Sea Grant Special Bulletin #4

Exchange Processes in Shallow Estuaries

Thomas N. Lee and Claes Rooth

University of Miami Sea Grant Program (NOAA Sea Grant No. 2-35147)
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PREFACE

The Sea Grant Colleges Program was created in 1966 to stimulate research, instruction, and extension of knowledge of marine resources of the United States. In 1969 the Sea Grant Program was established at the University of Miami.

The outstanding success of the Land Grant Colleges Program, which in 100 years has brought the United States to its current superior position in agricultural production, was the basis for the Sea Grant concept. This concept has three objectives: to promote excellence in education and training, research, and information services in the University's disciplines that relate to the sea. The successful accomplishment of these objectives will result in material contributions to marine oriented industries and will, in addition, protect and preserve the environment for the enjoyment of all people.

With these objectives, this series of Sea Grant Special Bulletins is intended to convey useful research information to the marine communities interested in resource development.

While the responsibility for administration of the Sea Grant Program rests with the Department of Commerce, the responsibility for financing the program is shared by federal, industrial and University of Miami contributions. This study, Exchange Processes in Shallow Estuaries, is published as a part of the Sea Grant Program.

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EXCHANGE PROCESSES IN SHALLOW ESTUARIES

ABSTRACT

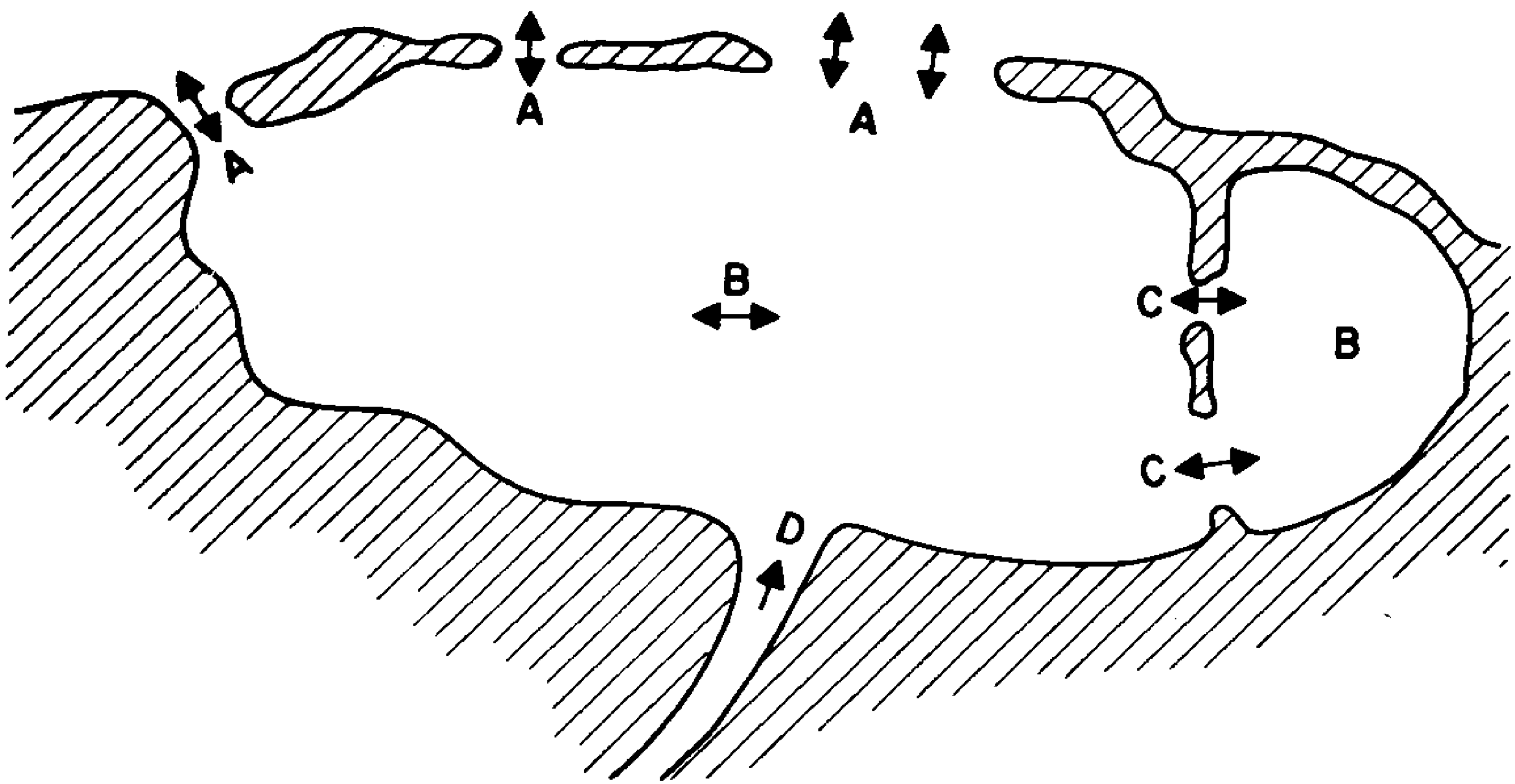
A modular approach to the analysis of mixing and flow characteristics in shallow tidal estuaries is presented using South Florida's Biscayne Bay as an example. The method depends on isolating relatively simple characteristic flow regimes in different parts of an estuary. These can be considered as building blocks, which when recombined in different configurations are capable of yielding a qualitative model for a specific estuary. Such models are of immediate value in preliminary assessments of estuarine water quality and interaction problems. This method can provide an effective base for further studies where more precise information is needed.

INTRODUCTION

Historically, studies of estuarine circulation have been conducted in deep openings to the ocean with large river outflows. Exchange processes were considered to be governed by the relative volumes of fresh water outflow vs. tidal prism. Solutions for surface elevation and flow are usually obtained either analytically with simplified versions of the linear hydrodynamic equations of motion, or numerically by fitting a grid to the estuarine geometry and finding stepwise solutions within each grid mesh. Unfortunately, these approaches have led to the belief that solutions obtained for a particular estuary are not applicable to other areas, thus requiring a complete study of each estuary of interest. Much of this complexity of estuarine circulation models arises from the necessity to cope with different dynamic regimes in different parts of any specific estuary. It is often possible, however, to divide an estuary into subregions, each with different but simpler dynamic characteristics. A set of dynamic models for such homogeneous subregions can be looked upon as a kit of building blocks. Suitably combined, these building blocks will yield a qualitative model for any specific estuary. The practical advantage of this method is that identification of flow regimes can be accomplished with a minimal observational program. Preliminary estimates of exchange rates and assessments of water quality can thus be made with modest efforts and expenditures. Where more precise information is needed, the method supplies an effective base for detailed studies.

METHOD OF ANALYSIS

Estuarine exchange processes are typically controlled by the forces of tides, winds, river run-off, evaporation,and precipitation. These forces display large variations in time and space between different estuaries and within any given estuary due to the geographic locations and complexities of shapes. The modular approach separates an estuary into characteristic regions as shown in FIGURE 1. The building blocks consist of regions of direct exchange between an estuary and the coastal waters (A), interior basins (B), the regions of exchange between basins (C), and the region of river influence (D). There is a large amount of literature on inlet hydraulics and effects of river run-off; therefore, the emphasis of this paper will be on mixing processes within the interior of estuaries.



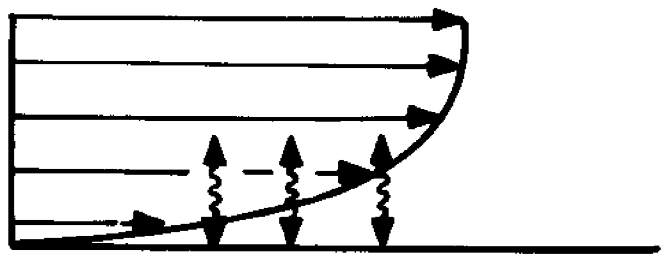
Estuarine Model
FIG. 1

Tidal Exchange in the Interior of Estuaries

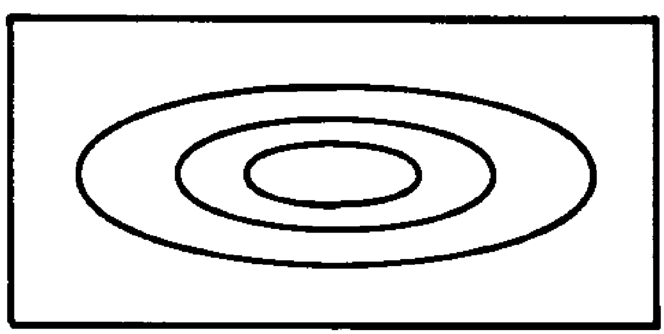
Tidal currents in the interior regions of estuaries will normally show a simple longitudinal reversing pattern aligned with the major axis of the basins. Theoretical tidal analysis suggests that this simple asymmetric tidal flow should result in substantial anisotropic mixing conditions, with mixing being enhanced in the direction of flow.

