is,

$$D_T = (F_H)_T = (\tau_\lambda)_T = (\tau_\phi)_T = 0$$

Terms are partitioned into mean and covariance contributions, primarily for two reasons: (1) to gain additional insight into the processes by which the balances are satisfied, and (2) to partition the various terms into a portion that can in principle be evaluated from the aerological data and a portion which cannot be adequately evaluated from these data alone. For example, the mean vertical transport $\overline{\overline{\omega}^* \overline{X}}$ can, in theory, be computed while only the larger scale eddy vertical transport $\overline{\omega^{*\prime\prime} X^{\prime\prime}}$ can be evaluated from the available aerological data.

VI.1 Mass Continuity

$$\frac{\epsilon}{A} \int_0^{p_T^*} [V_n] \, dp^* = - \overline{\overline{\omega}_T^*} \tag{36}$$

VI.2. Zonal Momentum

$$\frac{1}{g} \frac{\partial}{\partial t} \int_0^{p_T^*} \overline{u} \, dp^* = - \frac{\epsilon}{Ag} \int_0^{p_T^*} \left\{ [u] \; [V_n] + [u' V_n'] \right\} dp^* - \frac{\overline{\overline{u}_T \overline{\omega}_T^*}}{g} - \frac{\overline{(u'' \omega^{*\prime\prime})}}{g}$$

$$+ \frac{1}{ag} \int_0^{p_T^*} \overline{uv \tan\phi} \, dp^* + \frac{1}{g} \int_0^{p_T^*} \overline{fv} dp^* - \frac{\epsilon}{A} \overline{z}_T \, [p_o \vec{i} \cdot \vec{n}] - \overline{z_T'' \left(\frac{\partial p_o}{a \cos\phi \partial\lambda} \right)''}$$

$$- \frac{\epsilon}{A} \int_0^{p_T^*} [z \vec{i} \cdot \vec{n}] \, dp^* - (\overline{\tau}_\lambda)_o \; . \tag{37}$$

VI.3. Meridional Momentum

$$\frac{1}{g}\frac{\partial}{\partial t}\int_0^{p_T^*} \bar{v} dp^* = -\frac{\epsilon}{Ag}\int_0^{p_T^*}\left\{ [v]\ [V_n] + [v'V_n'] \right\} dp^* - \frac{\bar{v}_T\bar{\omega}_T^*}{g} \quad \frac{\overline{(v''\omega^{*''})}_T}{g}$$

$$-\frac{1}{ag}\int_0^{p_T^*}\overline{u^2 \tan\phi}\ dp^* - \frac{1}{g}\int_0^{p_T^*}\overline{fu}\ dp^* - \frac{\epsilon}{A}\bar{z}_T\ [p_o\vec{j}\cdot\vec{n}] + \bar{z}_T\frac{\overline{p_o \tan\phi}}{a} - \overline{z_T''\left(\frac{\partial p_o}{a\partial\phi}\right)''}$$

$$-\frac{\epsilon}{A}\int_0^{p_T^*}[z\vec{j}\cdot\vec{n}]\ dp^* - \frac{1}{a}\int_0^{p_T^*}\overline{z \tan\phi}\ dp^* - (\bar{\tau}_\phi)_o\ . \tag{38}$$

VI.4. Mechanical Energy

$$\frac{1}{g}\frac{\partial}{\partial t}\int_0^{p_T^*}\bar{K}\ dp^* = -\frac{\epsilon}{Ag}\int_0^{p_T^*}\left\{[K] + [\Phi'']\right\}\ [V_n]\ dp^*$$

$$-\frac{\epsilon}{Ag}\int_0^{p_T^*}\left\{[K'V_n'] + [\Phi'V_n']\right\} dp^* - \frac{\bar{K}_T\bar{\omega}_T^*}{g} - \frac{\overline{(k''\omega^{*''})}_T}{g} - \frac{\epsilon}{Ag}[p_o'']\int_0^{p_T^*}[\alpha V_n]\ dp^*$$

