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GENERATION OF GATE SHIP SPEED DATA BY VARIATIONAL TECHNIQUE

Jerry Sullivan

Center for Experiment Design
and Data Analysis
Washington, D.C.
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GENERATION OF GATE SHIP SPEED DATA BY VARIATIONAL TECHNIQUE[1]

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ABSTRACT. During GATE [GARP Atlantic Tropical Experiment (GATE); Global Atmospheric Research Program (GARP)], June to September 1974, ship velocities were required to process and correct surface and upper air wind velocities acquired by the research ships. In a ship maneuver, unfortunately, only ship-heading data were reliable and continuously available. The sensors measuring speed relative to water functioned erratically throughout the experiment. A technique to generate reasonable speeds during the maneuver was developed. The generated speeds are consistent with ship positions, times, and velocities of the beginning and end of the maneuver together with the heading data. The speeds are obtained by requiring that the sum of squares of the changes in speeds be a minimum. Finding the minimum is brought about by using the method of the discrete calculus of variations. This method is sketched, and a simple case is worked out as an illustration. The general method is then applied to a series of test cases. Finally, modifications of the technique needed to handle inconsistent real data are stated.

I. INTRODUCTION

During GATE conducted from June to September 1974, ship velocities were required to process and correct surface and upper air wind velocities acquired by the research ships. The primary source of ship-velocity information was derived from navigation radar fixes on a radar marker buoy anchored at each ship's station. Range and bearing from the ship to the buoy were obtained roughly every 15 min. This information then was used to calculate the ship's velocity for periods when the ship was steaming at constant velocity or drifting. These data, however, were too coarse to resolve ship velocities when the ship was maneuvering (i.e., the ship was accelerating or decelerating, and ship speeds exceeded 1 kt).

Ship speeds and directions of motion were required to obtain ship velocities in a maneuver. As a first approximation to the direction of motion, the ship heading was used. Headings from strip charts were sampled at 10-s intervals during maneuvers. Ship speeds relative to water were measured on each ship, but the speed sensors performed erratically throughout the experiment. A technique was needed and subsequently was developed to generate reasonable speeds during a maneuver. These speeds are consistent not only with the known headings but also with the known ship positions, times, and velocities of the beginning and end of the maneuver. The explanation and illustration of this technique is the subject of this memorandum.

II. A REASONABLE CRITERION

We assumed that the set of N+1 unknown speeds $V_0, V_1, V_2, \ldots, V_N$ is compatible with the set of known headings $\theta_0, \theta_1, \theta_2, \ldots, \theta_N$ in the sense that the ship should start at the known beginning position and stop at the known end position. This means that

$$\sum_{n=0}^{N-1} V_n \sin(\theta_n)\Delta t_n = X \qquad (1)$$

and

$$\sum_{n=0}^{N-1} V_n \cos(\theta_n)\Delta t_n = Y \qquad (2)$$

where X and Y are the east-to-west and north-to-south distance intervals from the beginning to the end location and Δt_n is the

[1]GARP Atlantic Tropical Experiment (GATE); Global Atmospheric Research Program (GARP)

time interval between heading readings. This condition by itself does not determine a unique path. Some additional restriction(s) must be placed on the set of speeds.

The first restriction was that the initial unknown speed equal the known speed $V0$ at the maneuver beginning ($V_0 \equiv V0$) and the same hold for the last unknown speed; $V_N \equiv VN$, the known speed at the maneuver end. The second restriction chosen was to minimize the sum of squares of speed changes, that is,

$$\sum_{n=0}^{N-1} \left[\frac{V_{n+1} - V_n}{\Delta t_n} \right]^2 = \text{minimum} . \qquad (3)$$

This is physically plausible since ship speeds usually do not change too rapidly from one heading reading to the next. The time intervals, Δt_n, in the denominator function correctly as weights. Suppose one time interval, Δt_k, is much smaller than the others. Then $V_{k+1} - V_k \simeq 0$ is necessary to keep the sum in eq (3) small. This implies that, for readings taken very close together in time, nearly identical speeds would be obtained, which makes sense. Finally, as will be shown in the next topic, the minimizing procedure is related to the curvature of the speed profile as a function of time. This curvature or smoothness is the quantity of interest in hand-smoothing analysis.