It is well-known from studies of exchange in the atmospheric boundary layer, as well as in shallow estuaries, that the combination of vertical shear in the mean velocity field and vertical turbulent diffusion leads to a horizontal dispersion effect, which can be described as an anisotropic diffusion process. As shown in a detailed treatise by Bowden (1965), this induced diffusivity is dependent on the velocity shear and on the vertical diffusion time scale in such a way that a stable stratification enhances the effect. If a diffusing property (pollutant being discharged into an estuary) has little effect on density, then asymmetric diffusion will result in a distribution of the property which resembles a simple advective outflow.

These mixing processes are displayed graphically in FIGURES 2 and 3. FIGURE 2 shows the combination of vertical shear and vertical turbulent diffusion. The effect of these conditions on the dispersion of a symmetrical dye patch is displayed in FIGURE 3. The vertical shear enhances mixing in the direction of flow, producing the elongated dye patch. If the variations in Richardson number are not negligible, an asymmetry ensues, with enhanced horizontal mixing effects during the stably stratified phase. If one is concerned with diffusion of the density determining property, the most important aspect of



Velocity Profile and Vertical Diffusion
FIG. 2



Dispersion of a Symmetrical Dye Patch
FIG. 3

this is the possibility of a breakdown of the uniform turbulent regime. Since the vertical mixing time must be much shorter than the tidal period for these arguments to apply, one must exercise some care in the extension of the simple mixing theory to cases involving substantial density fluxes.

The diffusion in the crosscurrent direction is given solely by the turbulence in the stream. In the neutral case, symmetry between vertical and cross-stream mixing is to be expected, but stable stratification will lead to enhancement of the lateral mixing due to the horizontal spreading of decaying eddies. Bowden estimates the cross-stream diffusivity at most a few percent of the streamwise diffusivity as long as stratification is subcritical.

It can be shown by an extension of Bowden's analysis (See APPENDIX) that the tidal-induced diffusivity in the downstream direction D_T is:

$$D_T = \mu \frac{L_T H}{T_o} \qquad 1)$$

where L_T = tidal excursion length

H = mean depth of the basin

T_o = tidal period

μ = proportionality constant, mainly dependent on bottom roughness and stability. For moderate to weak stratification, μ is approximately unity.

This leads to a simple rule of thumb for tidal-induced mixing in the interior: <u>The equivalent diffusivity is approximately one tidal prism per tidal cycle.</u>

The tidal mixing time scale (τ_T) for a basin of length L_B is given by:

$$\tau_T = \frac{L^2_B}{\pi^2 D_T} \qquad 2)$$

substituting for D_T:

$$\tau_T = \frac{L^2_B T_o}{\pi^2 L_T H} \qquad 3)$$

The tidal excursion length can be found from:

$$L_T = \int_o^{T_o/2} \frac{dx}{dt}\,dt = \int_o^{T_o/2} U_T \sin \omega t\,dt \qquad 4)$$

$$L_T = \frac{2U_T}{\omega} \qquad 5)$$

$$L_T = \frac{U_T T_o}{\pi} \qquad 6)$$

where $\frac{dx}{dt} = \underline{U}$ = tidal velocity aligned with major axis of basin X.

U_T = maximum tidal velocity

$$\omega = \frac{2\pi}{T_o}$$

substituting equation 6) into 3) gives:

$$\tau_T = \frac{L^2_B}{U_T H \pi} \qquad 7)$$

An estuary with a central basin 5 nm long and 10 feet deep and with a maximum tidal current of 0.5 knots would take about 400 days to exchange its water by tidal mechanisms alone.

If an estuary consists of several interconnected basins, then due to the preferred direction in tidal mixing, exchange of interior water will be predominantly between the different basins. Direct exchange with the ocean will be restricted to the regions in the vicinity of the tidal inlets.

Tidal Exchange in the Vicinity of Inlets

The regions of direct coastal-estuarine interaction are confined to the vicinities of the seaward openings, coupling the estuary to the sea. Flow through these openings will normally be dominated by tidal forces. The astronomical tidal wave progressing through the coastal waters produces a slope in the water surface through the opening. The magnitude of the resulting flow will be directly dependent upon the height of the slope. The volume of fluid that can be exchanged through an opening depends also on its width and depth. The shapes of openings can vary from narrow inlets to broad shallow flats.

The region of the estuary which undergoes direct exchange with the ocean will depend upon the shape of the seaward opening. In the vicinity of inlets narrow enough to produce noticeable fluid jetting, an induced mixing due to inertial effects on the flow will arise. The incoming tide may give rise to a jet-like motion, while on the falling tide the flow towards the inlet is similar to an ideal potential flow toward a sink. The net result, when averaged over a complete tidal cycle, is a mean circulation corresponding to a double vortex sheet along the edge of a widening jet. Using the standard type linear entrainment law, which has been found experimentally to work in most cases of turbulent jets (See APPENDIX), a first order estimate of the jet's penetration distance into the basin can be found. This penetration distance was determined to correspond to a semicircle with a radius approximately 500 times the mean depth of the inlet.

The region of influence from river run-off is dependent upon the volume rate of river discharge. If the volume of discharge during a tidal cycle is very small in comparison to the tidal prism, as is the case in most of the shallow embayments along the southeast Atlantic Coast and Gulf Coast, then the river influence on interior exchange will have a secondary effect. River run-off may be negligible, or at most seasonal, at which times it biases the tidal excursion by the mean river velocity and will aid in flushing the estuary. A convenient by-product of river run-off is that the low salinity discharge produces large salinity contrast and provides a useful tag for investigating estuarine exchange.

The complicated shapes of estuaries can be approximated by combining basins of various dimensions which interact with each other across shallow flats or narrow channels. The flow and exchange properties of these communication regions can be investigated separately. Both tides and wind forces can be important to the flow in these regions. Tidal motion will become less significant the further removed the area is from the ocean. A tidal wave progressing through an estuary, consisting of a number of interconnecting basins, will be frictionally damped by each successive shoal or channel.

Wind Influence on Estuarine Exchange

The importance of wind stress on estuarine exchange cannot be overemphasized. As indicated previously, shallow estuaries are very poorly flushed by tidal mechanisms alone. Wind influence can greatly reduce the flushing time by setting up a mean circulation which can advect interior water into the vicinity of tidal inlets where direct exchange with the ocean takes place. The relative effects of winds and tides on estuarine exchange can be characterized by the ratio, σ, of the tidal mixing time scale τ_T to the wind mixing time scale τ_W. If a mean circulation with a characteristic velocity U_W is induced by winds, the circulation time scale is:

$$\tau_W = \frac{L_W}{U_W} \qquad 8)$$

where L_W is between $2L_B$ and $4L_B$ dependents on the basin's slope and circulation pattern. Taking τ_T from equation 7),

$$\sigma = \frac{\tau_T}{\tau_W} = \frac{LB}{\pi HC} \frac{U_W}{U_T} \approx \frac{LB}{10H} \frac{U_W}{U_T} \qquad 9)$$

For the example previously given of a basin 5 nm in length and 10 feet deep, this becomes:

$$\sigma \approx \frac{LB}{10H} \frac{U_W}{U_T} \approx 300 \frac{U_W}{U_T} \qquad 10)$$

Thus, a wind-induced current with a magnitude of $\frac{1}{300}$ of the maximum tidal current will flush the estuary in a similar time span. If one takes the drag coefficients as equal for the wind action on the surface as for the bottom stress, the maximum drift velocity induced by local wind is about 3% of the wind speed. If a 10 knot wind produces a 0.3 knot drift current in the basin given above, then the residence time of the estuary would be 2.2 days. In most estuaries, significant set up develops soon after the onset of wind. A sustained circulation can arise, either due to partial sheltering,or due to irregular bottom topography, and is thus not likely to amount to more than a modest fraction of the simple estimate above. If one assumes a 10% value for the ratio of circulation velocity to maximum drift velocity, our quantitative example above would yield an exchange time with a 10 knot wind of 22 days. If a wind-induced circulation becomes as much as 50% of the maximum drift velocity, then the exchange time reduces to 4.4 days, showing that the tidal exchange time is nearly two order of magnitudes greater than wind exchange time.

Direct wind forcing in tidal inlets can usually be neglected due to the small surface areas involved. However, wind-induced set up in the interior can bias the head across the inlet, thus biasing the transport to sea and further augmenting the estuarine exchange with the ocean.

In regions of exchange between basins of an interconnected estuary, the effect of wind will be due to the set up within the basin. The set up will behave in the tidal inlet to bias the head between basins, producing a pressure gradient flow that may be of comparable magnitude to the tidal flow.

Wind influence may be the most important factor in flushing shallow estuaries; therefore, its effect should be isolated from tidal forcing. Then wind data can be used to estimate the frequency of such wind-flushing events.

APPLICATION OF METHOD

The modular approach to understanding estuarine exchange is applied to a particular estuary by segmenting it into regions of characteristic flow and exchange properties which, when understood, are recombined to form a qualitative model of estuarine interactions.

