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FATIGUE CRACK PROPAGATION IN 0°/90° E-GLASS/EPOXY COMPOSITES

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ABSTRACT

The mode of fatigue crack growth is described for a 0°/90° E-glass/epoxy laminate under cyclic tension - tension loading. Crack growth appears to occur in a stepwise fashion with the crack remaining stationary for many cycles before each step of growth, whereupon a ligament of longitudinal ply at the crack tip is broken. A simple theory is described which assumes that the ligament at the crack tip is fatigued according to the S-N curve of the unnotched material. Using an assumed stress field and cumulative damage law, the number of cycles for initial growth from a notch and the rate of crack growth thereafter is predicted, and good agreement is demonstrated with experimental data.

INTRODUCTION

The growth of cracks under the influence of cyclic loading is a problem of great engineering importance in structural applications involving relatively brittle materials. The process of fatigue failure in metals has proven difficult to explain in fundamental terms; one significant area of progress has been the description of crack growth under cyclic loading, where the rate of crack extension is typically proportional to some power of the stress intensity factor, K_I [1]. Knowledge of the stress intensity factor-crack growth rate relationship, coupled with an estimate of the critical crack length for unstable crack propagation taken from fracture toughness data, has led to accurate fatigue life predictions for sharply notched metal specimens [1].

Previous work [2, 3] has indicated that many varieties of fiber reinforced plastic laminates under monotonic loading conditions behave in a crack-sensitive manner which can be described by the fracture mechanics parameters found useful for homogeneous materials. The basic mechanism of crack propagation resistance for several varieties of glass fiber reinforced epoxy and polyester with fibers parallel and perpendicular to the load was found to be crack blunting by the growth of secondary subcritical splits perpendicular to the direction of main crack propagation. The fracture surface work was found to be approximately equivalent to the elastic energy dissipated upon failure of a ligament of longitudinal material along the secondary split which was partially isolated from the bulk laminate. The work of fracture was proportional to the length of the splits over a wide range for crossplied laminates; the split length was very sensitive to ply stacking configuration [3].

The study reported in this paper extends the understanding and description of the fracture process to include cyclic loading effects. A simple theory is presented which it appears may be used to predict the rate of crack propagation for the particular laminate type investigated under sinusoidal tension loading conditions.

MATERIALS AND TEST METHODS

Material

The material investigated was an unwoven, unidirectional-ply laminate of Scotchply Type 1002 E-glass/epoxy (3-M Company), five plies thick, of ply stacking configuration $(90^\circ/0^\circ/90^\circ/0^\circ/90^\circ)$, where 0° is the direction of loading, and 90° is the main crack direction. Each ply of the laminate had a nominal thickness of 0.01 inches and a fiber volume fraction of 0.50. This ply stacking sequence exhibits a lower fracture toughness in monotonic tests than do other possible variations due to the constraint of the 90° ply on each side of each 0° ply [3]; the monotonic fracture behavior under uniaxial loading in the 0° direction can be described by classical fracture mechanics parameters, as will be illustrated later.

Test Procedure

Test specimens of the two types shown in Figure 1 were cut from the laminates; the notches were cut with a 0.011 inch thick diamond edged wheel which gives a sufficiently sharp notch for valid fracture toughness testing for this class of materials [3,4]. The specimens were clamped in pin-loaded grips and subjected to either monotonic or pulsating uniaxial tension. The fatigue specimens were cycled between a low tensile stress ($\cong$ 1.0 ksi) and some maximum tension at five cycles per second as shown in Figure 2. Testing was restricted to this frequency due to significant heat build-up at higher frequencies, particularly at the crack tip. The low stress fatigue tests ($>$1000 cy.) were run on an Instron Model 1211 dynamic cycler; the higher stress fatigue and monotonic tests were run on either an Instron Model 1251 or an Amsler servo-hydraulic tester. Monotonic fracture and strength tests were conducted at a displacement rate of 20 inches per minute which coincides with the average displacement rate of the fatigue tests at five cycles per second. The monotomic fracture tests for comparison of fracture toughness values for various specimen geometries were conducted on a standard Instron TT tester at a displacement rate of 0.05 inches per minute.

MODE OF CRACK PROPAGATION

Fatigue crack propagation for this class of materials appears to occur in the absence of any peculiar mechanism of growth as typically is observed in metals, where the crack extends by a small increment on each cycle [1]. As indicated schematically in Figure 3 and in the micorgraph of Figure 4, the crack in the 0^{o} plies is observed to extend by the following repetitive process:

1. With the crack stationary and terminated by a subcritical split perpendicular to the main crack, the ligament of 0^{o} ply at the crack tip is fatigued by the local stress field.
2. After some number of cycles dependent on the stress intensity, the ligament at the crack tip fails, and the main crack extends by one ligament width.
3. The main crack remains stationary at the new length as steps (1) and (2) are repeated.

Thus, the main crack appears to extend by successively fatiguing to failure small ligaments of the 0^{o} ply at the crack tip, and the ligamented appearance of the fracture region is identical to that observed in monotonic fracture tests [3]. The ligament width, d, in this case is approximately 0.01 inches, encompassing a region containing approximately 700 fibers in each 0^{o} ply. The 90°plies simply crack between fibers as the main crack extends, and are not believed to contribute substantially to the fracture resistance [3]. Local delamination between plies at the crack tip is also observed, and the effect of this on the crack growth rate is not clear.

For purposes of this discussion, crack propagation will be defined as extension of the main crack by fracture of fibers in the 0^o plies, as distinct from splitting parallel to the fibers in the 0^o or 90^o plies or delamination between plies.

THEORETICAL PREDICTION OF FATIGUE CRACK GROWTH RATE

Initial Extension from Precut Notch

The experimentally observed characteristics of fatigue crack growth suggest the following simple theoretical model:

> The material adjacent to the subcritical split in the 0^o plies at the crack tip is fatigued to failure according to the fatigue life (S-N) curve of an unnotched strip of material, but at the local stress level.

Thus, by estimating the local stress field acting on the material at the crack tip, initial extension of the main crack colinear with the precut notch can be predicted through the tensile S-N curve of the unnotched material.

The local stress field at the crack tip is simplified by considering only the local stress in the load direction, normal to the axis of the main crack, termed σ_{ij} for the stress on the i^{th} ligament from the original crack tip, with the main crack located at the edge of the j^{th} ligament, where the $(j-1)^{st}$ ligament is broken. For the case of the precut notch with no prior crack extension, the stress at the notch tip, σ_{11}, at a maximum stress intensity factor of K_I, is assumed to be

$$\sigma_{11} = \sigma_f \left(\frac{K_I}{K_Q}\right) \qquad (1)$$

where σ_f is the ultimate strength of the unnotched material under uniaxial stress, and K_Q is the candidate opening mode critical stress intensity factor for the material, both at the appropriate strain rate. Equation (1) derives from the observation that the material at the crack tip in the 0^o ply must reach its local ultimate tensile strength simultaneously as K_I reaches K_Q in a monotonic test, and from the assumption that the local stress at the crack tip increases in a linear fashion with the applied stress. The validity of the latter assumption is difficult to establish since the subcritical split in the 0^o ply at the crack tip extends gradually under increasing number of cycles, beginning at some lesser length than that at fracture in the monotonic test, and extending to a greater length before main crack extension occurs. Since the 0^o subcritical split has the effect of blunting the main crack, its extension during cycling should diminish somewhat the local stress σ_{11}. Equation (1) is equivalent to the assumption of a stress concentration at the crack tip of

$$\frac{\sigma_{11}}{\sigma} = \frac{\sigma_f Y \sqrt{c}}{K_Q} \qquad (2)$$

where σ is the maximum applied stress, and $Y\sqrt{c}$ is from the classical fracture mechanics relationship [5, 6]

$$K_I = \sigma Y \sqrt{c} \tag{3}$$

for a crack of length c.