$$-\frac{\epsilon}{Ag}\left[p_o'\int_0^{p_T^*}(\alpha V_n)'dp^*\right] + \overline{\frac{p_o''}{g}\int_0^{p_T^*}(V\cdot\alpha\vec{V})''dp^*} + \frac{1}{g}\int_0^{p_T^*}\overline{\Phi''(\nabla\cdot\vec{V})''}\ dp^*$$

$$-\frac{1}{g}\int_0^{p_T^*}\overline{\dot{Q}_F}dp^* - \overline{\vec{\tau}_o\cdot\vec{V}_o} \tag{39}$$

VI.5. Latent Energy

$$\frac{L}{g}\frac{\partial}{\partial t}\int_0^{p_T^*}\bar{q}\ dp^* = -\frac{\epsilon}{A}\frac{L}{g}\int_0^{p_T^*}\left\{ [q]\ [V_n] + [q'V_n'] \right\} dp^* - \frac{L}{g}\bar{q}_T\bar{\omega}_T^*$$

$$-\frac{L}{g}\overline{(q''\omega^{*''})}_T - \frac{L}{g}\int_0^{p_T^*}(\overline{C-e})\ dp^* + L\bar{E}\ . \tag{40}$$

VI.6. Enthalpy

$$\frac{1}{g}\frac{\partial}{\partial t}\int_{o}^{p_T^*}\overline{H}\,dp^* = -\frac{C}{Ag}\int_{o}^{p_T^*}\left\{[H]\;[V_n] + [H'V_n']\right\}\;dp^* - \frac{\overline{H}_T\overline{\omega_T^*}}{g} - \frac{\overline{(H''\omega^{*''})_T}}{g}$$

$$+\left\{\overline{(F_{RAD})_o} - \overline{(F_{RAD})_T} + \overline{(F_H)_o}\right\} + \frac{1}{g}\int_{o}^{p_T^*}\overline{\dot{Q}_F}dp^*$$

$$+\frac{L}{g}\int_{o}^{p_T^*}\overline{(C-e)}\;dp^* + \overline{z}_T\frac{\partial\overline{p}_o}{\partial t} + \overline{z_T''\frac{\partial p_o''}{\partial t}} + \frac{C}{Ag}\,[p_o'']\int_{o}^{p_T^*}[\alpha V_n]\;dp^*$$

$$+\frac{C}{Ag}\left[p_o'\int_{o}^{p_T^*}(\alpha V_n)'dp^*\right] - \overline{\frac{p_o''}{g}\int_{o}^{p_T^*}(\nabla\cdot\alpha\vec{V})''dp^*} - \frac{1}{g}\int_{o}^{p_T^*}\left\{\overline{\alpha}\;\overline{\omega} + \overline{\alpha''\omega''}\right\}dp^*. \qquad (41)$$

VI.7. Total Energy

$$\frac{\partial}{\partial t}\frac{1}{g}\int_{o}^{p_T^*}\left\{\overline{H} + \Phi + L\overline{q} + K\right\}dp^* = -\frac{C}{Ag}\int_{o}^{p_T^*}\left\{[H] + [\Phi] + L[q] + [K]\right\}\;[V_n]\;dp^*$$

$$-\frac{C}{Ag}\int_{o}^{p_T^*}\left\{[H'V_n'] + [\Phi'V_n'] + L[q'V_n'] + [K'V_n']\right\}\;dp^*$$

$$-\frac{1}{g}\left\{\overline{H}_T + \overline{\Phi}_T + L\overline{q}_T + \overline{K}_T\right\}\;\overline{\omega_T^*} - \frac{1}{g}\left\{\overline{(H''\omega^{*''})}_T + \overline{(\Phi''\omega^{*''})}_T + L\overline{(q''\omega^{*''})}_T + \overline{(K''\omega^{*''})}_T\right\}$$

