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ESTIMATION OF DYNAMIC CHARACTERISTICS OF DEEP OCEAN TOWER STRUCTURES

by

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Administrative Statement

E. H. Vanmarcke and R. N. Iascone have extended the original analysis methodology developed by Vanmarcke under sponsorship of the National Science Foundation to an analysis and demonstration of results based on offshore platform response record obtained during the 1971 San Fernando earthquake. The authors present a significant improvement in the means by which the total damping, both structural and hydrodynamic, can be more accurately estimated for use by the design engineer.

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Dr. Alfred H. Keil
Director

July 1972

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INTRODUCTION

Successful design of deep offshore structures critically depends on the availability to the designer of an adequate description of the forces to which the structure is likely to be subjected, and on his ability to model the structure for the purpose of evaluating pertinent response measures. The motion of deep fixed offshore tower structures under the action of wind-induced ocean waves or earthquakes results from the complex interaction of inertial and drag forces (Morison, et al., 1950). Nath and Harleman (1967) found that for structures built in deep water, the drag component in the Morison wave force equation is negligible compared to the inertia component, and they express the structural response as a linear function of the wave heights. Foster (1967) and Malhotra and Penzien (1969) use an equivalent linearization procedure to treat, in an approximate way, system non-linearities due to drag forces and wave-structure interaction. The linearized system is then analyzed using the mode-super position procedure (Biggs, 1964; Gaither, 1964). The response at some critical point in the tower structure is represented by the first few modes of vibration. The motion is often predominantly in the fundamental mode.

Each mode is characterized by its natural period and damping, both very important factors in design. Very little is known about the damping of actual full-scale structures. In the case of offshore towers, the total damping is made up of both structural and hydrodynamic components which depend importantly on the level of motion. The designer is primarily interested in the value of damping to be used in dynamic response analyses for design level inputs. Specifically, provided linear dynamic analysis procedures are used, the quantity of most practical value is the total equivalent viscous damping at design level amplitudes. Natural frequencies can also change with motion intensity, but estimates derived from small amplitude sinusoidal or ambient vibration tests often provide sufficient information.

This report deals with the determination of structural dynamic parameters from field measurements of, say, platform motion. It is shown how actual response records can be used to estimate equivalent modal periods and equivalent total damping values. The proposed procedure is based on recent results (Vanmarcke, 1971) which relate periods and dampings to the moments and the "partial" moments of the spectral density function of the structural response to wide-band excitation. Simultaneous records of the exciting forces (generated by ocean waves or by earthquake-induced motion of the ocean floor) need not be available. For cases in which only the first mode is activated, the proposed procedure calls for the computation of the first three moments (i.e., area, first and second moment) of the estimated response spectral density function in a frequency band which includes the fundamental frequency. The same method may also be used to estimate properties of higher modes in multi-degree-of-freedom systems by successively isolating portions of the power spectrum which contain higher mode peaks.

Other methods for determination of structural dynamic properties by using response records have been suggested. Auto-correlation function techniques are discussed by Cherry and Brady (1965). For a white noise excited simple oscillator, the damping is estimated from the decay rate of the auto-correlation function. Spectral methods have the advantage that the dynamic properties for different modes can be obtained separately, simply by focusing attention on different portions of the output spectral density function. A well-known "spectral" method is to determine the damping ratio from the half-power bandwidth, i.e., the absolute value of the difference between the frequencies at which the spectral density is equal to one-half times its maximum value. When bandwidths are narrow, the accuracy of this measurement is often not satisfactory. It is difficult to estimate single spectral ordinates with high confidence, since the goals of high resolution and small statistical uncertainty are in basic conflict (Blackman and Tuckey, 1959; Jenkins, 1961).

PART I

ESTIMATION BASED ON SPECTRAL MOMENTS

Records of wind velocity, wave height or earthquake acceleration have the appearance of sample functions of random processes. The displacements, stresses, etc. induced into structures by wind, wave, or earthquake action are also random processes. It is often reasonable to assume that the intensity and the frequency content of the fluctuating part of the motion at hand (excitation or response) do not change predictably during the time interval of interest, and that the mean value equals zero. In short, the motion can often be viewed as a stationary random process (with zero mean, without loss of generality) and can be represented by (Rice, 1944)

$$X(t) = \sum_{i=1}^{n} A_i \sin(\omega_i t + \phi_i) \qquad (1)$$

where A_i = the random amplitude, ϕ_i the random phase angle and ω_i = the frequency of the i^{th} contributing sinusoid. Successive values of ϕ_i are mutually independent (in a statistical sense) and uniformly distributed between 0 and 2π. Successive values of A_i are also mutually independent, do not depend on the phase angles, and have mean zero and mean square (or variance) $\overline{A_i^2}$. The average total power, the mean square, or the variance of X(t) is $\sum_{i=1}^{n} \overline{A_i^2}$. Assume now that the frequencies ω_i in Eq. 1 are chosen to lie at equal intervals $\Delta\omega$. Fig. 1 shows a function $G(\omega)$ whose value at ω_i is equal to $\overline{A_i^2}/\Delta\omega$, i.e., $G(\omega)\Delta\omega = \overline{A_i^2}$. Allowing the number of sinusoids in the motion to become very large, the variance will become equal to the area under the continuous function $G(\omega)$ which is known as the (one-sided) spectral density function. It expresses the relative importance, i.e., the relative contribution to the mean square of X(t), of sinusoids with frequencies within some specified frequency band.

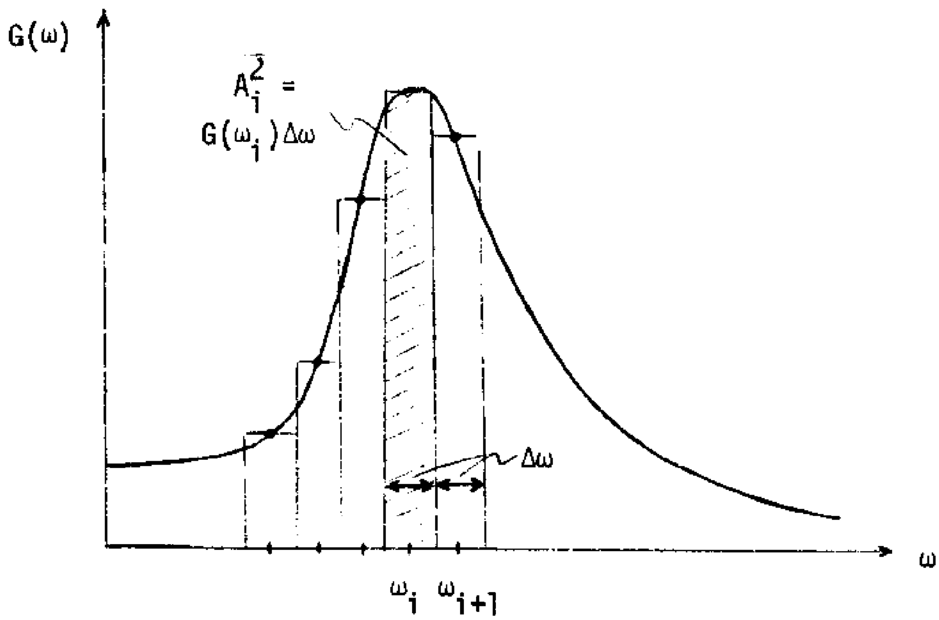


Fig. 1 Definition of the Spectral Density Function

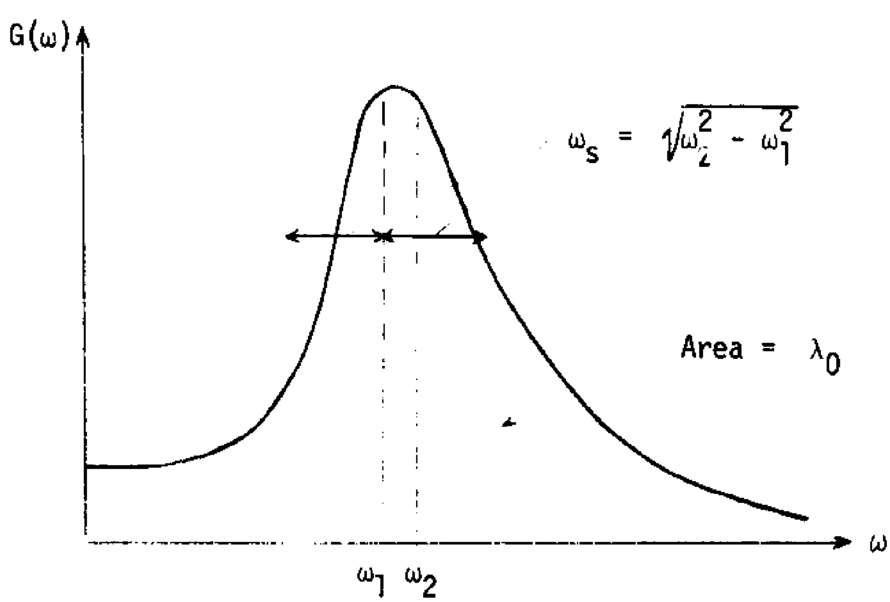


Fig. 2 Moment-Based Spectral Parameters:
λ_0, ω_2 and $q = \omega_s/\omega_2$

I.1 SPECTRAL MOMENTS AND RELATED SPECTRAL PARAMETERS

The moments of the spectral density function $G(\omega)$ of a stationary random process are

$$\lambda_i = \int_0^\infty \omega^i \; G(\omega) d\omega \tag{2}$$

The related quantities

$$\omega_i = (\lambda_i/\lambda_0)^{1/i}, \quad i = 1, 2 \tag{3}$$

have the dimension of circular frequency. Note that ω_1 may be interpreted as the distance from the "centroid" of the "spectral mass" $G(\omega)$ from the frequency origin. See Figure 2. An important unitless spectral parameter is (Vanmarcke, 1969, 1971)

$$q = \left(1 - \frac{\lambda_1^2}{\lambda_0 \lambda_2}\right)^{1/2} \tag{4}$$

From Schwarz' inequality, $0 \leq \lambda_1^2/\lambda_0\lambda_2 \leq 1$, and hence, $0 \leq q \leq 1$. The factor q depends only on the degree of dispersion of spread about the central frequency. To see this, note that q can be expressed as follows:

$$q = \frac{\omega_s}{\omega_2} \tag{5}$$

where

$$\omega_s = \sqrt{\omega_2^2 - \omega_1^2} \tag{6}$$

Note that ω_s may be interpreted as the "radius of gyration" of the "spectral mass" $G(\omega)$ about its "centroid." The factor q is a measure of the variability in the frequency content of a random process. It equals zero for a pure sinusoid and takes a value much smaller than one for narrow-band processes.

The spectral parameters λ_0, ω_2 and q all have a simple interpretation in the time domain. The following relations are well known

$$\lambda_0 = \sigma_X^2 \qquad (7)$$

$$\omega_2 = \sigma_{\dot{X}}/\sigma_X \qquad (8)$$

σ_X and $\sigma_{\dot{X}}$ denote the r.m.s. values of the random process X(t) and its time derivative $\dot{X}(t)$, respectively. The first moment λ_1 and the spectral parameter q are intimately related to the properties of the envelope R(t) of the random process X(t). When the envelope definition used is that due to Rice (1944) or Cramer and Leadbetter (1967), it may be shown that (Vanmarcke, 1972)

$$q = \sigma_{\dot{R}}/\sigma_{\dot{X}} \qquad (9)$$

The factor q is equal to the ratio of the r.m.s. value $\sigma_{\dot{R}}$ of the slope of the envelope R to the r.m.s. value $\sigma_{\dot{X}}$ of the slope of the process X. This result is illustrated in Figure 3. When the process X(t) is Gaussian, then many other statistical properties which are often of practical interest, e.g., average barrier crossing rates, mean clump sizes, and maximum response statistics can be approximately expressed in terms of λ_0, ω_2 and q. (Vanmarcke, 1972).