Segmentation of an estuary can be approximated at first by using existing bathymetric and tidal charts if available and any other forms that may exist, such as temperature or salinity charts. Since estuaries are traditionally drainage basins for the surrounding land mass, they will usually display large salinity variations within the estuary and sharp contrasts from the coastal waters. During the periods of low run-off, estuaries with weak exchange with the ocean will produce high salinities by evaporation. During high run-off periods, the interiors of estuaries will display low salinities. The net result is that salinity is a very useful tracer of estuarine interaction. Therefore, salinity measurements can be used to separate an estuary into characteristic flow regimes. Salinity measurements should be conducted as rapidly as possible at maximum and minimum run-off and wind stress conditions. Local airports and weather stations can be used to obtain historical meteorological data which will aid in selection of observational periods.

Southeast Florida's Biscayne Bay Estuary Example

Biscayne Bay and Card Sound (FIGURE 4) are excellent locations to initiate a modular approach to understanding estuarine mixing processes, due to the diversity of flow regimes present. North Biscayne Bay represents a highly polluted region where man's impact has been intense. The Miami River discharges into this region with a heavy load of industrial and domestic wastes. Dredge and fill operations associated with Miami Harbor's deep water port, land development, and construction of causeways have greatly altered the bottom topography. The creation of bulkhead lines and construction of seawalls have completely inundated the natural low-lying shoreline. Middle Biscayne Bay, South Biscayne Bay, Card Sound, Little Card Sound,and Barnes Sound are still relatively unspoiled. This system of basins separated from each other by shallow shoals and from the ocean by the northward extension of the Florida Keys extends approximately 45 miles along its major axis, with widths ranging from 8 to 3 miles. Water depths in the interior of the basins vary from 6 to 10 feet. The embayments communicate with each other across shallow shoals and through dredged navigational channels. Exchange with the ocean takes place through a complicated network of tidal inlets and across a broad shoal (Safety Valve), honeycombed with narrow flow channels (See FIGURE 13).

Semidiurnal tidal forcing at the seaward entrances to the embayments is approximately in phase because the length of the major axis is small in comparison to the tidal wave length. The wave that enters the interior of North Biscayne Bay progresses south with decreasing amplitude and increasing lag due to frictional dampening by the shallow shoals (Schneider 1969).

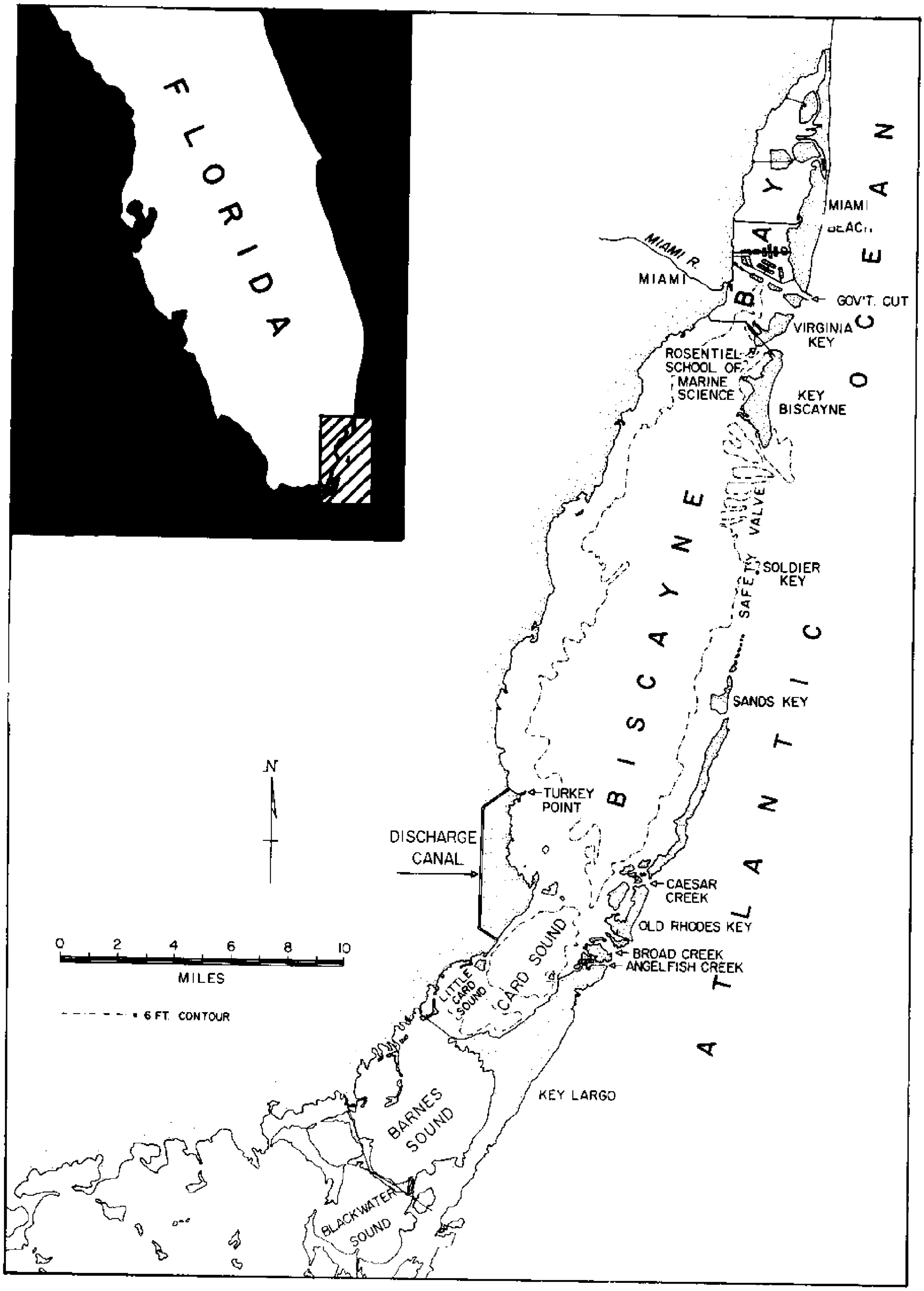

FLORIDA
BISCAYNE BAY
ATLANTIC OCEAN
MIAMI BEACH
MIAMI R.
MIAMI
GOV'T. CUT
VIRGINIA KEY
ROSENTIEL SCHOOL OF MARINE SCIENCE
KEY BISCAYNE
SAFETY VALVE
SOLDIER KEY
SANDS KEY
TURKEY POINT
DISCHARGE CANAL
CAESAR CREEK
OLD RHODES KEY
BROAD CREEK
ANGELFISH CREEK
CARD SOUND
LITTLE CARD SOUND
KEY LARGO
BARNES SOUND
BLACKWATER SOUND
N
0 2 4 6 8 10
MILES
6 FT. CONTOUR