III. MINIMIZING THE SUM

The formal name for the technique used to minimize the sum in eq (3) is the discrete calculus of variations, often used by Sasaki (1970). A very clear description of the technique is given by Feynman et al.(1966). It will be sketched here since the discrete version of the technique is not too familiar.

The way to obtain a practical algorithm for minimizing the sum in eq (3) is first to assume that the correct set of speeds V_1, $V_2, \ldots,$ V_{N-1} which minimizes the sum is known. Then choosing any other set of speeds such as $V_1 + \delta V_1$ and $V_2 + \delta V_2$ will increase the sum no matter what δV_1, $\delta V_2, \ldots,$ δV_{N-1} are. (Note that $\delta V = \delta V_N = 0$ since the initial and final speeds are fixed.) This means that

$$\sum_{n=0}^{N-1} \left[\frac{(V_{n+1} + \delta V_{n+1}) - (V_n + \delta V_n)}{\Delta t} \right]^2 - \sum_{n=0}^{N-1} \left[\frac{V_{n+1} - V_n}{\Delta t} \right]^2 \geq 0$$

for any choice of δV_n, δV_{n+1}. In the specific problem at hand, all time intervals were equal to $\Delta t = 10$ s. This eventually simplifies the mathematics but does not change the idea behind the minimization algorithm. Squaring the terms in the left-hand sum and performing the subtraction gives

$$\sum_{n=0}^{N-1} \left[\frac{V_{n+1} - V_n}{\Delta t^2} \right] (\delta V_{n+1} - \delta V_n) + \frac{1}{2} \sum_{n=0}^{N-1} \left[\frac{\delta V_{n+1} - \delta V_n}{\Delta t} \right]^2 \geq 0.$$

Since the quantities $(\delta V_{n+1} - \delta V_n)^2$ are always ≥ 0, they will be ignored in attempting to satisfy the inequality. (This leads to the same answer as a more rigorous analysis.) When the remaining sum is expanded and similar terms are collected, the final inequality that must be satisfied for any set of δV_n's is

$$- \sum_{n=1}^{N-1} \left[\frac{V_{n+1} - 2V_n + V_{n-1}}{\Delta t^2} \right] \delta V_n \geq 0. \qquad (4)$$

If the δV_n's were completely free choices, the only way to guarantee that the inequality *always* will hold is to set each term in brackets in eq (4) equal to zero. The set of δV_n's is not completely arbitrary since eq (1) and (2) still must be satisfied. To take these into account, we use the method of Lagrangian multipliers. Instead of minimizing eq (3), we minimize the sum

$$\sum_{n=0}^{N-1} \left[\frac{V_{n+1} - V_n}{\Delta t} \right]^2 + \lambda_x \left(\sum_{n=0}^{N-1} V_n \sin(\theta_n) \Delta t - X \right) + \lambda_y \left(\sum_{n=0}^{N-1} V_n \cos(\theta_n) \Delta t - Y \right)$$

where λ_x and λ_y are the Lagrangian multipliers. The minimizing algorithm now takes the form

$$\frac{V_{n+1} - 2V_n + V_{n-1}}{\Delta t^2} = \lambda_x \sin(\theta_n) \Delta t + \lambda_y \cos(\theta_n) \Delta t. \qquad (5)$$

The Lagrangian multipliers are determined by requiring that the solution to eq (5) also satisfies the distance conditions, eq (1) and (2).

The quantity on the left of eq (5) is a discrete analog to d^2V/dt^2. The second derivative, in turn, is proportional to the curvature or smoothness of the curve. Consider how a hand analysis of the missing speeds problem would proceed. A chain of N vectors would be drawn from the beginning to the end position of the maneuver. The nth vector would have the direction θ_n and length $S_n \Delta t$ where S_n is the estimated speed. Upon inspection, the estimated lengths would be adjusted until, eventually, the change in adjacent lengths (speeds) would be smooth rather than abrupt. Thus the minimization procedure, as embodied in eq (5), provides one way of automating the hand analysis.