The number of cycles for initial extension of the main crack can then be determined from the assumed value of σ_{11} and the S-N curve for the material. If, as in the present case, the S-N curve can be approximated by the linear relationship

$$\log N = \frac{\sigma_f - \sigma}{S} \tag{4}$$

where N is the number of cycles to failure, σ is the maximum applied stress, and S is the absolute value of the slope of the S-N curve, then the number of cycles to initial crack extension will be given by substituting σ_{11} from Equation (1) for σ in Equation (4):

$$\log N_i = \frac{\sigma_f}{S} \left(1 - \frac{K}{K_Q}\right) \tag{5}$$

where N_i is the number of cycles for initial extension of the main crack.

Fatigue Crack Growth Rate

The rate of fatigue crack growth can be predicted by the same model if additional assumptions are made as to the distribution of stress ahead of the main crack and the cumulative damage law for failure of the ligaments under several cyclic stress levels. Figure 5 gives the assumed variation in local stress in load direction ahead of the main crack; the stress at the crack tip is assumed to be given by Equation (1) and the variation along a line colinear with the main crack is assumed to follow an inverse relationship with $\sqrt{r}$, where r is the distance from the crack tip. This distribution is essentially that of the sharp crack problem for orthotropic materials [5] and for isotropic materials [6] which reduce to the same relationship directly ahead of the crack tip, and ignoring higher order terms

$$\sigma_{yy} = \frac{K_I}{(2\pi r)^{1/2}} \qquad (6)$$

where σ_{yy} is the local stress in the load direction; near the crack tip, the stress field is truncated by Equation (1). A more accurate description of the stress field awaits solution of the problem of a crack terminated by splits in the 0^o plies which may soon be available [4]. An additional assumption is Miner's cumulative damage relationship [7] which states that

$$\Sigma \frac{n_k}{N_k} = 1 \qquad (7)$$

for failure of the material, where n_k is the number of cycles experienced at the k^{th} stress level and N_k is the number of cycles which could be withstood for cycling at the k^{th} stress only. Although Miner's equation is not in good agreement with data for composites of this type [8], no better theory is available at this time.

The number of cycles for the first several ligaments of crack extension is given by Equation (7) as

$$n_{11} = N_{11}$$

$$n_{22} = \left(1 - \frac{n_{11}}{N_{21}}\right) N_{22} \qquad (8)$$

$$n_{33} = \left(1 - \frac{n_{11}}{N_{31}} - \frac{n_{22}}{N_{32}}\right) N_{33}$$

$$\vdots$$

where the first subscript gives the number of the ligament from the original crack tip, and the second subscript gives the location of the crack. Thus, N_{32} is the number of cycles for the third ligament with the crack at the edge of the second ligament, the first ligament having been broken.

Following the previous section, the number of cycles to first crack extension will be given by Equation (3), and

$$n_{11} = \exp\left[2.3 \frac{\sigma_f}{S} \left(1 - K_I/K_Q\right)\right] \qquad (9)$$

so the crack growth rate for the first ligament will be

$$\frac{dc}{dN} = d \,/\, \exp\left[2.3 \frac{\sigma_f}{S} \left(1 - K_I/K_Q\right)\right] \qquad (10)$$

The stress on the second ligament for the unextented crack is given by

$$\sigma_{21} = \frac{K_I}{\left[2\pi (r_o + d)\right]^{1/2}} \qquad (11)$$

where $r_\circ$ is defined in Figure 5. Substitution of σ_{21} into Equation (4) gives

$$N_{21} = \exp\left[2.3\,\frac{\sigma_f}{S}\left(1 - \frac{K_I}{\sigma_f\sqrt{2\pi(r_\circ + d)}}\right)\right] \qquad (12)$$

Since $N_{22} = N_{11}$, n_{22} reduces to

$$n_{22} = \exp\left[2.3\,\frac{\sigma_f}{S}\left(1 - \frac{K_I}{K_Q}\right)\right] - \exp\left[2.3\,\frac{\sigma_f}{S}\left(1 - \frac{2K_I}{K_Q} + \frac{K_I}{\sigma_f\sqrt{2\pi(r_\circ + d)}}\right)\right] \qquad (13)$$

and $dc/dn = d/n_{22}$ for the crack growth rate of the second ligament.

The same procedure can be used to obtain the crack growth rate for succeeding ligaments, with the calculations most conveniently carried out by computer. Although the ligament width, d, is experimentally determined to be approximately 0.01 inches for the laminate considered here, this dimension may vary for other laminate constructions. Figure 6 gives the crack length as a function of the number of cycles under constant K_I conditions for three ligament widths. These crack growth curves indicate the stepwise growth of the crack at an approximately constant rate after the first, slower step. The difference in number of cycles to fail the first ligament and some later ligament is a measure of the importance of the cumulative damage law; the cumulative damage has a significant effect on the 0.001 inch wide ligaments, but on wider ligaments the local stress so diminishes from the crack tip to the far side of the first ligament that damage to the second and succeeding ligaments away from the crack tip is not significant. The differences between the ligament widths, although strongly evident in Figure 6, fade in importance on typical crack growth rate plots which cover many orders of magnitude of rate. For finite width specimens under constant load amplitude, the value of K_I will vary as the crack extends; Figure 7 indicates the crack length as a function of the number of cycles for the three inch wide specimen with 0.60 inch notches used in this study, with a ligament width of 0.01 inches as experimentally observed.

EXPERIMENTAL RESULTS AND DISCUSSION

Experimental Results

The purpose of the experimental program, in addition to describing the proposed mode of crack propagation, was to measure the rate of fatigue crack growth and to establish the necessary material properties for prediction of the rate of growth by the proposed theory. As a preliminary step, it was also necessary to establish the constancy of K_Q for a range of specimen sizes and crack lengths.

The validity of classical fracture mechanics for this class of composite materials has been discussed elsewhere [3,4]. Additional evidence for the particular ply configuration under consideration is given in Figure 8, where the fracture toughness, K_Q, calculated from Equation (3) using the K-calibration for isotropic materials [9] is plotted against crack length to specimen width ratio for several specimen sizes. The data confirm an approximately constant value of K_Q for all specimen sizes and crack lengths tested, indicating that K_Q is a valid material property. The use of K_Q in predicting the rate of fatigue crack growth also requires that the tests be conducted at the rate of deflection experienced in the fatigue test. The frequency of five cycles per second used for the fatigue tests translates to a deflection rate of approximately 20 inches/minute for the monotonic fracture test; as Table 1 indicates, K_Q at this rate is significantly higher than that for the 0.05 inches/minute tests represented in Figure 8, an increase from 16.3 to 24.0 ksi$\sqrt{\text{in.}}$ for the notched specimen configuration shown in Figure 1.