Example: Wave Spectrum

Based on records of wave heights during storms in the North Atlantic, the following functional form was suggested by Pierson and Moskowitz (1963) for the spectral density function of the wave heights, or the wave spectrum, which has units feet^2 - sec.

$$G(\omega) = \left(\frac{\alpha g^2}{\omega^5}\right) e^{-\beta(\omega_0/\omega)^4} \qquad (10)$$

where

α = 8.10×10^{-3}

β = 0.74

g = acceleration of gravity (ft/sec^2)

ω_0 = g/V (radians/sec)

V = wind speed reported by weather ships (ft/sec)

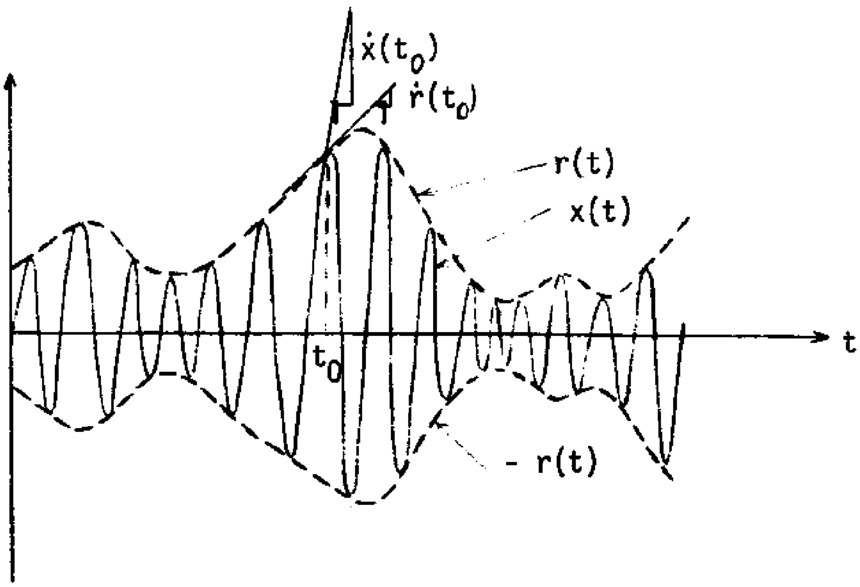


Fig. 3 Sample Functions of X(t) and R(t). Time Domain Interpretation of Spectral Parameters
$\lambda_0 = \sigma_X^2$, $\omega_2 = \sigma_{\dot{X}}/\sigma_{\dot{X}}$ and $q = \sigma_{\dot{R}}/\sigma_{\dot{X}}$

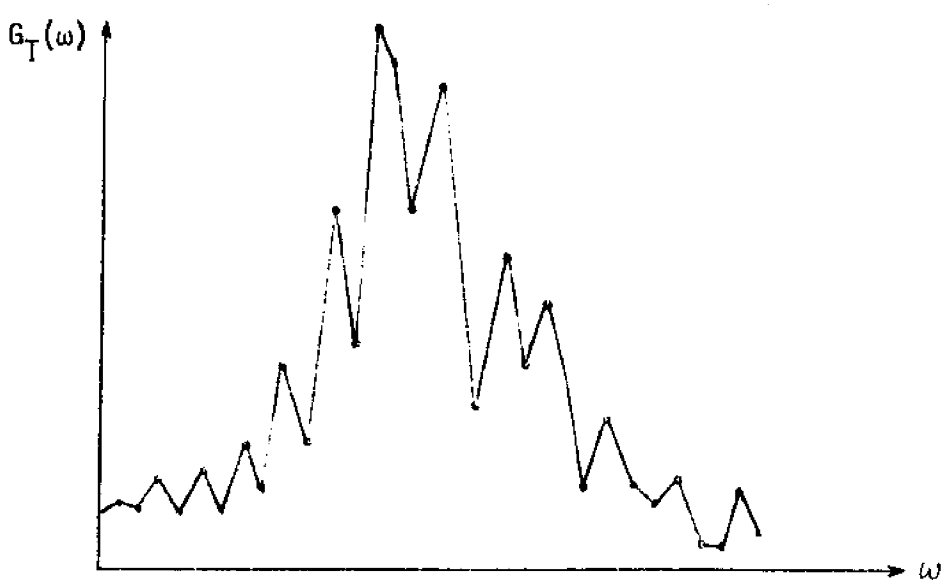


Fig. 4 Estimated Spectral Density Function $G_T(\omega)$

The values of λ_0, ω_2 and q are

$$\lambda_0 = \frac{\alpha}{4\beta}\frac{V^4}{g^2} \qquad (11)$$

$$\omega_2 = \omega_0 \left[\sqrt{\beta} \; \frac{\Gamma(0.5)}{\Gamma(1)} \right]^{1/2} = \omega_0(1.77 \sqrt{\beta})^{1/2} \qquad (12)$$

$$q = \left[1 - \frac{\Gamma^2(0.75)}{\Gamma(1)\Gamma(0.5)} \right]^{1/2} = 0.395 \qquad (13)$$

The value q = 0.395 is typical for spectra with moderate to large bandwidths. Spectral density functions with narrow bandwidths, e.g., those of the response of a lightly damped one-degree-of-freedom system to wide-band input, have q factors which are much closer to zero. Spectra whose power lies predominantly in the very low frequency range tend to have q factors close to one.

I.2 ESTIMATION OF SPECTRAL PARAMETERS. ADVANTAGES OF SPECTRAL MOMENT-BASED ESTIMATION

The spectral moments and parameters just defined can also be computed for the estimated spectral density $G_T(\omega)$ obtained from a recorded trace x(t) which can be regarded as a single sample function of the stationary random process X(t). Assume that the record x(t) is digitized into a sequence of n observations at equidistant intervals Δt. The <u>estimated spectral characteristics</u> are denoted by $\lambda_{i,T}$, $\omega_{i,T}$ and q_T, T being the length of the record. If the spectral density $G_T(\omega)$ is evaluated at equidistant frequencies $\omega = \omega_k$, k = 1, 2, ... , m (where the choice of the frequency interval depends on T), then (see Fig. 4)

$$\lambda_{i,T} = \sum_{k=1}^{m} \omega_k^i \; G_T(\omega_k)\Delta\omega \qquad (14)$$

The estimates $G_T(\omega_k)$ can be obtained either from a Fourier transformation of the estimated autocorrelation function, or directly from the squared norm of the Fourier transform of the digitized record x(t)

(Blackman and Tuckey, 1959; Jenkins, 1961; Tuckey, 1967). The total number, m, of frequencies depends on the upper bound cut-off frequency $\omega_c = m\Delta\omega$, which can be determined from the length Δt of the time interval between points at which x(t) is specified. ω_c is the so-called Nyquist frequency (Jenkins, 1961) which is defined by the relationship $2\Delta t = 2\pi/\omega_c$. It follows that $m = \pi/\Delta t\Delta\omega$.

Owing to statistical variation, the sample values of the spectral parameters, e.g., $\lambda_{i,T}$, obtained from different records of length T, would generally differ from each other and from the "true" value λ_i itself. When the process is ergodic, and the available record x(t) is sufficiently long, however, the difference between $\lambda_{i,T}$ and λ_i should become very small and it is expected that

$$\lim_{T\to\infty} \lambda_{i,T} \to \lambda_i \tag{15}$$

Similarly, for other spectral parameters, e.g., q

$$\lim_{T\to\infty} q_T = \lim\left[1 - \frac{\lambda_{1,T}^2}{\lambda_{0,T}\lambda_{2,T}}\right]^{1/2} \to q \tag{16}$$

It is important to note that the high and low frequency ends of the estimated spectral density are unreliable due to finite record length, digitization of the record ("aliasing"), non-stationary effects, measuring equipment limitation, etc. (Blackman and Tuckey, 1959; Crandall, 1963; Tuckey, 1967). "Aliasing" is the effect whereby the part of the spectrum corresponding to frequencies in excess of the Nyquist frequency ω_c are "folded" into the frequency range $0 \leq \omega \leq \omega_c$ and added to the spectral density in this range.

Other errors in $G_T(\omega_k)$ result from the fact that a basic conflict exists between the goals of high resolution (i.e., small $\Delta\omega$) and small statistical uncertainty. The smaller the frequency interval $\Delta\omega$, the greater the length of the record required to obtain spectral density measurements $G_T(\omega_k)$ of a given reliability (Jenkins, 1961).

For a given record length T, estimates of spectral moments may be expected to be much more reliable than those of individual spectral ordinates. This follows essentially from the fact that $\lambda_{i,T}$, given by Eq. 14, is the sum of a large number of statistically independent contributions $\omega_k^i\, G_T(\omega_k)\Delta\omega$. This argument is formally developed in the next few paragraphs.

The mean and variance of the estimate, $G_T(\omega_k)$, of the individual spectral ordinates are approximately given by (Jenkins, 1961)

$$E[G_T(\omega_k)] \simeq G(\omega_k) \tag{17}$$

$$Var[G_T(\omega_k)] \simeq \frac{2\pi}{n\Delta\omega}\, G^2(\omega_k) \tag{18}$$

The variance of $G_T(\omega_k)$ is inversely proportional to the product of the record length and the bandwidth. The fluctuation of $G_T(\omega_k)$ may be measured by its coefficient of variation or the ratio of the standard deviation to the mean value. Using Eqs. 17 and 18, one obtains

$$\frac{Var[G_T(\omega_k)]^{1/2}}{E[G_T(\omega_k)]} \simeq \left(\frac{2\pi}{n\Delta\omega}\right)^{1/2} \tag{19}$$

In Eq. 14, the contributions $\omega_k^i\, G_T(\omega_k)\Delta\omega$ to the estimated moment $\lambda_{i,T}$ are statistically independent, and hence

$$E[\lambda_{i,T}] = \sum_{k=1}^{m} \omega_k^i\, E[G_T(\omega_k)]\Delta\omega \simeq \sum_{k=1}^{m} \omega_k^i\, G(\omega_k)\Delta\omega \simeq \lambda_i \tag{20}$$

$$Var[\lambda_{i,T}] = \sum_{k=1}^{m} \omega_k^{2i} (\Delta\omega)^2\, var[G_T(\omega_k)] \simeq \frac{2\pi}{n} \sum_{k=1}^{m} \omega_k^{2i}\, G^2(\omega_k)\Delta\omega \tag{21}$$

The ratio of the standard deviation to the mean value of $\lambda_{i,T}$ is

$$\frac{(Var[\lambda_{i,T}])^{1/2}}{E[\lambda_{i,T}]} \simeq \left(\frac{2\pi}{n}\right)^{1/2} \left[\int_0^\infty \omega^{2i} G^2(\omega)d\omega\right]^{1/2} / \left[\int_0^\infty \omega^i G(\omega)d\omega\right] \tag{22}$$

For example, when $G(\omega)$ is constant in the frequency range $0 \leq \omega \leq \Omega$, then Eq. 22 becomes

$$\frac{(Var[\lambda_{i,T}])^{1/2}}{E[\lambda_{i,T}]} \simeq \left(\frac{2\pi}{n}\right)^{1/2} \frac{1}{\Omega^{1/2}} \frac{i+1}{(2i+1)^{1/2}} \; ; \quad i = 0, 1, 2 \qquad (23)$$

Assuming that $\Omega \gg \Delta\omega$, it is obvious that the coefficient of variation of the estimated spectral moments will be considerably smaller than that of the individual spectral ordinates. (Compare Eqs. 19 and 23).