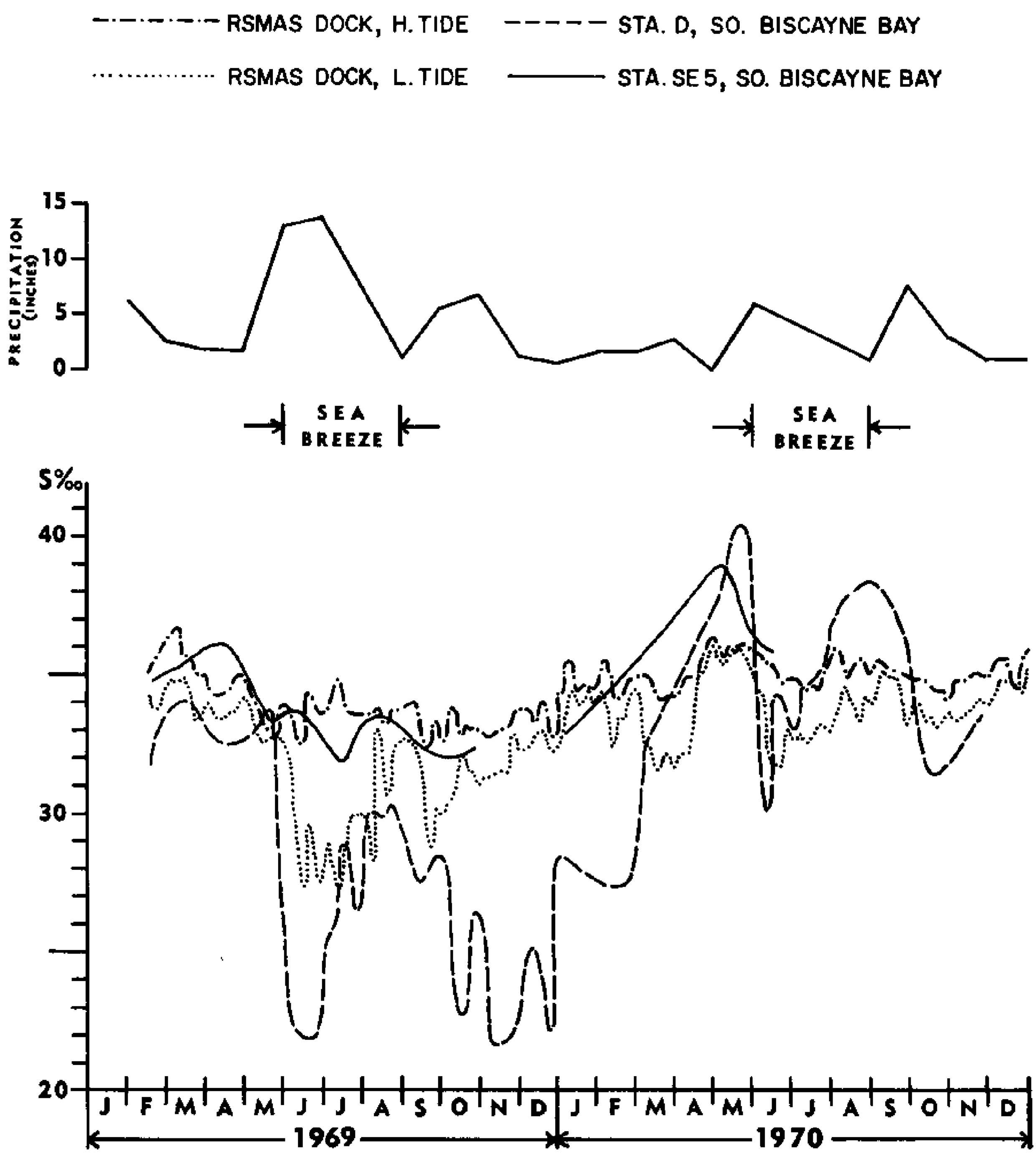
FIG. 4

Climatologically, wind forcing occurs mainly from the passage of winter cold fronts with accompanying strong northerly winds. Salinity conditions are forced by the pronounced wet-dry seasons of southeast Florida. The rainy periods are usually in late May and September, with dry conditions prevailing during the winter.

A striking demonstration of the weakness of tidal exchange in Biscayne Bay is provided by the seasonal salinity contrasts between the different basins and the ocean, which develops in response to these climatic conditions. A comparison of salinities taken in Bear Cut, which opens into Middle Biscayne Bay, with samples from South Biscayne Bay are shown in FIGURE 5. The two annual salinity minima are clearly shown at the time of rainfall maxima in spring and late summer. The influence of sea breeze circulations in mixing the estuaries is indicated from the two annual salinity minima by a somewhat smaller salinity contrast between the basins during the periods of maximum sea breeze development. Low tide salinities do not stabilize with the coastal waters (high tide) but rather continue to increase, indicating poor coastal-estuarine exchange.

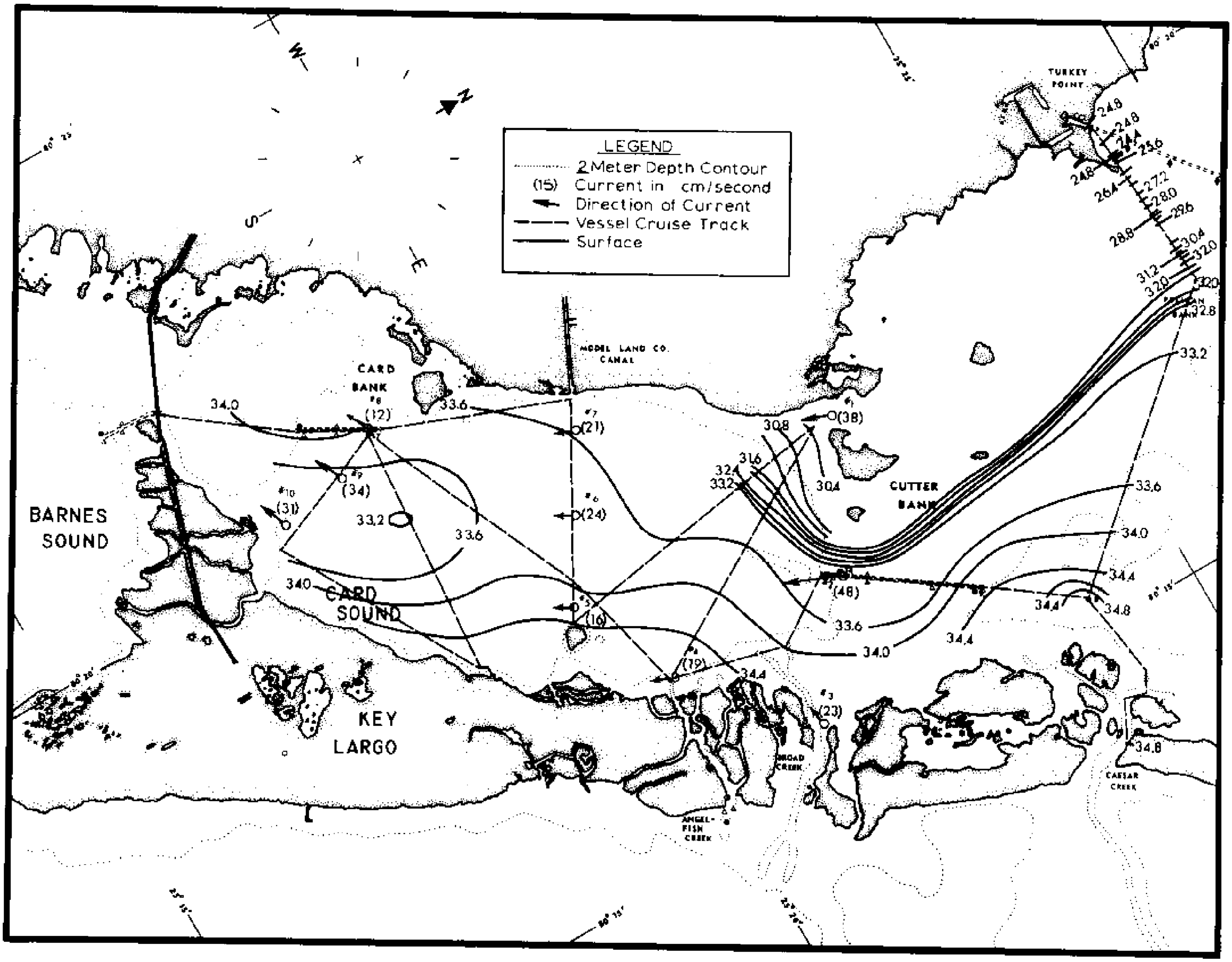
In order to facilitate study in greater detail of these large salinity contrasts, a system was developed for rapid mapping of surface temperature and salinity. A flow-through pumping system onboard a rapid boat feeds a Bissett-Berman thermosalinograph, which produces a continuous analog record of temperature and salinity.

FIGURE 6 shows the result of surface salinity mapping in the interiors of South Biscayne Bay and Card Sound with low wind effect. The fall wet



Precipitation and salinity relation in Biscayne Bay for 1969-1970.

FIG. 5



Isohalines (ppt) in Card Sound for 0900-1250 hrs. on 11 November, 1970. Wind 270°T at 3 m/sec. Flood Tide.