Figure 9 gives the fatigue life data for the unnotched specimen shown in Figure 1; the data are approximated by a line having a slope of 6.16 ksi per decade of cycles, and monotonic strength of 60.7 ksi, as determined by a least-squares curve fit. These data, along with the data prviously described for K_Q and the observed ligament size at the crack tip are sufficient to allow prediction of initial crack growth from the notch and the rate of crack growth by the theory presented in the previous section; Table 1 summarizes these properties.

Figure 10 gives the number of cycles to produce first crack extension from the precut notch as a function of the stress intensity factor. The data show good agreement with the theoretical prediction of Equation (5) over the complete range.

The rate of fatigue crack growth was measured for the first 0.05 inches of crack extension by observing the location of the terminal subcritical split at the crack tip in the 0^{o} ply at low magnification. Data was taken only when the length of both cracks in the specimen were within 0.02 inches of each other, and the crack length was measured for the 0^{o} ply with the largest crack for the uncommon cases where the two 0^{o} plies showed unequal crack growth.

Figure 11 indicates good agreement between the experimental data and theoretical predictions based on Equations (10), (13) and an average of the predicted growth rate for the first five ligaments over several decades of crack growth rate. Even the simple relationship of Equation (10) which ignores cumulative damage effects accurately predicts both the trend of dc/dN vs. K_I and the absolute value of the data.

Discussion

The geometry of the test specimen used to measure the crack growth rate is not convenient, as the crack rapidly becomes unstable after a relatively short length in which growth rates can be measured. The specimen is well suited to thin laminates, however, and provides two propagating cracks for growth measurements. It is thought that the measurement of only small amounts of crack extension from the precut notch does not severely restrict the data because the stress field very local to the crack tip dominates the crack growth characteristics, so that the failure of each ligament is almost independent of the stress history before the crack tip reaches the ligament. Thus, the predicted growth rate for the first few ligaments of growth in Figure 6 is almost identical to that for subsequent ligaments which have a longer stress history; for ligaments 0.01 inches wide or greater, a sufficiently accurate prediction of crack growth rate can be obtained without consideration of the cumulative damage law or the stress distribution away from the immediate crack tip. Recent unpublished results using thicker specimens confirm this approach for more standard cleavage type specimens where crack growth can be observed over greater length.

The exponential relationship between K and dc/dN results in a rapid variation in crack growth rate with increasing load equivalent to a high exponent if the relationship were linear on a log-log plot as is commonly the case the metals. A linear approximation to the data in Figure 11 gives approximately

$$\frac{dc}{dN} \propto (K_I)^{11} \qquad (14)$$

which is a considerably higher exponent than is commonly observed for metals [1]. This exponent is not necessarily typical of composites in general, as an exponent of approximately 5.0 has been reported for chopped E-glass mat/polyester laminates [10].

Although the data given in Figure 8 are consistent with the assumptions of linear elastic fracture mechanics, they prove no more than an inverse relationship between applied stress at fracture and the square root of the notch depth. The data are equally consistent with a stress concentration factor approach which may be more appropriate in view of the apparent bluntness of a natural crack, particularly a fatigue crack, in these materials. As noted previously, the local stresses assumed in the theoretical treatment would also apply to a stress concentration factor approach without significant alteration, and the initial crack extension and crack growth rate predictions would remain unchanged.

The application of the theory presented in this paper to other composite materials should depend on whether the local mode of crack propagation is similar. The mode of crack propagation for monotonic fracture tests is known to be of this type for woven fabric and unwoven $0^{\circ}/90^{\circ}$ crossplied laminates of other ply stacking arrangements [3], and recent work indicates a similar mode for some angle-ply composites, although Mode II and interlaminar mode failures may also occur in angle-ply composites in monotonic [4] and fatigue loading [11], and the approach taken here could not be expected to apply to such cases. Laminates which show a tendency to form longer splits in the 0° plies (see Ref. [3]) may also shift to notch-insensitive behavior if the splits extend sufficiently in fatigue.

The work at this stage is clearly restricted to uniaxial tension-tension fatigue. The uniaxial S-N curve has been used as a basic material property to predict behavior under multiaxial stress conditions at the crack tip, and this approach needs further investigation. The applicability of the theory to tension-compresssion fatigue or other load conditions is not clear at this time.

CONCLUSIONS

Fatigue crack propagation appears to occur by the successive failure of ligaments at the crack tip. The rate of crack growth can be predicted by a model based on the assumption that the ligaments follow the S-N curve of the unnotched material. The predicted growth rate is sensitive to the assumed stress at the crack tip, but relatively insensitive to the assumed distribution of stress ahead of the crack tip or to the cumulative damage law used.