Furthermore, for reasons stated earlier in this section, partial spectral moments, obtained by integrating $G_T(\omega)$ over a reasonably wide frequency range which does not include the very high and very low frequency ends, will often be even more reliable than complete moments (i.e., for which integration is over all frequencies).

PART II

RELATIONSHIP BETWEEN STRUCTURAL DYNAMIC PROPERTIES AND RESPONSE SPECTRAL MOMENTS

It is quite straightforward to compute the spectral parameters λ_0, ω_2 and q for any given spectral density function. An illustration has already been given in Section I.1 (Eqs. 10-13). When the steady-state motion of interest is the response of a linear dynamic system to random input, then the moments of the response spectral density function, $G_1(\omega)$, which is given by

$$G_1(\omega) = |H(\omega)|^2 \; G_0(\omega) \tag{24}$$

will depend on the parameters of both the input spectral density function $G_0(\omega)$, and the system amplification function, $|H(\omega)|$. When the input is an ideal white noise, however, (i.e., $G_0(\omega) = G_0$) then the spectral parameters ω_2 and q will depend on the system properties only. The case studied first is that of a viscously damped one-degree-of-freedom oscillator excited by a force or by a motion at its base. In Section II.1, the relative displacement response to white noise is studied. Acceleration output spectral parameters are computed in Section II.2. Wide band input spectra such as those characterizing wave forces acting on structures or earthquake motions, are discussed in Section II.3. Finally, in Section II.4, an extension of the analysis to multi-degree-of-freedom systems is presented.

II.1 RELATIVE DISPLACEMENT RESPONSE TO WHITE NOISE OF A ONE-DEGREE STRUCTURE

a. Complete Moments

The amplification function of a single-degree-of-freedom system with base acceleration input and relative displacement output is given by

$$|H(\omega)|^2 = \frac{1}{(\omega_n^2 - \omega^2)^2 + 4\zeta^2\omega_n^2\omega^2} \tag{25}$$

where ω_n is the natural frequency and ζ is the ratio of critical damping. Taking $G_0(\omega) = G_0$ in Eq. 24, and substituting $G_1(\omega)$ for $G(\omega)$ in the expressions for λ_1, ω_2 and q (Eqs. 2, 3 and 4), one obtains:

$$\lambda_0 = \frac{\pi G_0}{4\zeta\omega_n^3} \tag{26}$$

$$\omega_2 = \omega_n \tag{27}$$

and

$$q^2 = 1 - \frac{1}{1 - \zeta^2}\left\{\left(1 - \frac{1}{\pi}\tan^{-1}\frac{2\zeta\sqrt{1 - \zeta^2}}{1 - 2\zeta^2}\right)^2\right\}$$

$$\simeq \frac{4\zeta}{\pi}\,[1 - 1.1\zeta + 0(\zeta^2)] \tag{28}$$

Note that q depends only on the damping ratio ζ. For small values of damping, q^2 can be approximated by

$$q^2 \cong \frac{4\zeta}{\pi} \tag{29}$$

A plot of the exact and approximate expressions for q^2 is shown in Fig. 5.

When the damping is light, the system properties ω_n and ζ and the excitation spectral density G_0 can be expressed in terms of the spectral moments by combining Eqs. 3, 4 and 26-29. The result is:

$$\omega_n = (\lambda_2/\lambda_0)^{1/2} \tag{30}$$

$$\zeta \simeq \frac{\pi}{4}q^2 = \frac{\pi}{4}(1-\lambda_1^2/\lambda_0\lambda_2) \tag{31}$$

$$G_0 \simeq \frac{4\zeta\omega_n^3\lambda_0}{\pi} \simeq \lambda_0(\lambda_2/\lambda_0)^{3/2}(1-\lambda_1^2/\lambda_0\lambda_2) \tag{32}$$

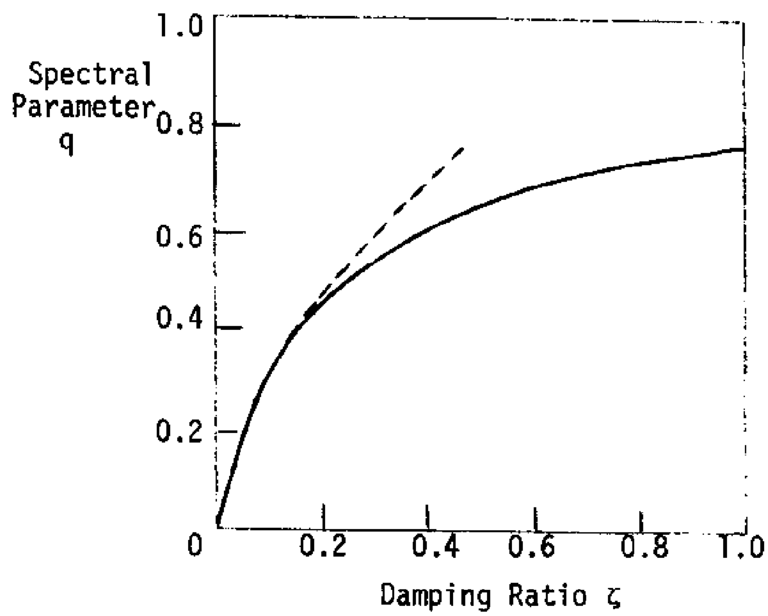


Fig. 5 The Factor q for the Response to Ideal White Noise of a Viscously Damped Linear Oscillator

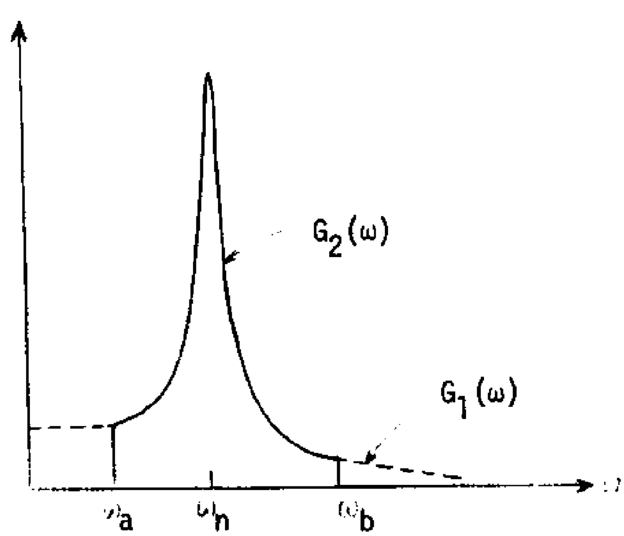


Fig. 6 The Spectral Density Functions $G_1(\omega)$ and $G_2(\omega)$

The above expressions, with the moments λ_i substituted by the sample moments $\lambda_{i,T}$ estimated from a recorded trace, may be regarded as estimators of ω_n, ζ and G_0.

b. Partial Moments

It is of considerable practical interest to study the effect (on the spectral moments and moment-based parameters) of neglecting the contributions due to components with frequencies outside a band of frequencies (ω_a, ω_b) centered around the natural frequency ω_n. These partial moments can be visualized as the complete moments of a hypothetical spectral density function, $G_2(\omega)$, which is equal to $G_1(\omega)$ within a limited band of frequencies and is assumed to be zero elsewhere. See Figure 6.

It is convenient to introduce the unitless band limits

$$\Omega_a = \frac{\omega_a}{\omega_n} \qquad\qquad \Omega_b = \frac{\omega_n}{\omega_b} \tag{33}$$

which must lie between 0 and 1. Crandall and Mark (1963) studied the influence of Ω_a and Ω_b on the area under $G_2(\omega)$. Neglecting terms of higher order in ζ, and higher than third order in Ω_a and Ω_b, the moments of $G_2(\omega)$, or the partial moments of $G_1(\omega)$ become

$$\lambda_0 \simeq \frac{\pi G_0}{4\zeta\omega^3}\left\{1 - \frac{4\zeta}{\pi}\left(\Omega_a + \frac{2}{3}\Omega_a^3 + \frac{1}{3}\Omega_b^3 \;.\;.\;.\;\right) + 0(\zeta^3)\right\} \tag{34}$$

$$\lambda_1 \simeq \frac{\pi G_0}{4\zeta\omega_n^2}\left\{1 - \frac{2\zeta}{\pi}\left(1 + \Omega_a^2 + \Omega_b^2 \;.\;.\;.\;\right) + 0(\zeta^2)\right\} \tag{35}$$

$$\lambda_2 \simeq \frac{\pi G_0}{4\zeta\omega_n}\left\{1 - \frac{4\zeta}{\pi}\left(\Omega_b + \frac{2}{3}\Omega_b^3 + \frac{1}{3}\Omega_a^3 \;.\;.\;.\;\right) + 0(\zeta^3)\right\} \tag{36}$$

The parameters ω_2 and q^2 are

$$\omega_2 \simeq \omega_n \left\{1 - \frac{4\zeta}{\pi} (\Omega_a - \Omega_b + \frac{1}{3}\Omega_a^3 - \frac{1}{3}\Omega_b^3)\right\} \tag{37}$$

$$q^2 \simeq \frac{4\zeta}{\pi} \left\{1 - \Omega_a + \Omega_a^2 - \Omega_a^3 - \Omega_b + \Omega_b^2 - \Omega_b^3\right\} \tag{38}$$

Introducing the series

$$(1+x)^{-1} = 1 - x + x^2 - x^3 + \ldots \tag{39}$$

into Eq. 38, one finds

$$q^2 \simeq \frac{4\zeta}{\pi} (\frac{1}{1 + \Omega_a} + \frac{1}{1 + \Omega_b} - 1) \tag{40}$$

The above result should be compared with Eq. 29. The ratio of the values of q corresponding to the spectra $G_2(\omega)$ and $G_1(\omega)$, respectively, is denoted by R_q, a reduction factor to be applied to the value of q given by Eq. 29. We have

$$R_q \simeq (\frac{1}{1 + \Omega_a} + \frac{1}{1 + \Omega_b} - 1)^{1/2} \tag{41}$$

For Ω_a and $\Omega_b \rightarrow 0$, $R_q \rightarrow 1$. If both Ω_a and Ω_b are very close to one, then the frequency band is very narrow; Eq. 41 indicates that R_q approaches zero in that case. Note that, in the case when $\Omega_a^q = \Omega_b$ is chosen, Eqs. 37 and 40 become

$$\omega_2 \simeq \omega_n \tag{42}$$

$$q^2 \simeq \frac{4\zeta}{\pi} (\frac{1 - \Omega_a}{1 + \Omega_a}) \tag{43}$$

Note that the system properties ω_n and ζ can be again expressed as simple functions of the spectral moments, now the partial, rather than the complete moments.

$$\omega_n \simeq (\lambda_2/\lambda_0)^{1/2} \tag{44}$$

$$\zeta \simeq \left(\frac{1+\Omega_a}{1-\Omega_a}\right) \frac{\pi}{4} \left(1 - \frac{\lambda_1^2}{\lambda_0\lambda_2}\right) \tag{45}$$

Having obtained ω_n and ζ, one can then use Eq. 34 to estimate the excitation spectral intensity G_0. When ζ is small and Ω_a is not too close to one, the following expression may be expected to be sufficiently accurate.