IV. EXAMPLE OF METHOD

To illustrate the methods of the difference calculus and the role of Lagrangian multipliers, consider the simple case in which all headings are *due north*. For this case, $\cos(\theta_n) = 1$ and $\sin(\theta_n) = 0$ so that eq (5) reduces to

$$V_{n+1} - 2V_n + V_{n-1} = \lambda_y \Delta t^3.$$

Working by analogy with differential equations, we need to find a function with its second difference being a constant. The function, which can be verified by substitution, is

$$V_n = a + bn + (\lambda_y \Delta t^3/2)n^2.$$

The two constants a, b are determined by the boundary conditions $V_0 \equiv V0$ and $V_N \equiv VN$. Using these conditions gives

$$V_n = V0 + n(VN - V0)/N - (\lambda_y \Delta t^3/2)n(N-n).$$

The first part of the solution gives a linearly interpolated speed profile, which will still remain in a more general solution.

The second, parabolic part of the solution contains the still undetermined Lagrangian multiplier. Now, it is determined by the distance condition

$$\sum_{n=0}^{N-1} V_n \Delta t \equiv Y.$$

To clarify matters even more, let $VN=V0$ and make the approximation that, for large N,

$$\sum_{n=0}^{N-1} n(N-n) \rightarrow N^3/6.$$

Then using the distance condition and setting $T \equiv N\Delta t$, the total maneuver time, the result is

$$V_n = V0 - 6(n/N)*[1-(n/N)]*(V0 - Y/T). \quad (6)$$

This solution is physically reasonable. The quantity $6(n/N) * [1-(n/N)]$ is a parabola in n, having a maximum of 1.5 at $n=N/2$ and going to 0 at $n=0$ and $n=N$. If $(V0 - Y/T) > 0$, then V_n is less than $V0$ during the maneuver. In this case, $V0*T > Y$; and there must be an overall deceleration, or the ship would overshoot the known distance Y. In the opposite case where $V0*T < Y$, an overall acceleration is needed and is provided by the solution. The Lagrangian multiplier is proportional to the difference between the known travel distance and the distance the ship would cover traveling at an "average" speed.

V. GENERAL CASE

For the general two-dimensional case, eq (5) must be solved. It can be solved by using a method of the difference calculus analogous to the variation of parameters method in the differential calculus. It can be solved more simply by treating it as a recursion relation. This method is shown in appendix I. Whichever approach is used, the result is

$$\begin{aligned} V_n = & [V0 + n(VN - V0)/N] \\ & + \lambda_x \Delta t^3 [S(n) - nS(N)/N] \\ & + \lambda_y \Delta t^3 [C(n) - nC(N)/N] \end{aligned} \quad (7)$$

where

$$S(n) \equiv \sum_{k=1}^{n} (n - k) \sin(\theta_k)$$

and

$$C(n) \equiv \sum_{k=1}^{n} (n - k) \cos(\theta_k).$$

This solution also must satisfy the distance conditions of eq (1) and (2). Substituting eq (7) in these results in two linear equations in the two unknown parameters λ_x and λ_y. The parameters are obtained by solving the pair of linear simultaneous equations.

To check the algorithm, we used a series of test cases of increasing complexity. For the test cases, sets of known headings and speeds were chosen. The distances X and Y were calculated by substituting the known headings and speeds into eq (1) and (2). Using X and Y, the known initial and final speeds $V0$ and VN, the known headings, and known Δt, from eq (7) we computed the speed profiles generated by the minimization algorithm. These were compared with the original chosen speeds.