ACKNOWLEDGEMENTS

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TABLE I

Mechanical Properties Assumed in Fatigue Crack Growth Theory

Fracture Toughness, K_Q = 24.0 ksi$\sqrt{in}$

Ultimate Strength, σ_f = 60.7 ksi

Slope of S-N Curve, S = 6.16 ksi/decade

Ligament Width, d = 0.01 inches

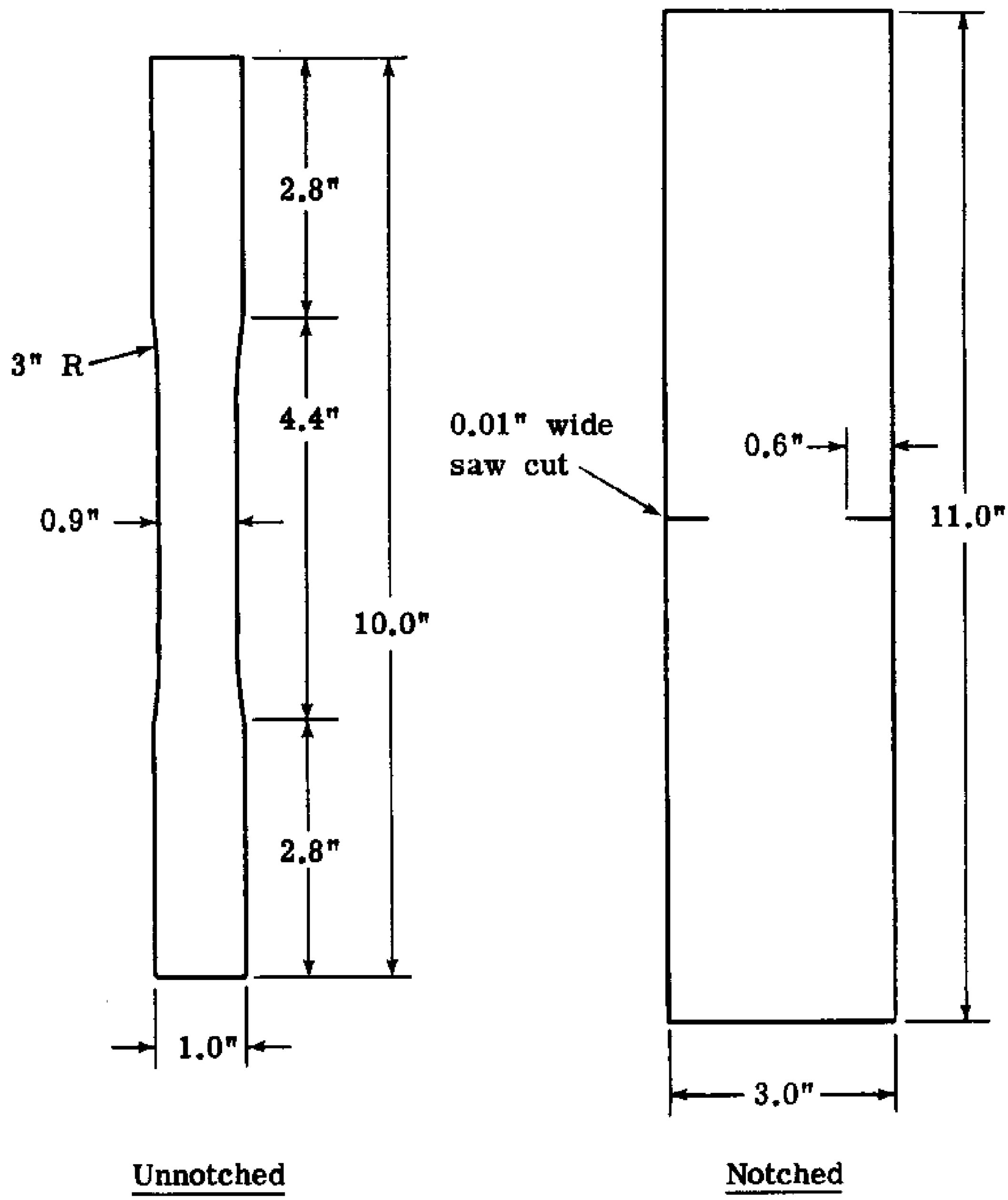

FIGURE 1.

TEST SPECIMENS.

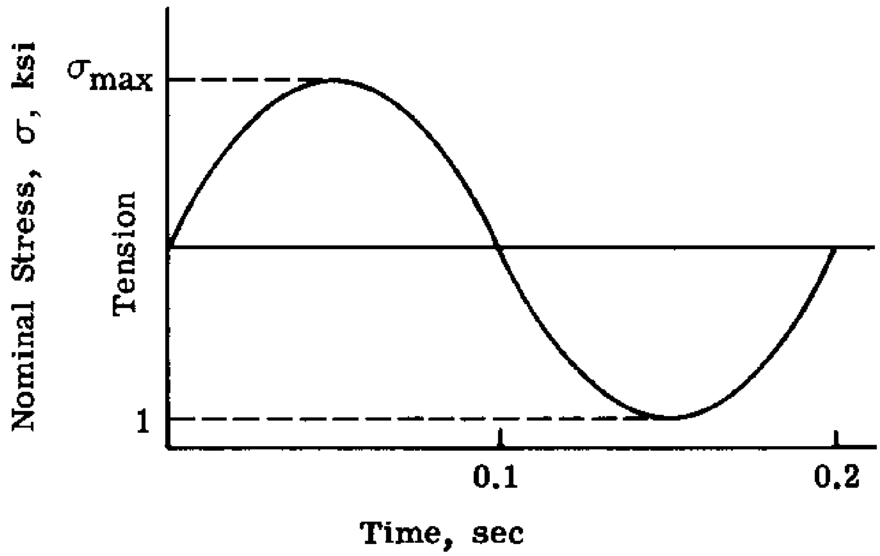

FIGURE 2.

STRESS vs. TIME FOR FATIGUE TESTS.

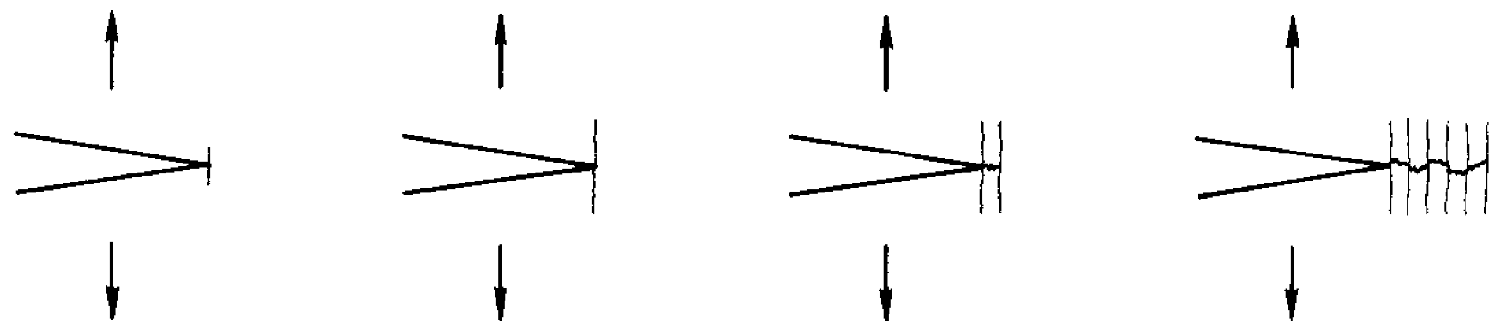

FIGURE 3.

MODE OF FATIGUE CRACK GROWTH IN 0° PLY OF (90°/0°/90°/0°/90°) LAMINATE, FIBERS PERPENDICULAR TO MAIN CRACK.

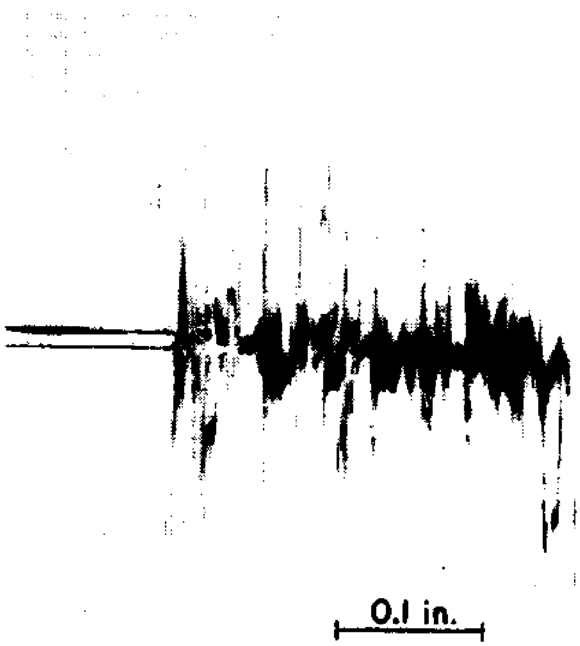

FIGURE 4.

FATIGUE CRACK GROWING FROM NOTCH (LEFT) IN 0° PLY OF (90°/0°/90°/0°/90°) GLASS-EPOXY LAMINATE. FIBERS ARE PERPENDICULAR TO NOTCH AXIS.