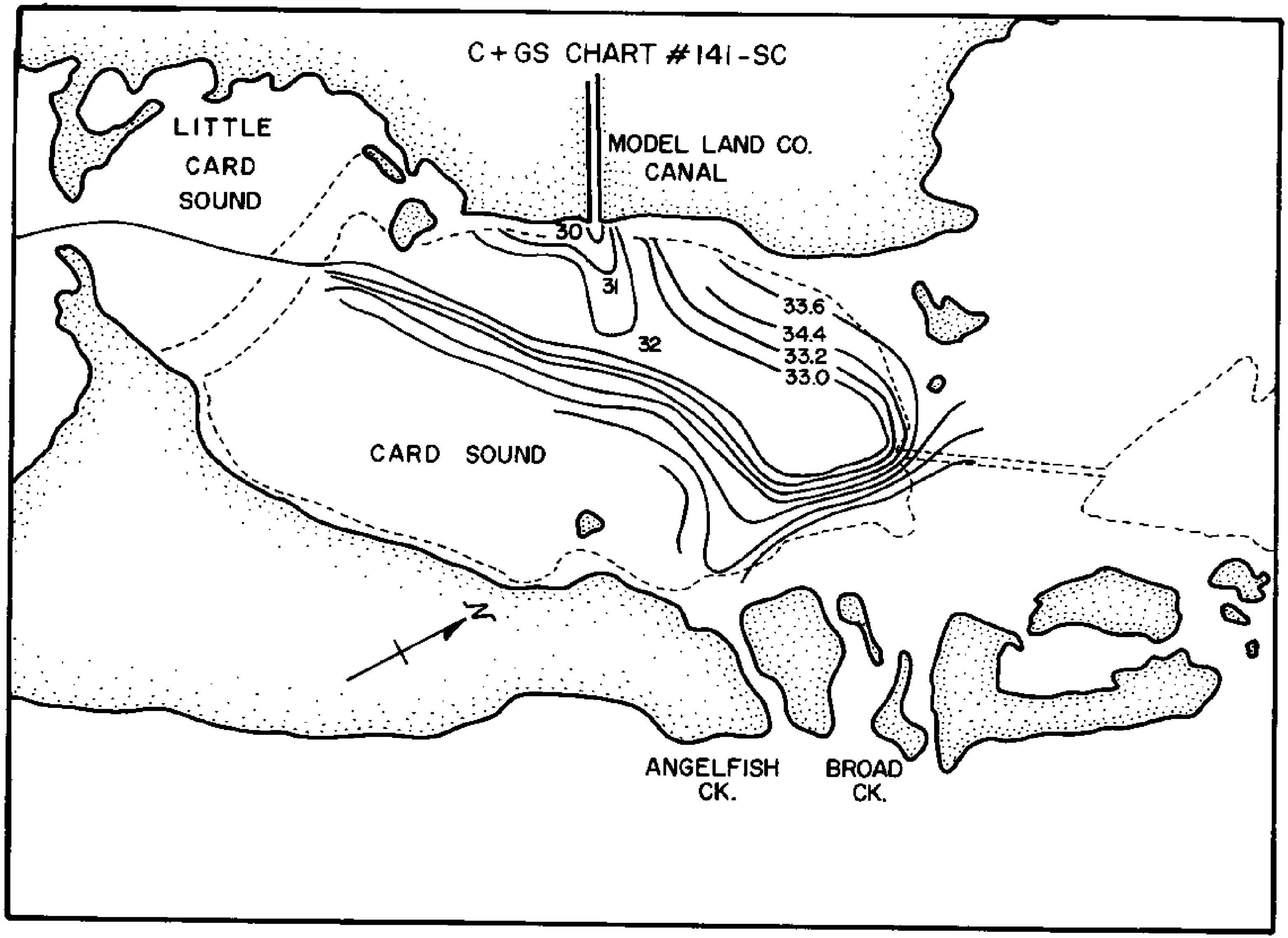
FIG. 6

season shown in FIGURE 5 is evident in the low salinities along the western shores of the basins. Synoptic measurements of surface currents were conducted on a tidal time scale by aerial photography of dye releases. Tidal currents in the interior show a simple longitudinal reversing pattern aligned with the major axis of the basins. In agreement with our previous analysis which suggests a preferred mixing in the downstream direction, the isohaline patterns are found to be aligned in the direction of flow.

The result of a anisotropic, tidal-induced mixing with a weak run-off is shown in FIGURE 7. The distribution of salinity resembles an advective outflow; however, we believe that this pattern is the result of several days of mixing. The stability of the mixing conditions during weak wind periods is shown in FIGURE 8, an aerial photograph of the western section of Card Sound. The parallel rows aligned in the direction of tidal flow are free, bottom-drifting Laurencia poitea.

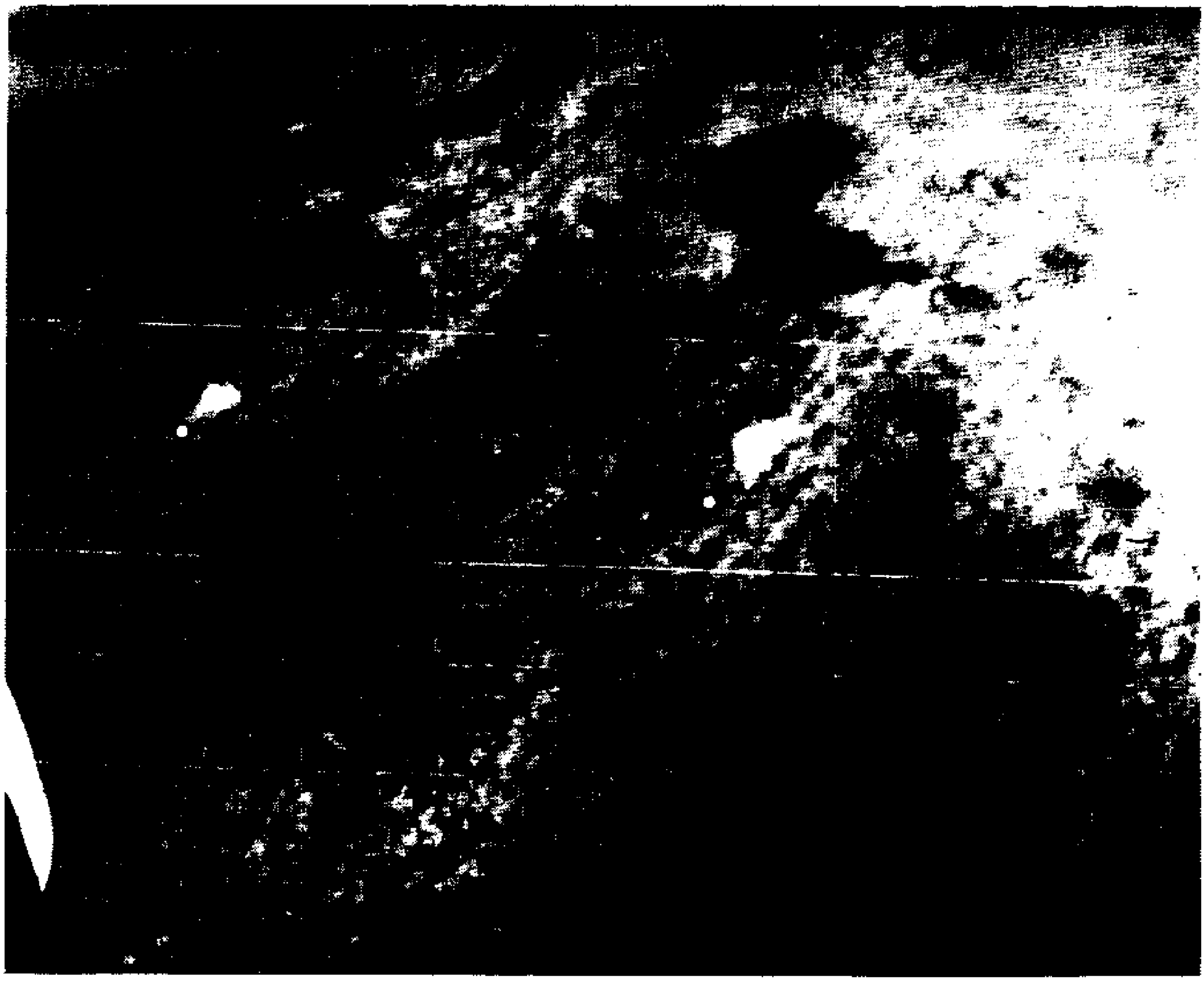
FIGURE 9 clearly shows the penetration of a low salinity inertial jet into Card Sound from the Broad and Angelfish Creek inlet complex. The observations were made at the start of ebb tide and therefore represent the maximum estuarine region of direct tidal exchange with the ocean during low wind periods. The inertial jetting analysis predicts the penetration distance of this region to be approximately 6000 feet, which corresponds quite well with FIGURE 9.

The wind influence on exchange in Card Sound can be seen in FIGURE 10. A cold front with wind speeds of 20 knots out of the north passed through the area one day prior to the field observations. The low salinity jet