$$+\left\{(\overline{F}_{RAD})_o - (\overline{F}_{RAD})_T + (\overline{F_H})_o\right\} + \overline{z}_T\frac{\overline{\partial p_o}}{\partial t} + \overline{z_T''\frac{\partial p_o''}{\partial t}} - \overline{\tau_o\cdot\vec{V}_o} + \frac{1}{g}\int_{o}^{p_T^*}\frac{\partial\overline{\Phi}}{\partial t}dp^* + L\overline{E}\;.$$

$$(42)$$

VI.8. Water Budget

As a companion to equation (40), we can write a budget equation for the water substance in liquid and solid form:

$$\frac{\partial}{\partial t}\frac{1}{g}\int_{o}^{p_T^*} \overline{q}_L \; dp^* = -\frac{\epsilon}{Ag}\int_{o}^{p_T^*} [q_L V_n] \; dp^*$$

$$-\frac{\overline{(q_L\omega^*)}_T}{g} - \overline{P}_o + \overline{P}_T + \frac{1}{g}\int_{o}^{p_T^*} (\overline{C-e})\, dp^* \; . \qquad (43)$$

Solving for

$$\frac{1}{g}\int_{o}^{p_T^*} (\overline{C-e})\, dp^*$$

from equation (40) and substituting for this term in (43) gives the budget equation for water vapor:

$$\frac{\partial}{\partial t}\frac{1}{g}\int_{o}^{p_T^*} \overline{q}\; dp^* = -\frac{\epsilon}{Ag}\int_{o}^{p_T^*} \left\{[q]\;[V_n] + [q'V_n']\right\}\; dp^* - \frac{\epsilon}{Ag}\int_{o}^{p_T^*} [q_L V_n]\, dp^*$$

$$-\frac{\overline{q}_T\overline{\omega}_T^*}{g} - \frac{\overline{(q''\omega^{*''})}_T}{g} - \frac{\overline{(q_L\omega^*)}_T}{g} - \frac{\partial}{\partial t}\frac{1}{g}\int_{o}^{p_T^*} \overline{q}_L\; dp^* + \overline{E} + \overline{P}_T - \overline{P}_o \; . \qquad (44)$$

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APPENDIX A

Effect of Assuming g Constant

Effect of ignoring the vertical variation of g

Typical values of g in the latitudes of the BOMEX array vary from around 978 cm sec^{-2} at 1000 mb to 976 cm sec^{-2} at 500 mb.

Consider the bias introduced in ω(the vertical "p" velocity) if the vertical variation of g is ignored. From the mass continuity equation, we have

$$\int_{500\ \text{mb}}^{P_o} \frac{\partial \omega}{\partial p} \frac{dp}{g} = \int_{500\ \text{mb}}^{P_o} \nabla \cdot \vec{V} \frac{dp}{g} .$$

The left side of this equation can be written

$$\int_{500}^{P_o} \frac{\partial \omega}{\partial p} \frac{dp}{g} = \int_{500}^{P_o} \frac{\partial(\omega/g)}{\partial p} dp - \int_{500}^{P_o} \omega \frac{\partial(1/g)}{\partial p} dp$$

$$= - \left(\frac{\omega}{g}\right)_{500} - \int_{500}^{P_o} \omega \frac{\partial(1/g)}{\partial p} dp \quad . \qquad \text{(A-1)}$$

The second term represents the error in $(\omega/g)_{500}$ resulting from the neglect of the vertical variation of g.

To compute an order of magnitude estimate for this second term, a rough estimate of the mean values of ω as a function of p is needed. Assume a profile which is approximated by the relationship

$$\omega = k_1 + k_2/g \ ,$$

where k, and k_2 are constants for any given profile. (A-2)