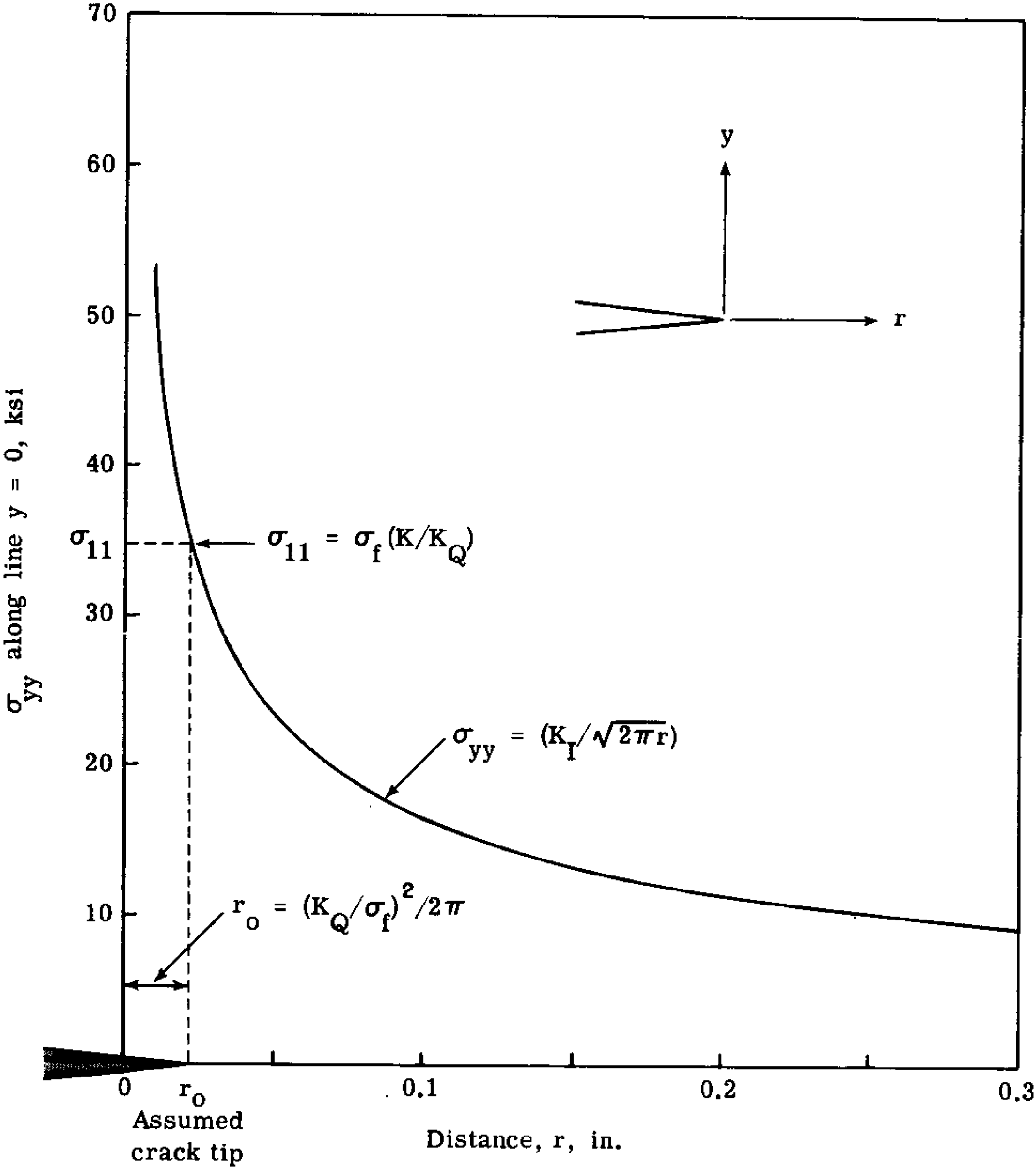

FIGURE 5.

ASSUMED STRESS DISTRIBUTION AHEAD OF CRACK FOR $K_I = 13.0$ ksi $\sqrt{in.}$, $\sigma = 8.32$ ksi.

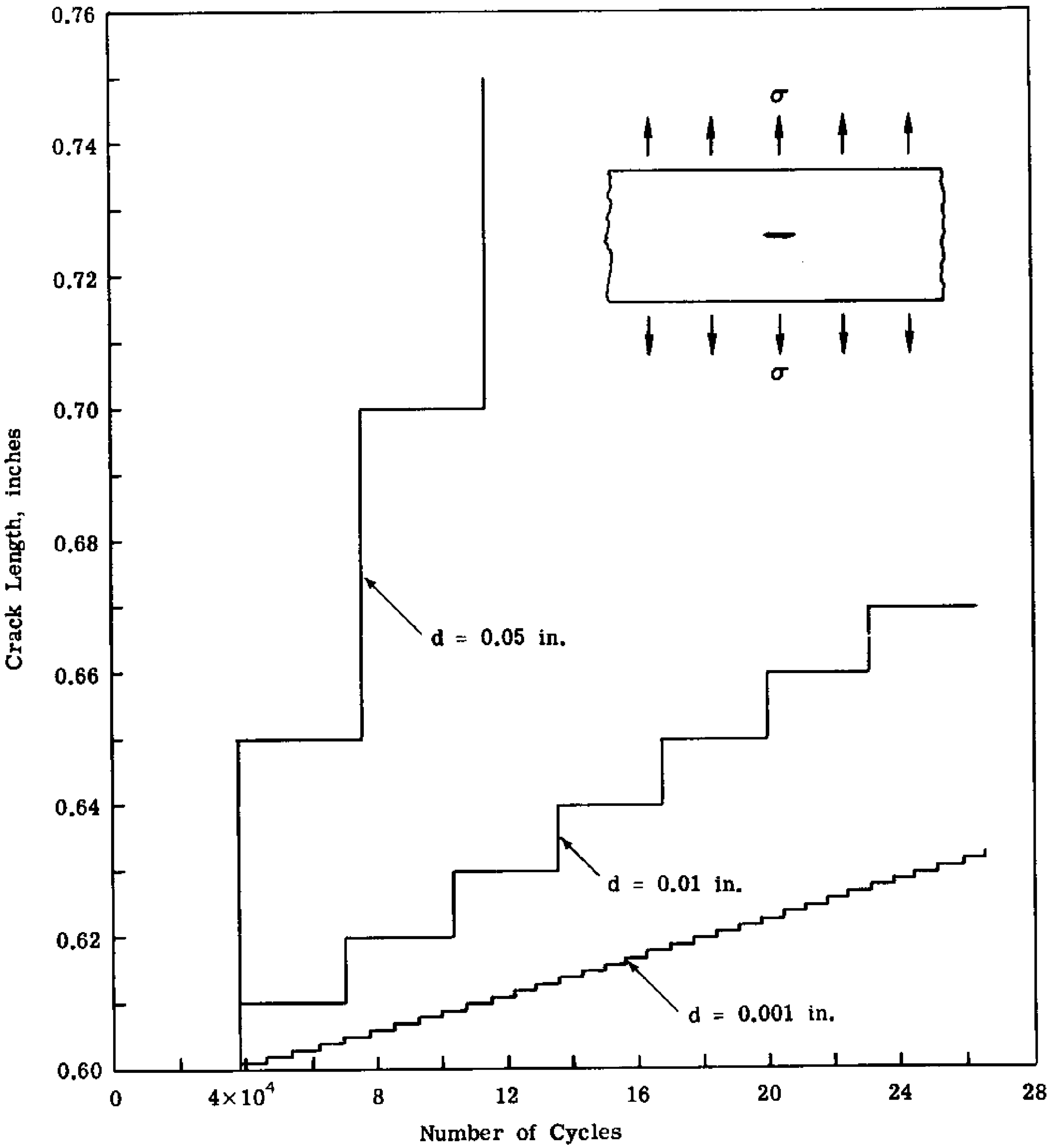

FIGURE 6.