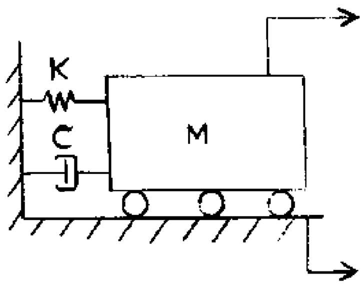
$$G_0 \simeq \left(\frac{1+\Omega_a}{1-\Omega_a}\right) \lambda_0 \left(\frac{\lambda_2}{\lambda_0}\right)^{3/2} \left(1 - \frac{\lambda_1^2}{\lambda_0\lambda_2}\right) \tag{46}$$

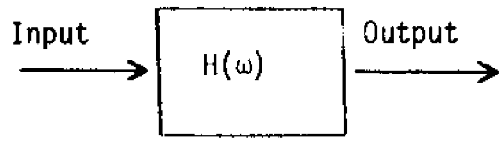
II.2 ABSOLUTE ACCELERATION RESPONSE TO WHITE NOISE INPUT OF A ONE-DEGREE STRUCTURE

The results just presented are valid when the input to the one-degree system is a force and the output a displacement, or when the input is a support acceleration and the output a relative displacement. Relations between partial moments and structural parameters can also be computed for various other combinations of input and output. In fact, absolute acceleration output is of most interest here, since structural response is usually measured by an accelerometer. Fortunately, for lightly damped systems, Eqs. 41-45 can be shown to remain approximately valid for all common one-degree system transfer functions if the reduced frequency limits Ω_a and Ω_b are chosen sufficiently close to one. For example, consider a one-degree-of-freedom system excited by a support acceleration and whose response is an absolute acceleration. (See Figure 7). The modulus squared of the complex frequency response $H'(\omega)$ is

$$|H'|^2 = (\omega_n^4 + 4\zeta^2\omega_n^2\omega^2)|H|^2 \tag{47}$$

in which $|H|^2 = |H(\omega)|^2$ is given by Eq. 25.





Equation: $\ddot{y} + 2\zeta\omega_n \dot{y} + \omega_n^2 y = -\ddot{z}$

Input	Output	$\lvert H(\omega)\rvert^2$
$\ddot{z}$	y	$\lvert H\rvert^2 = [(\omega_n^2 - \omega^2)^2 + 4\zeta^2\omega_n^2\omega^2)]^{-1}$
$\ddot{z}$	$\ddot{x}$	$(\omega_n^4 + 4\zeta^2\omega_n^2\omega^2)\lvert H\rvert^2$

Fig. 7. Some 1-D.O.F. System Amplification Functions

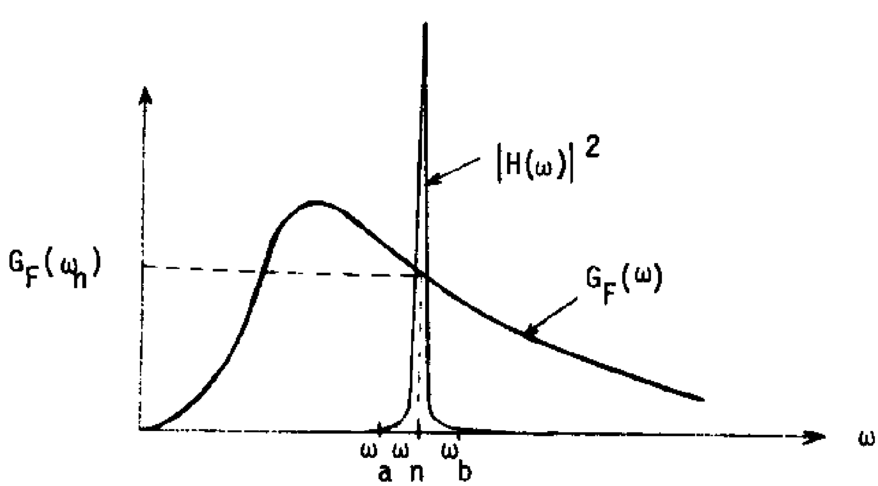


Fig. 8. Typical Wave Force Spectrum $G_F(\omega)$ and Squared Amplification Function $|H(\omega)|^2$

Assuming that the input is a stationary random process with intensity $G_0(\omega)$, the i^{th} partial moment of the acceleration response spectral density is

$$\lambda_i' = \int_{\omega_a}^{\omega_b} \omega^i \ (\omega_n^4 + 4\zeta^2\omega_n^2\omega^2) \ |H|^2 \ G_0(\omega)d\omega \tag{48}$$

It follows that the acceleration spectral moments can be expressed in terms of the displacement spectral moments:

$$\lambda_i' = \omega_n^4 \ \lambda_i + 4\zeta^2 \ \omega_n^2 \ \lambda_{i+2} \tag{49}$$

It will be assumed in the following analysis that the limits of integration, ω_a and ω_b, used in determining the partial spectral moments λ_i and λ_i', are chosen so that $\Omega_a = \Omega_b$, and that $G_0(\omega) = G_0$ in the frequency interval (ω_a, ω_b). Taking i = 0 in Eq. 49 and inserting Eq. 44, one obtains

$$\lambda_0' = \omega_n^4\lambda_0 + 4\zeta^2\omega_n^2\lambda_2 \simeq \omega_n^4 \ \lambda_0 \ (1 + 4\zeta^2) \tag{50}$$

The first spectral moment is

$$\lambda_1' = \omega_n^4\lambda_1 + 4\zeta^2\omega_n^2\lambda_3$$

$$= \omega_n^4\lambda_1 + 4\zeta^2\omega_n^2 \left(\omega_n^2\lambda_1 + G_0 \ \frac{4\zeta}{\pi} \left[\ell n\Omega\right]_{\Omega_b}^{\Omega_a}\right)$$

$$= \omega_n^4\lambda_1 \left\{ 1 + 4\zeta^2 - \frac{128\zeta^4}{\pi^2} \ \ell n\Omega_a \ \left[1 + \frac{2\zeta}{\pi}(1+2\Omega_a^2+\ldots)\right]\right\} \tag{51}$$

Use has been made of an exact expression for λ_3 given by Pulgrano and Ablowitz (1969). Note also that $\Omega = \omega/\omega_n$ and $\Omega_a = \Omega_b$. Similarly, the second acceleration response spectral moment is

$$\lambda_2' = \omega_n^4\lambda_2 + 4\zeta^2\omega_n^2\lambda_4$$

$$= \omega_n^4\lambda_2 + 4\zeta^2\omega_n^2 \left(\omega_n^2\lambda_2 + G_0 \frac{4\zeta}{\pi}\omega_n \left[\Omega\right]_{\Omega_b}^{\Omega_a}\right)$$

$$= \omega_n^4\lambda_2 \left[1 + 4\zeta^2 + \frac{64\zeta^4}{\pi^2}\left(\frac{1}{\Omega_a} - \Omega_a\right)\left(1 + \frac{4\zeta}{\pi}\Omega_a + \ldots\right)\right] \qquad (52)$$

Using Eqs. 40-52, it is easy to verify that for small values of ζ and for values of Ω_a larger than, say, 0.5, the acceleration and displacement spectral moments are approximately related as follows

$$\lambda_i' \simeq \omega_n^4 \lambda_i \qquad i = 0, 1, 2 \qquad (53)$$

It follows that the approximate expressions for ω_n and ζ derived earlier in terms of the displacement <u>partial</u> spectral moments (Eqs. 44 and 45) remain valid if every λ_i is substituted by λ_i'. The new expression for G_0 becomes

$$G_0 \simeq \left(\frac{1 + \Omega_a}{1 - \Omega_a}\right) \lambda_0' \left(\frac{\lambda_2'}{\lambda_0'}\right)^{1/2} \left(1 - \frac{\lambda_1'^2}{\lambda_0'\lambda_2'}\right) \qquad (54)$$

It should be noted that the complete acceleration spectral moments λ_1' and λ_2' become infinitely large when the input is ideal white noise. This can be seen by allowing Ω_a to become zero in Eqs. 51 and 52.

II.3 RESPONSE OF ONE-DEGREE STRUCTURES TO WAVE AND EARTHQUAKE FORCES

A commonly used expression for the spectral density function of ocean wave heights (the wave spectrum) is given by Eq. 10.

$$G(\omega) = \left(\frac{\alpha g^2}{\omega^5}\right) e^{-\beta(\omega_0/\omega)^4} \qquad (10)$$

The meaning of the symbols has been given earlier. Nath and Harleman (1967) found that for tower-like bottom-supported structures standing in deep water, wave forces on the cylindrical members can be approximately linearly related to wave heights. Similarly, the response (e.g., platform deflection) of a linear elastic structure can be expressed as a linear function of the wave forces. If the structure is modeled as an equivalent one-degree system with spring constant K, natural frequency ω_n and damping ζ, then the platform deflection spectral density function is

$$G_d(\omega) = \frac{\omega_n^4}{K} \frac{1}{(\omega_n^2 - \omega^2)^2 + 4\zeta^2\omega_n^2\omega^2} G_F(\omega) \qquad (55)$$

$G_F(\omega)$ is the wave force spectrum which is in turn related to $G(\omega)$ (Eq. 10). The exact relationship depends on the structural configuration (Nath and Harleman, 1967). A typical force spectrum is shown in Fig. 8. In the present context, the important point is that the force spectrum has a wide-band character and is smoothly varying compared to the structural amplification function for light damping values. The forcing spectrum parameter of prime interest is $G_F(\omega_n)$, the value at the structure's natural frequency. The spectral parameters of $G_d(\omega)$, based on partial moments obtained by integrating over frequencies which lie within a relatively narrow band centered around ω_n, will be the same as those given earlier (Eqs. 42 and 43).

$$\omega_i = \left(\frac{\lambda_2}{\lambda_0}\right)^{1/2} \simeq \omega_n \qquad (42)$$

$$q^2 = \left(1 - \frac{\lambda_1^2}{\lambda_0\lambda_2}\right) \simeq \frac{4\zeta}{\pi} \left(\frac{1 - \Omega_a}{1 + \Omega_a}\right) \qquad (43)$$

where

$$\lambda_i = \int_{\omega_a}^{\omega_b} \omega^i \, G_d(\omega) \, d\omega$$

$$\Omega_a = \frac{\omega_a}{\omega_n} = \frac{\omega_n}{\omega_b}$$

When the band limits ω_a and ω_b do not satisfy the above relationship, then the more general results, Eqs. 37 and 40, can be used. The above approximate relations remain valid when the response is the platform acceleration rather than the platform deflection, for reasons explained in Section II.3.

Similar conclusions are reached when sea-based fixed structures are subjected to earthquake ground motion, or to the combined action of wave and earthquake forces. Earthquake motions on firm ground or on rock are characterized by wide-band spectra; even those on soft soil tend to have spectra which vary sufficiently smoothly for the results to hold, at least for structures with damping ratios less than about 0.05.