This relationship allows one to specify a mean ω profile that is reasonable because ω may be set equal to zero at p_o and its maximum (or minimum) value can be set at 500 mb. This particular relationship was chosen in order to conveniently integrate the last term of (A-1). Using this relationship, we have

$$\omega \frac{\partial(1/g)}{\partial p} = k_1 \frac{\partial(1/g)}{\partial p} + \frac{\partial}{\partial p}\left[\frac{k_2}{2}(1/g)^2\right]$$

$$\int_{500}^{p_o} \omega \frac{\partial(1/g)}{\partial p}\, dp = k_1\left[\frac{1}{g(p_o)} - \frac{1}{g(500)}\right] + \frac{k_2}{2}\left[\left(\frac{1}{g(p_o)}\right)^2 - \left(\frac{1}{g(500)}\right)^2\right] \quad \text{(A-3)}$$

Using the typical values of g previously stated gives

$$1/g_o \approx .0010225$$

$$1/g_{500} \approx .0010246$$

$$1/g_o - 1/g_{500} \approx -2.1 \times 10^{-6} \sec^2 \text{ cm}^{-1}$$

$$(1/g_o)^2 - (1/g_{500})^2 \approx -4.3 \times 10^{-9} \sec^4 \text{ cm}^{-2} .$$

For the average value of $\nabla \cdot \vec{V}$ between the surface and 500 mb, assume a typical 24-hour mean value of 10^{-6} $\sec^{-1}$ for the BOMEX array scale. Assuming a surface pressure of 1000 mb, we obtain

$$\omega_{500} \approx 5 \times 10^{-4} \text{ mb sec}^{-1}$$

$$\omega_{1000} = 0.0$$

Using these values of ω, we can solve for k_1 and k_2 in (A-2), giving

$$k_1 = -000.243 \text{ mb sec}^{-1} ,$$

$$k_2 = 238.000 \text{ cm mb sec}^{-3} .$$

Note that the signs of k_1 and k_2 depend on the assumption of mean convergence or divergence. Substitution in (A-3) gives

$$k_1 \left(\frac{1}{g_{1000}} - \frac{1}{g_{500}} \right) = 5.103 \times 10^{-7} \text{ gm } (\text{cm}^2\text{sec})^{-1}$$

$$\frac{k_2}{2!} \left[\left(\frac{1}{g_{1000}} \right)^2 - \left(\frac{1}{g_{500}} \right)^2 \right] = -5.117 \times 10^{-7} \text{ gm } (\text{cm}^2\text{sec})^{-1}$$

$$\therefore \int_{500}^{1000} \omega \frac{\partial(1/g)}{\partial p} dp \sim -10^{-9} \text{ gm } (\text{cm}^2\text{sec})^{-1}$$

In summary, the assumed mean flux through the top of the BOMEX cube is

$$\sim 5 \times 10^{-7} \text{ gm } (\text{cm}^2\text{sec})^{-1} ,$$

$$\sim 5 \times 10^{-2} \text{ gm } (\text{cm}^2\text{day})^{-1} .$$

The computed error in the mean flux through the top of the BOMEX cube is

$$\sim 10^{-9} \text{ gm } (\text{cm}^2\text{sec})^{-1} ,$$

$$\sim 10^{-4} \text{ gm } (\text{cm}^2\text{day})^{-1} .$$

Thus, it appears that the vertical variation of g can be neglected. In any event, the above analysis can be repeated after the mean ω profile has been computed from the actual data, and, if necessary, minor adjustments can be applied to the computed values.

Effect of ignoring the horizontal variation of g

In the absence of longitudinal variations in g, this approximation will only affect the evaluation of the meridional flux component. The following analyses are therefore for a north-south strip 1 cm wide and 500 km long. Following Hess (1959), we assume a typical variation of g between 978.53 and 978.69 cm sec^{-2}.

(a) Water vapor:

Assume

$$\int_{0}^{p_T^* = 500 \text{ mb}} qv \frac{dp^*}{g} \approx 10^3 \text{ gm (cm sec)}^{-1} .$$

This value for the vertically integrated meridional flux is probably an order of magnitude larger than the mean value that will be found in the BOMEX array.