In the first test, we used a set of constant input speeds, $V0 = V_1 = \ldots, V_N$ and different sets of headings. For these cases, the generated speeds also should equal the same constant since this will make the sum in eq (3) equal to zero, an absolute mini-

Table 1.--Comparison of speeds generated by the minimization algorithm with test speeds for a roughly semicircular maneuver

Heading (deg.*)	Test speed (m/s)	Generated speed (m/s)
60.00	1.00	1.00
63.00	1.50	1.50
65.73	2.00	2.09
68.43	2.70	2.75
70.99	3.50	3.44
73.62	4.20	4.16
76.34	5.00	4.87
79.01	5.50	5.55
81.98	6.00	6.18
84.84	6.80	6.74
87.76	7.20	7.19
90.49	7.50	7.51
93.14	7.70	7.68
95.96	7.80	7.68
98.50	7.50	7.48
101.41	7.00	7.07
104.00	6.50	6.41
106.83	5.50	5.50
109.50	4.50	4,30
112.00	2.50	2.81
114.59	1.00	1.00

*Degree

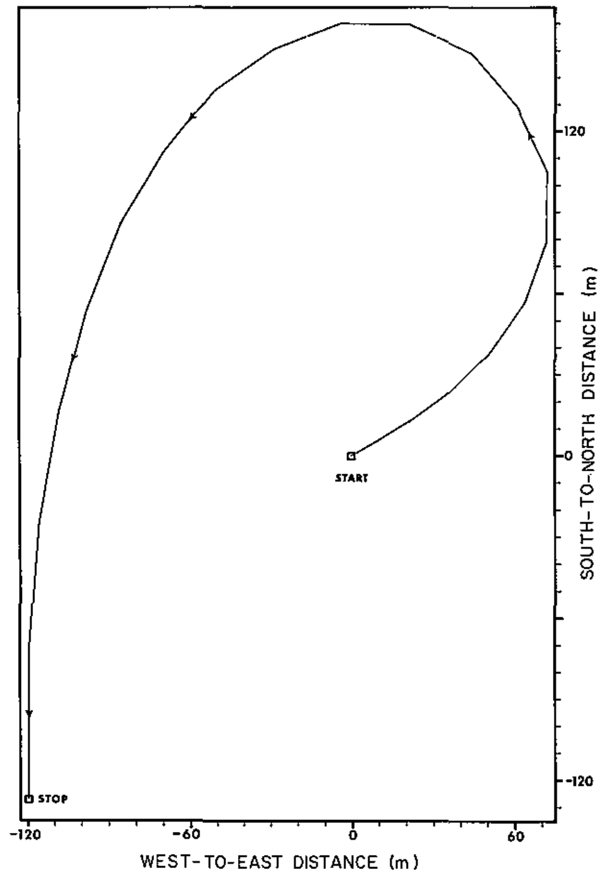

Figure 1.--Path of a trial ship-maneuver that was used to check the method. The arrows indicate the direction of travel.

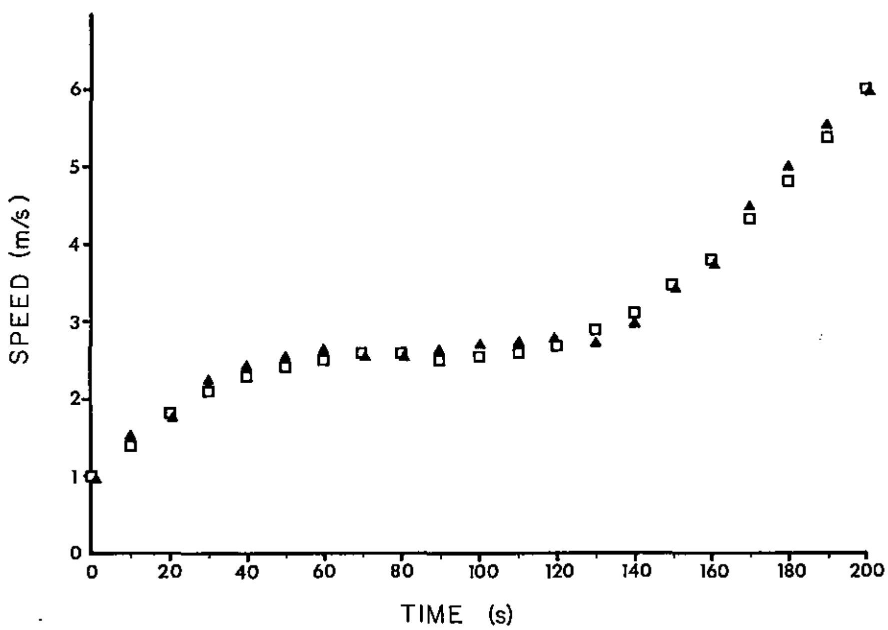

Figure 2.--Plot of the ship's speed as it travels the path shown in figure 1. Triangles denote actual speed data; squares indicate speeds generated by the variational technique.