II.4 RESPONSE OF MULTI-DEGREE-OF-FREEDOM SYSTEMS

A similar procedure may also be used to estimate modal frequencies and damping values in multi-degree-of-freedom systems by successively isolating portions of the estimated spectral density $G_T(\omega)$ which contain individual modal peaks. It will be assumed that the spectral density $G_0(\omega)$ of the stationary random wave forces $Z(t)$ is reasonably flat in the vicinity of each of the modal frequencies. It is further assumed that the structural response $X(t)$ can be expressed in terms of the normal modes Φ_j and the modal coordinates $X_j(t)$, $j = 1, 2, \ldots n$.

$$X(t) = \sum_{j=1}^{n} \Phi_j X_j(t) \qquad (56)$$

Each component $X_j(t)$ represents the output of a single-degree-of-freedom system, characterized by its modal frequency ω_j and the modal damping ratio ζ_j, and excited by the modal forcing function $\Gamma_j Z(t)$, Γ_j being the participation factor of the j^{th} mode. The j^{th} decoupled equation has the form

$$\ddot{x}_j + 2\zeta_j\omega_j\dot{x} + \omega_j^2 x_j = (\Gamma_j/m_j)z(t) \qquad j = 1, \ldots, n \qquad (57)$$

The system transfer function $H(\omega)$ may be expressed in terms of the transfer functions $H_j(\omega)$ of the individual modes.

$$H(\omega) = \sum_{j=1}^{n} \frac{\Phi_j \Gamma_j}{m_j} H_j(\omega) = \sum_{j=1}^{n} c_j H_j(\omega) \tag{58}$$

where $c_j = \Phi_j \Gamma_j / m_j$, and

$$H_j(\omega) = \frac{1}{\omega_j^2 - \omega^2 + 2i\zeta_j \omega_j \omega} \tag{59}$$

The response spectral density is

$$G(\omega) = G_0(\omega)|H(\omega)|^2 = G_0(\omega) \sum_{j=1}^{n} \sum_{k=1}^{n} c_j c_k H_j(\omega) H^*(\omega) \tag{60}$$

where $H^*(\omega)$ is the complex conjugate of $H_k(\omega)$. It has been shown (Vanmarcke, 1971) that Eq. 60 can be rewritten as follows

$$G(\omega) = G_0(\omega) \sum_{j=1}^{n} |H_j(\omega)|^2 \left\{ \sum_{k=1}^{n} c_j c_k \left[A_{jk} - \left(1 - \frac{\omega_j^2}{\omega^2} \right) B_{jk} \right] \right\} \tag{61}$$

where A_{jk} and B_{jk} are constants which depend on the frequency ratio $r = \omega_k / \omega_j$ and on the damping ratios in modes j and k.
For lightly damped systems whose modal frequencies are well separated, all cross-terms vanish and the i^{th} spectral moment λ_i is then approximately equal to the sum of the "pure" terms

$$\lambda_i \simeq \sum_{j=1}^{n} c_j^2 \, G_0(\omega_j) \int_{\omega_a}^{\omega_b} \omega^i |H_j(\omega)|^2 dw = \sum_{j=1}^{n} c_j^2 \lambda_{i,j} \tag{62}$$

When the above approximation is reasonably accurate, this provides a straightforward way of computing the damping ratio of the j^{th} mode. First, two frequency limits, ω_a and ω_b, which isolate the j^{th} modal peak in the estimated response spectral density $G_T(\omega_k)$ are chosen. The major contribution to the estimated partial moments

$\lambda_{i,T}$ will be that due to the j^{th} mode, i.e., $c_j^2\lambda_{i,j}$ is roughly estimated by $\lambda_{i,T}$. The frequency and damping estimation can then proceed as indicated below.

II.5 PROPOSED ESTIMATION PROCEDURE

To evaluate the equivalent natural frequency and the equivalent damping ratio of a structure that can be modeled as a one-degree-of-freedom system, the procedure requires the computation of the first three moments (i.e., area, first and second moment) of the estimated spectrum (of the response) in a frequency band (ω_a, ω_b) which includes the fundamental frequency ω_n. Actually, the value of ω_n is unknown, but can be estimated reliably for lightly damped structures. The first estimate, ω_n', might be the frequency at which $G_T(\omega)$ is maximum (See Fig. 9a). It is convenient to choose ω_a and ω_b so that $\omega_a/\omega_n' = \omega_n'/\omega_b$. A new estimate ω_n'' can now be obtained from Eq. 44, and an estimate of the damping from Eq. 45. If ω_n'' and ω_n' are sufficiently close, no iteration will be required. Otherwise, new values will be needed for the reduced band limits Ω_a and Ω_b, either by choosing a new frequency range over which to integrate (while keeping $\Omega_a = \Omega_b^{-1}$), or by taking $\Omega_a = \omega_a/\omega_n''$ and $\Omega_b = \omega_n''/\omega_b$ (in which case $\Omega_a \neq \Omega_b^{-1}$, so that Eqs. 37 and 40 need to be used).

The same method can in some cases be used to estimate the period and damping of higher modes in multi-degree-of-freedom systems by successively isolating portions of the response spectrum which contain higher mode peaks, as suggested in Fig. 9b. But to account properly for the contribution, due to adjacent modes to the various computed partial moments, several cycles of iteration may be required.

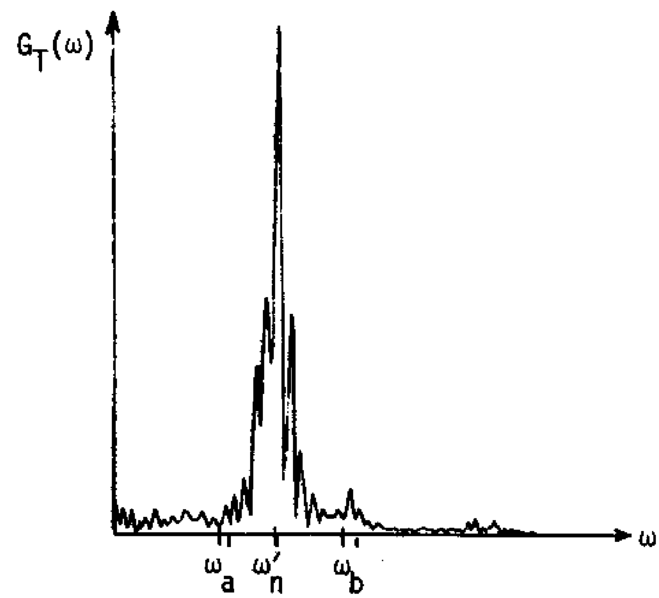


Fig. 9a. Estimation of One-Degree-of-Freedom System Properties

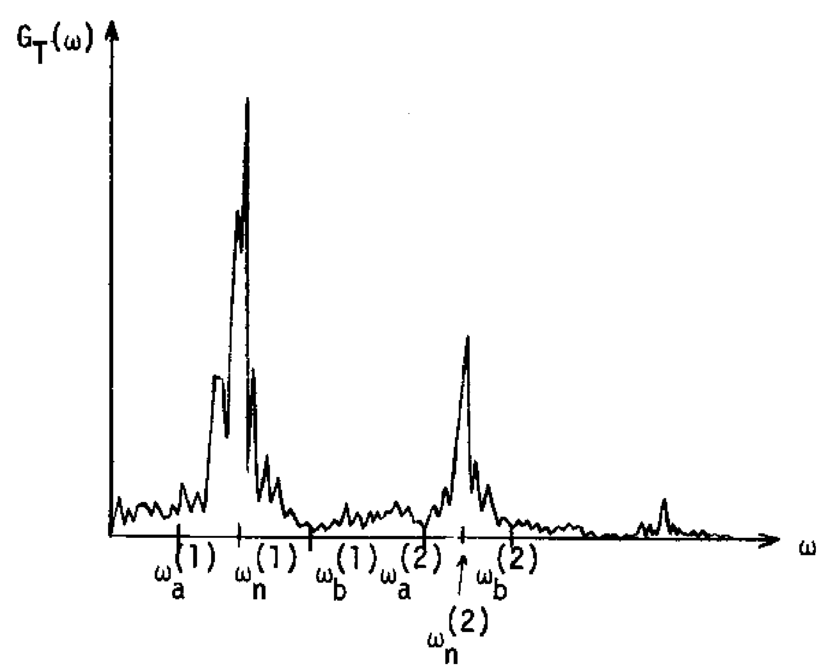


Fig. 9b. Estimation of Multi-Degree-of-Freedom System Properties

PART III

APPLICATION TO AN OFFSHORE TOWER

III.1 PRELIMINARY SPECTRAL ANALYSIS

This part is devoted to an analysis of the recorded response of an offshore bottom-supported tower to the February 9, 1971, San Fernando earthquake. The structure and its instrumentation are described in Appendix A. The response is in the form of a digitized time history of acceleration, measured at two points on a platform which is at the top of the structure. One point is at the center of mass of the platform and the other point is located at an edge of the platform. Preliminary analysis indicated that it is reasonable to assume that the center of mass and the center of stiffness of the platform are both located at the same point. The accelerations at each point are measured along two orthogonal axes and are denoted by $\ddot{u}$ along the x-axis and $\ddot{v}$ along the y-axis. The subscripts "A" and "B" denote the two points on the platform, "A" being the edge point and "B" the center point. The first step is to obtain an estimate of the frequency content of each record, by computing the squared norm of the Fourier Transform. Figures 10 to 13 are plots of these spectra. They are characterized by a large spike at a frequency which is close to the system's natural frequency, which is yet to be determined. Also, since most of the response is concentrated around a single frequency we can infer that the structure responds primarily in just one mode of vibration and can be modeled by an equivalent single-degree-of-freedom system. An analysis aimed at isolating the rotational component of motion is presented in Section III.3.

III.2 TRANSLATIONAL MOTION

We focus attention here on the primary mode of vibration, presumably the first translational mode, which appears to have a natural frequency of 3.75 rad/sec. Inspection of the spectra shown in Figs. 10 - 13 indicates that reduced band limits $\Omega_a = \Omega_b = 0.8$ (i.e., initial

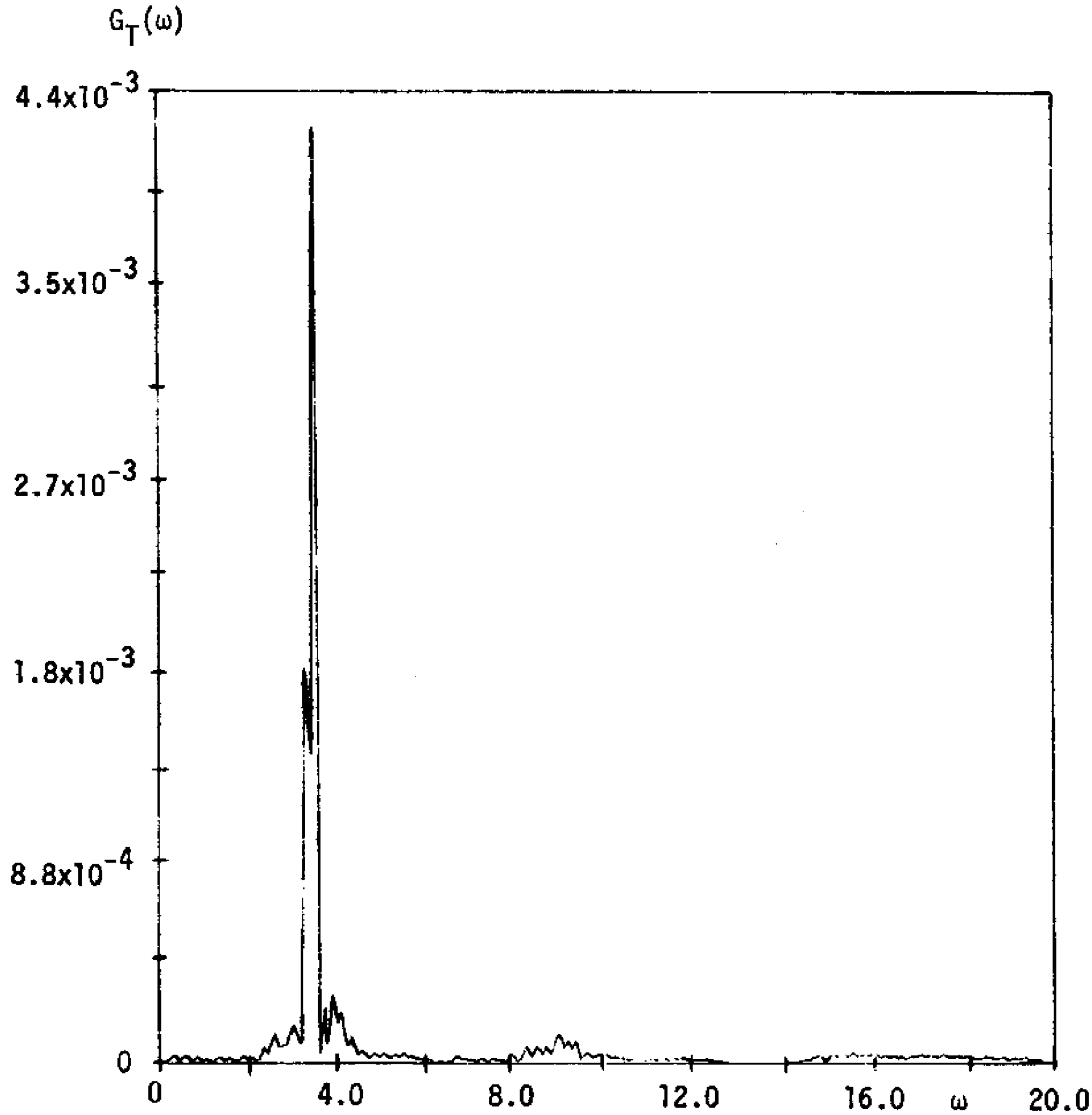


Fig. 10. Spectrum of Platform Center Motion in y-Direction ($\ddot{v}_B$)