Using this figure, we can compute the error in flux divergence arising from neglect of the horizontal variation of g as

$$\begin{aligned}
\text{Divergence Error} &= \frac{\text{Error in flux through the boundary}}{5 \times 10^7 \text{ cm}} \\
&= \frac{10^3 \dfrac{978.53}{978.69} - 10^3}{5 \times 10^7} \\
&= \frac{0.3 \text{ gm (cm sec)}^{-1}}{5 \times 10^7 \text{ cm}} \\
&\approx 6 \times 10^{-9} \text{ gm (cm}^2 \text{ sec)}^{-1} \\
&\approx 6 \times 10^{-4} \text{ gm (cm}^2 \text{ day)}^{-1} .
\end{aligned}$$

A comparison of this value with typical evaporation rates (0.2 - 0.6 cm day^{-1}), indicates the error to be negligible.

(b) Enthalpy:

Repeating the above analysis, using

$$\int_{o}^{p_T^* = 500} vH \frac{dp^*}{g} \approx 10^7 \text{ joules (cm sec)}^{-1}$$

as a reasonable value for the meridional enthalpy flux, gives

$$\text{Error} \approx 10^{-4} \text{ joules } (\text{cm}^2\text{sec})^{-1}$$

$$\approx 2.5 \text{ cal cm}^{-2}\text{day}^{-1} \ .$$

In terms of average heating rate this amounts to around 5×10^{-3} ^{o}K day^{-1} and thus appears negligible.

APPENDIX B

Effect of Assuming L Constant

From Hess (1959, p. 46), we have

$$\frac{dL}{dT} \text{ (evaporation)} = -\ 0.566 \text{ cal gm}^{-1}\ {}^{\circ}\text{C}^{-1} .$$

At 0°C,

$$L \text{ evaporation} = 597.3 \text{ cal gm}^{-1} ,$$

$$L \text{ sublimation} = 677.0 \text{ cal gm}^{-1} ,$$

$$L \text{ melting} = 79.7 \text{ cal gm}^{-1} .$$

Assuming $T_o \sim 30°C$, L at the surface will be

$$L_o = 597.3 - 0.566(30) = 580.3 .$$

Assuming temperatures in the upper portions of the box of around -5°C gives a variation in L of

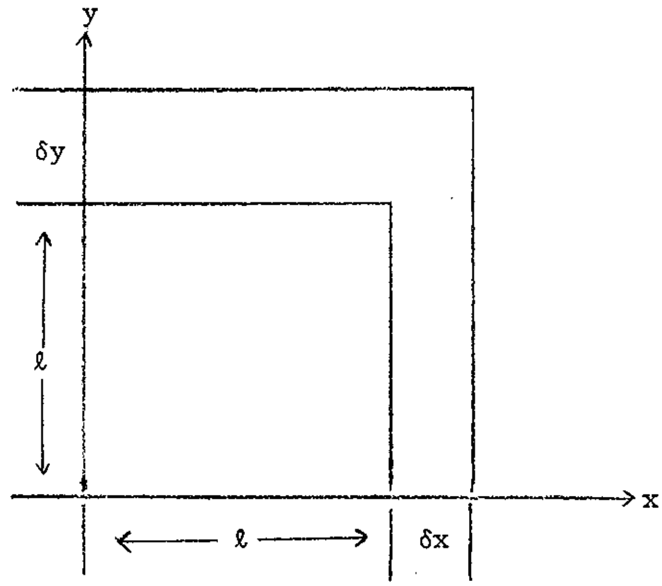
$$\frac{\Delta L}{L} \sim \frac{20}{600} \sim 4 \text{ percent.}$$

Since our ability to determine condensation as a function of p* is limited, it seems consistent to ignore this 4-percent variation in L. Alternately, we may assume a constant, but slightly different, value of L for surface evaporation and for atmosphere condensation.

APPENDIX C

Effect of Assuming Fixed Lateral Boundaries

For purposes of discussion, consider the following figure, which represents the lateral boundaries of the BOMEX box:



We assume a basic area ℓ^2. We assume observations at 0,0; 0,ℓ+δy; ℓ+δx, ℓ+δy; and ℓ+δx,0. In the computation of divergence, the observations are assumed to be taken at 0,0; 0,ℓ; ℓ,ℓ; and ℓ,0.