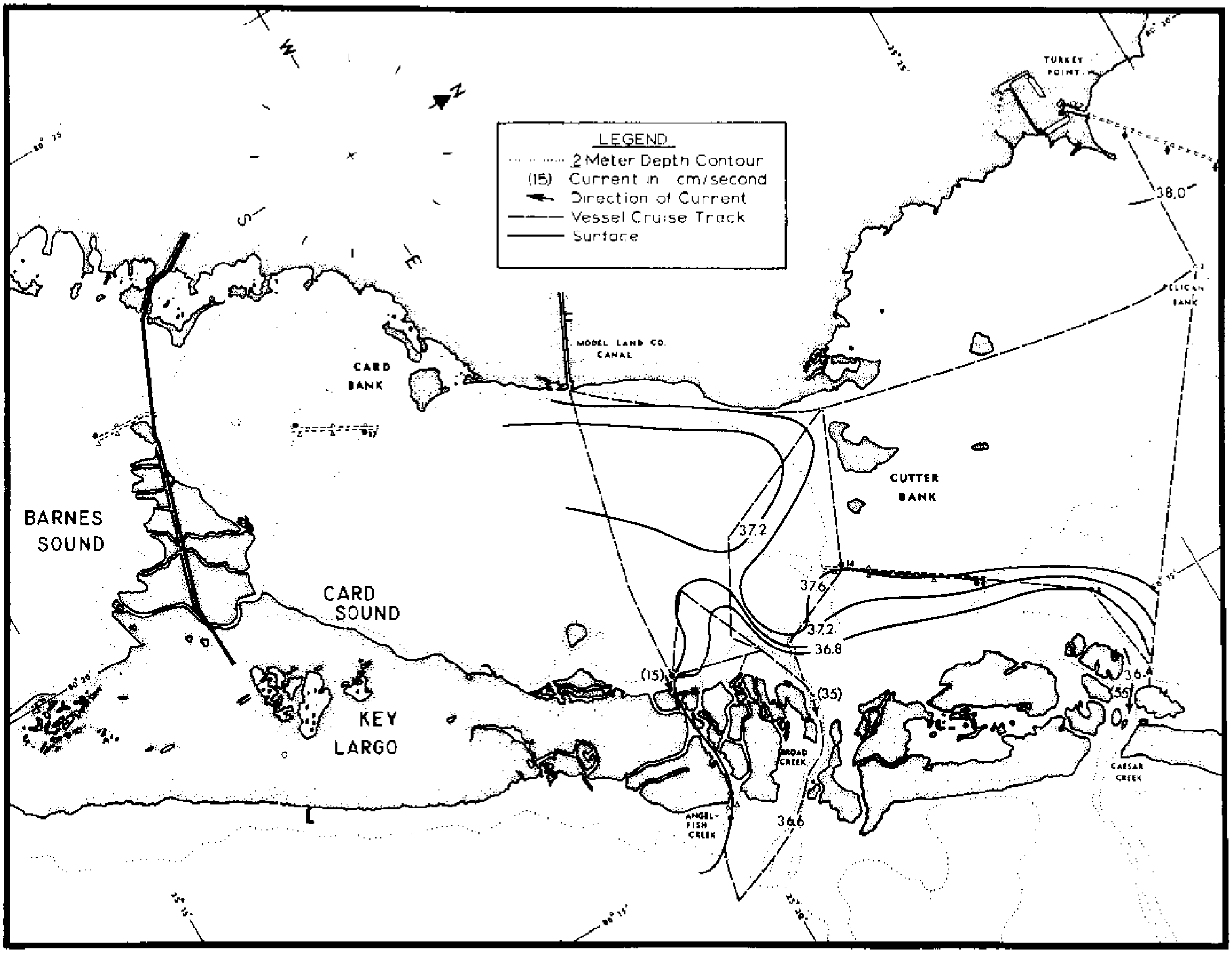
Isohalines in Card Sound on 25 June 1970.

FIG. 7



Parallel rows of Laurencia poitei in Card Sound indicating long-shore current patterns.

FIG. 8

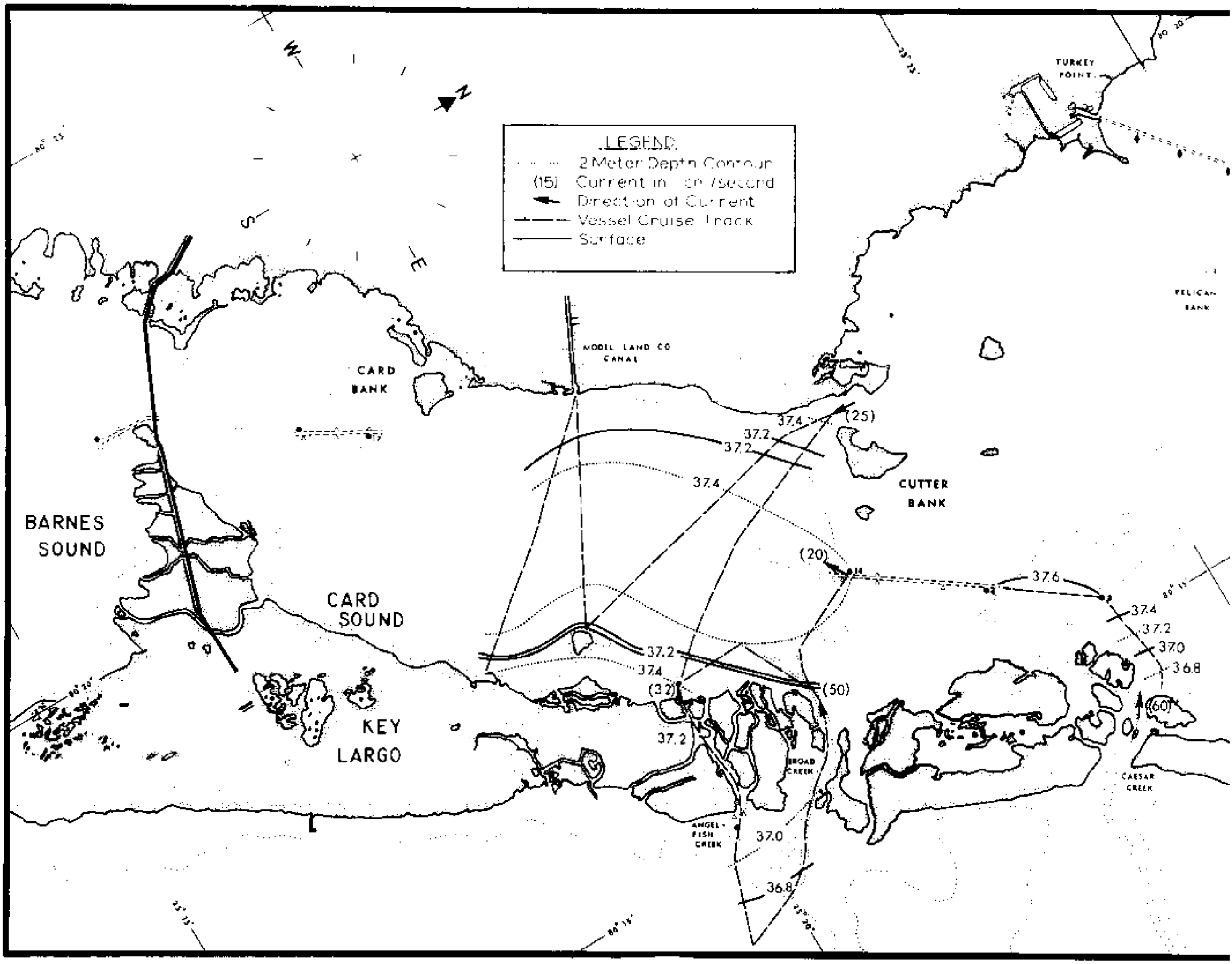


Isohalines (ppt) in Card Sound for 0726-1115 hrs. on 5 February, 1971. Wind 140°T at 5 m/sec. Ebb Tide.

FIG. 9

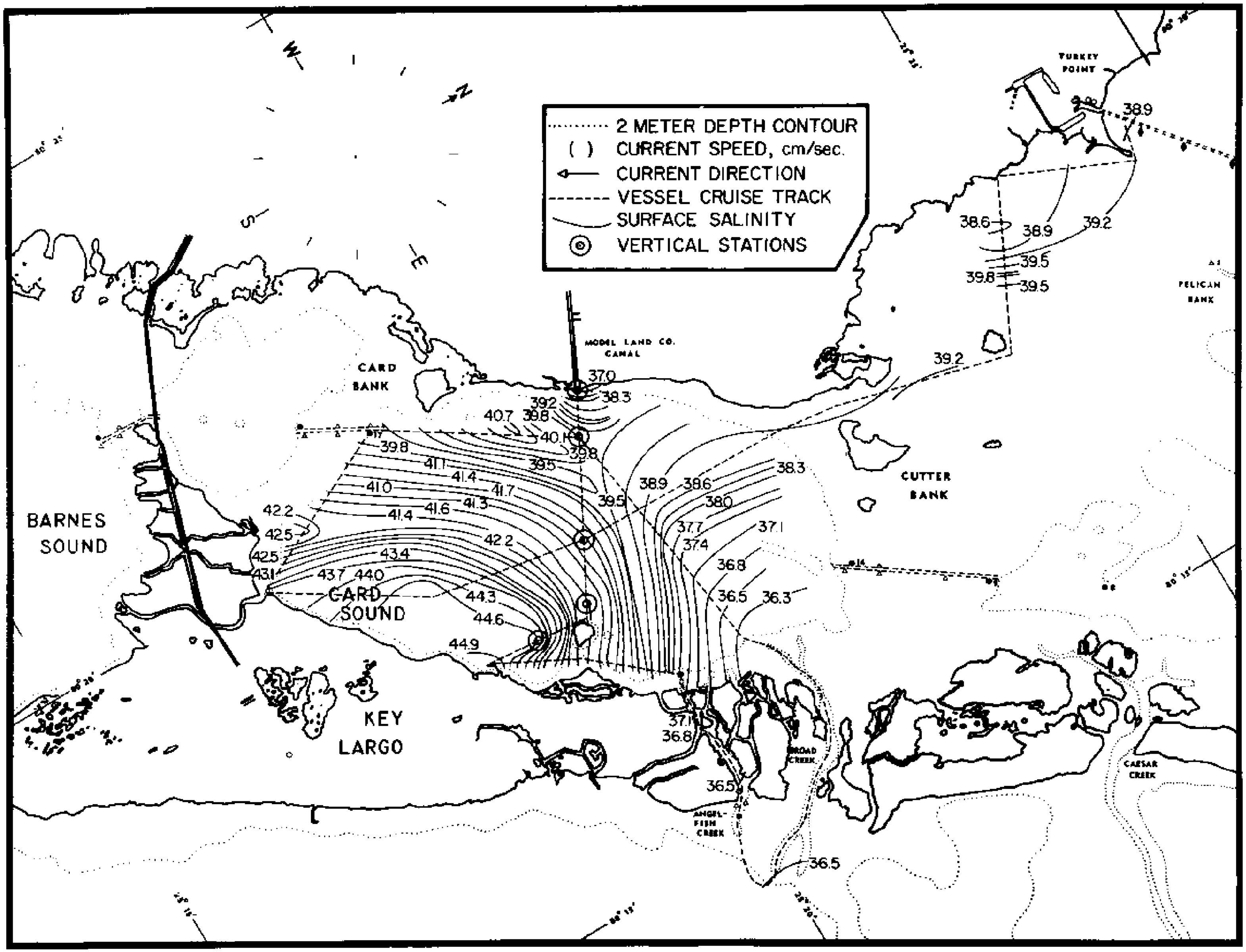
which is so apparent in FIGURE 9 has disappeared even though both observations were made at a similar stage of the tide. Also the interior of Card Sound is well-mixed and the salinities in Broad and Angelfish Creeks have increased 0.4 ^{o}/oo to be comparable to estuarine salinities. The above differences in the two figures suggest a mean wind-induced circulation which is transporting interior water to the region of direct exchange with the ocean, thereby biasing the flow from the inlets.