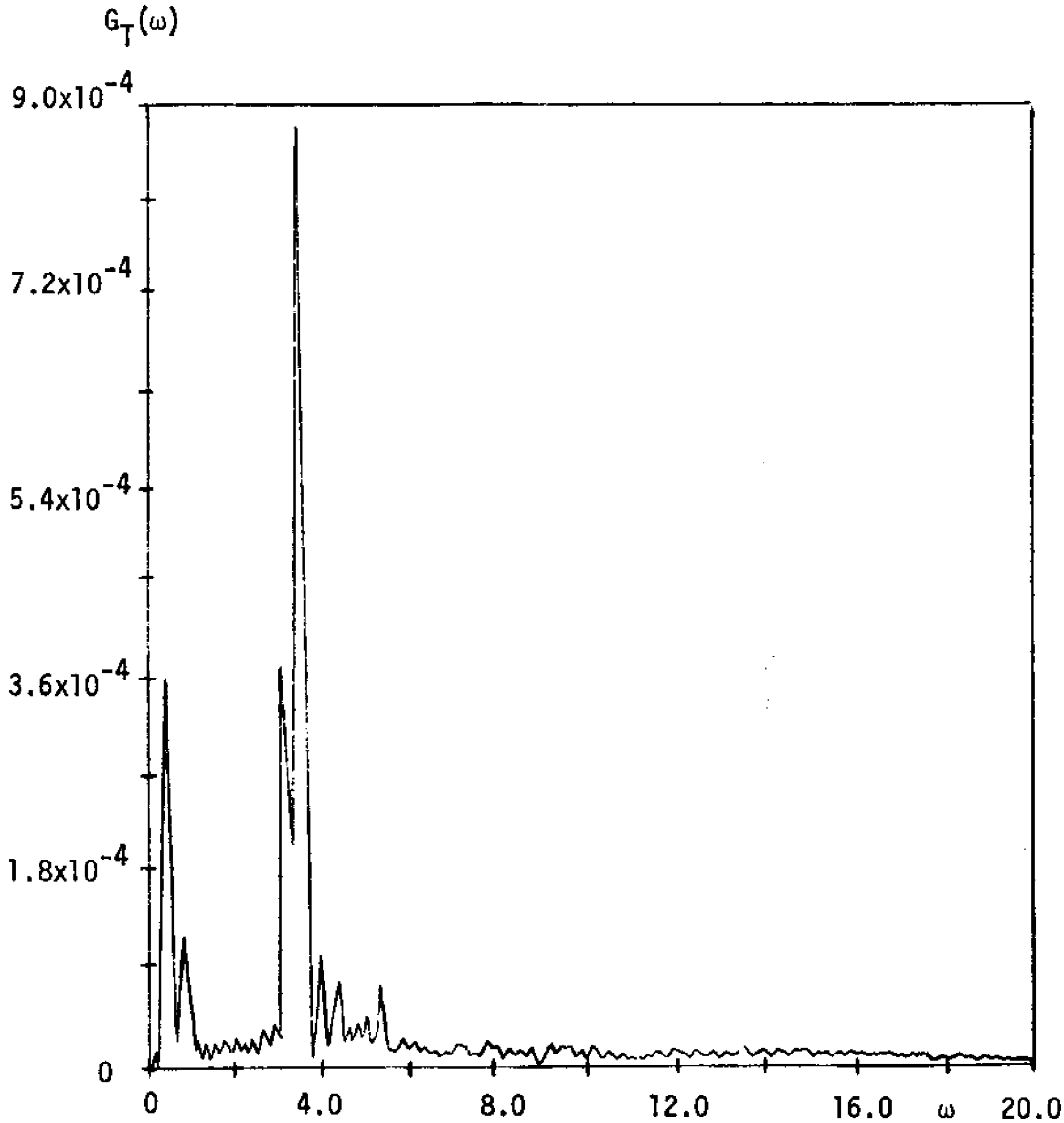


Fig. 11. Spectrum of Platform Center Motion in x-Direction ($\ddot{u}_B$)

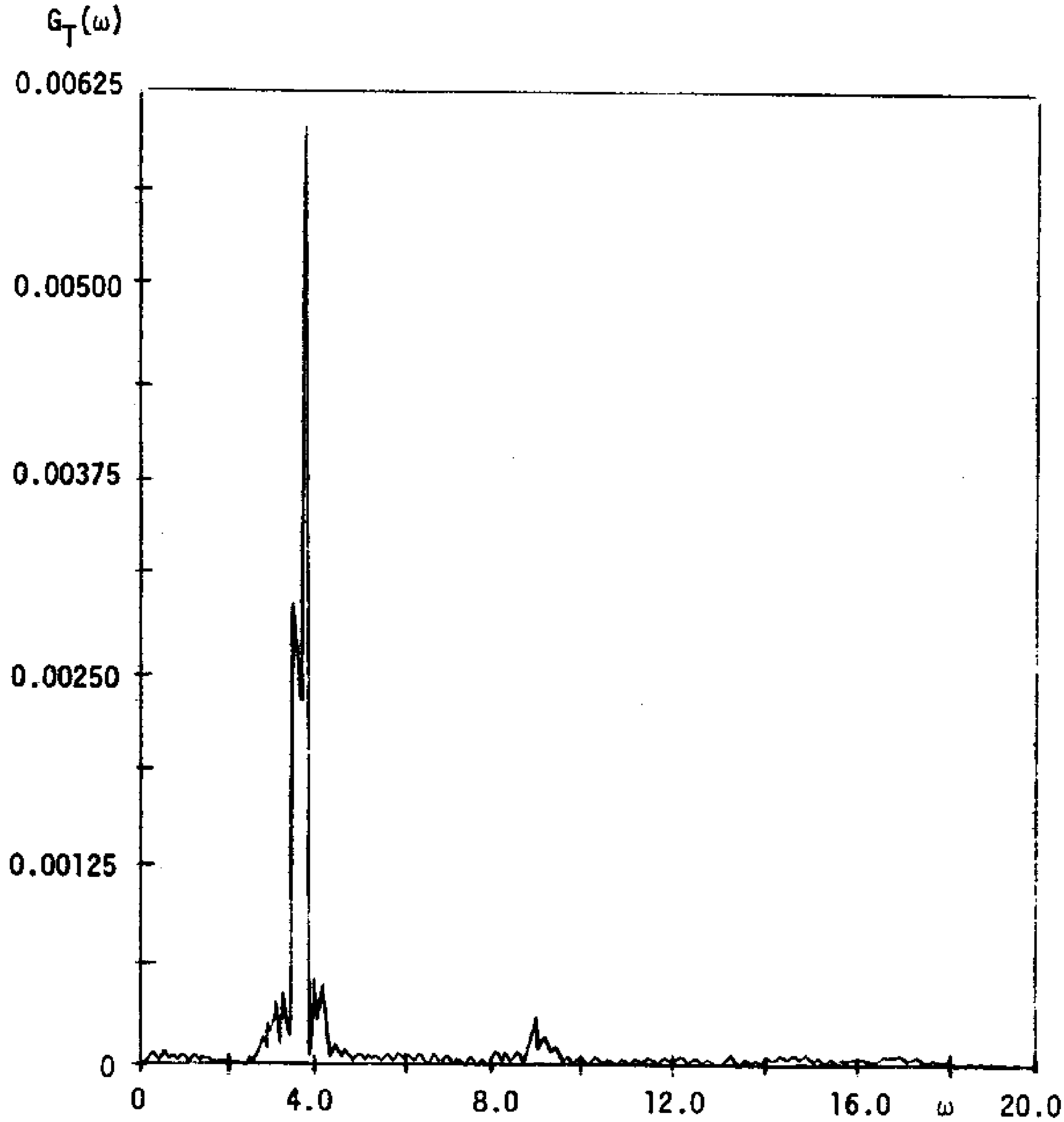


Fig. 12. Spectrum of Platform Edge Motion in y-Direction ($\ddot{v}_A$)

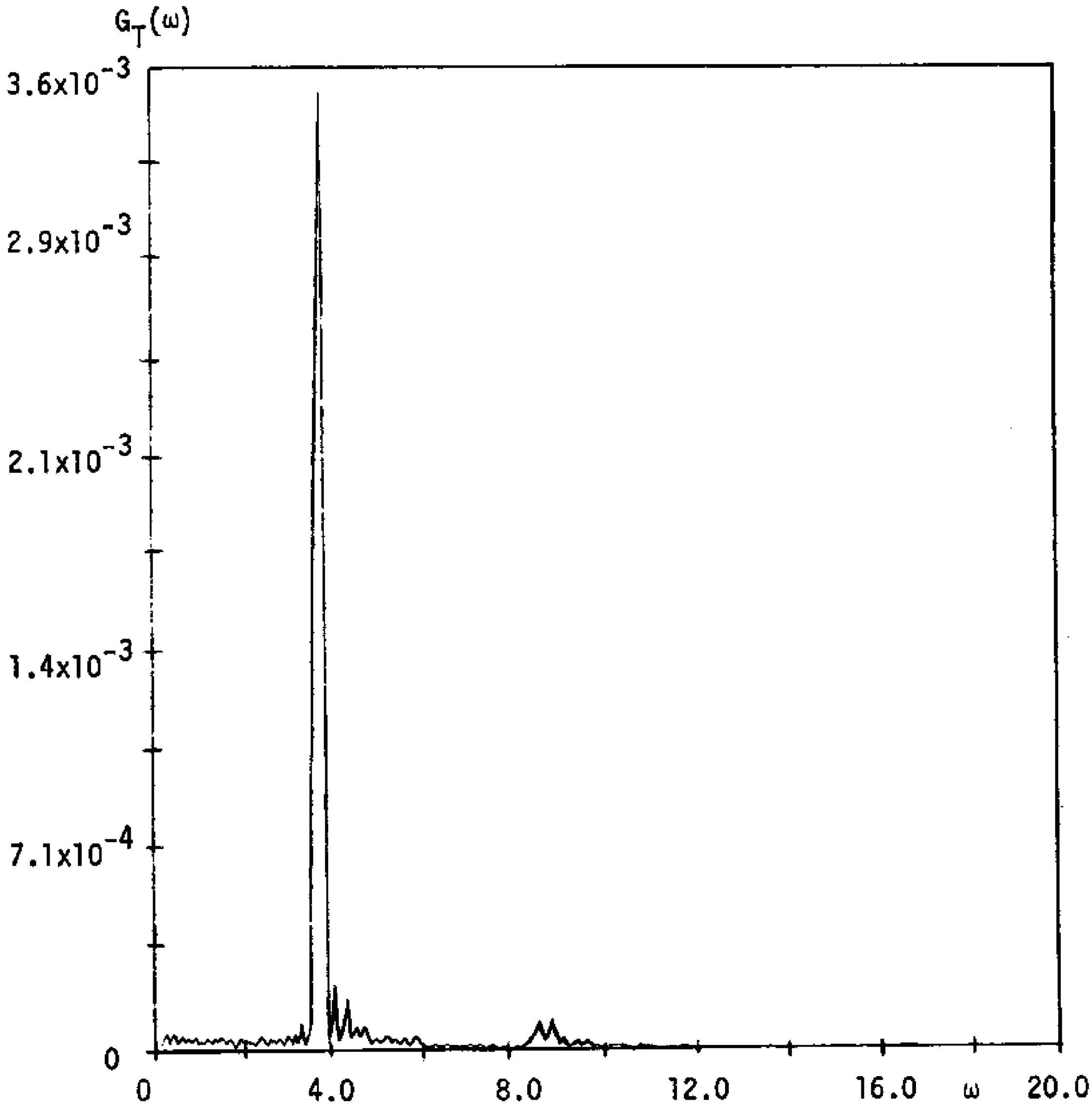


Fig. 13. Spectrum for Platform Edge Motion in x-Direction ($\ddot{u}_A$)