mum. These cases actually are a verification that the algebraic steps in the algorithm have been handled correctly. All the generated speeds were constant for these cases. In the second test, we used a roughly semicircular path with varying speeds. The results are shown in table 1; they are encouraging. In the third test, we used the maneuver shown in figure 1. The results are plotted in figure 2; and, once again, the agreement is good.

VI. MODIFICATIONS

The results of the test cases showed the feasability of the method. It then was tried on the actual GATE data, and these trials showed that two modifications were necessary. The first, oddly enough, occurred in the case in which the maneuver was almost a constant one. In this case, the general two-dimensional solution [eq (7)] was sensitive to measurement errors in the path distances X and Y. This was corrected easily by stipulating that, if the total sum of the absolute values of heading changes was $\lesssim 3°$, the maneuver then would be treated as a true constant heading case.

The second modification was necessitated by the fact that ship-heading data are not always representative of the actual ship-motion directions. This is due to such effects as ocean currents and side slipping by the ship. In those instances when the ship headings are not accurate enough to bring the ship to the end point of the maneuver, the following procedure is used. The computer program rotates each of the headings by a constant angle until a reasonable path is reached. The criterion for reasonableness is, once again, the minimization of the sum of squares, eq (3). The program tries a set of different rotation angles and calculates eq (3) for each angle (each different path). It settles on the path for which the minimum is an absolute minimum.

These modifications were needed for only a small minority of the maneuvers that were evaluated. If the headings are reasonably consistent with the beginning and end positions, the minimization method does give a reasonable set of speeds.

ACKNOWLEDGMENT

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APPENDIX I

Equation (5) is a recursion relation in which V_{n+1} is given in terms of V_n, V_{n-1}, and the known heading θ_n. The initial speed is known, $V_0 \equiv V0$. If V_1 were known, V_2 could be obtained from V_1 and $V0$. Then, V_3 could be obtained, for example, from V_2 and V_1. Unfortunately, V_1 is not given. A second boundary condition is available, namely $V_N \equiv VN$. V_N can be obtained in terms of V_1, $V0$, and the set of known headings. Setting V_N equal to VN, we then obtain V_1 in terms of known quantities. The recursion relation now generates the rest of the speeds. Using eq (5), we obtain the following set of equations:

$$V_2 = 2V_1 - V0 + \lambda_x \Delta t^3 \sin(\theta_1) + \lambda_y \Delta t^3 \cos(\theta_1),$$
$$= V0 + 2(V_1 - V0) + \lambda_x \Delta t^3 \sin(\theta_1) + \lambda_y \Delta t^3 \cos(\theta_1),$$
$$V_3 = V0 + 3(V_1 - V0) + \lambda_x \Delta t^3 [2\sin(\theta_1) + \sin(\theta_2)] + \lambda_y \Delta t^3 [2\cos(\theta_1) + \cos(\theta_2)],$$
$$\vdots$$
$$VN = V0 + N(V_1 - V0) + \lambda_x \Delta t^3 [(N-1)\sin(\theta_1) + (N-2)\sin(\theta_2) + \ldots, \sin(\theta_{N-1})]$$
$$+ \lambda_y \Delta t^3 [(N-1)\cos(\theta_1) + (N-2)\cos(\theta_2) + \ldots, \cos(\theta_{N-1})].$$

Using the definitions of $S(n)$ and $C(n)$ results in

$$V_1 = V0 + (VN - V0)/N - \lambda_x \Delta t^3 S(N)/N - \lambda_y \Delta t^3 C(N)/N.$$

Substituting V_1 into the recursion relation eq (5) gives the solution shown in eq (7).