PREDICTED CRACK GROWTH IN INFINITE PLATE FOR SEVERAL LIGAMENT SIZES (K_I = 13.0 ksi $\sqrt{\text{in.}}$, INITIAL CRACK LENGTH = 0.60 inch).

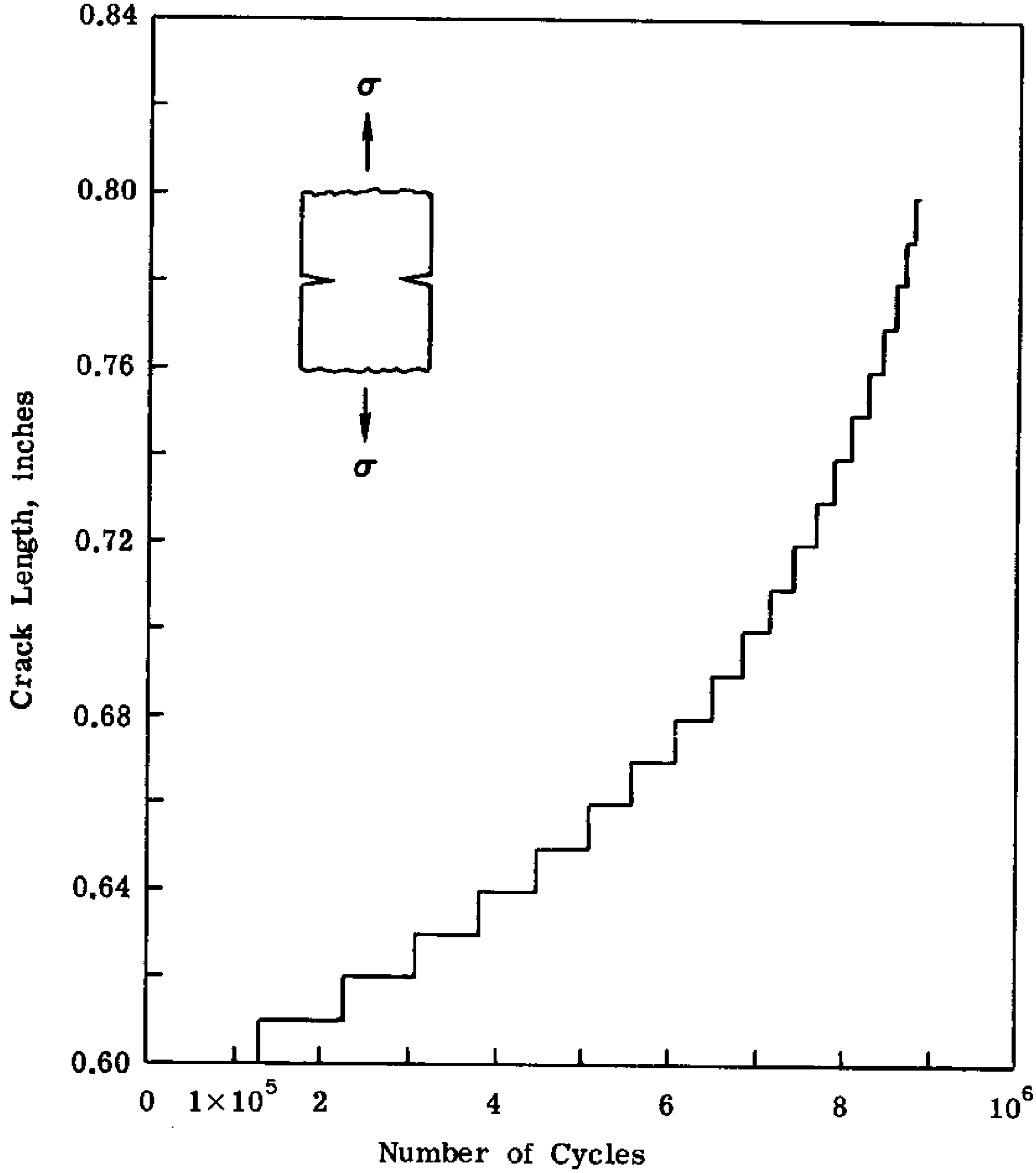

FIGURE 7.

PREDICTED CRACK GROWTH FOR 3-INCH WIDE SAMPLE WITH INITIAL CRACK LENGTH OF 0.60 INCH (σ_{max} = 7.57 ksi, d = 0.01 inch).