band limits are $\omega_a = 0.8 \times 3.75$ and $\omega_b = 3.75/0.8$) might lead to suitable damping estimates. From the computed partial moments, and by iteration using Eqs. 44 and 45, the equivalent one-degree-of-freedom system parameters ω_n and ζ are estimated for the four response components (two orthogonal accelerations at points "A" and "B"). The results of the calculations are summarized in Table 1.

$\Omega_a = 0.8$		
Point A	ω_n(rad/sec)	ζ (%)
X-Dir.	3.81	1.92
Y-Dir.	3.69	2.70
Point B		
X-Dir.	3.85	4.05
Y-Dir.	3.71	2.56

Table 1

To determine the sensitivity of these results to the choice of frequency interval used to compute partial moments, two other values of Ω_a were tried. Tables 2 and 3 show the results corresponding to $\Omega_a = 0.7$ and $\Omega_a = 0.9$, respectively. Summary statistics for the damping estimates are given in Table 4.

$\Omega_a = 0.7$		
Point A	ω_n(rad/sec)	ζ (%)
X-Dir.	3.84	2.48
Y-Dir.	3.70	3.10
Point B		
X-Dir.	3.86	3.75
Y-Dir.	3.70	1.97

Table 2

$\Omega_a = 0.9$		
Point A	ω_n(rad/sec)	ζ(%)
X-Dir.	3.80	1.51
Y-Dir.	3.71	2.56
Point B		
X-Dir.	3.82	3.14
Y-Dir.	3.73	3.19

Table 3

Point	Direction	Damping ζ(%)				
		$\Omega_a = 0.7$	$\Omega_a = 0.8$	$\Omega_a = 0.9$	Average	Minimum
A	X	2.48	1.92	1.51	1.97	1.51
	Y	3.10	2.70	2.56	2.79	2.56
B	X	3.75	4.05	3.14	3.65	3.14
	Y	1.97	2.56	3.19	2.57	1.97

Table 4

III.3 ROTATIONAL MOTION

The motion at the center of the platform (point "B") is assumed to be due to pure translation, i.e., the platform twists about its center. Any contribution due to twist to the total motion can be identified by isolating the rotational components at point "A". It is assumed that the platform acts as a rigid diaphragm and that there is no in-plane distortion at the platform elevation. Therefore, any difference between the accelerations at points "A" and "B" is due solely to platform rotation. Shown in Fig. 14 is a plan view of the platform with an arbitrarily imposed displacement u_B, v_B and θ, where

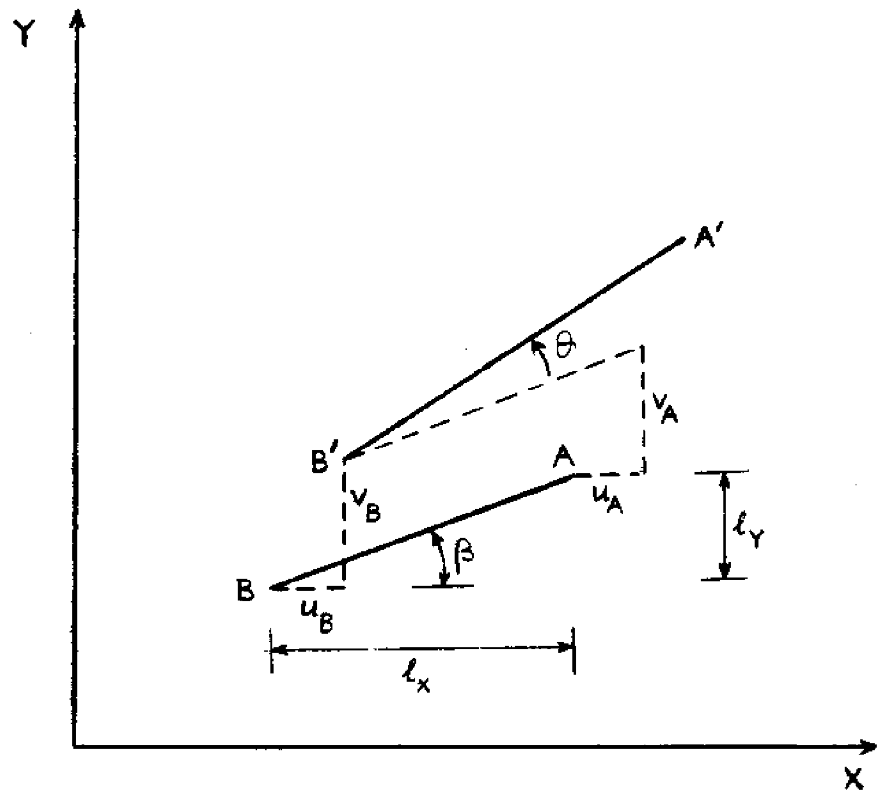


Fig. 14. Platform Deformation

u_B and v_B are the displacements of the center and θ is the rotation of the platform. From the figure one can determine that

$$\tan(\beta + \theta) = (\tan\beta + \tan\theta)\left(\frac{1}{1 - \tan\beta\ \tan\theta}\right) \tag{64}$$

For small θ, $\tan\theta \cong \theta$. Expanding Eq. 64 and neglecting terms of second or higher order in θ, one obtains:

$$\tan(\beta + \theta) \cong \tan\beta + \theta(1 + \tan^2\beta) \tag{65}$$

and

$$\theta \cong \frac{\tan(\beta = \theta) - \tan\beta}{1 + \tan^2\beta} = \left\{\frac{\ell_x + v_A - v_B}{\ell_x + u_A - u_B} - \frac{\ell_y}{\ell_x}\ \frac{1}{1 + \tan^2\beta}\right\} \tag{66}$$

We wish to work with the rotational acceleration $\ddot{\theta}$. Differentiating twice with respect to time:

$$\ddot{\theta} \cong \frac{1}{1 + \tan^2\beta}\left[\frac{\ell_x(\ddot{v}_A - \ddot{v}_B) - \ell_y(\ddot{u}_A - \ddot{u}_B)}{\ell_x^2}\right] \tag{67}$$

By inserting the known values of $\ddot{v}_A$, $\ddot{v}_B$, $\ddot{u}_A$, and $\ddot{u}_B$ and taking the Fourier Transform resulting rotational motion record, one obtains the plot shown in Fig. 15. Note that the maximum ordinate of the spectrum of the rotational accelerations is much smaller than any of the maximum ordinates of the translational spectra. The estimated values of the natural frequency and damping for rotational motion are shown in Table 5 below.

	$\Omega_a = 0.8$	
Platform	ω_n(rad/sec)	ζ(%)
Rotation	3.87	6.42

Table 5

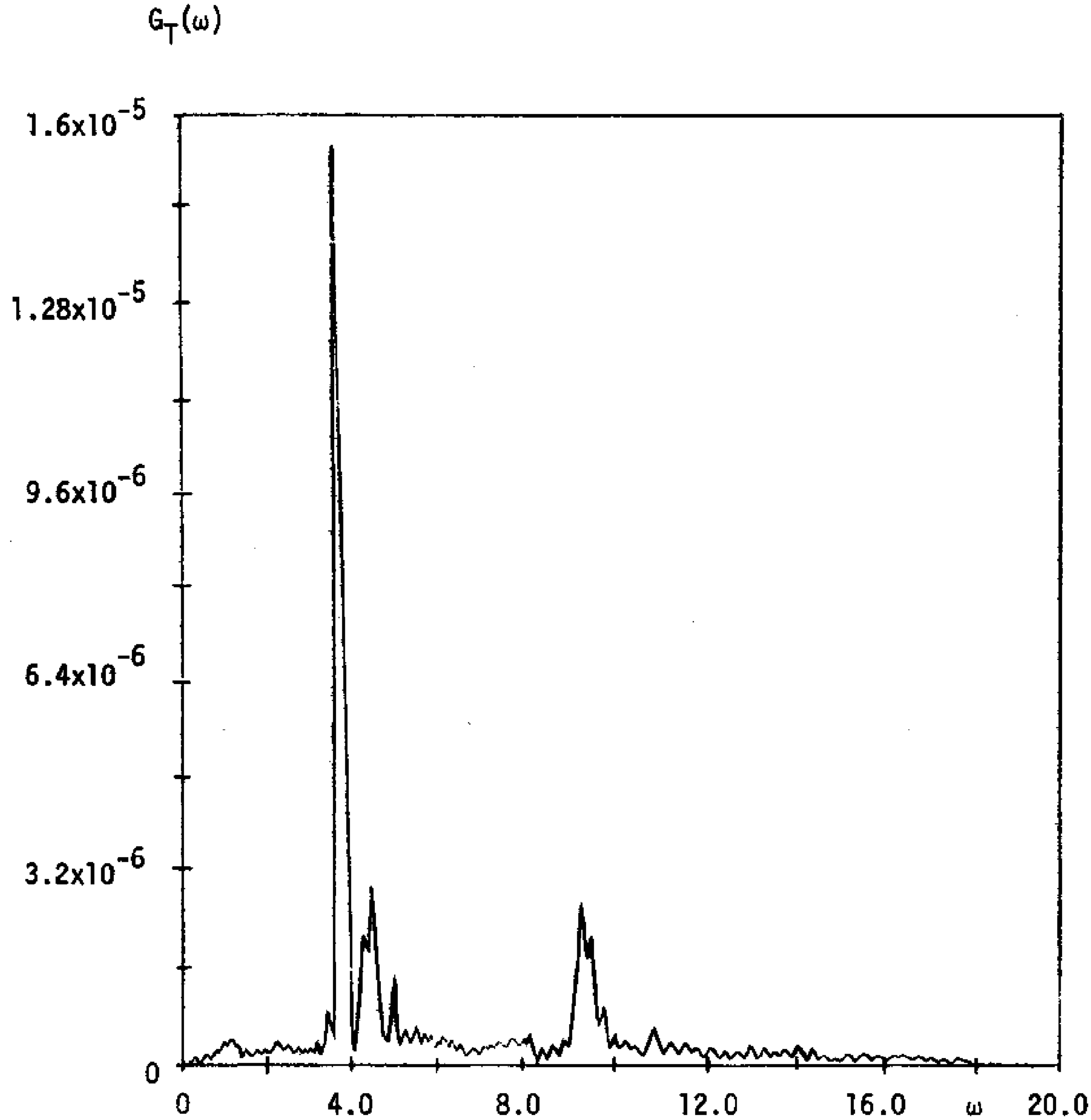


Fig. 15. Spectrum of Platform Rotation

CONCLUSIONS

A method has been presented for determining the equivalent linear dynamic properties of bottom-supported ocean structures from records of their response to random water waves or earthquakes. From a Fourier decomposition of the vibration, an estimate of the spectral density function can be determined. For a single-degree-of-freedom system, the proposed procedure requires the computation of the first three moments of the estimated response spectral density in a frequency band which includes the fundamental frequency.