Very complex mixing patterns are produced when the forces that control exchange processes in shallow estuaries, such as wind stress, tidal motion, evaporation, and precipitation interact with comparable magnitudes. FIGURES 11 and 12 show the result of interaction of these mixing forces. These measurements were obtained at the end of flood tide, during a period of predominant easterly winds. FIGURE 11 indicates that the wind influence has increased the size of the region that undergoes direct exchange with the ocean through the tidal inlets. This is shown in the northeast section of Card Sound as salinities of coastal water origin from Broad and Angelfish Creeks. Exchange in the interior of the basin appears to be dominated by preferred longitudinal tidal mixing which produces isohalines aligned with the major axis of the sound. The extremely high salinities are the result of vigorous evaporation during a long South Florida drought. An area of very low exchange is indicated by the salinity maximum (>45^{o}/oo) in the southeast section of Card Sound. A weak run-off from recent rains has produced a salinity minimum in the vicinity of the Model Land Company Canal. These mixing processes are also revealed in the vertical section from Model Land Company Canal east to Pumpkin Key (FIGURE 12). In the interior of the

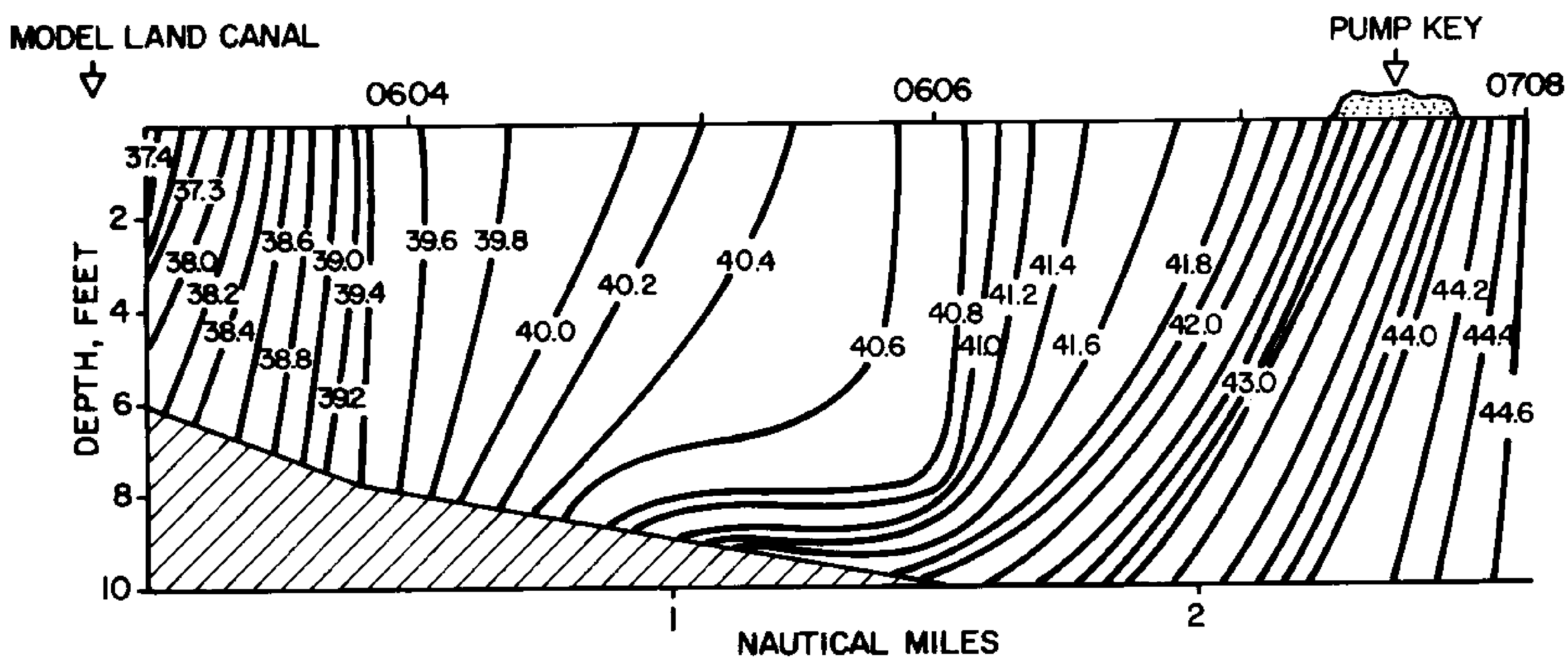


Isohalines (ppt) in Card Sound for 1057-1427 hrs. on 15 February, 1971. Wind 090° T at 1.5 m/sec. Flood Tide.

FIG. 10



Isohalines (ppt) in Card Sound for 1024 to 1349 hrs. on August 19, 1971
Wind 130° at 2 m/sec. Flood Tide
FIG. 11



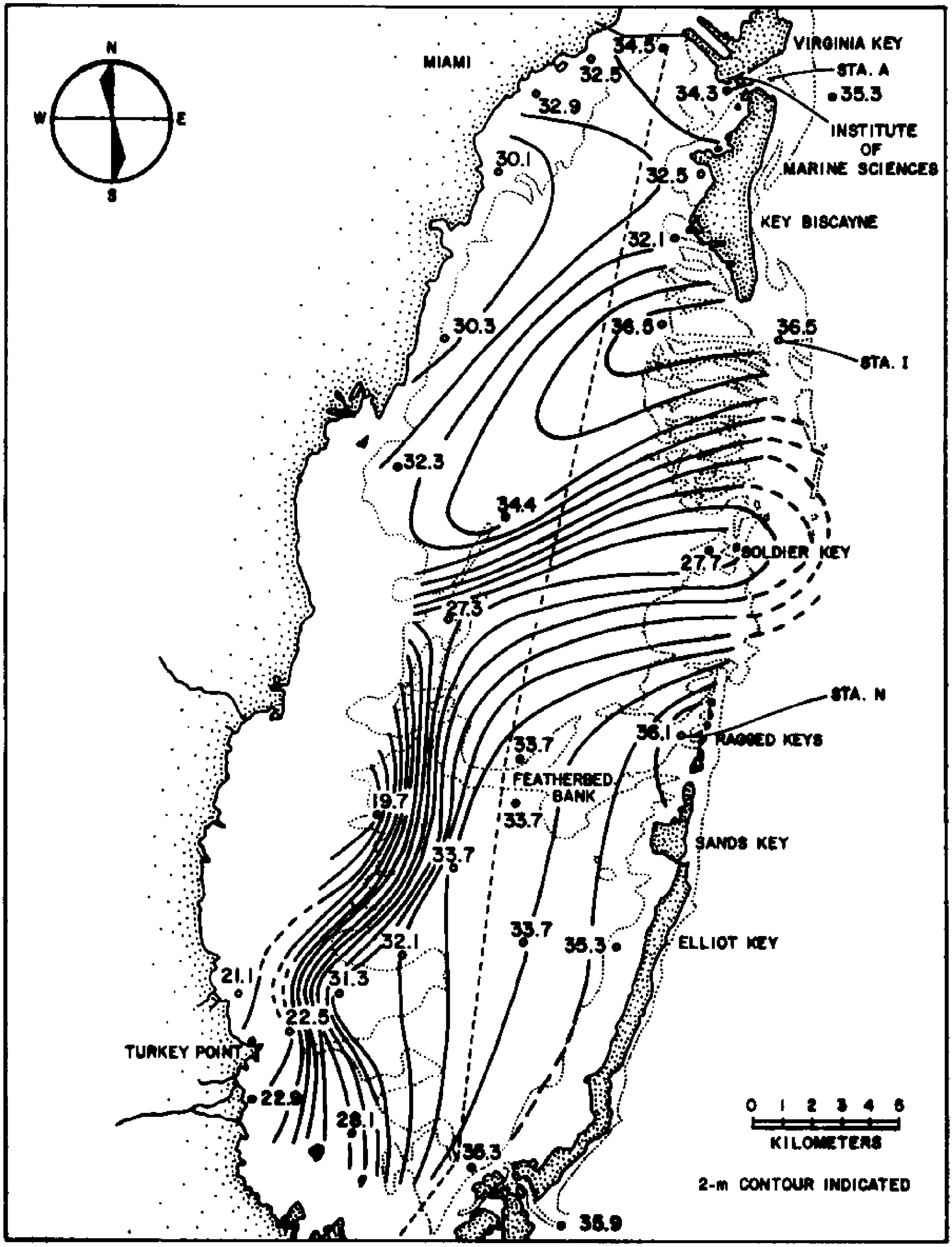
Vertical Salinity Contours, Card Sound
19 August, 1971
FIG. 12