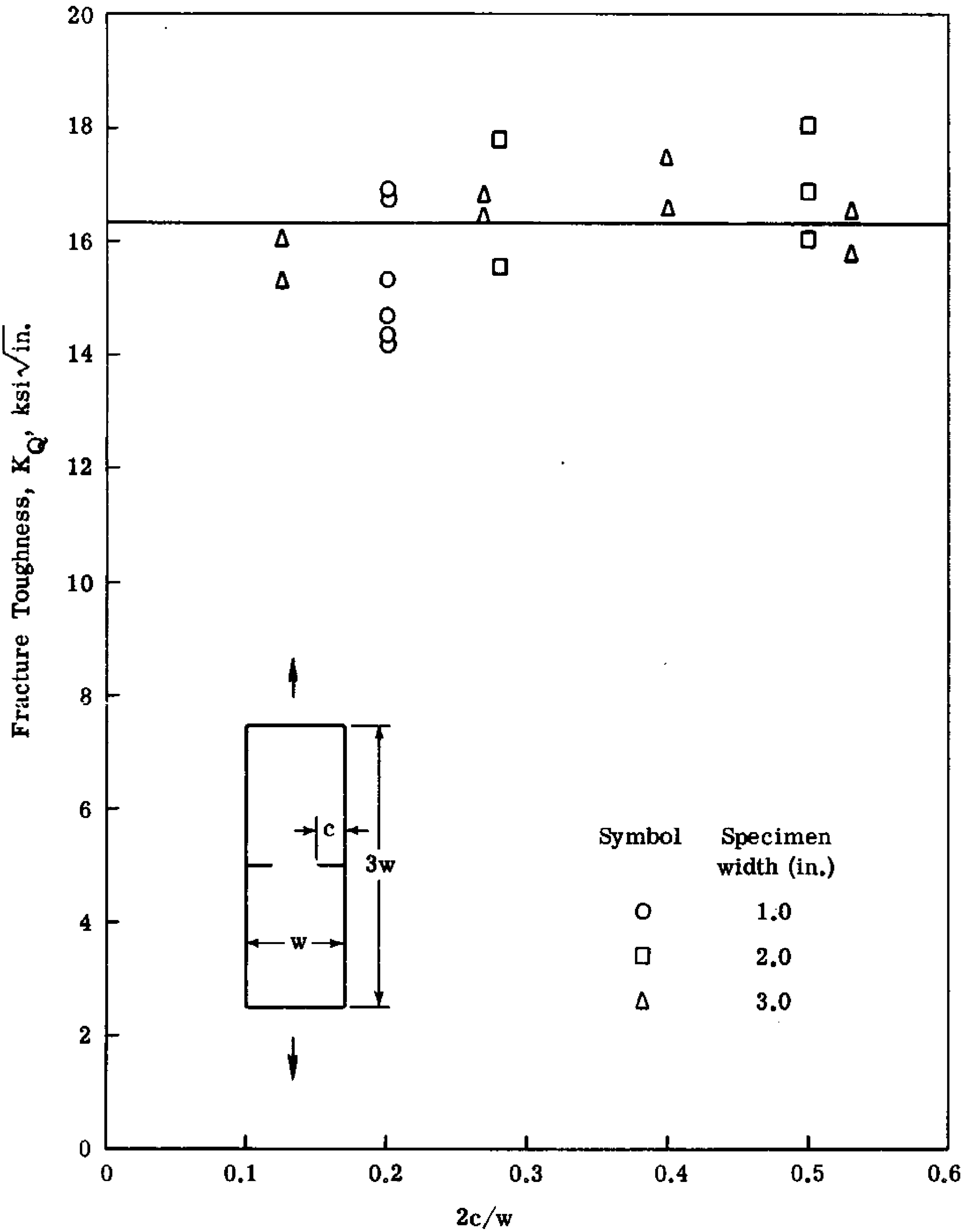

FIGURE 8.

FRACTURE TOUGHNESS FOR VARIOUS SPECIMEN SIZES AND CRACK LENGTHS, 90°/0°/90°/0°/90° SCOTCHPLY, DISPLACEMENT RATE = 0.05 in./min.

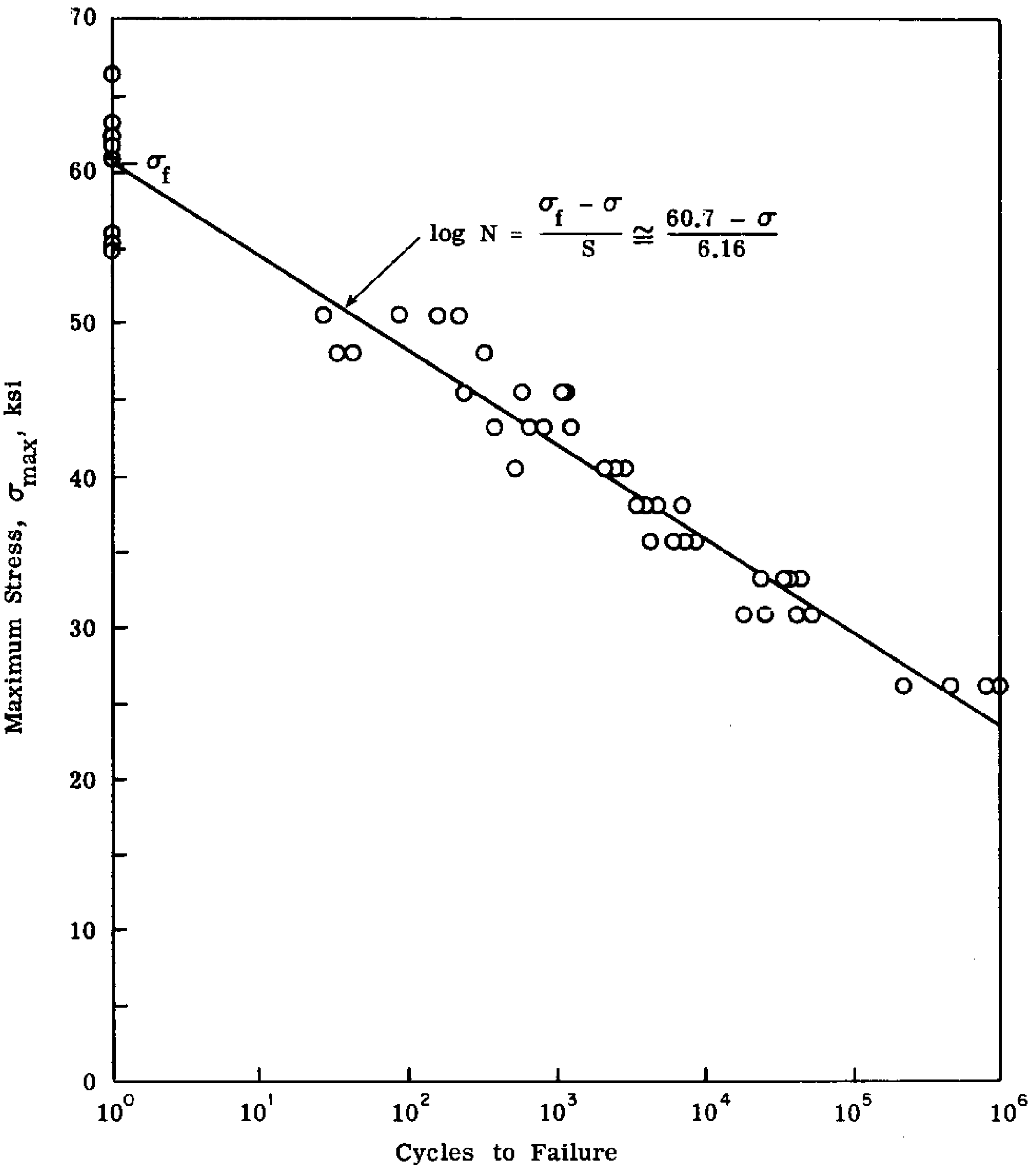

FIGURE 9.

FATIGUE LIFE (S-N) CURVE FOR UNNOTCHED E-GLASS/EPOXY, PLY CONFIGURATION (90°/0°/90°/0°/90°).