The method has been applied to the records of the motion of an offshore platform during the February, 1971 San Fernando earthquake. The damping values obtained might be used in the design of similar ocean towers to resist strong earthquakes. The rotational components of motion appear to be relatively unimportant. Of course, the conclusions cannot be generalized to different load conditions (e.g., ocean waves only) or, for the same load condition, to very different response amplitudes. Valuable information can be gathered, however, by repeating the analysis whenever new motion records become available, and by comparing the frequency and damping estimates obtained.

The proposed damping estimation procedure has some distinct advantages over other methods, e.g., the half-power bandwidth method (Tanaka et al, 1969). Particularly notable features are:

(i) Low and high frequency portions of the estimated spectra, which are unreliable due to time interval size, finite record length and non-stationary effects, measuring equipment limitations, etc., can be eliminated.

(ii) The record length needed to obtain reasonably stable estimates of partial spectral moments is relatively small.

(iii) Smoothing of the "raw" spectral estimates is unnecessary; estimated spectral moments and parameters based upon them may be expected to change very little as a result of smoothing.

The method could also be used to estimate wave height spectral density parameters. It suffices to equate computed spectral parameters and their corresponding estimated values obtained from a recorded trace.

APPENDIX A

Description of Platform and Instrumentation

The data used in the final part of the report was provided by engineers at Standard Oil of California. It was recorded during the February 9, 1971, San Fernando Valley Earthquake on Platform Hope, which is located off the coast of California near Carpinteria. It is a development and production platform built in 1965. The earthquake-wave instrumentation system was installed in December, 1969. The platform has a capacity for 60 wells located in three rows parallel to the west plane of the platform. The locations of the accelerometer packages are shown in Figures A.1,A.2, and A.3. The coordinates of the packages are shown in Fig. A.3. Package C is located in a conductor pipe at the mudline, while packages A and B are attached to the main deck support beams. The data recorded at location C were not used in this report. The positions of the sensors A and B were selected to indicate translational and torsional motion (Titlow, 1971).

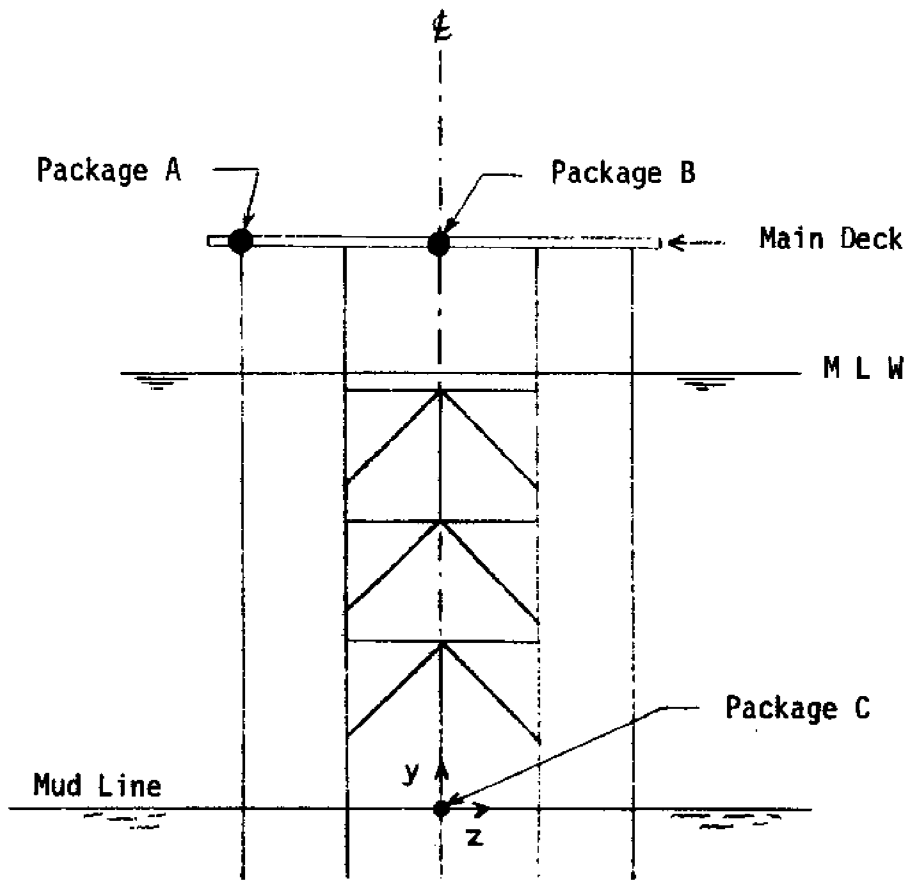


Fig. A.1 Platform Hope, West Elevation
(From Titlow, 1971)

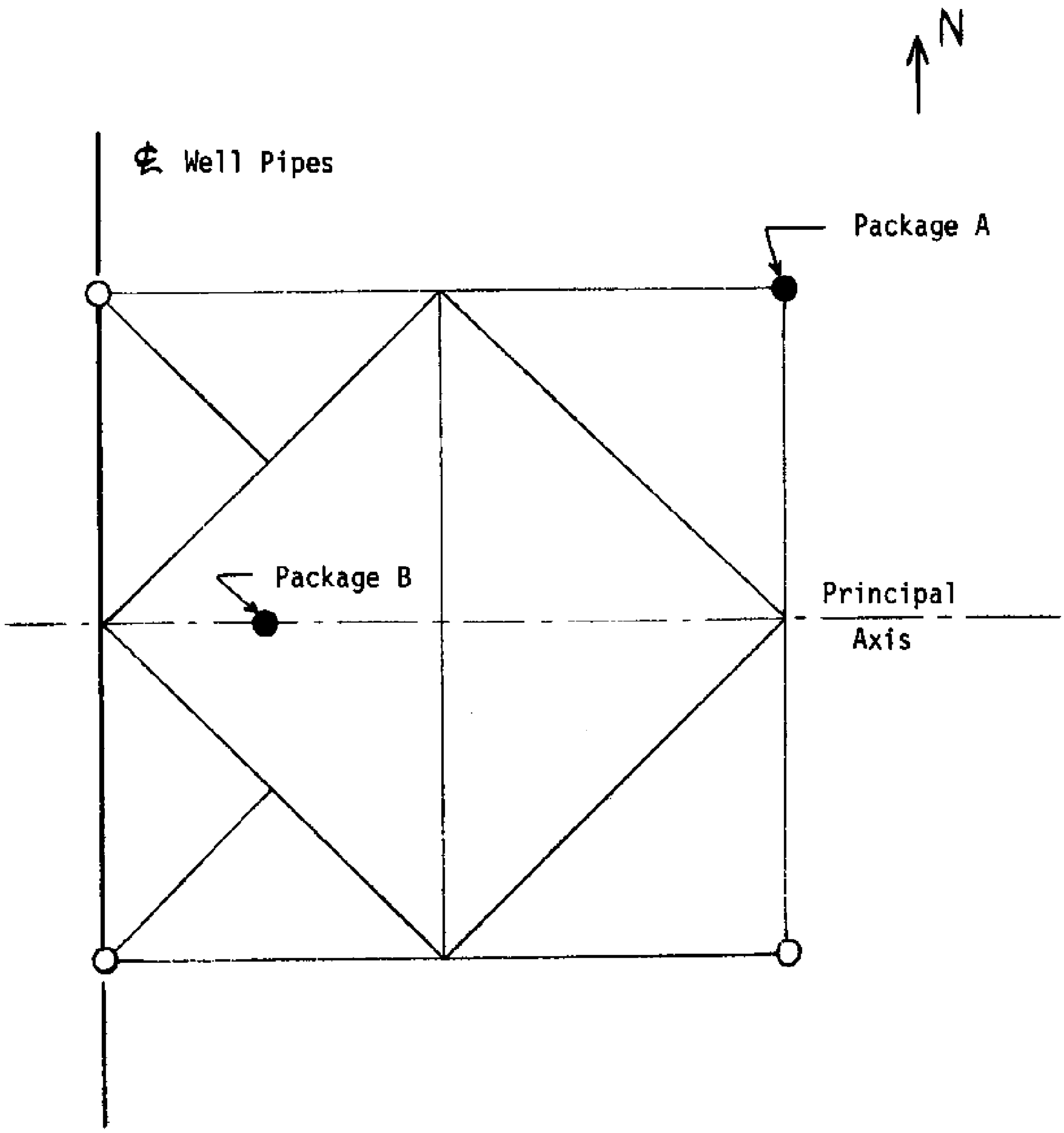
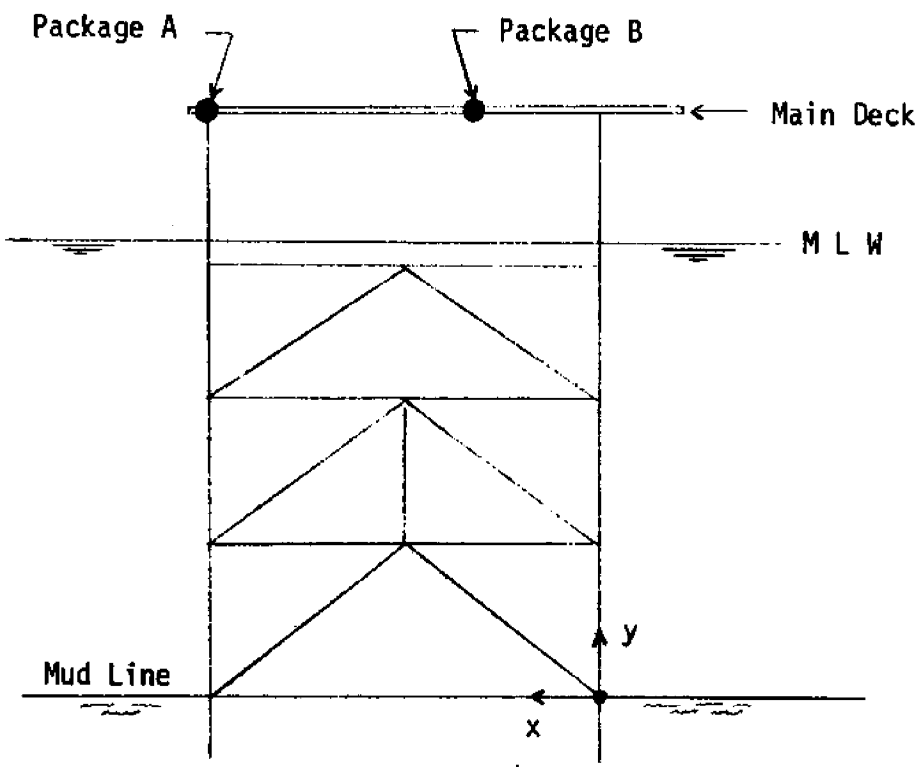


Fig. A.2 Platform Hope, Tower Plan View
(From Titlow, 1971)



Sensor Locations

Package	x-Coord.	y-Coord.	z-Coord.
A	+85	+184	-56
B	+28	+184	0

Fig. A.3 Platform Hope, North Elevation and Sensor Coordinates (From Titlow, 1971)

APPENDIX B

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