basin there is sufficient turbulence generated by longitudinal tidal flow to mix the water column vertically. However, mixing is weak in the low exchange region of southeast Card Sound which enables stratification to develop. As the salinity of this region increases, a thermohaline motion is set up which extends high salinity water along the bottom into the interior. Thus, an equilibrium is created which attempts to balance the turbulent tidal flow with the salinity produced thermohaline motion.

The mixing processes of Card Sound are typical of the basins in Biscayne Bay as can be seen by the similarity of surface salinity patterns in upper Biscayne Bay (FIGURE 13). This figure was taken from a progress report to E.P.A. by D. de Sylva (1970). There appears to be pronounced inertial jetting of ocean water into the Bay from the Safety Valve region. The interior of the Bay exhibits isohaline patterns aligned with the major axis of the Bay, similar to the pattern found in Card Sound.

SUMMARY

A modular approach to the analysis of mixing and flow characteristics in shallow tidal estuaries is presented using as an example the South Florida Biscayne Bay estuary. The method depends on isolating relatively simple characteristic flow regimes in different parts of an estuary. These can be considered as building blocks which, when recombined in different configurations, are capable of yielding a qualitative model for any specific estuary. In Card Sound and South Biscayne Bay, tidal-induced mixing is separated into two flow regions: the interior of the basins and the regions in the vicinity of the tidal inlets. Asymmetric, reversing tidal flow in the interior produces a



Distribution of Surface Salinity, $^{o}/oo$, 2/2-9/70
FIG. 13

preferred longitudinal mixing which enhances downstream diffusion, resulting in isohaline patterns aligned with the direction of flow. This effect inhibits direct exchange with the ocean, producing very long flushing times in the interior. Direct exchange between the basins and the ocean takes place in a region near the inlets defined by a semicircle with a radius approximately 500 times the depth of the inlets. Wind effects were found to have a great influence on exchange processes in this estuary. Wind effects mix the estuary horizontally and vertically and can set up a mean circulation that advects interior water into the direct exchange region of the tidal inlets, thereby substantially decreasing the basins flushing time.

The generalization of this approach to other specific estuaries is straightforward. One would begin by considering the gross geometry of basins and their connections, attempting to identify à priori the dominant processes in each. Such identification serves to facilitate planning of an efficient minimal observation program.

REFERENCES

Bowden, K. F. 1965: Horizontal mixing in the sea due to a shearing current. J. Fluid. Mech., 21(2): 83-95

de Sylva, D. P. 1970: Ecology and distribution of postlarval fishes of southern Biscayne Bay, Florida. Univ. of Miami, RSMAS, Prog. Rep. to E.P.A., 198 pp.

Schneider, J. J. 1969: Tidal relations in the south Biscayne Bay area, Florida. Open File Rep. U. S. Geol. Surv. Mimeo Rep., 16 pp.

APPENDIX

For a detailed account of the problem of effective horizontal exchange in a shallow shear flow, the reader is referred to Bowden's (loc. cit.) treatise; here only the essential argument involved will be described. Consider a fluid layer of depth H, within which a turbulent shear flow of characteristic velocity U maintains a mixing process, defined by a vertical mixing time scale t_v. Let the distribution of a conservative property be defined by a scalar measure of concentration S. Suppose the vertically averaged concentration (subscript S_o) is a linear function, $S_o = sx$. Now, the advection of the property is given by:

$$\int_o^H USdz = H\{U_oS_o + cov(U,S)\} \qquad 1)$$

U_o is the vertically averaged horizontal mean velocity. The covariance of U and S depends on the profile shapes, but the way the problem variables enter can be deduced by elementary considerations. Firstly, the characteristic fluctuations in S must clearly be of order SU_ot_v. If furthermore, the vertical exchange time scale to be determined is taken as the depth divided by the frictional velocity, the following estimate is obtained:

$$cov(U,S) = -\gamma C_D^{-\frac{1}{2}} |U_o| HS \qquad 2)$$

Here C_D is a coefficient dependent on such aspects of the physical situation as bottom roughness, Richardson number, etc. The covariance term 1) is thus seen to have the form of a diffusive flux, driven by the gradient S, with a diffusion coefficient: (See next page)

$$K_x = \gamma C_D^{-\frac{1}{2}} |U_o| H = D|U_o|H \qquad 3)$$

In Bowden's (loc. cit.) study the combined coefficient D in the above expression was found to range between 0.26 and 1.11, dependent on the assumptions made about the profiles, implying values of γ between 0.01 and 0.04.

If in equation 1) U_o is purely an oscillatory function of time, with zero mean value, only the diffusion term remains. If the diffusing property has a significant effect on the density, an asymmetric diffusing effect will ensue. From the form of the expression 2) it can be seen that the maximum displacement, an easily observed quantity, may be used to provide a first estimate of the mean diffusion coefficient. Denoting the displacement by L and tidal period by T gives:

$$T^{-1} \int_o^T |U_o| dt = 2LT^{-1} \qquad 4)$$

This estimate will not be degraded significantly by the fact that the assumption of profile similarity is likely to break down for low values of the velocity. Using D = 0.6 as a reasonable value based on Bowden's results, the following expression for the apparent time averaged diffusion coefficient follows:

$$K_{av} = 1.2LHT^{-1} \qquad 5)$$

Now LH is the water volume displaced between ebb and flood, or tidal prism per unit coastline length. Thus, the simple rule of thumb for tidally induced mixing results: The equivalent diffusivity is approximately one tidal prism per tidal cycle.

Another type of induced mixing due to inertial effects on the flow will arise in the vicinity of inlets. The incoming tide may give rise to a jet-like motion, while on the falling tide the flow towards the inlet is likely to be similar to an ideal potential flow towards a sink. The net results, when averaged over a complete tidal cycle, is a mean circulation corresponding to a double vortex sheet along the edge of the widening jet. A first order estimate of the penetration distance of such a jet can be made in the following manner.

Let M_0 be the momentum flux through the mouth of the inlet. Let further the width of the jet be L, its characteristic velocity U, and the bottom friction $C_d\, U^2$. Using further the standard type of linear entrainment law, which has been found experimentally to work in most cases of turbulent jets, gives the following equations governing mass and momentum flux in the jet:

$$\frac{\partial}{\partial X} LU = \varepsilon U \qquad 6)$$

$$\frac{\partial}{\partial X} LU^2 = - C_D H^{-1} LU^2 \qquad 7)$$

H is again the depth of the fluid, X is a horizontal coordinate in the shear direction and ε is the entrainment constant. These equations can be immediately integrated, to give: (See next page)

$$L = (L_o t \frac{\varepsilon H}{C_D} \exp(C_D X H^{-1}) - \varepsilon H C_D^{-1} \qquad 8)$$

where L_o is the initial width of the current. Note that the result is independent of the momentum flux. It is, however, naturally dependent on the momentum flux exceeding some critical value required to provide a significant inertial effect. It is concluded that significant inertial effects will be limited to a region of approximate radius of about 500 H inward from an inlet. The strength of the induced circulation is defined by the asymptotic transport rate estimate:

$$LU \to L_o U_o (1 + \varepsilon H C_D^{-1} L_o^{-1})^{\frac{1}{2}} \qquad 9)$$

By setting C_D^{-1} at about 100, and the inlet width L_o at about 20 H, the asymptotic transport will be nearly two and a half times the discharge. This appears approximately as a doublet field with a source sink separation of the same order as the penetration distance of the jet. Its effects will be limited to a region of the similar linear dimensions. Thus, a semicircle is roughly defined with a radius of 500 H as the region of inertial mixing at each inlet narrow enough to produce noticeable jet effects.