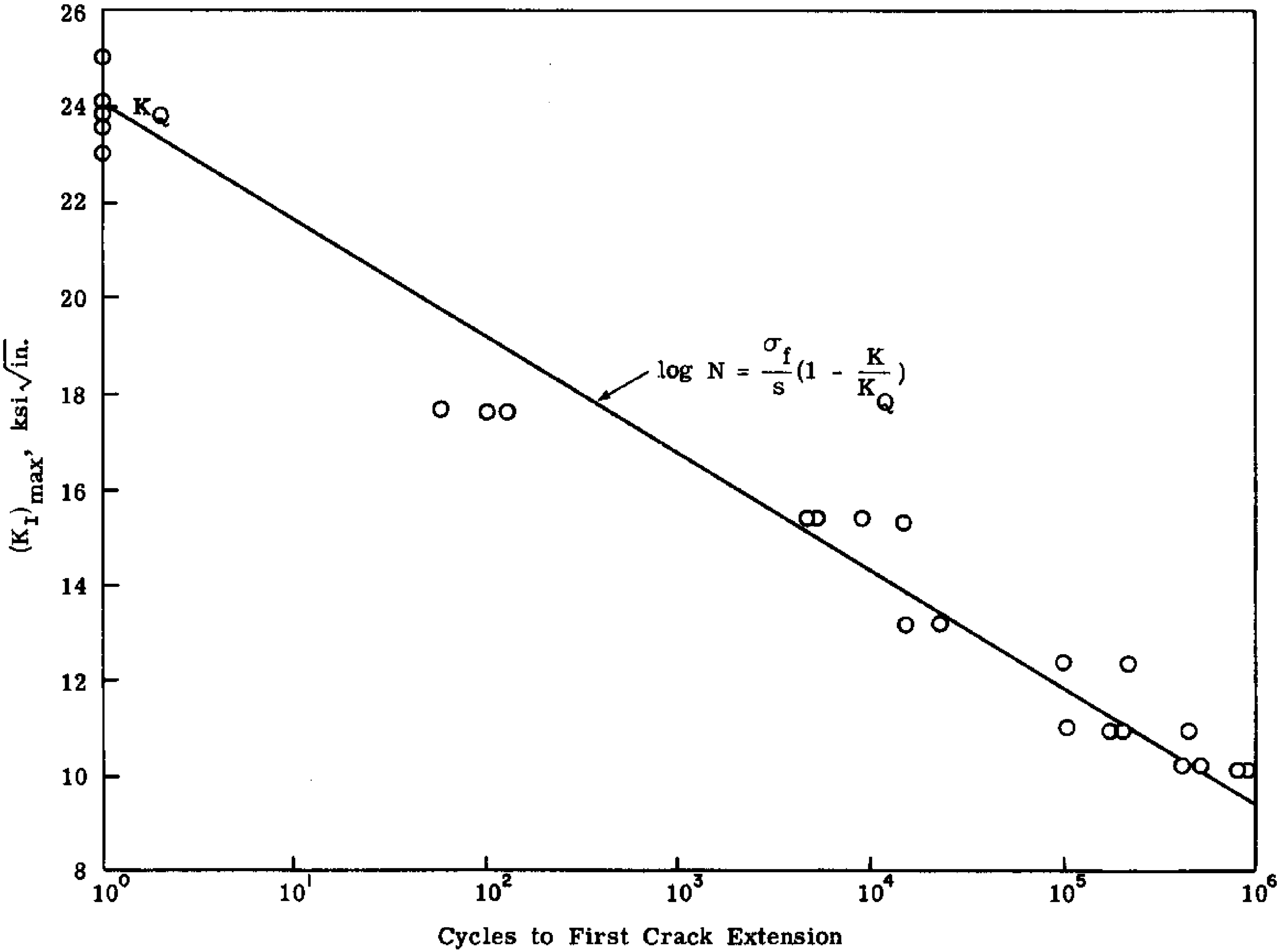

FIGURE 10.

THEORETICAL vs. EXPERIMENTAL NUMBER OF CYCLES FOR FIRST CRACK GROWTH FROM NOTCH.

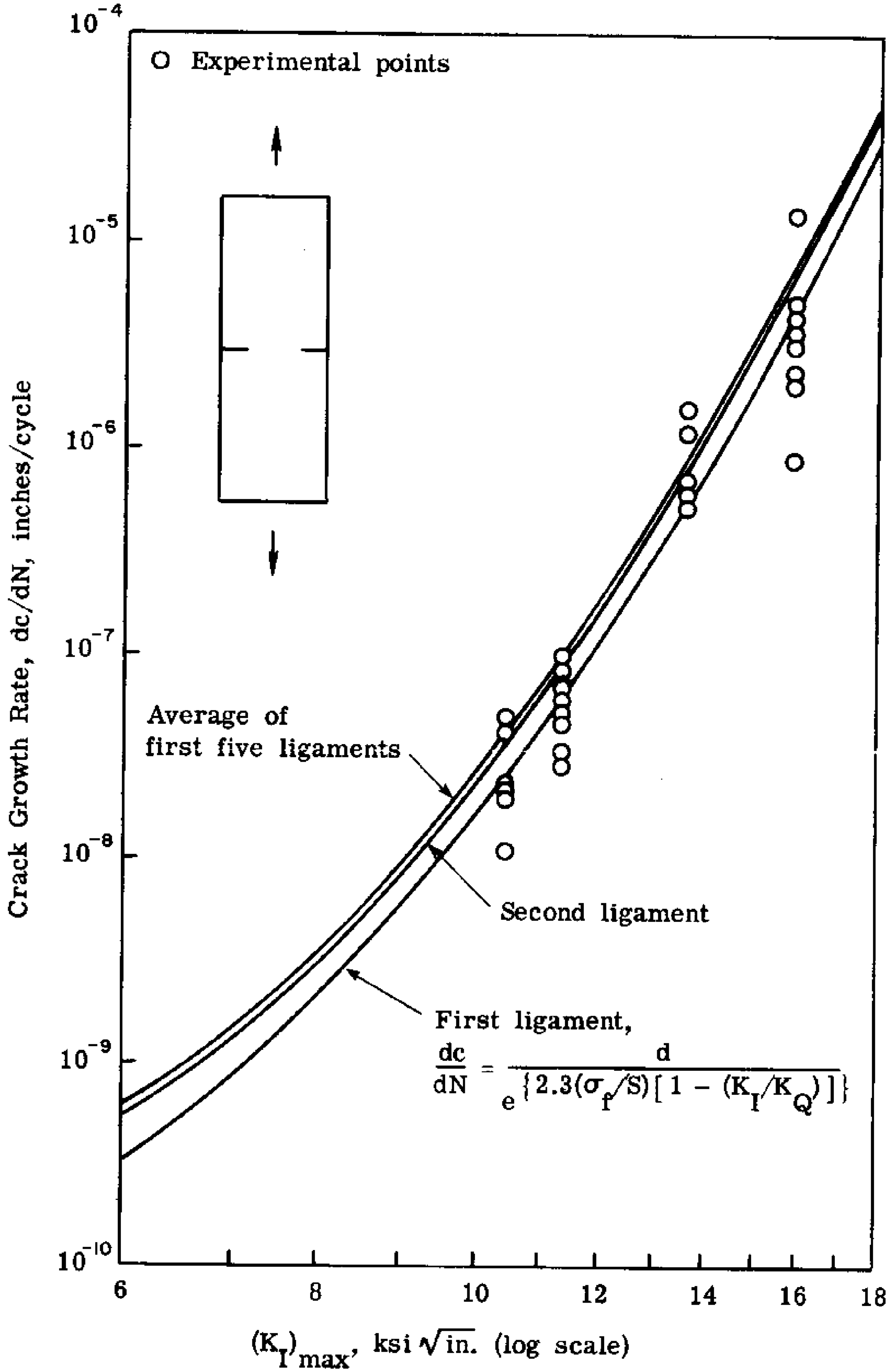

FIGURE 11.

THEORETICAL vs. EXPERIMENTAL FATIGUE CRACK GROWTH RATE, E-GLASS/EPOXY PLY CONFIGURATION (90°/0°/90°/0°/90°), TENSION-TENSION FATIGUE.