Assume a linear variation in the wind components, that is,

$$\frac{\partial u}{\partial x} = C_1; \quad \frac{\partial v}{\partial y} = C_2; \quad \frac{\partial u}{\partial y} = \frac{\partial v}{\partial x} = 0$$

$$\therefore \nabla \cdot \vec{V} = C_1 + C_2 \ .$$

Also, assume

$$u(\ell+\delta x,0) = u(\ell+\delta x,x+\delta y) = u(\ell,0) + C_1 \ \delta x = u(\ell,\ell) + C_2 \ \delta x,$$
$$v(0,\ell+\delta y) = v(\ell+\delta x,\ell+\delta y) = v(0,\ell) + C_2 \ \delta y = V(\ell,\ell) + C_2 \ \delta y.$$

The divergence error would then be

$$\frac{(C_1\delta x + C_2\delta y)}{\ell} \ .$$

Let us consider the error arising from two combinations of C_1, C_2, δx, and δy.

First,

$$\delta y = \delta x$$

$$\text{Error} \quad \frac{\delta x \ (C_1 + C_2)}{\ell} \ .$$

Typical values of δx are 10 km. Thus, typical percentage errors in divergence will be (10/500) × 100 = 2 percent.

Second,

$$\delta y = -\delta x$$

$$C_2 = -C_1 \sim 10^{-5}$$

This situation will arise if the wind field has a substantial deformation component, and δy and δx are of opposite sign. Under these conditions,

$$\text{Divergence} \approx \frac{2\delta x}{\ell} \ 10^{-5}$$

$$\text{Error} \approx 4 \times 10^{-7}$$

This figure can be considered as an estimate of the order of magnitude of the maximum error arising from the approximation of fixed lateral boundaries. Thus, under circumstances where (1) the wind field has a large deformation component, (2) the error in the fixed-ship positions is of opposite sign for the two components, and (3) the magnitude of the errors in fixed-ship positions exceeds 20 km, the divergence error could conceivably approach 10^{-6}. Consequently, it would be well to generate as part of the computations a first-order estimate of this error, based on observed values of δx and δy, and on values of C_1 and C_2 derived from the computations. If cases arise in which the error introduced by this approximation is deemed unacceptable, a slight correction based on computed gradients of the wind components over the area can be applied to the shipboard observations to arrive at better estimates of the parameters at the corners of the array.

APPENDIX D

Computational Requirements

A list of those quantities in the proposed budget equations (36) - (44) that can be computed from aerological observations of u, v, p, T, z, q, and q_L follows.

Quantity	Budget equation	
$\bar{u}$	37	These quantities to be computed as functions of p*.
$\bar{v}$	38	
$\overline{\omega^*}$	37, 38, 39, 40, 41, 42, 44	
$\bar{K}$	39, 42	
$\bar{T}$	41, 42	
$\bar{q}$	40, 42, 44	
$\bar{z}$	37, 38, 42	
$\bar{p}_o$	39, 41, 42	
u''	37	These quantities to be computed, to the extent possible, as functions of λ, ϕ, and p*.
v''	38	
$\omega^{*''}$	37, 38, 39, 40, 41, 42, 44	
K''	39, 42	
T''	41, 42	
q''	40, 42, 43	
z''	37, 38, 39, 41, 42	
p_o''	37, 38, 39, 41, 42	Computed as a function of λ, ϕ.
$[u]$	37	These quantities to be computed as functions of p*.
$[v]$	38	
$[V_n]$	36, 37, 38, 39, 40, 41, 42, 44	
$[K]$	39, 42	
$[T]$	41, 42	
$[q]$	40, 44	
$[z]$	39, 42	
u'	37	These quantities to be computed as functions of p* and position on the boundary.
v'	38	
V_n'	37, 38, 39, 40, 41, 42, 44	
K'	39, 42	
T'	41, 42	
q'	40, 42, 44	
z'	39, 42	